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Tidal resonance in icy satellites with subsurface oceans

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Abstract Tidal dissipation is a major heat source for the icy satellites of the giant planets. Several icy satellites likely possess a subsurface ocean underneath an ice shell. Previous studies of tidal dissipation on icy satellites, however, have either assumed a static ocean or ignored the effect of the ice lid on subsurface ocean dynamics. In this study, we examine inertial effects on tidal deformation of satellites with a dynamic ocean over lain by an ice lid based on viscoelasto-gravitational theory. Although ocean dynamics is treated in a simplified fashion, we find a resonant configuration when the phase velocity of ocean gravity waves is similar to that of the tidal bulge. This condition is achieved when a subsurface ocean is thin (<1 km). The enhanced deformation (increased $h_2$ and $k_2$ Love numbers) near the resonant configuration would lead to enhanced tidal heating in the solid lid. A static ocean formulation gives an accurate result only if the ocean thickness is much larger than the resonant thickness. The resonant configuration strongly depends on the properties of the shell, demonstrating the importance of the presence of a shell on tidal dissipation.

1. Introduction

Tidal dissipation is one of the major heat sources for the evolution of planetary bodies, particularly the satellites of the giant planets [e.g., Schubert et al., 1986, 2010]. Most of the satellites of Jupiter and Saturn are covered by an icy shell because of the low surface temperature. Based on internal thermal and structural modeling, large icy satellites, such as Europa and Titan, are expected to possess an internal ocean underneath an ice shell [e.g., Hussmann et al., 2007]. This expectation is supported by observational data by the Galileo and Cassini spacecraft and by the Hubble Space Telescope [e.g., Kivelson et al., 2000, 2002; Iess et al., 2012; Saur et al., 2015]. Observations by the Cassini spacecraft further suggest that even small satellites of Saturn, such as Enceladus and perhaps Mimas, have a subsurface ocean [e.g., Iess et al., 2014; Tajeddine et al., 2014]. Moreover, model calculations suggest that Pluto may also possess a subsurface ocean [e.g., Schubert et al., 2010; Robuchon and Nimmo, 2011], and tidal dissipation due to the orbital motion of its largest satellite, Charon, may have heated Pluto in the past [Barr and Collins, 2015]. Thus, a detailed investigation of the evolution of planetary bodies in the outer solar system should consider tidal dissipation using an interior model consisting of an outer solid layer, internal liquid layer(s), and a solid (or liquid) core.

Previous studies of tidal dissipation in icy satellites using spherically symmetric models can be classified into two types: those considering tidal dissipation in the solid part and those considering tidal dissipation in the ocean. The former studies consider deformation of a solid planetary body due to periodic tidal force [e.g., Moore and Schubert, 2000; Tobie et al., 2005; Roberts and Nimmo, 2008; Beuthe, 2015a]. In such studies, the equation system based on elasto-gravitational theory (widely used to investigate free oscillation problems at seismic frequencies [e.g., Love, 1911; Alterman et al., 1959; Takeuchi and Saito, 1972]) are used because this equation system can be easily applied to a viscoelastic body. An application of Fourier transformation to the equations for tidal deformation on a viscoelastic body leads to the same form of the equations for deformation on an elastic body (i.e., the correspondence principle) [e.g., Zschau, 1978]. Although this formulation has been applied to many satellites and planets, the effect of an internal liquid layer on tidal dissipation has usually been treated in a simplified fashion, as we discuss below.

The other type of studies, on the other hand, investigate tidal dissipation considering ocean dynamics using the Laplace tidal equations [e.g., Tyler, 2008; Chen et al., 2014]. Such studies reveal that a thin ocean (i.e., <1 km for most cases) in icy satellites leads to a resonance, resulting in potentially large heat production [e.g., Tyler, 2011; Matsuyama, 2014]. This finding is important not only because the ocean thickness in icy satellites is...
poorly constrained but also because the ocean thickness may vary largely with time. One important assumption in these models is that the surface topography follows an equipotential surface. This requires either that the ocean is at the surface or that the ice shell overlying the ocean is soft. In reality, none of the icy satellites has a surface ocean, and in some cases (such as Titan [e.g., Hemingway et al., 2013]) the ice shell may not be sufficiently soft because the viscosity of ice is very high at low temperatures [e.g., Schulson and Duval, 2009]. Prior to the work presented here, the influence of an overlying solid lid (i.e., the top icy shell) on ocean tidal dissipation has not been investigated.

In this study, we revisit the viscoelasto-gravitational theory and obtain a comprehensive equation system that can account for a thin subsurface ocean in viscoelastic planets (and satellites) (section 2). We then apply our theory to icy satellites and investigate the effect of an icy shell on the resonant configuration (sections 3 and 4). Because we are most interested in identifying what factors control the resonant response, our satellite models are deliberately simplified. However, the use of more realistic structural models will not affect our conclusions in a qualitative sense.

2. Theory

We follow the well-established elasto-gravitational theory considering deformation of a spherically symmetric, nonrotating, elastic, and isotropic body [e.g., Love, 1911]. In this theory, three equations are solved: the equation of momentum conservation, the Poisson equation for the gravitational field, and the constitutive equation. This theory can be applied not only for an elastic body but also for a viscoelastic body by adopting an appropriate constitutive equation. First, the equation of momentum conservation ignoring centrifugal and Coriolis forces is given by

\[ \frac{d^2u}{dt^2} = \nabla \cdot \sigma + \rho F. \]  

(1)

where \( u \) is the displacement vector, \( \sigma \) is stress tensor, \( F \) is the sum of all the forces per unit mass acting on the body, \( t \) is time, and \( \rho \) is density, respectively. Second, the Poisson equation for the gravitational field is given by

\[ \nabla^2 \Phi = -4\pi G \nabla \cdot (\rho u). \]  

(2)

where \( \Phi \) is gravitational potential and \( G \) is the gravitational constant, respectively. Third, the constitutive equation for a Maxwell body is given by

\[ \frac{d\sigma_{ij}}{dt} + \frac{\eta}{\mu} \left( \sigma_{ij} - \frac{\sigma_{kk}}{3} \delta_{ij} \right) = \lambda \frac{de_{ik}}{dt} \delta_{ij} + 2\mu \frac{de_{ij}}{dt}, \]  

(3)

where \( e \) is strain tensor, \( \delta_{ij} \) is the unit diagonal tensor, \( \eta \) is viscosity, \( \mu \) is shear modulus, and \( \lambda \) is the first Lamé’s parameter, respectively.

To calculate periodic deformation, a Fourier transform is applied to the three equations above assuming that the unknown variables \( (\mathbf{u}, \sigma, \Phi) \) oscillate with a frequency of \( \omega \); \( d/dt \) is replaced with \( i\omega \) where \( i \) is the imaginary number. Then, a spherical harmonic expansion is applied to such Fourier transformed equations, leading to a six-component differential equation system. In this study, we follow the formulation by Takeuchi and Saito [1972]. The equation system for a compressible solid layer is given by

\[ \frac{dy_1}{dr} = -\frac{2}{(\lambda + 2\mu)r} y_1 + \frac{1}{\lambda + 2\mu} y_2 + \frac{n(n+1)}{\lambda + 2\mu} \frac{y_3}{r}. \]  

(4)

\[ \frac{dy_2}{dr} = \left\{ -\omega^2 \rho + \frac{12x\mu}{(\lambda + 2\mu)r^2} - \frac{4\rho g}{r} \right\} y_1 - \frac{4\mu}{(\lambda + 2\mu)} y_2 + \frac{n(n+1)}{r} y_4 + \frac{(n+1)\rho}{r} y_5 - \rho y_6. \]  

(5)

\[ \frac{dy_3}{dr} = -\frac{1}{r} y_1 + \frac{1}{\mu} y_3 + \frac{1}{\lambda} y_4. \]  

(6)
The differential equations above are integrated numerically upward assuming a small homogeneous core (see Appendix B). The boundary conditions between solid-liquid interfaces can be found in Takeuchi and Saito [1972].

The coefficients for the solutions are determined from the boundary condition at the surface, which is given by

\[ y_2 = 0, \quad y_4 = 0, \quad y_6 = \frac{2n+1}{r_s} \Phi, \]
Figure 1. Interior structure models adopted. An incompressible five-layer and three-layer model is used for Ganymede and Enceladus, respectively.

Figure 2 shows the absolute value of the degree 2 Love number, \( h_2 \), as a function of ice viscosity, \( \eta_{\text{ice}} \). Here \( h_2 \) is calculated from \( h_2 = y_1(r,g(r)/\Phi_t) \). Results obtained using a dynamic formulation and a static formulation are shown in Figures 2a and 2b, respectively. Values of \( h_2 \) for \( H_{\text{ocean}} = 0 \) (i.e., no ocean), 20, and 200 km reported by Moore and Schubert [2003] are in agreement with those obtained from our code both for Figures 2a and 2b. This result not only demonstrates the validity of our code but also indicates that a static formulation leads to accurate results when there is no ocean or when the ocean is thick (i.e., \( \geq 20 \) km). On the other hand,
Table 1. Model Parameters for Ganymede

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_s )</td>
<td>Radius of Ganymede</td>
<td>2638</td>
<td>km</td>
</tr>
<tr>
<td>( r_m )</td>
<td>Radius of the mantle</td>
<td>1745</td>
<td>km</td>
</tr>
<tr>
<td>( r_c )</td>
<td>Radius of the core</td>
<td>710</td>
<td>km</td>
</tr>
<tr>
<td>( \rho_{H_2O} )</td>
<td>Density of ice and ocean</td>
<td>1050</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>Density of the mantle</td>
<td>3100</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>Density of the core</td>
<td>5150</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>( \mu_s )</td>
<td>Rigidity of the shell</td>
<td>10</td>
<td>GPa</td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>Rigidity of the mantle</td>
<td>100</td>
<td>GPa</td>
</tr>
<tr>
<td>( \mu_c )</td>
<td>Rigidity of the core</td>
<td>0</td>
<td>GPa</td>
</tr>
<tr>
<td>( \eta_m )</td>
<td>Viscosity of the mantle</td>
<td>(10^{20})</td>
<td>Pa s</td>
</tr>
<tr>
<td>( \eta_c )</td>
<td>Viscosity of the core</td>
<td>0</td>
<td>Pa s</td>
</tr>
<tr>
<td>( \omega_{orb} )</td>
<td>Orbital frequency</td>
<td>(1.016 \times 10^{-5})</td>
<td>rad s(^{-1})</td>
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when the ocean is thin (i.e., \(0 < H_{\text{ocean}} \leq 1\) km), a large difference in \(h_2\) between Figure 2a and Figure 2b can be seen, indicating that a static formulation can lead to large errors even when the frequency is low.

Figure 3 shows the real part of \(h_2\) as a function of ocean thickness, \(H_{\text{ocean}}\). This figure clearly shows that a dynamic formulation leads to a resonance when \(H_{\text{ocean}} \sim 0.1\) km. We found that this thickness is close to the thickness that results in a phase velocity of gravity waves \(v_{\text{grav}}\) similar to the phase velocity of degree 2 tidal deformation \(v_{\text{tidal}}\). The phase velocity for a shallow surface water wave is given by \(v_{\text{grav}} \approx \sqrt{g H_{\text{ocean}}}\) when curvature is neglected. The phase velocity of degree 2 tidal deformation, on the other hand, is defined as \(v_{\text{tidal}} = \frac{\omega_{\text{orb}}}{k}\), where \(\omega_{\text{orb}}\) is the orbital frequency and \(k\) is wave number, respectively. The wave number \(k\) for degree 2 deformation is given by \(k = (n+1/2)/r\), where \(n=2\) is spherical harmonic degree [Dahlen and Tromp, 1998]. Consequently, a resonant thickness \(H_{\text{res}}\) is given by

\[
H_{\text{res}} \approx \frac{r^2 \omega_{\text{orb}}^2}{(n+1/2)^2 g} \approx 81.8\;\text{m.} \tag{18}
\]

Here we use the central radius of the ocean (i.e., a depth of 145 km) of 2493 km and a gravitational acceleration at the ocean center \(\approx 1.25\;\text{m s}^{-2}\). A more exact result is given in equation (124) of Beuthe [2015b]. While the resonance occurs when \(H_{\text{ocean}} \sim H_{\text{res}}\), an increase in the real part of \(h_1\) can be seen even if the ocean thickness is several kilometers (Figure 3). Thus, a static formulation would give a sufficiently
accurate result only if $H_{\text{ocean}} \gg H_{\text{res}}$; a dynamical formulation should be used unless $H_{\text{ocean}} \gg H_{\text{res}}$. We note that calculations using a dynamical formulation are stable even if $H_{\text{ocean}} \gg H_{\text{res}}$, indicating that a wider range of parameters values can be adopted when the dynamic formulation is used.

The above result highlights the fact that a static formulation implicitly requires that the subsurface ocean is much thicker than the resonant thickness. This assumption, however, may not be satisfied for small icy satellites of giant planets. Figure 4 shows the resonant thickness $H_{\text{res}}$, for major icy satellites, calculated from equation (18). Here we use the surface values of $r$ and $g$, and the radius and mean density are taken from Chen et al. [2014]. The resonant thickness $H_{\text{res}}$ for Mimas, Tethys, Miranda, and Enceladus is $\sim 588, 457, 298$, and $253$ m, respectively. Although the minimum ocean thickness that gives sufficiently accurate results under a static formulation depends on properties of satellites (i.e., orbital period, gravity, and physical properties), the above results imply that such a critical thickness would be at least 10 km for these satellites. This result suggests that an application of a dynamic formulation may lead to a result different from previous studies using a static formulation for thin ocean models [e.g., Roberts and Nimmo, 2008; Shoji et al., 2014], though a detailed comparison with such studies is beyond the scope of this study.

We note that a resonant behavior caused by an internal liquid layer is similar to the free core nutation [e.g., Van Hoolst et al., 2003], though the time evolution of rotational axis is not considered in this study. Further discussions will be given in section 5.

### 4. Effect of a Lid on Tidal Resonance

The results presented above suggest that the solid lid plays an important role in the tidal response of the ocean and the body as a whole. Below we investigate the effect of varying lid parameters on the resonant configuration. In order to emphasize the effect of the solid lid on resonance, we use a
Table 2. Model Parameters for Enceladus

<table>
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<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
<td>( r_s )</td>
<td>Radius of Enceladus</td>
<td>252.1 km</td>
<td></td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>Mean density</td>
<td>1610 kg m(^{-3})</td>
<td></td>
</tr>
<tr>
<td>( \rho_{H_2O} )</td>
<td>Density of the shell and ocean</td>
<td>1000 kg m(^{-3})</td>
<td></td>
</tr>
<tr>
<td>( \mu_s )</td>
<td>Rigidity of the shell</td>
<td>4.00 GPa</td>
<td></td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>Rigidity of the mantle</td>
<td>6.78 GPa</td>
<td></td>
</tr>
<tr>
<td>( \omega_{orb} )</td>
<td>Orbital frequency</td>
<td>5.31 ( \times 10^{-5} ) rad s(^{-1})</td>
<td></td>
</tr>
</tbody>
</table>

simple three-layer Enceladus model, consisting of a solid shell, a liquid ocean, and a solid mantle (Figure 1, right). For our nominal case, an incompressible limit (i.e., \( \lambda \to \infty \)) is assumed. The icy shell and mantle are assumed to be a Maxwell viscoelastic body and a Hookean elastic body, respectively. Free parameters are the viscosity \( \eta_{ice} \) of the top icy shell, the thickness \( D_{shell} \) of the top shell, and that \( H_{ocean} \) of the subsurface ocean.

Table 2 summarizes parameter values, mainly adopted from Matsuyama [2014]. The density of the mantle is determined from the mean density (i.e., 1610 kg m\(^{-3}\)) and its radius, which is a function of \( D_{shell} \) and \( H_{ocean} \). In what follows, we refer to situations in which the effect of the lid is ignored as a “surface ocean” case.

Figure 5 shows the absolute value of the gravitational Love number for degree 2, \( k_2 \), as a function of ocean thickness. Here \( k_2 \) is calculated from \( k_2 = y_2(r_s)/\Phi_1 - 1 \). This figure demonstrates the effect of a lid overlying an ocean on the tidal resonance. First, no sharp increase in \( |k_2| \) is found when \( \eta_{ice} \) is extremely low (i.e., \( 10^{11} \) Pa s) and \( D_{shell} \) is large. Under this condition, the icy shell behaves as a thick fluid layer. As a result, the “effective ocean thickness” of the satellite is given by \( D_{shell} + H_{ocean} \), which is not small. Consequently, even if \( H_{ocean} \)
Figure 6. Absolute value of the degree 2 gravitational Love number \( k_2 \) as a function of ocean thickness for an incompressible Enceladus model. Results for a shell viscosity \( \eta_{\text{ice}} \) of \( 10^{15} \) Pa s and a shell thickness \( D_{\text{shell}} \) of 1 km are shown. The vertical dashed lines indicate the resonant thickness \( H_{\text{res}} \approx 253 \) m estimated from equation (18).

where \( T \) is surface tension [e.g., Lamb, 1932]. Here we use a shallow water approximation: \( \tanh(kH) \approx kH \) (\( kH \ll 1 \)). This equation may be compared with equation (123) of Beuthe [2015b]. Equation (19) indicates that in order to produce the same \( v_{\text{grav}} \), an increase in \( T \) requires a decrease in \( H_{\text{ocean}} \). In a similar fashion, in our results the effect of this “membrane” increases with increasing \( D_{\text{shell}} \); a thicker lid requires a thinner ocean thickness for a resonance to occur. Similarly, this membrane effect increases with increasing rigidity \( \mu_s \) of a shell, leading to a thinner resonant ocean thickness as shown in Figure 6. The amount of this shift in the resonant thickness can be very large, even by a factor of \( >100 \).

We also investigated the effect of compressibility. Figure 7 shows results for different values of the first Lamé’s parameter for the shell (\( \lambda_s \)) and illustrates that an increase in \( \lambda_s \) leads to a smaller resonant thickness and a smaller value of the Love number (\( k_2 \)). This trend is similar to that seen in Figure 6 where we change rigidity \( (\mu_s) \), supporting a surface tension interpretation for the effect of a lid on tidal resonance.

5. Discussion

As discussed in section 4, as the role of the lid becomes more important, the resonant ocean thickness becomes smaller. This indicates that our estimate of the resonant thickness using equation (18)—which ignores lid effects—should be considered as the upper limit. We found only one resonant configuration (i.e., one ocean thickness) for a given set of shell properties. In contrast, previous studies with more complex ocean dynamics, but neglecting the role of the lid, found several resonant configurations [Tyler, 2011; Matsuyama, 2014]. The major difference between such studies and our surface ocean case is whether Coriolis forces are taken into account or not. In order to examine
Figure 8. Energy flux due to tidal dissipation in the ocean as a function of ocean thickness for an incompressible Enceladus model. Results are obtained using a code developed by Matsuyama [2014], though Coriolis forces are removed. A quality factor $Q$ for the ocean of 100 is adopted. The vertical dashed lines indicate the resonant thickness $H_{res} \approx 253\, \text{m}$ estimated from equation (18). The solid and dashed curves show results including and excluding the effects of ocean loading, self-attraction, and deformation of the solid regions, respectively. The resonant thickness for the former and the latter is $\sim 430\, \text{m}$ and $\sim 270\, \text{m}$, respectively.

In addition, even if a subsurface ocean is global, lateral variations in physical properties in the shell may have an effect on tidal response; in such cases, components other than degree 2 would appear [e.g., A et al., 2014]. To quantify the effect of such lateral variations, 3-D modeling is necessary. Furthermore, large-amplitude deformation of the kind which arises near resonance violates the assumption of small-amplitude deformation on which our analysis is based.

Nevertheless, these caveats do not imply that tidal resonance is not important. As noted above, a large increase in tidal heating rate is expected near the resonant configuration. Such a strongly varying tidal heating rate is more likely to give rise to periodic behavior than smoothly varying cases. For example, a case in which no heating at all occurs until the tidal stresses exceed a threshold produces strongly periodic behavior [Stevenson, 2008]. Thus, tidal resonance may not only qualitatively but also quantitatively affect the evolution of icy satellites possessing a subsurface ocean [Tyler, 2008; Matsuyama, 2014].

While Coriolis forces, ocean loading, self-attraction, and deformation of the solid regions do affect the major resonant configuration, the difference in terms of resonant ocean thickness are small (i.e., by a factor $< \sim 2$) [Matsuyama, 2014]. In contrast, as shown above, the effect of a lid can be much larger; the resonant thick-
ness can differ by a factor of $>100$ if the lid is thick and stiff, demonstrating the importance of a lid on tidal resonance. Nevertheless, our model may overestimate the effect of a lid since we assume that the shell has a uniform viscosity structure for simplicity. The viscosity of an icy shell of actual satellites, however, would vary largely with depth [e.g., Hussmann et al., 2007]; the viscosity of the near-surface layer would be very high while that of the base of the shell would be much lower. Consequently, the thickness of a stiff lid may be much thinner than that of the icy shell. In addition, we assume a zero viscosity ocean; tidal dissipation in the ocean is not modeled in this study. In an actual body with a subsurface ocean, however, tidal dissipation occurs in both the solid and liquid parts. Further studies considering a depth-dependent viscosity structure and combining tidal dissipation both in the solid and liquid would be important to better understand the role of tidal heating on planetary evolution.

6. Conclusion

In this study, we revisited the formulation for tidal deformation based on viscoelasto-gravitational theory and examined inertial effects. We found that a dynamic ocean formulation leads to a resonance while a static ocean formulation does not. This resonance would be important for a satellite with a thin (i.e., $<1$ km) subsurface ocean since it would lead to significantly enhanced tidal heating in the solid lid. The static ocean formulation, which has been used in previous tidal dissipation studies, would give an accurate Love number only if the ocean thickness is larger than the resonant thickness by a factor of several 10 (or $>1$ km). The resonant configuration strongly depends on properties of the shell; a higher shell viscosity leads to a sharper resonant configuration, and a thicker or more rigid shell leads to a thinner resonant ocean thickness. For tidal dissipation in the ocean, addition of the Coriolis force introduces additional minor resonant configurations and slightly shifts the major resonant configuration. This shift is much smaller than that caused by the presence of a thick or rigid shell. These results highlight the importance of the effects of a solid lid for tidal dissipation in icy satellites with subsurface oceans.

Appendix A: Governing Equation System Under the Incompressible Limit

The equation system for a solid layer is given by

$$\frac{d^2 y_1}{dr^2} = -\frac{2}{r} y_1 + \frac{n(n + 1)}{r^2} y_3. \quad (A1)$$

$$\frac{d y_2}{dr} = \left( -\omega^2 \rho + \frac{12 \bar{\mu}}{r^2} - \frac{4 \rho g}{r} \right) y_1 + \frac{n(n + 1)}{r} \left( \rho g - \frac{6 \bar{\mu}}{r} \right) y_3 + \frac{n(n + 1)}{r} y_4 + \frac{(n + 1) \rho}{r} y_5 - \rho y_6, \quad (A2)$$

$$\frac{d y_4}{dr} = \left( \rho g - \frac{6 \bar{\mu}}{r^2} \right) y_1 - \frac{1}{r} y_2 + \left\{ -\omega^2 \rho + \frac{4 n(n + 1) \bar{\mu}}{r^2} - \frac{2 \bar{\mu}}{r^2} \right\} y_3 - \frac{3}{r} y_4 - \frac{\rho}{r} y_5. \quad (A3)$$

The others remain the same (see equations (4)–(9)).

The equation system for a liquid layer is given by

$$\frac{d y_1}{dr} = \left\{ -\frac{2}{r} + \frac{n(n + 1)}{\omega^2 r^2} \right\} y_1 - \frac{n(n + 1)}{\omega^2 \rho^2} y_2 - \frac{n(n + 1)}{\omega^2 r^2} y_5. \quad (A4)$$

The others remain the same (see equations (11)–(14)).

Appendix B: Initial Values

The differential equation systems are solved numerically assuming solutions of a homogeneous sphere for initial values $y_i$ at a small value of $r$. 

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B1. Solid Core

The solution for spheroidal deformation of a compressible homogeneous solid sphere is given by a superposition of three solutions [e.g., Love, 1911; Takeuchi and Saito, 1972]. Because only the ratios between \( y_i \) are needed to be specified, we can use

\[
y_1 = -\frac{f}{r} z_n(x),
\]

(B1)

\[
y_2 = -\rho f a^2 k^2 + \frac{2\tilde{\mu}}{r^2} \left( 2f + n(n+1) \right) z_n(x),
\]

(B2)

\[
y_3 = \frac{1}{r} z_n(x),
\]

(B3)

\[
y_4 = \mu k^2 - \frac{2\tilde{\mu}}{r^2} (f+1) z_n(x),
\]

(B4)

\[
y_5 = 3f - h \left( n\gamma - \alpha^2 \right),
\]

(B5)

\[
y_6 = \frac{2n+1}{r} y_5
\]

(B6)

for the first two sets of solutions and

\[
y_1 = \frac{n}{r},
\]

(B7)

\[
y_2 = \frac{2\tilde{\mu}n(n-1)}{r^2},
\]

(B8)

\[
y_3 = \frac{1}{r},
\]

(B9)

\[
y_4 = \frac{2\tilde{\mu}(n-1)}{r^2},
\]

(B10)

\[
y_5 = n\gamma - \alpha^2,
\]

(B11)

\[
y_6 = \frac{2n+1}{r} y_5 - \frac{3n\gamma}{r}
\]

(B12)

for the third solution where

\[
k^2 = \frac{1}{2} \left\{ \frac{\alpha^2 + 4\gamma}{a^2} + \frac{\alpha^2}{\beta^2} \pm \sqrt{ \left( \frac{\alpha^2}{\beta^2} - \frac{\alpha^2 + 4\gamma}{a^2} \right)^2 + \frac{4n(n+1)\gamma^2}{a^2 \beta^2}} \right\},
\]

(B13)

\[
z_n(x) = \frac{x j_{n+1}(x)}{j_n(x)}, \quad x = kr, \quad f(k) = \frac{\beta^2 k^2 - \alpha^2}{\gamma}, \quad h = f - (n+1),
\]

(B14)

\[
\alpha^2 = \frac{\gamma + 2\tilde{\mu}}{\rho}, \quad \beta^2 = \frac{\tilde{\mu}}{\rho}, \quad \gamma = \frac{4\pi}{3} G\rho.
\]

(B15)
and $j_n$ is the spherical Bessel function of degree $n$. Negative $k^2$ values are accepted [see Love, 1911, chapter VII]. The factor $z_n(x)$ is calculated from the recursion formula,

$$z_{n-1}(x) = \frac{x^2}{(2n+1) - z_n(x)},$$  \hspace{1cm} (B16)

and the calculation should be start from a sufficiently large $n$ [Takeuchi and Saito, 1972]. The initial value of $z_n(x)$ for a large $n$ is given by $z_n(x) = x^2/(2n + 3)$.

When the medium is incompressible (i.e., $\lambda \to \infty; \alpha^2 \to \infty$), we find that $f(k_+^+ \to 0$ and $f(k_-)\alpha^2 k_+^2 = n(n+1)\gamma$. Consequently, the first set of initial values is given by

$$y_1 = 0,$$

$$y_2 = n(n+1) \left\{ -\rho_\gamma + \frac{2\hat{\mu}}{r^2} z_n(x) \right\},$$  \hspace{1cm} (B17)

$$y_3 = \frac{1}{r} z_n(x),$$  \hspace{1cm} (B18)

$$y_4 = \hat{\mu} \left\{ \frac{\alpha^2}{\rho^2} - \frac{2}{r^2} z_n(x) \right\},$$  \hspace{1cm} (B19)

$$y_5 = (n+1) \left( n\gamma - \alpha^2 \right),$$  \hspace{1cm} (B20)

$$y_6 = \frac{2n+1}{r} y_5.$$  \hspace{1cm} (B21)

Similarly, we find that $\alpha^2 k_-^2 \to \alpha^2 + 4\gamma - n(n+1)\gamma^2/\alpha^2$, $f(k_-) \to -\alpha^2/\gamma$, $z(x) \to 0$. Consequently, the second set of initial values is given by

$$y_1 = 0,$$

$$y_2 = \rho \left\{ \frac{\alpha^2}{\gamma} \left( \alpha^2 + 4\gamma \right) - n(n+1)\gamma \right\},$$  \hspace{1cm} (B22)

$$y_3 = 0,$$

$$y_4 = 0,$$

$$y_5 = (h-3) \alpha^2 - nh\gamma,$$  \hspace{1cm} (B23)

$$y_6 = \frac{2n+1}{r} y_5.$$  \hspace{1cm} (B24)

The third set of initial values remains the same.

**B2. Liquid Core ($\mu \to 0$)**

In this case, two sets of initial values for $y_i$ ($i = 1, 2, 5, 6$) are required. We find that the form

$$y_1 = -\frac{f}{r} z_n(x),$$  \hspace{1cm} (B25)

$$y_2 = -\rho \left\{ f \left( \alpha^2 + 4\gamma \right) + n(n+1)\gamma \right\},$$  \hspace{1cm} (B26)

$$y_5 = 3\gamma f - h \left( n\gamma - \alpha^2 \right),$$  \hspace{1cm} (B27)

$$y_6 = \frac{2n+1}{r} y_5.$$  \hspace{1cm} (B28)
for the first set of initial values and

\[ y_1 = \frac{n}{r}, \quad (B33) \]
\[ y_2 = 0, \quad (B34) \]
\[ y_5 = n\gamma - \omega^2, \quad \text{and} \quad (B35) \]
\[ y_6 = \frac{2n + 1}{r} y_5 - \frac{3n\gamma}{r} \quad (B36) \]

for the second set of initial values, respectively. Here

\[ k_2^2 = \frac{1}{\alpha^2} \left\{ \omega^2 + 4\gamma - \frac{n(n+1)\gamma^2}{\alpha^2} \right\}, \quad f = -\frac{\alpha^2}{\gamma}, \quad h = f - (n+1), \quad \alpha^2 = \lambda/\rho. \quad (B37) \]

When \( \alpha^2 \to \infty \), we find \( k_2^2 \to 0 \). Thus, for the first set of initial values, \( y_1 = 0 \), and the others remain the same. The second set of initial values also remains the same.

**Appendix C: Profiles of \( y \) Functions**

Here we provide an example of profiles of \( y \) functions. Results for the top 2 km of an incompressible Enceladus model are shown in Figures C1 (real part) and C2 (imaginary part). Calculation conditions are \( \eta_{\text{ice}} = 10^{15} \) Pa s, \( \mu_s = 4 \) GPa, \( D_{\text{shell}} = 1 \) km, and \( 10^{-2} \leq H_{\text{ocean}} \leq 1 \) km. Absolute values of \( y \) increase as the ocean thickness
H_{ocean} approaches the resonant ocean thickness (\approx 140 m). In particular, y_3 in the ocean becomes very large at the resonant configuration. This results suggests that tidal dissipation in the ocean as well as friction at the solid-liquid boundary would lead to a large heat production when the ocean thickness is close to the resonant configuration.

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