学 位 論 文 内 容 の 要 旨

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学 位 論 文 題 名

Low Dimensional Homology of Artin Groups

（アルティン群の低次ホモロジー）

The study of *braid groups* is an active topic in diverse areas of mathematics and theoretical physics. In 1925, E. Artin introduced the notion of braids in a geometric picture. Fox and Neuwirth showed that the configuration space of unordered n-tuples of distinct points in $\mathbb{C}$ is a classifying space of the braid group $Br(n)$. This led to extensive investigations of the cohomology of braid groups by Arnol'd, Fuks, Vainstein et al.

In this work, we first review the basic definitions of *Artin groups* and *Coxeter groups*. The relation between them is a generalization of that between braid groups and symmetric groups. For a Coxeter graph Γ and the associated Coxeter system $(W(\Gamma), S)$, we associate an Artin group $A(\Gamma)$ obtained by, informally speaking, dropping the relations that each generator has order 2 from the standard presentation of $W(\Gamma)$. The braid group $Br(n)$ is the Artin group of type A_{n-1} and the symmetric group $\mathfrak{S}_n$ is the Coxeter group of type A_{n-1}. The Coxeter group $W(\Gamma)$ can be realized as a (in general non-orthogonal) reflection group acting on a convex cone U (called Tits cone) in $\mathbb{R}^n$ with $n = \#S$ the rank of W. Let $\mathcal{A}$ be the collection of reflection hyperplanes. Consider the complement

$$M(\Gamma) = (\mathrm{int}(U) + \sqrt{-1}\mathbb{R}) \setminus \bigcup_{H \in \mathcal{A}} H \otimes \mathbb{C}$$

and the $W(\Gamma)$-action on $M(\Gamma)$, the resulting quotient space $N(\Gamma) = M(\Gamma)/W(\Gamma)$ has fundamental group isomorphic to $A(\Gamma)$. However, it is only conjectured that $N(\Gamma)$ is a $K(A(\Gamma), 1)$ space in general.

The most effective tool in the computation of cohomology of Artin group is the so-called *Salvetti complex* introduced by Salvetti. He associated a CW-complex (known as Salvetti complex) to each real hyperplane arrangement which has the homotopy type of the complement to the complexified arrangement. Later, Salvetti and De Concini applied the construction of Salvetti complex to Coxeter arrangements and obtained a very useful algebraic complex that computes the (co)homology of the quotient space $N(\Gamma)$. Whenever $N(\Gamma)$ is known to be a $K(\pi, 1)$ space, this provides a standard method to compute the (co)homology of the Artin group $A(\Gamma) \cong \pi_1(N(\Gamma))$ over both trivial and twisted coefficients.

Existing results about (co)homology of Artin groups all rely on the truth of the $K(\pi, 1)$ conjecture, since the computations are actually the (co)homology of the quotient space $N(\Gamma)$. In Section

4, we attempt to compute low dimensional homology of arbitrary Artin groups, regardless of the truth of the $K(\pi,1)$ conjecture. It is well-known that low dimensional homology of groups has pure (combinatorial) group-theoretic descriptions. For example, the first homology of a group is isomorphic to its abelianization. Since Artin groups have nice presentations, it makes sense to apply group-theoretic arguments. Our main tool is the classical Hopf's formula on the second homology (or Schur multiplier) of groups, together with Howlett's theorem on the second homology of Coxeter groups. We are primarily inspired by Pitsch's paper *Un calcul élémentaire de* $H_2(\mathcal{M}_{g,1},\mathbb{Z})$ *pour* $g \geq 4$ *(C. R. Acad. Sci. Paris, t. 329, Série I, p. 667-670, 1999)* and Korkmaz-Stipsicz's paper *The second homology groups of mapping class groups of orientable surfaces (Math. Proc. Camb. Phil. Soc. (2003), vol. 134, p. 479-489)*, where the authors computed the second homology groups of mapping class groups of orientable surfaces using Hopf's formula. Section 4 is based on joint work with Professor Toshiyuki Akita.

Our main result is a formula for the second mod 2 homology of arbitrary Artin groups. In fact, we shall prove that for any Artin group $A(\Gamma)$, the second integral homology fits into the following sequence

$$\mathbb{Z}^{p(\Gamma)+q(\Gamma)} \twoheadrightarrow H_2(A(\Gamma);\mathbb{Z}) \twoheadrightarrow \mathbb{Z}_2^{p(\Gamma)+q(\Gamma)}$$

where all maps are surjective, $p(\Gamma)$ and $q(\Gamma)$ are nonnegative integers associated to the Coxeter graph Γ. By taking tensor product with $\mathbb{Z}_2$ for this sequence, we derive that

$$H_2(A(\Gamma);\mathbb{Z}_2) \cong \mathbb{Z}_2^{p(\Gamma)+q(\Gamma)}.$$

As a corollary, we obtain a sufficient condition of the triviality of the Hurewicz homomorphism

$$h_2 : \pi_2(N(\Gamma)) \to H_2(N(\Gamma);\mathbb{Z}).$$

Furthermore, we conclude that the induced Hurewicz homomorphism

$$h_2 \otimes \mathrm{id}_{\mathbb{Z}_2} : \pi_2(N(\Gamma)) \otimes \mathbb{Z}_2 \to H_2(N(\Gamma);\mathbb{Z}) \otimes \mathbb{Z}_2$$

is always trivial. This provides affirmative evidence for the $K(\pi,1)$ conjecture.

In the last section, we present a computation of the cohomology ring structure of 2-dimensional Artin groups. Our computation relies on a suitable Δ-complex structure of the classifying space.