Young children and Mathematics: A Relook at Mathematical Development from Sociocultural Perspectives

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Abstract

The primary goal of this article is to explore the sociocultural context of young children's mathematical development by reviewing the psychological theories and research, and propose future directions for research on this topic. I begin with a brief history of the early theories of mathematical learning and development, including one of the earliest and influential developmental theories – Piaget's theory of cognitive development. I will then look at two areas of study addressing young children's mathematical development that challenged Piaget's theory. One examined the mathematical competencies in infants and young children, while the other investigated the linkage between sociocultural influences and young children's mathematical development. As this article focuses on the sociocultural context of young children's mathematical development, I will, therefore, concentrate on the latter area of study.

In terms of the latter area of study, I will take a closer look at studies that seek to identify and explain the cross-cultural differences in mathematical development between East Asian and Western children, and also another group of studies that addressed the development of mathematical cognition in the context of everyday practices.

Finally, in order to advance the study of young children's mathematical development through the use of sociocultural perspective, I proposed a number of future directions for research on this topic.

Keywords

Sociocultural context, young children, mathematical learning and development.

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1. INTRODUCTION

Mathematics is an inseparable part of our everyday life and whether at work, at home, playing sports, or even doing groceries we all use mathematics. In fact, no matter where we are as well as whatever we are doing, mathematics exits among us. Mathematics is so important that the whole world will come to a pause if it disappears completely for a second.

Since the past century, psychologists, and educators began to examine mathematics using various perspectives, for instance, cognitive or developmental, educational and sociocultural aspects. They were interested in issues, such as cognitive representations and process for numbers (Butterworth, 1999; Dehaene, 1997), learning and development of numerical skills (Huttenlocher, Jordan, & Levin, 1994; Starkey & Cooper, 1995), sociocultural influences on mathematical cognition (Guberman, 2004; Saxe, 1991), and learning and performance disabilities in mathematics (Bull & Johnston, 1997; Jordan & Montani, 1997). Amongst these studies, an area which is particularly worth investigating is the link between sociocultural influences and young children's mathematical development. And it will be the primary focus in this article.

In fact, over the years, researchers had conducted numerous studies on the development of mathematical concepts and skills in young children and infants, and even the origin of human mathematical ability (Bryant & Nunes, 2002; Gelman & Gallistel, 1978; Ginsburg, Klein, & Starkey, 1998; Fuson & Hall, 1983; Gelman & Meck, 1983; Silverman & Rose, 1980; Wynn, 1990, 1998). The importance of this field has been further proven by the findings of recent studies revealing that the development of early math skills plays a vital role in the children's later mathematical development and other related developments (Classens & Engel, 2013; Duncan, Claessens, Magnuson, Huston et al., 2007). However, similar to the studies on other intellectual development, these studies assumed that the stages of cognitive development which all children pass through are universal, therefore focused mainly on individual children's development and often ignore the importance of sociocultural influences on young children's mathematical development.

In contrast, the assumption of universality in children's cognitive development have been challenged by other researchers, that cognitive development involves the shaping of human's basic biological capabilities in order to fit with the social context in which they are been used. Many found inspirations in Vygotsky's cultural-historical theory and used it as their theoretical guidance to further develop other theoretical frameworks that examine how sociocultural influences play their roles in shaping human cognition, and also the sociocultural conditions in which human cognition develops. For instance, in the topic of children's mathematical development, studies that examine everyday mathematics (Guberman, 2004; Saxe, 1991; Taylor, 2013) and cross-cultural comparison of mathematical competencies between East Asian and Western children (Han & Ginsburg, 2001; Miller, Smith, & Zhang, 2004; Stevenson & Lee, 1998) have all provided compelling evidences that sociocultural influences are closely interwoven with young children's mathematical development.

In view of these consideration, it is evident children's mathematical development is no longer regarded as a private, solitary process, rather an active process of individuals as they engage in sociocultural activities with other members in their own cultures. Since every culture has its own goals for mathematical activities, not only do the types of these activities vary across cultures, children also participate in these activities following their own cultural paths (Saxe, 1991; Guberman, 2004). In other words, although children are innately endowed with certain level of mathematical ability (Antell & Keating, 1983; Wynn, 1996), later development of this ability rely greatly on the types of sociocultural experiences that one has during their participation in mathematical activities. There are thus good theoretical grounds for examining young children's mathematical development from a sociocultural perspective.

For this reason, the primary goal of this article is to explore the sociocultural context of young children's mathematical development by reviewing the psychological theories and research, and propose future directions for research on this topic. I begin with a brief history of the early theories of mathematical learning and development, including one of the earliest and influential developmental theories – Piaget's theory of cognitive development. I will then look at two areas of study addressing young children's mathematical development that challenged Piaget's theory. One examined the mathematical competencies in infants and young children, while the other investigated the linkage between sociocultural influences and young children's mathematical development. As this article focuses on the sociocultural context of young children's mathematical development, I will, therefore, concentrate on the latter area of study.

In terms of the latter area of study, I will take a closer look at studies that seek to identify and explain the cross-cultural differences in mathematical development between East Asian and Western children, and also another group of studies that addressed the development of mathematical cognition in the context of everyday practices.

Finally, in order to advance the study of young children's mathematical development through the use of sociocultural perspective, I proposed a number of future directions for research on this topic.

2. EARLIER THEORIES OF MATHEMATICAL LEARNING AND DEVELOPMENT

While studies on young children's mathematical development continue to attract the interest from cognitive, developmental psychologists or educators, the origins of this field can be traced back to more than a century ago to one of the most influential educator and philosopher of the twentieth century, John Dewey who developed theories of children's mathematical development that centered on linking mathematics to human activities. Central to Dewey's theory was the notion that the need to solve human problems gave rise to mathematical development. As Stemhagen & W.Smith (2008) stated:

> Dewey's descriptive account of how children come to know mathematical concepts centered on the mental activities of children as they encountered various empirical situations. The psychological processes he detailed explained how it is that ideas of "much" and "many" might lead to the more refined notions of "how much?" and "how many?" Dewey (1895) indicted that his simple sense of quantity originated from the human need to measure in order to live more efficient and better lives. (p.33)

Over the years, the contemporaries of Dewey began to extend his work by examining young children's mathematical development from developmental or cognitive perspectives, however, this interest gradually faded away as Thorndike's behavioural theories of learning were adopted by the mainstream educators. During this period of time, most educators believed that learning requires one to keep repeating and practising certain skills till these skills become automatic. However, this trend took a new turn by the middle of the twentieth century as the theories of children's mathematical development were popularized by Jean Piaget's theory of cognitive development which examined children's mathematical cognition from developmental or cognitive perspective.

3. PIAGET'S THEORY ON CHILDREN'S MATHEMATICAL DEVELOPMENT AND ITS LIMITATIONS

Piaget developed the first major theory of cognitive development, and had conducted one of the most systematic study of children's thinking. Since the middle of the twentieth century his theory has generated numerous key questions, issues and criticisms about cognitive development in children, and even today many researchers still continue to address them.

Piaget's constructivist treatment of cognitive development has its roots in a neo-Kantian epistemology which assumed that knowledge is assimilated by intellectual structures. However, instead of adopting Kant's monism, Piaget took a development perspective and developed a systematic sequence of four stages of cognitive development from infancy to adolescence. He emphasized that cognitive development is driven by the process of reestablishing equilibrium in order to resolve cognitive conflicts with other same status peers. In other words, as the children reestablish the equilibrium, they revise their existing ways of thinking to better fit with the new information.

Over the years, although researchers attempted to refute Piaget's (1952) theory, his theory still remains as one of the few theories that provides a more systematic explanation about the steps that children take to overcome the complexities of mathematics. For Piaget, young children do not understand the meaning of number words even though they often count objects and actions in their everyday life. And in order for them to grasp these meanings, they would have to understand the cardinal and ordinal properties of numbers. In one of Piaget's most famous experiments that examined the concept of conservation, not only did he demonstrated that young children have no grasp of this numerical concept, but also the fact that mental logical structure of children vary across the stages in his theory. According to Piaget, conservation is a logical thinking ability to determine that quantity remains unchanged despite a mere change in physical appearance. In the task, he showed two rows of same quantity of objects to the children, one right above the other. After confirming with the children that the two rows contained the same quantity of objects, he began to spread out one of the rows and then asked the children whether both rows have the same quantity of objects. The older children were capable of conserving the quantity, however, the younger children replied that there were more objects in the longer row. Based on the findings from the experiment, Piaget argued that the ability to conserve numbers does not develop until the child reaches the concrete operational stage, and preoperational children do not understand the concept of one-to-one correspondence. Besides Piaget's conservation experiments, his other experiments that examined children on their understanding of transitivity, seriation, and cardinality had all reported that young children have no grasp of these concepts. Therefore, he concluded that mathematical knowledge would only emerge when a child reaches the concrete operational period at around age 6-7 years (Piaget, 1952).

For several years, many psychologists and educators came under the influence of Piaget and his theory of cognitive development, and thus the issue of mathematical development during infancy and early childhood appeared to be of little, if any, value for the researchers to explore. However, this trend took a new turn as researchers began to find loopholes in Piaget's findings, arguing that he might have negatively characterized the mathematical competencies of infants and young children. And at the same time, they raised various key questions to challenge Piaget's theory, such as, are humans innately endowed with certain level of mathematical ability, if so, how does this ability further develop during the infancy period, how do infants represent quantities and numbers since they are still in their preverbal stage, and how do the minds of young children work as they perform arithmetical operations? (Bisanz, Sherman, Rasmussen, & Ho, 2005; Geary, 1994; Ginsburg, Klein, & Starkey, 1998; Haith & Benson, 1998; Wynn, 1990, 1998). This new trend had led numerous researchers to relook at the treatment of children's mathematical cognition and generated many interesting new findings, paradigms, and discoveries on early mathematical competencies. This area of research will only be briefly discussed in the next section since it is not directly relevant to the primary focus of this article.

A second challenge to Piaget's theory is the lack of attention to the role of sociocultural influences in children's mathematical development. Although Piaget's theory has its roots in a neo-Kantian episotemology, his constructivist treatment of cognitive development was also greatly influenced by the British empiricist philosophers like Locke and Hume, who assumed that cognitive and perpetual processes are generally presumed to be universal in all human groups (Nisbett, Peng, Choi, & Norenzayan, 2001). In addition, Piaget believed that equilibration, which consists of assimilation and accommodation, is a main mechanism of children's developmental process. He further stressed that social interaction with same status peers bring about cognitive conflicts, and children attempt to resolve these conflicts by achieving equilibrium in their understanding. Although Piaget stressed that equilibration requires the children to interact with other individuals in their environment in order to develop cognitively, he paid very little attention to the link between sociocultural influences and cognitive development in most of his work.

In contrast, researchers began to question about this underdeveloped aspect of Piaget's work, and search for theoretical construct that could enhance their understanding of how sociocultural experiences and children's mathematical cognition are interwoven. Many found inspiration in Vygotsky's sociocultural theory, which stated cognitive development occurs as one engages in sociocultural activities. For this reason, these researchers took an approach different from Piaget and other traditional approaches in psychology by integrating human development, which includes cognitive processes into the sociocultural ones, and generated many interesting new findings, paradigms, and discoveries on young children's mathematical development that will be discussed later.

4. MATHEMATICAL COMPETENCIES IN INFANTS AND YOUNG CHILDREN

For Piaget, preoperational children do not understand the meaning of number words, and counting in them is done by rote. He stressed that in order for them to grasp these meanings, they would have to understand the cardinal and ordinal properties of numbers, and such ability would only emerge when a child reaches the concrete operational period at around age 6-7 years (Piaget, 1952). Other contemporary empiricists who shared the same notion with Piaget, that children develop from perpetual to the abstract for all concept domains, suggested that young children could have connected their meaningless count words to perpetual representations when they first count (Baroody & Wilkins, 1999; Mix, Levine, & Huttenlocher, 2002).

On the other hand, other researchers were eager to refute Piaget's negative characterization of the preoperational children's mathematical cognition as they searched for the ontogency of human's cognition, which includes the origins of mathematical knowledge. As a result, they studied younger and younger children, and eventually preverbal infants. Through their experiments, they found that mathematical abilities begins during infancy. And this discovery had led them to develop a new treatment and paradigm for mathematical competencies in preverbal infants and young children. As Cordes & Gelman (2005) stated:

> Accounts of early counting differ in the degree of conceptual competence granted to the young child, as well as whether there are ontogenetic and/or phylogenetic continuities. Much of the debate is centered on whether various counting tasks license the conclusion that young learners understand the cardinal counting principle, that the last word in a count list represents the cardinal value of a collection. (p.127)

For many of these researchers, counting is a very important fundamental ability which lays the foundation for many other later mathematical abilities. Further, they also believed that counting is closely interwoven with arithmetic since execution of a counting involves yielding a cardinal value which also involves the arithmetic operations. In fact, many shared this same notion with Gelman and Gallistel (1978), who were the first to develop the five basic principles for counting, which includes one to one correspondence, stable ordering, cardinality, order irrelevance and item-kind irrelevance. Gelman and Gallistel (1978) also stressed that these principles are innate, and therefore it is also known as the "Principles-before theory". Gelman and Gallistel's five counting principles provides one of the most coherent explanation of the different principles that are required to execute a competent counting, and is still very influential. For this reason, much of the work on mathematical ability in infants and young children involves the use of counting tasks and arithmetic tasks.

4.1 The Counting Tasks

"How many?" (HM) is one of the most popular counting tasks, which requires the children to determine the quantity of objects that are shown to them. The findings revealed that the performance of the tasks was positively correlated with the age of the child (Fuson, 1988). In addition, younger children were likely to repeat the last count word for each HM question more than older children. Further, younger children were also more likely to perform better in the tasks if they were told to touch the objects, or counting of small sets (Gelman & Tucker, 1975). All these findings have clearly explained the function of biological development in children.

In other tasks, such as "What is on the card?" (WOC) task (Gelman, 1993), most of the children between the ages of 2 ½ years and 3 ½ years were capable for stating cardinal value up to the set sizes of 4, and the older children tend to perform better in counting than their younger counterparts. Although the findings from the above experiments provide good grounds to believe that counting ability is already present as early as infancy, there is more to the story.

In another version of counting tasks, Wynn's "Give-N" tasks (1990, 1992), the children were asked to give a puppet certain number of small animals ranging from one to six. Unexpectedly, most of the children who are younger than 3 ½ years failed the tasks. Wynn's "Give-N" tasks were later replicated by Brannon and Van de Walle (2001) and their results were generally consistent with those of Wynn. However, they went further to observe the behavioural responses of these children and found that children tend to respond faster in the HM and WOC tasks than "Give-N" tasks, and at the same time, they displayed more signs of hesitation and confusion during their participation in the "Give-N" tasks. Therefore, they concluded that as compared to the rest of the counting tasks, "Give-N" tasks seem to be a relatively difficult counting task. This was because these tasks required the children to count and correspond the physical set of objects they had created to the numerical value in their memory, which involved more steps than the other tasks.

4.2 The Arithmetic Tasks

Besides the counting tasks, another commonly used tasks among these researchers to assess the infants' mathematical competence is the arithmetic tasks. For example, Hughes's (1981) study found that 3, 4, 5-year old children were capable of solving simple addition and subtraction problems involving set sizes less than 8. In other studies, infants as young as 1 year old could discriminate between sets that contain very small set sizes, such as 2 or 3 (Antell & Keating, 1983; Strauss & Curtis, 1981).

By 1990s, Wynn (1992) became the first to develop an experimental method to better examine arithmetical competencies in infants, which was later replicated by many other researchers. In her experiment, infants were presented with some dolls on the stage (usually 1 or 2), the experimenter then covered the display, and he will (a) either do nothing, thus the quantity of the dolls remains unchanged that is "possible" (arithmetically correct), or (b) secretly remove or add a doll, which resulted the set to become "impossible" (arithmetically incorrect). The display was then presented to the infants again and their visual behaviours were recorded. Wynn proposed that if the infants are capable of arithmetic, they will look longer at those "impossible" outcomes than those of "possible" outcomes since the former violated their expectations.

Wynn (1992) found out that 4 and 5 month-old infants spent more time looking at those "impossible" than "possible" outcomes. And therefore, she concluded that infants as young as 4 and 5 month-old are sensitive to simple arithmetic transformation. Similarly, those infants in the other replicated studies also generally exhibited the same visual behaviours as those of Wynn's experiments (Koechlin, Dehaene, & Mehler, 1997; Uler, Carey, Huntley-Fenner, & Klatt, 1999). Therefore, it is evident that preverbal infants possess the ability to understand simple arithmetic operations.

4.3 Conclusion

In view of the experimental findings from the above studies, there are good grounds to believe that infants as young as 4-month-old possess the capacity to count at least very small set sizes, and perform simple arithmetical operations. Therefore, at this point, it seems reasonable to assume that humans are innately endowed with some kind of basic mathematical ability. This new insight into the mathematical abilities in infants and young children has convinced us that Piaget's notion stating that young children are devoid of mathematical knowledge until they reach the concrete operational stage around 6 to 7-year-old is fundamentally incorrect.

In sum, the above studies have not only provided an insight into the innate mathematical ability in humans, they have also challenged us to think deeper into the topic of young children's mathematical development. For example, children have developed more sophisticated mathematical knowledge beyond their innate endowed ability since we began to see differences in children's mathematical development across cultures in numerous cross cultural comparison studies (Han & Ginsburg, 2001; Miller, Smith, & Zhang, 2004; Saxe, 1991). Although I do not deny the importance of biological development in children's mathematical ability, sociocultural forces serve as an even more important mechanism for mathematical development.

5. SOCIOCULTURAL INFLUENCES ON YOUNG CHILDREN'S MATHEMATICAL DEVELOPMENT

For many years, many psychologists and educators came under the influence of Piaget and his theory of cognitive development had paid little attention to the link between sociocultural influences and children's mathematical development. However, other researchers began to use Piaget's original tasks or making slight modification to these tasks, and administered to children from different cultural groups. These work were often related to the assessment of children's understanding of mathematical concepts such as conservation (Laurendeau-Bendavid, 1977; Opper, 1977), and classification (de Lacey, 1970, 1971). Besides focusing on Piaget's tasks, others also studied the relationships between engagement in practices, such as pottery making, economic trades and hunting, and development of conservation's concept (Price-Williams, Gordon, & Ramirez, 1967; Posner & Baroody, 1979; Dasen, 1975). Although, these work had shed some light on the role of sociocultral influences on children's mathematical development, they have failed to examine the intrinsic relation between sociocultural processes and cognitive developmental ones since they often treat sociocultural processes as an external aspect of the cognitive developmental ones. As Saxes (1991) stated:

> Across most research efforts, to the extent that sociocultural processes are addressed, the analytic approach is to dissect both cultural and cognitive phenomena into separate sets of elements in which the social properties of cognition are no longer recoverable. (p. 8)

> While there may be attempts to identify cultural influences on independently defined cognitions through the external connections of correlational analyses as we have seen in much of the Piagetian work, these efforts cannot lead to analyses of the intrinsic relations whereby cultural and cognitive developmental phenomena are constitutive of one another. (p. 9).

Researchers began to search for a “better” theoretical framework that could advance their understanding of how young children's mathematical cognition is closely interwoven with their sociocultural experiences. And at the same time, differs from how sociocultural processes are been analyzed in connection with cognitive developmental ones in Piagetian theory and related work. Many found inspiration in Vygotsky's work that explores the intrinsic relation between sociocultural variables and cognitive development. And Vygotsky's sociocultural theory, one of the most influential theories that has impacted many sociocultural studies, including those relating to children's mathematical development.

5.1 Vygotsky's Theories

Vygotsky, whose psychological work are rooted in the works of Marx and Hegel, adopted a sociocultural approach to explore children's cognitive development (Vygotsky, 1978, 1986). For Vygotsky, children's appropriation of culturally produced artifacts through their interactions with the world helps to liberate them from the direct control of their environment. In addition, he also stressed that cognitive development results from the process of learning of how to use cultural artifacts (such as languages and mathematics) with the assistance from those who are expert in these tools. Therefore, cultural artifacts and social interactions are central to Vygotsky's theories.

Amongst the cultural artifacts, the role of language caught the attention of Vygotsky the most. He divided language into three forms, namely social speech, private speech, and inner speech. Unlike Piaget who viewed private speech as egocentric and a sign of cognitive immaturity, Vygotsky stressed that private speech is critical in children's cognitive development because private speech occurs when thought and speech, which have separate roots, are interwoven with each other. According to Vygotsky, during early childhood, young children made use of private speech as a form of support to rehearse those contents in their minds during their problem solving processes. And as the children grow older, this inner speech will gradually go “hidden”, and became “inner speech”. Saxes (1991) explained this process in his book:

> Thus, what was once a social artifact external to the child, is gradually transformed by the

child, first into an external aid which helps organize problem solving and later into a core ingredient of conscious thought. (p. 9)

Vygotsky's concept of inner speech is often related to studies on literacy in children. Nonetheless, this concept can also be closely related to the topics addressing young children's mathematical development because it is not uncommon to see young children verbally counting to themselves or saying out the addends as they solve arithmetical problems in their everyday life. Though there are various studies that have linked private speech to children's mathematical development, I still see many potential unexplored issues in this topic.

Besides language, social interaction is another central tenet of Vygotsky's theories. For Vygotsky, cognitive development occurs as the children appropriate those cultural artifacts through social interactions with others, especially those who are more experienced in these artifacts and cultural organizations. In other words, he believed that cognitive development moves from social to the individual. Vygotsky argued that these social interactions lead to the creation of "zones of proximal development", and further stressed the important role which adults play in enhancing children's cognitive development. A zone of proximal development can be defined as the difference between what a child is capable to doing or accomplishing without any assistance and what he or she can do with assistance.

Over the years, various researchers had based on Vygotsky's zone of proximal development to examine the interactions between children and adults (Gray & Fledman, 2004; Hyde, Else-Quest, Alibali, Knuth, & Romberg, 2006; Rogoff, 1986, 1990; Rogoff, Ellis, & Gardner, 1984; Saxe, Gearhart, & Guberman, 1984; Wertsch, McNamee, McLane, & Budwig, 1980; Wood, Bruner, & Ross, 1976). These studies found that it is quite prevalent to see adults assisting children in problem solving to accomplish even higher level and more complex goals which is beyond the child's level of competencies.

For example, in one of the recent works by Hyde et al., (2006), they observed the interaction between mothers and their children during their mathematics homework practices. Their results revealed that the quality of mathematics content that the mothers conveyed to their children during the practices, the quality of scaffolding of the materials they had used were all important in enhancing the children's learning and performances. In fact, their results were consistent with Vygotsky's zone of proximal development because mothers who provided better quality mathematics contents, more mathematics preparation, and use better materials, their children turned out to be better in solving more challenging arithmetic problems and made more progress in understanding. In other words, children are not only capable of tasks that can be accomplished within their own constructive efforts, but through adult-child interactions, they are been driven to go beyond their original level of competence to challenge even more complex and higher level tasks, which in turn enhances their intellectual development significantly.

To sum up, we have seen that Vygotsky placed great emphasis on the social basis of mind. In his theory, he stressed that cognitive development occurs as one learns to use cultural tools which are created from cultural-historical activities, and this development is further enhanced through social interactions with others who are more proficient in using these tools and cultural organization. Therefore, we have good grounds to believe that children's cognitive development, including mathematical development is no longer regarded as a private, solitary process, rather an active process of individuals as they engage in shared endeavors with others. In other words, children's cognitive development is closely interwoven with his or her experiences within their own cultural institutions.

Although Vygotsky's theories are based solely upon Western children, and did not directly examine cultural practices in his works, researchers have continued to use Vygotsky's works as theoretical guidance to extend their works to other communities outside the Western world. Through their work, they have generated new findings that provide an even more detailed insight into social and culture-mathematical development relations, and lay the foundation for other later studies. In the next few sections, we will review some of the major findings of these studies. One focuses on the cross cultural differences in mathematical development, especially in the context related to schooling. Whereas, the other centered on mathematics in the context of everyday cultural practices, which had brought the concepts of mathematical cognition "located in" specific and familiar contexts into limelight.

5.2 Cross cultural differences in mathematical development between East Asian and Western children

In the late 1980s, as various cross cultural researchers began to investigate the mathematical competencies of the East Asian and Western children, they made a great discovery. Their findings revealed that not only did the level of competencies vary across these two groups of children , the variation was relatively substantial. For example, in one of the earlier studies, Miller & Stigler, (1987) reported that the American children tend to face more difficulties than the Chinese children during the counting tasks. And as a result, they fared worse than their Chinese counterparts. In another earlier study which examined the mathematical achievement of American and Japanese children residing in the United States, they also found the same tendency of East Asian children outperforming the Western children in terms of mathematical achievement (Miura, 1987). These new discoveries quickly drew the attention of more researchers to search for causes that could explain these substantial differences.

Over the years, a great deal of research relating to this area has been generated. Because mathematical development is closely interwoven with children's learning, researchers had categorized these causes into the mathematical contents of what the children learn and the contexts of mathematical learning.

5.2.1 Differences in the content of mathematics children learn

Formation of Number Names

Though a large portion of mathematical contents, including the Arabic numerals, basic counting concepts is universal in nature, some contents, such as number naming structures vary quite a bit across languages which have a great impact on young children's mathematical development. For example, number names from 1 to 10, both the English and Chinese systems (including the Japanese and Korean systems which were adopted from Chinese writing system) require one to memorize these number names by rote. However, these two types of languages represent the number names differently from 11 to 19. Take for instance, a literal translation of "十三: shi san" (13) into English will be "ten-three". However, the formation of this same number in English, which is "thirteen", does not have much relationship with corresponding number ten or three. The same formation applies to the rest of the number names from 11 to 19. Therefore, as compared to the Chinese system, the formation of 11 to 19 tend to be much more complicated, and difficult to memorize for the young English-speaking children.

A number of studies that investigated the counting abilities of Chinese and American young children had revealed that the differences in their number naming structures had lead these two groups of children to perform differently in the counting tasks during the experiments (Fuson & Kwon, 1991; Miller, Smith, & Zhang, 2004; . Miller, Smith, Zhu, & Zhang, 1995). For example, in one of the studies, mathematical tasks for counting were administered to Chinese and American children on a monthly basis in a longitudinal study. The findings revealed that there was no much difference between the two groups for the two-year-olds as most of them had difficulties in the counting tasks. However, as the children reached the age of four, the Chinese children were capable of counting up to a larger value numbers and progressed better in the learning process of counting as compared to the American children (Miller, Smith, & Zhang, 2004). Therefore, it is evident that the less complicated Chinese number naming structure plays a vital role in making young Chinese children a better counter.

Transparency of Base-Ten Structure

Base-ten structure is another concept which is closely connected with the number naming structures, and is critical for young children's mathematical development (Miura, Okamoto, Kim, Steere, & Fayol, 1993; Miura & Okamoto, 2003). As Miller, Kelly, & Zhou (2005) emphasized in their work:

> This base-ten structure is a feature of a particular representational system rather than a fundamental mathematical fact, but it is a feature that is incorporated into many of the algorithms children learn for performing arithmetic and, thus, is a powerful concept in early mathematical development. (p.170)

In fact, the transparency of base-ten structure varies across languages. For example, English number names show up a base-ten structure less consistent and much later as compared to the Chinese number names. Therefore, young East Asian children are more likely to use base-ten structure, and take advantage of it to help them to solve arithmetic problems than their same- age Western peers. In a study by Ho and Fuson (1998), they found that half of their Chinese-speaking preschoolers in Hong Kong who used the base-ten structure were able to correctly solve "10 + n = ?" related addition problems faster, whereas none of their British and American counterparts used it. In another study, Miller and his colleagues (Miller et al., 1995) also stated that the less obvious base-ten structure in English number names has put the English-speaking children in a less advantageous position as they do their counting or solving numerical problems. In contrast, Chinese children were able to utilise the base-ten structure more effectively and efficiently since it is more transparent in Chinese number names. And thus, this could also explain those previously reported differences in mathematical achievement in favour of East Asian children.

Ordinal Numbers

Similar to the formation of number names, English ordinals are also known to be more complex than those of Chinese ordinals. For example, English ordinals (first, second, third, etc) are represented differently from their corresponding cardinal number names. In contrast, Chinese ordinals are formed by adding an ordinal prefix (第:di) before the cardinal number names. And therefore, the complexity of English ordinals has again put the English-speaking children in a less advantageous position where counting or solving mathematical related problems are concerned. Miller and his colleagues (Miller, Major, Shu, & Zhang, 2000) assessed Chinese and American children in terms of their counting abilities of ordinal numbers. They found out that less than half of the American fourth grader could correctly count above "thirtieth". And overall, American children fared worse than those of Chinese children in counting ordinal numbers during the experiment. In one of the studies by Sakakibara (2008), she found that 3 to 5-years-old Japanese pre-schoolers might initially focus solely on numbers as they do their counting or solving arithmetic. However, as they learn ordinals, they were able to understand and use ordinal numbers without much difficulties. In other words, it is evident that the simplicity of Chinese and other East Asian number systems has helped East Asian children to grasp the concept of ordinal numbers faster and easier than their Western peers, which in turn enhances their overall mathematical achievement.

5.2.2 Differences in the contexts of learning mathematics

Besides the differences in the content of mathematics children learn, especially in the area of linguistic, children's mathematical development is also influenced by the contexts in which they learn mathematics. And these contexts of learning are often affected by various cultural factors such as parental beliefs of what the children should learn and when they should learn, and how early childhood mathematical learning should be organized.

Parental Beliefs of Early Mathematics

Mothers of kindergarteners in China and the United States were interviewed and asked to rate about the importance of mastering mathematical and literacy skills prior to their children's entry to first grade as that will lead to later academic success (Kelly, 2002). Mothers in China perceived that mastering both skills were equally important, neither of them should be neglected. In contrast, mothers in the United States tend to rate literacy skills higher than the mathematical skills. One reason for the differences in the ratings between the two groups could be due to the different levels of emphasis placed on mathematics. In China, and other East Asian societies, they place extremely strong emphasis on school mathematics, and future success is closely linked with good mathematical skills (Hatano, 1990; Stevenson & Lee, 1990). Contrary, in the Western societies, illiteracy is perceived to be more valued than mathematics. As what Young-Loveridge (2015) has written in in her article:

> The low value of mathematics can be seen in the way that (often Western) people brag about their personal lack of competence with mathematics. Whereas illiteracy is usually a source of deep shame to people and hidden at all costs, the preserve pride taken by people over their mathematical ignorance reflects a general failure to recognise how vital mathematics is for a successful life. (p. 1)

Therefore, this could also explain why those American mothers in Kelly (2002)'s study rated literacy skills as more important than mathematics skills. In fact, parent beliefs and children's mathematical abilities are closely related. For example, Kelly (2002) found that those Chinese children whose mothers valued mathematics skills more than reading were reported to have more advanced mathematical abilities than their counterparts. Whereas, for those mothers in the United States who valued reading skills more than mathematics skills, their children tend to possess higher level of reading skills.

Taken together, there is a relatively strong relationship between parent beliefs and children's mathematical competencies. However, parental beliefs alone will not have any effect on children's abilities unless they base on their beliefs to determine the types of practices they would like to engage with their children in order to achieve their pre-determined goals. Therefore, in the next section, I will discuss more about the differences of mathematical practices that the East Asian and Western children engage in.

Parental Practices relating to Mathematics

According to Blevins-Knabe and Musun-Miller (1996), the amount of time parents spent together with their pre-schoolers in mathematical activities is positively correlated to their children's mathematical competencies. In other words, higher parental involvement in children's learning process of mathematics has a positive effect in enhancing their mathematical competencies. This is especially true as the American parents are more likely to spend time reading to their children than to practice mathematics with them. In contrast, the East Asian parents are likely to do otherwise (Miller et al, 2005). For this reason, East Asian children who spent more time with their parents to practise mathematics performed better in mathematical tasks than their Western counterparts. In addition, mothers in Japan and Taiwan tend to believe that the road to success is through effort, whereas mothers in the United States believed that academic ability is innately endowed (Stevenson, Lee, & Stigler, 1986). Therefore, it is not uncommon to see parents in East Asian countries having higher level of educational involvement in their children's mathematics learning process than mothers in the United States. In another study that compared Chinese American and European American families also revealed that Chinese American parents allocated more time to teach their children, and adopted more formal methods in their teachings. Further, they were more likely to assign their own homework to their children. Therefore, their children performed better (including mathematics) than their counterparts.

In sum, it is evident that parental beliefs and practices related to mathematics greatly influence a number of factors, such as the types of activities in which the children engage in, the amount of time they spent in those activities, and all these will eventually contribute to the children's mathematical competencies. Therefore, in order to gain a better insight into young children's mathematical development from a sociocultural perspective, the context in which children learn mathematics is also another aspect that should not be ignored.

5.2.3 Conclusion

From the above, it is clear that these cross-cultural comparisons play an important role in our understanding of the nature of children's mathematical development. For example, in some of these studies, researchers looked for the aspects of mathematics which are influenced by language. Since linguistic representation varies across cultures and have different effects on mathematics, researchers will never be able to unveil these effects if they were to focus only on a monolingual group. Next, these cross-cultural comparisons have also brought some of the mathematical strengths and weakness of the Western and East-Asian children into limelight, which could serve as good references for early childhood educators when they design the mathematical learning curriculum for young children. For example, base-ten structure has been proven to be an "effective tool" in enhancing children's arithmetical performance since East-Asian children who always take advantage of it when solving arithmetic problems performed better than their Western counterparts (Miura et al., 1993; Miura & Okamoto, 2003). For this reason, researchers and educators began to ponder whether explicit teaching of base-ten structure help to improve the arithmetic performance of the children from non-East Asian cultural backgrounds. In one of the studies, Fuson and her colleagues (Fuson, Smith, & Lo Cicero, 1997) explicitly taught a group of Latino young children about the use of base-ten structure, and their end-of-year performance were almost on a par with those of East Asian children. Given the results from the above study (Fuson et al., 1997), it might have easily convinced us that explicit teaching of the base-ten structure to children from non-East Asian cultural backgrounds will enhance their arithmetic performance. However, I hold that this conclusion might not work all the time as there is more to the story.

Similar to other cognitive development, mathematical development involves the influence of a multitude of variables, including maturational factors, linguistic influences, schooling systems, parental beliefs and practices, and the mathematical content of learning. Therefore, by just "improving" on one of these variables is unlikely to achieve a desired level of mathematical development in young children. For example, in Fuson et al. (1997)'s study, those Latino young children might have gradually improved their arithmetic competence upon acquiring the concept of base-ten structure. However, if the parents of these Latino children continue to value other subjects more than mathematics, or if they do not spend as much time as those of East-Asian parents together with their children to practice arithmetic problems, these children might not be able to maintain their high level of mathematical competencies in the long run.

In our next section, I will continue to discuss mathematical development from a sociocultural perspective. However, I will move away from school mathematics and turn to mathematics that are evolved from cultural practices in everyday life, which is also known as everyday mathematics. Various researchers have discovered the strong link between everyday mathematics and mathematical development, and brought the concepts of mathematical cognition "located in" specific and familiar contexts into limelight. In adddition, they have also challenged the dominant notion about Western mathematics being one of the hallmarks of rationality, which have led many researchers to question about the nature of rationality.

5.3 Everyday Mathematics and Mathematical Cognition

In the last three decades, a group of researchers who were influenced by Vygotsky's works have been studying how individuals deal with mathematical problems in a great variety of mathematical practices in their everyday life, especially those in the non-Western 'traditional' cultures. And the type of mathematics that these groups of individuals used is often known as everyday mathematics (Lave, 1988). Unlike those Piagetian practice-based studies which we have mentioned earlier, these studies of everyday mathematics have provided more detailed insights into how individuals use their cognitive forms to accomplish cognitive functions that are related to their practices. In addition, they have also enhanced our understanding of the closely interwoven relationship between sociocultural and mathematical cognition processes.

These studies primarily focused on mathematical practices in non-Western traditional cultures ranging from measurement of rice in the farms (Gay and Cole, 1965) to tailoring work in Liberia (Reed & Lave, 1979). For example, Gay and Cole (1965) who investigated how Kpelle rice farmers in central Liberia measured their rice discovered that these farmers measured the rice using their own common unit of measure known as kopi, which differed greatly from the school mathematics. In other study, Lave (1988) reported that housewives in California used their own “invented” strategy to determine the best buys during their grocery shopping, and surprisingly they were able to determine which item to buy almost immediately without much effort. Scribner (1986) also learnt that those dairy workers made use of the arrangement of the crate to determine the quantity of milk to be retrieved, a mathematical strategy which was never taught in schools.

If we put these studies together, we began to see some of the common characteristics that are shared by them. Firstly, the calculations by the participants in these studies often involve small numbers and their answers were typically rounding numbers. Secondly, addition, subtraction and multiplication of small numbers were some of the most common arithmetic strategies that were reported in these studies. And finally, the calculations that we see in these studies differ greatly from school mathematics strategies (pencil-and-paper application). This is because the participants were likely to use mental arithmetic, which includes strategies such as estimation, rounding up, or approximating. Given these common characteristics, it is clear that the nature of everyday mathematics differs greatly from those of school mathematics, especially in terms of their mathematical computations, results, and also the contexts or purposes of the activities.

Besides bringing to light the nature of everyday mathematics and its differences from school mathematics, what is more important is the fact that these studies of everyday mathematics have a great impact on fields such as mathematics education, developmental psychology and cognitive science since they had led researchers to re-think about questions relating to the nature of mathematics, cognition and rationality (Greiffenhagen & Sharrock, 2008). For example, since the past, many hold the dominant view that (Western) mathematics represents one of the trademarks of rationality for two reasons. Firstly, mathematics is known to be a more superior form of human knowledge to others in terms of rationality.

> Problem solving through the use of mathematics has become a cultural symbol for human rationality and ‘right’ thinking and is often considered the underlying mechanism of thought itself. (de la Rocha, 1986, p. 12)

Next, Western cultures were seen as superior to other cultures without professional mathematics.

Nonliterate peoples are often explicitly characterized as simpleminded or childlike, as only capable of concrete thought and not of abstraction or generalization, as of lesser intelligence, as incapable of analytic thought, and as without formal reasoning or logic. In any context these descriptions are heavily judgemental; in the context of mathematics, they are condemning. (Ascher & Ascher, 1986, p. 128)

For the studies of everyday mathematics, researchers have attempted to challenge the notion that rationality only exists in Western cultures. And they have proven that individuals in non-Western 'traditional' cultures without professional mathematics are also capable of developing their own sets of mathematical procedures and systems to deal with practical problems during the course of their practices such as farming, trading or tailoring. For this reason, similar to those in Western cultures, individuals in the non-Western 'traditional' cultures are also 'rational thinkers', and their cultures are never cognitively inferior to Western cultures though the kind of mathematics they used differ greatly from professional mathematics.

Though these studies of everyday mathematics have provided us with an insight into the nature of another kind of mathematics, as well as the closely interwoven relationship between sociocultural and cognitive processes, they do not examine mathematical cognition from a developmental perspective. In fact, these researchers were more keen to examine the mathematical computations and procedures used by the individuals or their mathematical performances in their practices rather than their mathematical development. For this reason, they hardly studied individuals from different age groups, especially children or at different time period in their practices. Perhaps the main question here is how young children of different age groups construct mathematical forms as they engage in their practices with others, and use those mathematical forms to accomplish mathematical functions that are related to their practices. Despite the importance of this question, there is little empirical research that could provide us a detailed insight into the treatment of sociocultural processes in young children's mathematical development.

In the next section, amongst the studies of everyday mathematics, I will review Saxe's work on child street vendors in Brazil. His work is one of the few noteworthy empirical research that has provided us with a better theoretical guidance and understanding of the interplay between the mathematical forms and functions of everyday cultural practices and the development of children's mathematical cognition.

5.3.1 Saxe's Research: The Mathematics of Child Candy Sellers in Brazil

Saxe's work is one of the leading and influential examples among the studies of everyday mathematics. In his work, he has documented mathematical related activities that the Brazilian candy child sellers had engaged in, which include candy purchases at the wholesale stores, setting the retail prices for the candy, making adjustments to the frequent inflation in Brazil, negotiation of prices with the customers and wholesale clerks, and competing with other sellers. And in order to examine the interplay between the mathematical forms and functions of these activities and the mathematical developmental processes of those child street vendors, Saxe used a number of methodologies, such as structured interviews, ethnographic, experiments and quantitative methods in his study.

We have chosen this example, because his work has carefully focused and analysed specific problems encompassing how to account for the nature of everyday cultural practices that require mathematical knowledge. Furthermore, not only does his work made groundbreaking contributions to our understanding of the interplay between the forms of cultural practices and mathematical cognition, Saxe's Emergent Goals Framework has also proven to be useful in the description of various everyday cultural practices such as trade practices of Oksapmin villagers and Brazilian candy street vendors, American students' snack purchasing in local stores (Taylor, 2009) or the tithing practices amongst the Christian communities' children in the United States (Taylor, 2013).

In his work, Saxe challenged the inadequacies of both Piagetian and Vygotskian theories, and based on these inadequacies to develop his theoretical framework. For example, Saxe (1991) explained how his framework distinguishes from both Piagetian and Vygotskian theories:

> Unlike the Piagetian approach, my concern is to treat cognitive development on a level of analysis in which activity-in-sociocultural context is a critical focus and cognitive developmental processes are analyzed with reference to these contexted activities. Unlike Vygotskian writings, which do not develop core development and sociocultural theoretical constructs with reference to systematic analysis of core domains of knowledge, the present approach is concerned with a systematic analysis of mathematical cognition that integrates cognitive developmental and sociohistorical perspectives. (p. 14)

As we look deeper into Saxe's work, it is evident that his work shares the same notion with the works of Wertsch (1993, 1998) and Vygotsky (1962, 1978) that social interactions, cultural artifacts and individual's prior knowledge are the three main important variables required to construct knowledge in the social world's contexts. In his three-component framework, there are three analytic components: practice-linked goals, cognitive forms and functions, and the interplay between learning across contexts. And these components enhance our understanding of the relationship between sociohistorical processes and the development of mathematical cognition.

Practice-linked Goals

For Saxe, specific mathematical goals emerge as those young street candy sellers participated in the practices of candy selling. In other words, their construction of goals tend to be the function of those knowledges that are involved in the practices, and these vendors are constantly shaping their goals as these practices take form in everyday life (Saxe, 1991).

Within this component, Saxe developed the Emergent Goals Framework, which comprises four main parameters that are closely related to the emergence of individuals' goals. The first is the goal structures of the practices, there are the Sell phase, selling of candy to the customers by the sellers, and the Purchase phase, in which the sellers purchased their candy from the wholesales clerks. The second is social interactions where the sellers interacted with other individuals, and their goals were been transformed into cognitive forms during their practices. For example, in order to sell the candy as fast as possible to achieve their money-making motive, the sellers were reported to have constantly modifying their retail unit prices and negotiating with their customers. The third parameter includes the use of cultural artifacts (currency units, candy box), sign forms (Arabical numerals), and conventions (price ratio system). And the last parameter is the prior understanding that these sellers brought to the practices which will either constraint or enhance their emergent mathematical goals. For example, the young sellers were likely to use more simplified pricing and counting system, and received more help from the wholesales clerks when setting the unit prices. In addition, the younger sellers tend to arouse more customers' sympathy, which resulted in speedy sales in some cases (Saxe, 1991). Saxe primarily used ethnographic and structured interviews in this component.

Cognitive forms and functions

In order to document the cognitive forms and function that were used by the children in candy-selling practices, Saxe administered candy selling related tasks, standard orthography tasks, and arithmetical tasks to three groups of children. And these three groups of children comprise the urban candy sellers, urban non-sellers from the same economic level of community, and the rural non-sellers.

From the above tasks, Saxe found that candy sellers accomplish their goals in the candy selling practices by developing a specific kind of cognitive forms (e.g, currency as an alternative number representation) and functions (e.g, use of price ratio convention; determining the candy prices). Though these sellers had little knowledge of standard orthography, they made use of the currency related strategies to solve those arithmetical tasks during the experiments. In addition, Saxe also found out that the sellers performed better in those currency related tasks, and solved most of the tasks using price ratio convention, a strategy which they commonly used in their selling practices on the streets. In his study, Saxe concluded that there is a close relationship between the mathematical developments of these sellers and their candy selling practices, and the sellers and non-sellers developed their mathematical cognition differently. He further explained that due to the frequent and repeated exposure to candy selling linked mathematical problems, these sellers tend to develop familiarity and later expertise in solving mathematical tasks related to candy selling practices. And based on these findings, we are even more convinced that children's mathematical cognition is "located" in specific and familiar contexts rather than general abilities.

The interplay between learning across contexts

In the last component of his framework, Saxe examines the transfer of cognitive forms which are related to school mathematics (algorithmic multiplication procedures) to deal with practical mathematical problems in the candy selling practices. For Saxe, he views transfer "as a process of transforming prior cognitive forms into means of accomplishing new functions through a process of progressive specialization" (p.174) from a development perspective. In his study, he found that children were capable of using cognitive forms which were learnt in one practice to address problems in the other. Further, some sellers were reported to have made use of their school mathematical knowledge to develop new procedures to better accomplish some of the practical mathematical problems that they encountered during their candy selling practices. Based on the findings, Saxe pointed out that "some of the developmental processes that may be entailed in the act of transfer linked to cultural practices" (p. 179). And he further outlined that "the process of development and transfer (or, in terms more closely linked to Vygotsky's writings, learning and development) merge into a single process in which both are intrinsically linked to one another and both have intrinsic links to sociocultural life" (p. 179).

5.3.2 Conclusion

From above, we have seen Saxe investigating a new level of analysis for those activities within the sociohistorical context, in terms of how cognitive forms and functions are been developed as they participate in their cultural practices. His work has not only provided us a detailed descriptive analysis of candy selling practices by those young sellers, but also groundbreaking findings about how the emergence of mathematical goals for the candy sellers vary across their ages. And the most unique contribution of his work lies in his detailed analysis of the development of cognitive forms and functions through the candy selling practices, and also lead us to think deeper into the topics of mathematical cognition "located in" specific and familiar contexts rather than being general abilities.

6. FUTURE RESEARCH DIRECTIONS

From the above, we have looked at the two main areas of study that examined children's mathematical development from a sociocultural perspective. These studies have generated new findings that provide a detailed insight into sociocultural influences and mathematical development relations, and lay the foundation for other related future studies. However, we suggest that there are still some limitation and questions which are left unanswered in these studies, and should be addressed in future studies.

6.1 'Intermental' and 'intramental' functioning are mutually constituting

Studies that examine young children's mathematical development from a sociocultural perspective are commonly inspired by Vygotskian theories, and adopted his notion that cognition develops from 'intermental' functioning to 'intramental' functioning. In other words, many believed that development of mathematical cognition first takes place on social plane (among people during their engagement in joint sociocultural activities) and then on the individual plane (within the child). However, this notion has often constrained our concepts by separating 'intermental' functioning and 'intramental' functioning into two stand-alone entities, with sociocultural forces influencing individuals, and treating mathematical cognition as the "product" of these forces. For this reason, literature in early mathematics often focus on "what" sociocultural forces lead to changes in mathematical cognition rather than "how" sociocultural forces and human mind interplay over the course of mathematical development.

In contrast, I share the same notion with the works of Rogoff (2003) that human development should not be portray as an unidirectional causal process that progresses from 'intermental' functioning to 'intramental' functioning. According to Rogoff (2003):

> In the emerging sociocultural perspective, culture is not an entity that influences individuals. Instead people contribute to the creation of cultural processes and cultural processes contribute to the creation of people. Thus, individual and cultural processes are mutually constituting rather than defined separately from each other. (p. 51)

Although I also hold that the relationship between individual and cultural processes is mutually constituting, I suggest, this relationship might be even more mutually constituting than those suggested by Rogoff.

Because children can often be seen appropriating different aspects of the sociocultural activities as they engage these activities with other individuals, the interpersonal aspects of their functioning in these activities and individual aspects of development seem to occur and interact with each other simultaneously. In other words, the "boundaries" between individual and sociocultural processes has become so diffused to an extent that both processes interweave into a single "mechanism" once individuals participate in any forms of sociocultural activities. For this reason, future studies on early mathematical development might want to analyze the interwoven relationship of the individual mind and social world by avoiding the traditional notion that either is a stand-alone entity or that one brings about the other.

6.2 Appropriation process and mathematical development

The concept of appropriation is one of the central tenets of Neo-Vygotskian concept which has been used to depict how learning is shaped by social interactions with others and how children learn with the guidance and direction provided by the adults (Newman, Griffin, & Cole, 1989; Rogoff, 1990). According to Rogoff (1990), appropriation takes place as individuals share focus of attention, and meanings during their engagement in joint sociocultural activity. In the process of appropriation, information and skills are been transformed rather than transmitted, which means the learners are actively constructing new meanings for what they are been exposed to in joint activity. In other words, appropriation "should not be viewed as limited to the process by which the child (novice) learns from the adult (expert) via a static process of imitation, internalizing observed behaviors in an untransformed manner". (Brown et al., 1993, p. 193)

Although literature in early mathematics from a sociocultural perspective sometimes discuss the appropriation of mathematical practices and reveal the importance of appropriation in children's mathematical development (Saxe, 1991), it is always unclear what is appropriation and how does it function in detail for early mathematics topics. Therefore, in terms of future directions for research, I suggest, it is worth shifting our focus to examine how young children appropriate the different aspects of mathematical practices as they interact with other individuals, and how this social experience can serve as a mechanism for their mathematical development.

Next, by focusing only on the learners' competencies rather than failures in their activities is another limitation of work addressing early mathematics using the concept of appropriation. In order to better understand the link between appropriation and mathematical development, I suggest, it is important to examine not only the learners' competencies but also failures in their mathematical activities because encountering failures is also inevitable in these activities. In fact, by addressing both the learners' competencies and failures in future studies contribute to unveiling how social interactions enhance and constrain the process of young children appropriating the aspects of their mathematical activities, which is closely interwoven with their mathematical development.

6.3 Common psychological underpinnings addressing children's mathematical development

For studies addressing young children's mathematical development from a sociocultural perspective, there is a common trend amongst them, they were more interested in seeking answers to the differences in terms of mathematical performances and behaviours amongst children across cultures rather than examining the common psychological process which these differences are stem from. For example, researchers who compared the mathematical development of East Asian and Western children stressed on the differences among children across cultures instead of their commonalities in their findings. Similarly, in work related to everyday mathematics, researchers seem to be more keen to capture the attention and interest of other researchers and readers by extending their work to the non-Western “traditional” cultures such as child candy sellers in Brazil and Okaspmin children in Papua New Guinea, and portraying how different the mathematical performances and behaviours of these children can be from those of Western cultures.

In contrast, I suggest, although across a wide range of sociocultural contexts, the mathematical performances and behaviours of children appear different on the surface, these differences might actually stem from a common psychological process. In other words, children could have undergone a common psychological process over their course of mathematical development as development proceeds in a systematic fashion. And by understanding this common psychological process better, not only does it enriches theory, it may also be useful for psychologists to predict mathematical performances and behaviours of children outside the contexts in which observations occurred.

7. FINAL NOTE

In order to advance the study of young children's mathematical development through the use of sociocultural perspective, I have suggested three future directions for research on this topic. All this said, tough challenges remain. For example, it might not be easy for some researchers to break away from the commonly held notion that sociocultural forces influence mathematical cognition and adopt the notion that sociocultural process and the human mind are mutually constituting.

Next, it is not uncommon to see work addressing children's mathematical development from a sociocultural perspective, refuting the long-standing universality hypothesis by portraying and comparing the differences in mathematical behaviours, performances and development among children across cultures. I believe many of these researchers would not be keen on going one more step further in developing a theoretical framework, which explains the common psychological processes that are shared by children over the course of their mathematical development. This is because analysing commonalities amongst children across cultures gives the perception that these researchers have contradicted their own notion by “supporting” the concept of “universality”. Furthermore, findings related to the differences amongst children across cultures are more likely to capture the attention and interest of other researchers and readers than reports on commonalities amongst children. For this reason, researchers might find it more noteworthy to examine the former, and therefore they are likely to shift away from work related to the latter.

In sum, I hope that these researchers could provide us a more complete understanding of the sociocultural context of children's mathematical development in their future work. Besides addressing the above future directions, this effort can be expedited if the findings from their work regarding children's mathematical development from a sociocultural perspective are brought together with research findings in the same domain of study that adopted a non-sociocultural approach. For example, how do infants or young children's individual performances on the counting and arithmetic tasks in the work of Gelman (1993), and Wynn (1990, 1992) relate to the sociocultural contexts in which these skills develop? Even though these studies could have ignored the role of sociocultural experiences on children's mathematical development, sociocultural processes are still embedded in their performances.

Finally, I believe work addressing young children's mathematical development still plays a very important role in contributing interesting, unique and noteworthy insights into the discussion of human cognition. This topic continues to leave lots of room for developmental psychologists to debate.

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8. References

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695-701.

Ascher, M. & Ascher, R. (1986). Ethnomathematics. *In History of Science, 24*, 125-144.

Baroody, A. J. (1986). Counting ability of moderately and mildly handicapped children. Education and Training of the Mentally Retarded, 21, 289-300.

Baroody, A. J., & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. Copeley (Ed.), *Mathematics in the early years, birth to five*. Reston, VA: National Council of Teachers of Mathematics.

Bisanz, J., Sherman, J., Rasmussen, C., & Ho, E. (2005). Development of arithmetic skills and knowledge in preschool children. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 143-162). New York: Psychology Press.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5*, 35-45.

Brannon E-M, Van de Walle G-A (2001) The development of ordinal numerical competence in young children. *Cognitive Psychology, 43*(1), 53-81.

Brown, A. L., Ash, D., Rutherford, M., Nakagawa, K., Gordon A., &.Campione, J.C. (1993). Distributed expertise in the classroom. In G. Salomon, (Ed.), *Distributed Cognitions*. New York: Cambridge University Press.

Bryant, P., & Nunes, T. (2002). Children's understanding of mathematics. In U. Goswami (Ed.), *Blackwell handbook of childhood cognitive development* (pp. 412-439). Malden, The Netherlands: Blackwell.

Bull, R., & Johnston, R. S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65*, 1-24.

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Claessens, A., & Engel, M. (2013). How important is where you start? Early mathematics knowledge and later school success. *Teachers College Record, 115*(6), 1-29.

Cordes, S., & Gelman, R. (2005). The young numerical mind: When does it count? In: Campbell JID (Ed), *Handbook of Mathematical Cognition*. (pp. 127-142) New York, NY: Psychology Press.

Dasen, P. R. (1975). Concrete operational development in three cultures. *Journal of Cross-Cultural Psychology, 6*, 156-172.

de la Rocha, 0. (1986). Problems of sense and problems of scale: An ethnographic study of arithmetic in everyday life. (Doctoral dissertation, University of California, Irvine, 1986). *Dissertation Abstracts International, 47*, 4198A.

De Lacey, P. R. (1970). A cross-cultural study of classification ability in Australia. *Journal of Cross-Cultural Psychology, 1*, 293-304.

De Lacey, P. R. (1971). Classification ability and verbal intelligence among high-contact aboriginal and low socioeconomic white Australian children. *Journal of Cross-cultural Psychology, 2*, 393-396.

Dehaene, S. (1997). *The number sense*. New York: Oxford University Press.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2008). 'School readiness and later achievement': Correction to Duncan et al. (2007). *Developmental Psychology, 44*(1), 232-232.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Fuson, K. C., & Hall, J. W. (1983). The acquisition of early number word meanings: A conceptual analysis and review. In H. Ginsburg (Editor), *The Development of Mathematical Thinking*. (pp. 49-107) New York: Academic Press.

Fuson, K. C., & Kwon, Y. (1991). Chinese-based regular and European irregular systems of number words: The disadvantages for English-speaking children. In K. Durkin & B. Shire (Eds.), *Language in mathematical education* (pp. 211- 226). Milton Keynes, GB: Open University Press.

Fuson, K. C., Smith, S. T., & Lo Cicero, A. M. (1997). Supporting Latino first graders' tenstructured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*(6), 738-766.

Gay, J., & Cole, M. (1965). *The new mathematics in an old culture: A study of learning among the Kpelle of Liberia*. New York: Holt, Rinchart, & Winston.

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.

Gelman, R. (1993). A rational-constructivist account of early learning about numbers and objects. In D. Medin (Ed.), *The psychology of learning and motivation. Advances in research and theory* (pp. 69-96).

Gelman, R. & Gallistel, C. (1978) *The Child's Understanding of Number. Cambridge*, MA. Harvard University Press.

Gelman, R., & Meck, E. (1983). Preschooler's counting: Principles before skill. *Cognition, 13*, 343-359.

Gelman, R., & Tucker, M. F. (1975). Further investigations of the young child's conception of number. *Child Development, 46*, 167-175.

Ginsburg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In I. E. Sigel, K. A. Renninger (Vol. Eds.), & W. Damon (Series Ed.), *Handbook of child psychology: Vol. 4. Child psychology in practice* (pp. 401-476). New York: Wiley.

Gray, P. and Feldman, J. (2004). Playing in the zone of proximal development: Qualities of self-directed age-mixing between adolescents and young children at a democratic school. *American Journal of Education, 110*, 108-143.

Greiffenhagen, C. and W. Sharrock (2006). Mathematical relativism: logic, grammar, and arithmetic in cultural comparison. *Journal for the Theory of Social Behaviour, 36*(2), 97-117.

Guberman, S. R. (2004) A comparative study of children's out-of-school activities and arithmetical achievements. *Journal of Research in Mathematics Education, 35*(2), 117-150.

Haith, M. M., & Benson, J. B. (1998). Infant cognition. In W. Damon (Series Ed.) & D. Kuhn & R. S. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 2. Cognition, perception, and language* (5th ed., pp. 199-254). New York: Wiley.

Han, Y., Ginsburg, H. P. (2001). Chinese and English mathematics language: The relation between linguistic clarity and mathematics performance. *Mathematical Thinking and Learning, 3*. 201-220.

Hatano, G. (1990). Toward the cultural psychology of mathematical cognition. Comment on Stevenson, H. W., Lee, S.-Y. (1990). Contexts of achievement. *Monographs of the Society for Research in Child Development, 55* (1-2, Serial No. 221), 108-115.

Ho, C. S., & Fuson, K. C. (1998). Children's knowledge of teen quantities as tens and ones: Comparisons of Chinese, British, and American kindergartners. *Journal of Educational Psychology, 90*, 536-544.

Hughes, M. (1981). Can preschool children add and substract? *Educational Psychology, 1*, 207-219.

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General, 123*, 284-296.

Hyde, J. S., Else-Quest, N. M., Alibali, M. W., Knuth, E., & Romberg, T. (2006). Mathematics in the home: Homework practices and mother-child interactions doing mathematics. *Journal of Mathematical Behavior, 25*, 136-152.

Jordan, N. C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: A comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities, 30*, 24-634.

Kelly, M. K. (2002). Getting ready for school: A cross-cultural comparison of parent and child beliefs about and preparations for entry into first grade in China and the United States. *Dissertation Abstracts International, 63*(11-B), 5550 (University Microfilms No. 3070347).

Koechlin, E., Dehaene, S., & Mehler, J. (1997). Numerical transformations in five month old human infants. *Mathematical cognition, 3*, 89-104.

Laurendeau-Bendavid, M. (1977). Culture, schooling and cognitive development: A comparative study of children in French Canada and Rwanda. In P. R. Dasen (Ed.), *Piagetian psychology: Cross cultural contributions* (pp. 175-200). New York: Gardner.

Lave, J. (1988). *Cognition in practice: Mind, mathematics, and culture in everyday life.* Cambridge, England: Cambridge University Press.

Miller, K.F., Kelly, M.K., & Zhou, X. (2005). Learning mathematics in China and the United States: cross-cultural insights into the nature and course of mathematical development. In J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition* (pp. 163-173). New York: Psychology.

Miller, K., Major, S. M., Shu, H., & Zhang, H. (2000). Ordinal knowledge: Number names and number concepts in Chinese and English. *Canadian Journal of Experimental Psychology, 54*(2), 129-140.

Miller, K. F., Smith, C. M., & Zhang, H. (2004). *Language and number: A longitudinal study of learning to count in Chinese and English.* Unpublished manuscript, University of Illinois at Urbana-Champaign.

Miller, K., Smith, C., Zhu, J., & Zhang, H. (1995). Preschool origins of cross-national differences in mathematical competence: The role of number-naming system. *Psychological Science, 6*, 56-60.

Miller, K. F., & Stigler, J. W. (1987). Computing in Chinese Cultural variation in a basic cognitive skill. *Cognitive Development, 2*, 279-305.

Miura, I. T. (1987). Mathematics achievement as a function of language. *Journal of Educational Psychology, 79*, 79-82.

Miura, I. T., & Okamoto, Y. (2003). Language supports for mathematics understanding and performance. In A. Baroody & A. Dowker (Eds.), *The development of arithmetical concepts* (pp. 229-242). Mahwah, NJ: Lawrence Erlbaum Associates.

Miura, I. T., Okamoto, Y., Kim, C. C., Steere, M., & Fayol, M. (1993). First graders' cognitive representations of number and understanding of place value: Cross national comparisons. *Journal of Educational Psychology, 85*, 24-30.

Mix, K., Huttenlocher, J., & Levine, S.C. (2002). Multiple Cues for Quantification in infancy: Is number one of them? *Psychological Bulletin.128*(2), 278-294.

Newman, D., Griffin, P., & Cole, M. (1989). *The Construction Zone.* Cambridge: Cambridge University Press.

Nisbett, R. E., Peng, K., Choi, I., & Norenzayan, A. (2001). Culture and systems of thought: Holistic vs. analytic cognition. *Psychological Review, 108*, 291-310.

Opper, S. (1977). Piaget's clinical method. *Journal of Children's mathematical Behaviour*, 90-107.

Piaget, J. (1952). *The Child's Conception of Number. London*: Routledge and Kegan Paul Ltd.

Posner, J. K., & Baroody, A. (1979). Number conservation in two Western African Societies. *Journal of Cross- Cultural Psychology, 10*, 479-496.

Price-Williams, D., Gordon, W., & Ramirez, M. (1967). Manipulation and conservation: A study of children from pottery-making families in Mexico. In *Memorias del XI Congreso Interamericano de Psicologıa*. Mexico City.

Reed, H. J., & Lave, J. (1979). Arithmetic as a tool for investigating relations between culture and cognition. *American Ethnologist, 6*, 568-582.

Rogoff, B. (1986). Adult assistance of children's learning. In T. E. Raphael (ed.), *The contexts of school-based literacy* (pp. 27-40). New York: Random House.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.

Rogoff, B. (2003). *The cultural nature of human development*. New York. Oxford University Press.

Rogoff, B., Ellis, S., & Gardner, W. (1984). The adjustment of adult-child instruction according to child's age and task. *Developmental Psychology, 20*, 193-199.

Sakakibara, T. (2008). *Young children's mathematical development in the sociocultural context*. Kazama Shobo.

Saxe, G.B. (1991). *Culture and cognitive development: studies in mathematical development*. Hillsdale NJ: Lawrence Erlbaum Associates.

Saxe, G. B., Gearhart, M., & Guberman, S. R. (1984). The social organization of early number development. In B. Rogoff & J. V. Wertsch (Eds.), *Children's Learning in the Zone of Proximal Development* (pp. 19-30). San Francisco: Jossey-Bass.

Scribner, S. (1986). Thinking in action: some characteristics of practical thought. In R. Sternberg and R. Wanger (Eds.), *Practical intelligence: Nature and origins of competence in the everyday world* (pp. 13-30). Cambridge: Cambridge University Press.

Silverman, I. W., & Rose, A. P. (1980). Subitizing and counting skills in 3-year-olds. *Developmental Psychology, 16*, 539-540.

Starkey, P., & Cooper, R.G. Jr. (1995). The development of subitizing in young children. *British Journal of Developmental Psychology, 13*, 399-420.

Stemhagen, K., & Smith, J.W. (2008). Dewey, Democracy, and Mathematics education: Reconceptualizing the last bastion of curricular certainty. *Education and Culture, 24*(2), 25-40.

Stevenson, H.W., & Lee, S. (1990). *Contexts of achievement: A study of American, Chinese, and Japanese children*. Chicago: University of Chicago Press.

Stevenson, H.W., & Lee, S. (1998). An examination of American student achievement from an international perspective. In D. Ravitch (Ed.), *The state of student performance in American schools* (pp. 7-52). Washington, DC: Brookings.

Stevenson, H. W., Lee, S. Y., & Stigler, J. W. (1986). Mathematics achievement of Chinese, Japanese, and American children. *Science, 231*, 693-699.

Strauss M.S., & Curtis L.E. (1981).Infant Perception of Numerosity. *Child Development, 52*, 1146-1152.

Taylor, E. V. (2009). The purchasing practice of low-income students: The relationship to mathematical development. *Journal of the Learning Sciences, 18*, 370-415.

Taylor, E. V. (2013). The mathematics of tithing: A study of religious giving and mathematical development. *Mind, Culture, and Activity, 20*(2), 132-149.

Uller, C., Huntley-Fenner, G., Carey, S., & Klatt, L. (1999). What representations might underlie infant numerical knowledge? *Cognitive Development, 14*, 1-36.

Vygotsky, L. S. (1962). *Thought and language*. Cambridge MA: MIT Press.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin, Ed.). Cambridge, MA: MIT Press.

Wertsch, J. V. (1993). *Voices of the mind: A sociocultural approach to mediated action.* Cambridge, MA: Harvard University Press.

Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.

Wertsch, J. V., McNamee, G. D., McLane, J. B., & Budwig, N. A. (1980). The adult-dyad as a problem-solving system. *Child Development, 51*, 1215-1221.

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry and Allied Disciplines, 17*, 89-100.

Wynn, K (1990) Children's understanding of counting. *Cognition, 36*, 155-193.

Wynn, K. (1996). Infants' individuation and enumeration of action. *Psychological Science, 7*(3), 164-169.

Wynn, K. (1998). Psychological foundations of number: numerical competence in human infants. *Trends in Cognitive Science, 2*, 296-303.

Young-Loveridge, J. (2015). Guest Editor's Introduction. *Asia-Pacific Journal of Research in Early Childhood Education, 9*(3), 1-4.

幼児期と数

―社会文化的な観点から数的発達を再考する―

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【要旨】 本論文の目的は，社会文化的文脈における幼児の数的発達に関わる，心理学理論と研究についてレビューし，今後展開すべき研究の方向性について提案することであった。始めに，ピアジェの認知発達理論を含む，数的学習と発達に関わる理論の変遷を紹介した後，ピアジェ理論を批判した幼児の数的発達に関する研究を２つのグループに分類した。１つは乳児期と幼児期における数的発達に関する研究であり，もう１つは社会文化的文脈と幼児の数的発達との関連を探った研究である。本論文では目的達成のため，後者の研究である，東アジアと欧米の子どもの数的発達に関する比較研究や，日常生活に関わる数的活動と数的発達に関する研究について中心的に検討した。最後に，社会文化的な観点を用いた幼児の数的発達に関する研究を進展させるための，いくつかの方向性について提案した。

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