Tidal deformation of Ganymede: Sensitivity of Love numbers on the interior structure

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Abstract: Tidal deformation of icy satellites provides crucial information on their subsurface structures. In this study, we investigate the parameter dependence of the tidal displacement and potential Love numbers (i.e., $h_2$ and $k_2$, respectively) of Ganymede. Our results indicate that Love numbers for Ganymede models without a subsurface ocean are not necessarily smaller than those with a subsurface ocean. The phase lag, however, depends primarily on the presence/absence of a subsurface ocean. Thus, the determination of the phase lag would be of importance to infer whether Ganymede possesses a subsurface ocean or not based only on geodetic measurements. Our results also indicate that the major control on Love numbers is the thickness of the ice shell if Ganymede possesses a subsurface ocean. This result, however, does not necessarily indicate that measurement of either of $h_2$ or $k_2$ alone is sufficient to estimate the shell thickness; while a thin shell leads to large $h_2$ and $k_2$, independent of parameters, a thick shell does not necessarily lead to small $h_2$ and $k_2$. We found that to reduce the uncertainty in the shell thickness, constraining $k_2$ in addition to $h_2$ is necessary, highlighting the importance of collaborative analyses of topography and gravity field data.

1. Introduction

Investigating whether the icy satellites of giant planets possess subsurface oceans or not is crucial not only to reveal their thermal histories but also to understand their astrobiological potential. In terms of this, Ganymede, a Jovian icy satellite, is an interesting body. Theoretical studies have shown that the presence of a subsurface ocean in current Ganymede can be strongly controlled by the composition of the ocean; a pure H2O ocean would have frozen by the present [e.g., Spohn and Schubert, 2003; Bland et al., 2009; Kimura et al., 2009], while an ocean containing other components such as salts and/or ammonia may persist to the present because of its lower melting point [e.g., Sotin and Tobie, 2004]. The interior thermal state would be coupled with the orbital evolution because tidal heating is a potential internal heat source. Although the orbital state of present Ganymede leads to only a negligibly low tidal heating rate [e.g., Hussmann et al., 2010], that of past Ganymede may lead to a higher tidal heating rate; Ganymede may have passed through one or more Laplace-like resonances that were able to pump the orbital eccentricity, supporting the development of a subsurface ocean [Showman et al., 1997; Bland et al., 2009]. While theoretical studies propose different scenarios and different present-day interior structure models, previous observations support the presence of a subsurface ocean in Ganymede indirectly; magnetometer data from the Galileo spacecraft suggest the presence of a subsurface ocean with dissolved electrolytes at a depth of the order of 150 km [Kivelson et al., 2002]. Recent observations of auroral ovals of Ganymede using the Hubble Space Telescope also suggest the presence of an electrically conductive subsurface ocean [Saur et al., 2015]. Nevertheless, more direct observational evidence is necessary to definitively infer the presence/absence of a subsurface ocean in Ganymede and determine its depth and volume.

In 2030s, Jupiter Icy Moons Explorer (JUICE) will orbit Ganymede carrying multiple scientific instruments to reveal the environment, surface, and interior of Ganymede [e.g., Grasset et al., 2013]. During the mission lifetime, Ganymede Laser Altimeter (GALA) on board the JUICE spacecraft will make multiple measurements at many geographically fixed locations to provide a time series of surface displacement [Steinbrügge et al., 2015].
Considering Ganymede’s nonzero eccentricity, the temporal changes of the surface displacement are likely to be due to tidal deformation. In addition, precise radio tracking of the JUICE spacecraft would provide the tidal signal of Ganymede’s gravity field [Parisi et al., 2014]. Since the presence/absence of a subsurface ocean largely affects tidal response [e.g., Moore and Schubert, 2003], tidal deformation measurements through laser altimetry and the radio science experiment are key observations to constrain the interior structure of Ganymede [Grasset et al., 2013].

There are several issues that should be addressed in anticipation of such observations. First, what kind of data is the most effective to determine the presence/absence of a subsurface ocean? If Ganymede possesses a subsurface ocean, with what precision can we constrain the thickness of the ice shell? These issues require numerical studies using realistic interior models under a wide variety of parameter conditions. Although the tidal response of Ganymede has already been investigated by several authors, previous models generally made simplifying assumptions. For example, the models used by Moore and Schubert [2003] ignore the strong temperature dependence of ice viscosity, and those by Steinbrügge et al. [2015] assume a fully elastic model. Another study using 3-D modeling by A et al. [2014] adopts a depth-dependent viscosity profile, though the parameter ranges investigated are limited. While the factors which are not incorporated in previous studies may not be the dominant effects on the tidal response of Ganymede, such effects should be quantified using a realistic model in order to extract quantitative information on the interior structure from tidal deformation measurements. Furthermore, these previous studies consider the amplitude but not the phase lag of tidal response; whether additional constraints on the interior structure can be obtained from the latter quantity has not been examined hitherto.

In this study, we calculate the amplitude and phase lag of tidal deformation under a wide variety of parameter conditions (i.e., ice viscosity, ice rigidity, and shell thickness) and investigate the major controls on the tidal response of Ganymede. We consider cases with and without a subsurface ocean. Sections 2 and 3 describe the numerical calculation models and method adopted, respectively. Section 4 presents results obtained and discusses implications for future geodetic observations.

2. Model

In this study, we adopt a 1-D, fully differentiated Ganymede model consisting of an outer H₂O layer, a rocky mantle, and a metallic core. The H₂O layer is further divided into several layers depending on pressure, as discussed below. The solid H₂O layers and the rocky mantle are assumed to be Maxwellian viscoelastic bodies, and the subsurface ocean and the core are assumed to be inviscid fluids, respectively. Table 1 lists parameters adopted. Free parameters are the thickness of the ice Ih shell \( D_{sh} \), the rigidity of the shell \( \mu_{sh} \), the reference viscosity for the shell (i.e., the viscosity at the melting point) \( \eta_{ref} \), and the viscosity of high-pressure (HP) ices. Although lateral variations in physical properties are not considered, such variations have little effect on the degree-2 tidal deformation for Ganymede [A et al., 2014].

2.1. Density and Bulk Moduli

We model the density profile for the H₂O layer based on the thermodynamic model for water and ices by Choukroun and Grasset [2010]. We assume that the surface layer consists of ice Ih. Assuming that the satellite is in hydrostatic equilibrium, we integrate

\[
\frac{dP}{dr} = -\rho g(r)
\]

(1)

\[
\frac{dg}{dr} = 4\pi G \rho - \frac{2g(r)}{r}
\]

(2)

from the surface to the bottom of the ice Ih shell. Here \( P \) is pressure, \( \rho \) is density, \( g \) is gravitational acceleration, \( r \) is radial distance from the center of the satellite, and \( G \) is the gravitational constant, respectively. We ignore the temperature dependence of density for ice Ih while incorporating its pressure dependence since the former is extremely small while the latter can be up to several percent. The density profile gives the bulk modulus profile via the Adams-Williamson condition:

\[
\frac{d\rho}{dr} + \rho^2 \frac{g(r)}{\kappa(r)} = 0,
\]

(3)

where \( \kappa \) is bulk modulus.
The basal temperature (i.e., the melting point) $T_m$ of the ice I$_h$ shell can be determined from the pressure $P$ at the base of the shell. We assume that the subsurface ocean layer has an adiabatic temperature profile:

$$\frac{dT}{dr} = -\frac{\alpha_v(T)\varrho(T)}{C_p(T)} T,$$

where $T$ is temperature, $\alpha_v$ is thermal expansivity, and $C_p$ is specific heat, respectively. We integrate equation (4) as well as equations (1) and (2) in the ocean layer. The values for $\alpha_v(T)$ and $C_p(T)$ are calculated from the formulation by Choukroun and Grasset [2010]. We compare this $P$-$T$ curve for the ocean layer and the melting curve of HP ices and determine the thickness of the ocean. Results are shown in Figure 1, indicating that the temperature increase in the subsurface ocean can be up to >10 K and leads to a thicker ocean.

In a similar manner to the ice I$_h$ layer, we integrate equations (1) and (2) for HP ice layers and determine the phase boundary between ice III, ice V, and ice VI. Note that the presence of ice III and ice V layers depends on the $P$-$T$ curve of the ocean (thus depends on $D_{sh}$), while the ice VI layer appears under all calculation conditions (see Figure 1).

The determination of the depth of the boundary between the ice VI layer and the rocky mantle and that between the rocky mantle and the metallic core require additional assumptions. For our nominal model, we use a mantle density $\rho_m$ of 3222 kg m$^{-3}$, which is for olivine, and a core density $\rho_c$ of 5330 kg m$^{-3}$, which is for Fe-FeS [Sohl et al., 2002]. Under these assumptions, only two parameters remain to be determined: the radius of the silicate mantle and that of the metallic core. These values are chosen to satisfy two gravitational constraints: the mean density and the normalized moment of inertia given by Schubert et al. [2004] (see Table 1).
Figure 1. Phase diagram of H₂O [Choukroun and Grasset, 2010]. Colored lines show temperature profiles of our models for different shell thicknesses (D_sh). A thinner shell has a higher ocean temperature, leading to a thicker ocean.

Note that we ignore the pressure dependence of densities for mantle and core. To maintain consistency in our model, bulk moduli for these two deepest layers are assumed to be infinite (cf. equation (3)).

Figure 2 shows our interior structure model obtained through the procedures described above. The maximum shell thickness is found to be ∼155 km, which is much thinner than that modeled by Steinbrügge et al. [2015]. This is simply because the work by Steinbrügge et al. [2015] does not consider the phase diagram of H₂O in detail. The most uncertain parameter is the density of the core. We adopt a conservative value for core density, though the actual core density may be that of pure Fe: ∼8000 kg m⁻³ [e.g., Schubert et al., 2004]. Although such a large density leads to a significantly smaller core, as clearly seen in Figure 2, tidal response depends little on the deep interior structure. We found that the difference between Love numbers for our nominal model (i.e., ρ_c = 5330 kg m⁻³) and those using ρ_c = 7800 kg m⁻³ is of the order of 10⁻³. Such a small difference does not change our conclusions.

Figure 2. Our interior structure model for different shell thicknesses (D_sh). Solid and dashed curves show results for the core density ρ_c of 5330 and 7800 kg m⁻³, respectively. The difference in ρ_c strongly affects the radius of the core. See Table 1 for parameters adopted.
Although the uncertainty in the density of the silicate mantle is much smaller than that of the metallic core, we found that the Love numbers are most sensitive to the mantle density compared to other parameters describing the deep interior (i.e., the rocky mantle and the metallic core); a 200 kg m\(^{-3}\) difference in the mantle density can lead to differences in \(|h_2|\) and \(|k_2|\) up to \(\sim 0.11\) and \(\sim 0.043\), respectively. Since typical \(|h_2|\) and \(|k_2|\) are about 1 and 0.5, respectively, these differences are about 10% of the Love numbers. Such large differences, however, can be found only under specific conditions and thus do not change our conclusions (see Appendix A).

### 2.2. Rigidity

The rigidity of ices has been investigated experimentally by several studies. In this study, we assume that each layer has a uniform rigidity since the pressure and temperature dependencies of rigidity are much weaker than those of density and of viscosity [e.g., Schulson and Duval, 2009]. Nevertheless, rigidity of the ice Ih shell may be the major control on tidal deformation of icy satellites [Moore and Schubert, 2000], and a small change in rigidity due to porosity or contaminants may have nonnegligible effects [e.g., Hessinger et al., 1996]. To quantify such an effect, rigidity of the top shell (\(\mu_{sh}\)) is treated as a free parameter.

On the other hand, the rigidity of the rocky mantle is fixed in this study. We found that the use of different values of rigidity for the mantle (i.e., 50 GPa and 120 GPa) leads to differences in \(|h_2| < 0.01\) and that such a small difference does not change our conclusions.

It is also noted that we assume an inviscid fluid core; zero rigidity is assumed. We found that the use of different interior structure models having a purely elastic core, whose rigidity is 100 GPa, leads to the difference in \(|h_2| \sim 0.01\). Consequently, the use of a solid core model does not change our conclusions.

### 2.3. Viscosity

In this study, we assume a flow law of pure water ice and a thermally convective ice Ih shell. More specifically, the viscosity \(\eta\) of the ice Ih shell is calculated from the following equation:

\[
\eta (r) = \eta_{\text{ref}} \exp \left[ \frac{E_a}{R_g T_m(P)} \left( \frac{T_m(P)}{T_\text{ref}} - 1 \right) \right], \tag{5}
\]

where \(T_m(P)\) is the pressure-dependent melting point, \(E_a\) is the activation energy, and \(R_g\) is the gas constant, respectively [e.g., Tobie et al., 2005a]. We use \(E_a = 60\ \text{kJ mol}^{-1}\) for pure water ice [Goldsby and Kohlstedt, 2001]. The thermal profile is calculated using a parameterized convection model. Here we adopt a scaling law between the Rayleigh number (\(Ra\)) and Nusselt number (\(Nu\)) for a bottom heated spherical shell incorporating the effect of curvature and the temperature dependence of viscosity [Yao et al., 2014]:

\[
Nu = \frac{1.46Ra^{0.27}}{\gamma^{1.21}f^{0.78}}, \tag{6}
\]

where \(\gamma\) is a parameter describing the temperature dependence of viscosity given by

\[
\gamma = \frac{E_a}{R_g T_\text{c}^2} \left( \frac{T_m(P)}{T_\text{c}} - 1 \right) \tag{7}
\]

and \(f = 1 - D_{sh}/R_s\) is curvature, respectively. Here \(R_s\) is the radius of Ganymede, and \(T_\text{c}\) is the surface temperature. \(T_\text{c}\) is the temperature of the convective region calculated from

\[
T_\text{c} = T_m(P) - \frac{1.23R_g T_\text{c}^2}{f^{1.5}E_a}. \tag{8}
\]

Parameter values required for determining \(Ra\) and \(Nu\) are listed in Table 1. For cases without a subsurface ocean, we adopt a basal temperature of 250 K for the ice Ih shell and obtained the viscosity profile following the procedure described above. It is noted that we assume that the convective region has a constant temperature \(T_\text{c}\); a temperature increase with the adiabatic temperature gradient is ignored. Since, the adiabatic temperature gradient in the shell is sufficiently small (i.e., \(agT/C_p \sim 10^{-2}\) K/km), it has little effect on the temperature and the viscosity profile.

Figure 3 shows temperature and viscosity profiles of the ice Ih shell for \(D_{sh} = 100\ \text{km}\). For this case, thermal convection occurs if \(\eta_{\text{ref}} < 10^{19}\ \text{Pa s}\) because of the thick shell. The viscosity near the surface is much higher
Results for shell thickness $D_{sh} = 100$ km are shown. Because of the thick shell, thermal convection occurs for most cases, leading to a constant temperature and viscosity region.

than that for the convective region because of the low surface temperature. In this study, such strongly depth-dependent viscosity profiles for the ice Ih shell are employed.

On the other hand, we assume that each HP ice layers and the rocky mantle have a uniform viscosity profile. Since rheologies of HP ices under a low-stress condition are highly uncertain [e.g., Kubo et al., 2006], we consider a wide range of the viscosity for the HP ice layer (i.e., $10^{12} - 10^{17}$ Pa s). For cases without an ocean, the viscosity for ice III ($\eta_{III}$), ice V ($\eta_{V}$), and ice VI ($\eta_{VI}$) are assumed to be independent free parameters. On the other hand, for cases with an ocean, these viscosities are assumed to be the same and denoted as $\eta_{HP}$ ($= \eta_{III} = \eta_{V} = \eta_{VI}$).

It is noted that our nominal model assumes a viscosity of $10^{21}$ Pa s for the rocky mantle. We found that the use of different values of viscosity for the mantle (i.e., $10^{19}$ Pa s and $10^{23}$ Pa s) leads to differences in $|h_2| < 10^{-4}$ and that such a small difference does not change our conclusions.

2.4. Free Parameters

Here we summarize the free parameters in this study. For cases with a subsurface ocean, there are four free parameters: the thickness of the shell ($D_{sh}$), the rigidity of the ice Ih ($\mu_{sh}$), the reference viscosity of ice Ih ($\eta_{ref}$), and the viscosity of HP ices ($\eta_{HP}$). We use 155 different values for $D_{sh}$, ranging from 1 to 155 km; the interval is uniformly 1 km. For $\mu_{sh}$, we use 100 different values, ranging from 0.1 to 10 GPa; the interval is uniformly 0.1 GPa. For $\eta_{ref}$, we use 101 different values, ranging from $10^{12}$ to $10^{17}$ Pa s; the interval is uniformly 0.05 in terms of common logarithms. For $\eta_{HP}$, we use six different values, ranging from $10^{13}$ to $10^{17}$ Pa s; the interval is uniformly 1 in terms of common logarithms. Thus, we consider $155 \times 100 \times 101 \times 6 = 9,393,000$ cases.

For cases without a subsurface ocean, there are five free parameters: the rigidity of the ice Ih ($\mu_{sh}$), the reference viscosity of ice Ih ($\eta_{ref}$), and the viscosity of HP ices ($\eta_{III}, \eta_{V}, \eta_{VI}$). For $\mu_{sh}$, we use 34 different values, ranging from 0.1 to 10 GPa; the interval is uniformly 0.3 GPa. For viscosities, we use 26 different values, ranging from $10^{12}$ to $10^{17}$ Pa s; the interval is uniformly 0.2 in terms of common logarithms. Thus, we consider $34 \times 26 \times 26 \times 26 = 15,537,184$ cases.

It is noted that above numbers of cases are for our nominal model. As discussed above, we conduct sensitivity checks for fixed parameters, such as the density of the core. Such additional calculations are not counted in above numbers.

3. Love Number Calculation Method

In a Ganymede reference frame, the time-dependent tidal potential $\Phi_t$ to first order in the eccentricity is given by

$$\Phi_t (r, \theta, \phi, t) = r^2 \omega^2 e \left[ -\frac{3}{2} P_2^0 (\cos \theta) \cos \omega t \\
+ \frac{1}{4} P_2^0 (\cos \theta) \{ 3 \cos \omega t \cos 2\phi + 4 \sin \omega t \sin 2\phi \} \right],$$

Figure 3. (a) Temperature and (b) viscosity profiles of the top ice Ih shell for different reference viscosities ($\eta_{ref}$).
where $\omega$ is orbital angular frequency, $e$ is eccentricity, $t$ is time, $\theta$ is colatitude, $\phi$ is longitude with zero longitude at the sub-Jovian point, and $P_m^n$ is the associated Legendre polynomial of degree $n$ and of order $m$, respectively [e.g., Kaula, 1964]. Such a potential causes degree-2 deformation on a satellite. Thus, we calculate spheroidal degree-2 deformation using a spectral scheme which has been used in several previous studies [e.g., Tobie et al., 2005b; Wahr et al., 2009; Kamata et al., 2015]. In the following, we briefly summarize the method. See Kamata et al. [2015] for further details.

The governing equation system consists of three equations; the equation of momentum conservation for a self-gravitating body given by

$$\rho \frac{d^2\mathbf{u}}{dt^2} = \nabla \cdot \mathbf{\sigma} + \rho \nabla \Phi,$$

(10)

the Poisson equation for the gravitational field given by

$$\nabla^2 \Phi = 4\pi G \nabla \cdot (\rho \mathbf{u}),$$

(11)

and the constitutive equation for a Maxwell body given by

$$\frac{d\sigma_y}{dt} + \frac{\mu}{\eta} \left( \sigma_y - \sigma_{y3} \delta_{y3} \right) = \lambda \frac{d\epsilon_{y3}}{dt} + 2\mu \frac{d\epsilon_{y3}}{dt},$$

(12)

where $\mathbf{u}$ is the displacement vector, $\mathbf{\sigma}$ is the stress tensor, $\epsilon$ is the strain tensor, $\Phi$ is the gravitational potential, $\eta$ is viscosity, $\mu$ is rigidity (i.e., shear modulus), $\lambda(=\kappa - 2\mu/3)$ is the first Lamé's parameter, and $\delta_y$ is the unit diagonal tensor, respectively. The effect of bulk viscosity is not taken into account. For solid layers, applications of Fourier transformation in the time domain and spherical harmonic expansion in the spatial domain lead to a six-component first-order differential equation system:

$$\frac{dy_i(n,r)}{dr} = \sum_{j=1}^{6} A_{ij}(n,r)y_j(n,r) \ (i,j = 1, 2, \ldots, 6),$$

(13)

where $n$ is spherical harmonic degree, $y_i$ is the coefficient for the vertical displacement, $y_2$ is that for the vertical stress, $y_3$ is that for the tangential displacement, $y_4$ is that for the tangential stress, $y_5$ is that for the gravitational potential perturbation, and $y_6$ is that for the “potential stress.” The matrix $A$ is a function of the interior structure model, i.e., $\rho(r)$, $\mu(r)$, $\kappa(r)$, $\eta(r)$, and the frequency of potential perturbation, $\omega$. Similarly, a four-component first-order differential equation system is obtained for a liquid layer which has $\mu = 0$ [e.g., Takeuchi and Saito, 1972; Kamata et al., 2015]. One can integrate equation (13) using a numerical integration scheme and obtain linearly independent solutions. The coefficients for such solutions can be determined from the boundary conditions at the surface, layer boundaries, and the center.

Nondimensional complex Love numbers can be written using the solution $y$ at the surface; the degree-2 displacement Love number $h_2$ and potential Love number $k_2$ are given by

$$h_2 = g_y y_2(n=2, r=R_s) / \Phi_i(n=2)$$

(14)

$$k_2 = y_2(n=2, r=R_s) / \Phi_i(n=2) - 1$$

(15)

where $g_y$ is the gravitational acceleration at the surface and $\Phi_i(n)$ is the degree-$n$ coefficient of tidal potential. Note that $y$ (and thus $h_2$ and $k_2$) is a complex variable, and the absolute value and argument yield the amplitude and phase shift, respectively.

4. Results and Discussion

We first consider how to infer the presence/absence of a subsurface ocean based on Love numbers. We then investigate the major control on tidal response to understand what information we can obtain from Love numbers. We find that it is the thickness of the ice Ih shell if Ganymede possesses a subsurface ocean, though a precise estimate of the shell thickness may be difficult if we use either of $h_2$ or $k_2$ alone. Finally, we consider how well we can constrain the shell thickness if we use both $h_2$ and $k_2$.

4.1. The Presence/Absence of a Subsurface Ocean

Figure 4 shows the relation between the absolute values of $h_2$ and $k_2$. For cases with a subsurface ocean, $|h_2|$ and $|k_2|$ range about 1.1–1.7 and about 0.36–0.57, respectively. These values are consistent with those
obtained by previous studies [e.g., Moore and Schubert, 2003; Steinbrügge et al., 2015]. A model with a larger $|h_2|$ tends to have a larger $|k_2|$, indicating that such a model has a softer ice shell. This trend is also the case for models without a subsurface ocean. The ranges of Love numbers, however, are clearly different between cases with and without an ocean; the latter model covers much wider ranges, $|h_2|$ of 0.05–1.6 and $|k_2|$ of 0.03–0.57, respectively. This result is also consistent with those obtained by a previous study [Moore and Schubert, 2003]. Comparing these results, it can safely be said that if future geodetic observations yield $|h_2| < 1$ or $|k_2| < 0.35$, Ganymede does not possess a subsurface ocean. On the other hand, large Love numbers (i.e., $|h_2| > 1$ or $|k_2| > 0.35$) can be seen not only for cases with an ocean but also for those without an ocean (dashed ovals in Figure 4). Consequently, if a large tidal amplitude is observed, it would be difficult to infer whether Ganymede possesses a subsurface ocean or not in the absence of other constraints.

Nevertheless, this result does not necessarily indicate that geodetic measurements cannot infer the presence/absence of a subsurface ocean in Ganymede. We found that the phase lag of tidal deformation is a key to determine the presence/absence of a subsurface ocean. Figure 5 shows the relation between the amplitude and phase lag for $k_2$. A phase lag only as large as $12^\circ$ is found for cases with an ocean. On the other hand, cases without an ocean generally have large phase lags up to about $65^\circ$. While a small phase lag (i.e., $<10^\circ$) is also found for cases without an ocean, such a model mostly accompanies a small amplitude (i.e., $|k_2| < 0.1$). Such a small amplitude is not obtained for cases with an ocean. Consequently, measurements of the phase lag as well as the amplitude provide critical information for the presence/absence of a subsurface ocean.

Figure 4. Two-dimensional histogram of the tidal amplitude for (a) cases with a subsurface ocean and for (b) cases without a subsurface ocean. The horizontal and vertical axes represent the absolute value of the displacement Love number ($h_2$) and that of the potential Love number ($k_2$), respectively. The dashed oval represents the distribution of solutions for cases with an ocean.

Figure 5. Two-dimensional histogram of the gravitational tidal response for (a) cases with a subsurface ocean and for (b) cases without a subsurface ocean. The horizontal and vertical axes represent the absolute value and the argument of the potential Love number ($k_2$), respectively. The dashed oval represents the distribution of solutions for cases with an ocean.
Figure 6. Partial mean values of the absolute value of Love numbers for (a) cases with a subsurface ocean and for (b) cases without a subsurface ocean. Results for $|h_2|$ are shown using solid lines and closed circles, while those for $|k_2|$ are shown using dashed lines and open circles. Red and blue lines and symbols are variables for the ice Ih shell and for high-pressure ice layers, respectively. The factor that leads to the largest variation in partial means is $D_{sh}$ for Figure 6a and $\eta_{VI}$ for Figure 6b.

We note that this argument is also applicable to $h_2$; cases with an ocean have phase lags $< \sim 10^\circ$, and cases without an ocean having such small phase lags mostly accompany $|h_2| < 0.2$.

The detectability of phase lag is not clear particularly for $h_2$. However, Parisi et al. [2014] argue that gravity field measurements with a low altitude would provide $k_2$ with an absolute accuracy of about $10^{-3}$, both for real and imaginary parts. The imaginary part of $k_2$ is given by $-|k_2| \sin(\phi_{lag}) \approx -|k_2| \phi_{lag}$, where $\phi_{lag}$ is phase lag and is a small value. Consequently, if $|k_2| = 0.5$ and $\phi_{lag} = 1^\circ$, the imaginary part of $k_2$ is $\sim 0.01$. Thus, precise gravity field measurements can lead to the determination precision of the phase lag of the order of $1^\circ$ or better. We thus conclude that geodetic measurements during the JUICE mission would have a sufficient precision to infer the presence/absence of a subsurface ocean.

4.2. Major Control on Tidal Response

In addition to the presence/absence of a subsurface ocean, what kind of information can we obtain from the tidal response? In other words, what is the major control on the tidal response of Ganymede? To answer this question quantitatively, we analyze our results using partial means. Consider a variable $X$ which is a function of $Y$ and $Z$, and $Y$ is the major control on $X$. Under such a situation, the partial means of $X$ for a given $Y_i$, 

$$\bar{X}(Y_i) = \frac{1}{j_{max}} \sum_{j=1}^{j_{max}} X(Y_i, Z_j), \quad (16)$$

depend strongly on the choice of $Y_i$ because $Y$ is the major control on $X$. On the other hand, the partial means for a given $Z_i$, 

$$\bar{X}(Z_i) = \frac{1}{j_{max}} \sum_{j=1}^{j_{max}} X(Y_j, Z_i), \quad (17)$$

depend little on the choice of $Z_i$ because $Z$ is the minor control on $X$. Thus, the factor that leads to the largest variation in partial means is the major control.

Figure 6 shows variations in partial means for the absolute value of Love numbers. For cases with a subsurface ocean, we find that the thickness of the ice Ih shell ($D_{sh}$) gives the largest variation in partial means for $|h_2|$ and $|k_2|$. Consequently, the major control on tidal response is the thickness of the ice Ih shell. On the other hand, for cases without a subsurface ocean, we find that the major control is the viscosity of the ice VI layer, which is the deepest and thickest H$_2$O layer (see Figure 2).

It should be noted that the dependence of Love numbers on $D_{sh}$ is not linear; Love numbers decrease with increasing shell thickness if $D_{sh} \leq 152$ km, though Love numbers increase if $D_{sh} \geq 152$ km. The former trend indicates that a thicker shell is more resistant to deformation. The latter, counterintuitive result is caused by tidal resonance; a thin ocean having a gravity wave speed close to the phase velocity of degree-2 deformation enhances tidal deformation [e.g., Kamata et al., 2015].
4.3. Estimation of the Thickness of the Ice Ih Shell

Although the thickness of the ice Ih shell is the major control on the amplitude of tidal response (if Ganymede possesses a subsurface ocean), unfortunately this does not necessarily indicate that measurements of either of \(|h_2|\) or \(|k_2|\) alone are sufficient to estimate the thickness of the ice Ih shell precisely. Figure 7a shows the range of \(|h_2|\) as a function of \(D_{sh}\). If \(D_{sh}\) is small, the range of \(|h_2|\) is small independent of other parameters. As \(D_{sh}\) increases, the range of \(|h_2|\) increases, indicating that other parameters have a larger effect on the tidal response. If \(D_{sh} = 155\) km, the thickness of the subsurface ocean is <1 km, leading to an enhanced \(|h_2|\) due to tidal resonance [Kamata et al., 2015]. We found the same trends for \(|k_2|\): a small \(D_{sh}\) leads to \(|k_2| \sim 0.55\), while a large \(D_{sh}\) leads to 0.36 < \(|k_2| < 0.56\), and \(D_{sh} = 155\) km could lead to \(|k_2| > 0.56\).

In terms of shell thickness estimation based on Love numbers, these results indicate that (1) if a small tidal amplitude (for example, \(|h_2| = 1.1\)) is observed, the ice Ih shell is thick (i.e., \(D_{sh} \sim 150\) km), and that (2) if \(|h_2| > 1.6\) is observed, \(D_{sh} > 154\) km; a thin ocean beneath a thick shell leads to tidal resonance. The problem is that if we observe a relatively large amplitude, such as \(|h_2| \sim 1.5\), it is difficult to estimate the thickness of the ice Ih shell.

One might argue that such a large uncertainty could be reduced by using a linear combination of Love numbers. Wahr et al. [2006] consider tidal deformation on Europa and show that 1 + \(|k_2| - |h_2|\) linearly depends on shell thickness and thus could be a good index for shell thickness. Steinbrügge et al. [2015] show that this is also the case for Ganymede. These works, however, use fully elastic models with a given ice rigidity. We found that different viscosities and/or rigidities lead to a different 1 + \(|k_2| - |h_2|\) for a given \(D_{sh}\), as shown in Figure 7b. Thus, the use of this linear combination of Love numbers would not reduce the uncertainty in the shell thickness unless we assume the viscosity and rigidity of the ice Ih shell.

Nevertheless, the use of both \(|h_2|\) and \(|k_2|\) can reduce the uncertainty in the shell thickness. Figure 8 shows the relation between \(D_{sh}\) and \(|k_2|\) for a given \(|h_2|\). Results for all the ranges of \(\eta_{ref}\), \(\mu_{sh}\), and \(\eta_{HP}\) are shown. For a given \(\eta_{HP}\), \(|k_2|\) monotonically changes with increasing \(D_{sh}\) (if we fix \(|h_2|\)). See Appendix B for an explanation for the trends. Figure 8 indicates that if we can constrain the viscosity of HP ices, precise measurements of \(|h_2|\) and \(|k_2|\) enable us to determine the thickness

**Figure 7.** Two-dimensional histogram of (a) tidal amplitude (|\(h_2|\)) and of (b) a linear combination of Love numbers as functions of shell thickness (\(D_{sh}\)). A thin and a thick shell have small and large variations in |\(h_2|\) and 1 + |\(k_2| - |h_2|\).

**Figure 8.** The absolute value of \(k_2\) as a function of shell thickness (\(D_{sh}\)) for a given \(|h_2|\). If \(|h_2|\) and \(\eta_{HP}\) is given, \(|k_2|\) is a good index for \(D_{sh}\) independent of \(\mu_{sh}\) and \(\eta_{ref}\).
of the ice Ih shell precisely. Consequently, in addition to the precise determination of $|h_2|$ and $|k_2|$, studies of high-pressure ice rheologies and detailed thermal modeling of the deep interior of Ganymede are important for constraining the structure of the upper part of Ganymede. For ice II, the effective viscosity under a low-stress condition can be smaller even by several orders of magnitude than that for ice Ih [Kubo et al., 2006]. Our results indicate that whether a viscosity $<10^{15}$ Pa s is plausible for HP ices or not under the conditions relevant to Ganymede is an important subject to be investigated.

Actual tidal deformation measurements cannot avoid including some errors. Figure 9 shows the uncertainty in the shell thickness with plausible ranges of errors in $|h_2|$. As already seen in Figure 7a, the uncertainty in $D_{sh}$ is extremely large if $|h_2| \sim 1.5$. Consequently, a decrease in the error for $|h_2|$ does not lead to a decrease in the uncertainty in $D_{sh}$ (Figure 9a). However, as noted above, $|k_2|$ depends strongly on $D_{sh}$ for a given $|h_2|$. We found that a decrease in the error for $|h_2|$ leads to a decrease in the uncertainty in $D_{sh}$ if $|k_2|$ is constrained (Figure 9b). The nominal error in the $|h_2|$ measurements using Ganymede Laser Altimeter (GALA) on board the JUICE spacecraft is $\sim 2\%$ and decreases with increasing number of crossover points [Steinbrügge et al., 2015]. Thus, our results highlight the importance of an extended mission for altimetric measurements as well as of collaborative analyses of topography and gravity field data for constraining the interior structure of Ganymede.

4.4. Discussion

Our results indicate that the tidal amplitude does not have a one-to-one correspondence with the shell thickness; a thick shell leads to a wide range of $|h_2|$ and $|k_2|$ depending on the property of the ice Ih shell (section 4.3). One might argue that the tidal amplitude is controlled not by the thickness of the shell but by the thickness of the lithosphere or the heat flux. Figure 10 compares the tidal response and the thickness of the lithosphere (i.e., the layer with a viscosity higher than $10^{25}$ Pa s) and the surface heat flux, illustrating that the tidal amplitude does not have a one-to-one correspondence with these parameters either. Thus, it would be difficult to constrain the lithospheric thickness and the heat flux from the tidal amplitude. On the other hand, phase lag can be large only if the lithospheric thickness is $\sim 5$ km and the surface heat flux is $\sim 15$ mW m$^{-2}$. Consequently, if we observe a phase lag of $\sim 8^\circ$ and $|h_2|$ of $>1$, it would imply the presence of a subsurface ocean as well as a lithospheric thickness of $\sim 5$ km. Nevertheless, if we observe a small phase lag (i.e., $<2^\circ$), we can constrain neither of the thickness of the lithosphere nor the heat flux.

In the following, we consider the validity of our model assumptions. The major simplification in our model is that we assume a pure H$_2$O ocean. If the ocean contains other molecules, such as ammonia, the melting temperature (i.e., the basal temperature) is lower than our model [e.g., Choukroun and Grasset, 2010]. This would lead to a decrease in temperature and an increase in the viscosity of the ice shell. However, as discussed above, the viscosity of the ice Ih shell is not the major control on the tidal response of Ganymede. In addition to the melting temperature, the density of the ocean also depends on its chemical composition. Although its effect on the tidal response is not completely zero, it should be small since the properties of the ice Ih shell largely determine the tidal response. Thus, while the quantification of these effects requires further detailed modeling using a phase diagram of nonpure water ice and different thermophysical models, our results would not change significantly.
Another simplification adopted in this study is the rheology of the ice shell. The presence of other volatiles, such as CO₂ and CH₄, may change the viscosity of ice [e.g., Durham et al., 2010], though we explored a wide range of the reference viscosity and found that it is not the major control on tidal response.

In our model, an increased activation energy (leading to an enhanced deformation rate) at a high temperature (i.e., >258 K) seen in laboratory experiments [Goldsby and Kohlstedt, 2001] is not incorporated. Furthermore, the temperature dependence of thermal conductivity and other parameters for ice Ih are not considered. This is because the Ra-Nu scaling law we adopt is based on models which neglect these effects. For further detailed modeling, development of a Ra-Nu scaling law incorporating these effects is necessary.

5. Conclusion

We modeled the tidal response of Ganymede adopting a realistic interior structure model considering a convective ice shell and investigated its parameter dependence. Cases with a subsurface ocean lead to large tidal amplitudes, while those without a subsurface ocean can lead to a wide range of the tidal amplitude depending mainly on the viscosity of the ice VI layer. Because of this, it is difficult to infer whether Ganymede possesses a subsurface ocean or not based only on the amplitude of tidal deformation. In contrast, the phase lag is different between cases with and without an ocean; models with an ocean have phase lags smaller than models without an ocean. Consequently, the determination of the phase lag as well as the tidal amplitude is critical to infer the presence/absence of a subsurface ocean in Ganymede. If a subsurface ocean exists, the major control on tidal response is the thickness of the ice Ih shell. To estimate the thickness of this shell precisely, constraints on both of the Love numbers, \( h_2 \) and \( k_2 \), with a high accuracy is necessary. This result indicates the importance of an extended mission time as well as of data analyses using both of topography and gravity field data.
Appendix A: Different Rocky Mantle Densities

A change in the mantle density affects the depths of boundaries between the ice VI layer and the rocky mantle and between the rocky mantle and the metallic core. Consequently, the mantle density controls the thickness of the ice VI layer. This is because the mean density and the moment of inertia for our model need to match the observed values. As discussed in section 4.2, the viscosity of this deep ice layer is the major control on tidal response if a subsurface ocean does not exist. Because of this, the use of a different mantle density could lead to nonnegligible contributions to the tidal response for cases without an ocean.

However, the tidal response for cases with a subsurface ocean is almost independent of the mantle density. In addition, cases without an ocean and with a small phase lag (i.e., $<10^\circ$) have small difference in Love numbers (i.e., $\Delta |k_2| \sim 0.01$) between models using different mantle densities, as seen in Figure A1. Consequently, even if we adopt different mantle densities, solutions for cases with an ocean remain confined in the dashed region in Figure 5, while most solutions for cases without an ocean are distributed outside the dashed region in the figure. Thus, the use of different mantle densities does not affect our conclusion that the determination of the phase lag as well as the amplitude is a key to infer whether Ganymede possesses a subsurface ocean or not.

Appendix B: Trends of $|k_2|$ as a Function of $D_{sh}$ Under a Given $|h_2|$?

Here we examine what determines the trends of $|k_2|$ as a function of $D_{sh}$ under a given $|h_2|$ seen in Figure 8. Deformation of density boundaries produces gravity anomalies, but how it contributes to $|k_2|$ depends both on the amplitude and on the depth; a larger amplitude and a shallower boundary lead to a larger $|k_2|$.

Figure B1 shows the amplitudes of deformation at three different depths: surface, the base of the ice Ih shell, and the top of the high-pressure (HP) ice layer. Since we consider models which give $|h_2| = 1.5$, the amplitude of surface deformation is fixed. The amplitude of deformation at the base of the ice Ih shell is close to that at the surface and is nearly independent of $\eta_{HP}$ nor $D_{sh}$. While the amplitude does not vary significantly, its depth increases with increasing $D_{sh}$, leading to a smaller contribution to $|k_2|$ with increasing $D_{sh}$. This causes the decrease in $|k_2|$ with increasing $D_{sh}$ for $\eta_{HP} = 10^{13}$ Pa s (see Figure 8).
On the other hand, the amplitude of deformation at the top of HP ice layer depends on $\eta_{HP}$; if $\eta_{HP} \gtrsim 10^{13}$ Pa s, the HP ice layers are relatively highly viscous, so that the ocean-HP ice boundary does not deform significantly. In contrast, if $\eta_{HP} \lesssim 10^{12}$ Pa s, the ocean-HP ice boundary deforms because of a low HP ice viscosity. Since a thinner ocean leads to a stronger gravitational coupling between the ice Iih shell and the HP ice layers, a larger deformation occurs for a larger $D_{th}$. This causes the increase in $|k_2|$ with increasing $D_{th}$ for $\eta_{HP} = 10^{12}$ Pa s (see Figure 8).

**References**


