Direct Synthesis of Equivalent Circuits from Reduced FE Models using Proper Orthogonal Decomposition

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Abstract
Purpose – This paper proposes a fast synthesis method of the equivalent circuits of electromagnetic devices using model order reduction. Finite element method (FEM) has been widely used to design electromagnetic devices. For FE analysis of these devices connected to control and deriving circuits, FE equations coupled with the circuit equations have to be solved for many times in their design processes. If the FE models are replaced by equivalent circuit models, computational time could be drastically reduced.
Design/methodology/approach – In the proposed method, a reduced FE model is obtained using proper orthogonal decomposition (POD) in which the size of FE equation is effectively reduced so that the computational time for FE analysis is shortened. Then, the equivalent circuits are directly synthesized from the admittance function of the reduced system.
Findings – Accuracy and computational efficiency of the proposed method are compared with those of another POD-based method in which the equivalent circuits are synthesized from fitting of frequency characteristics using optimization algorithm. There are no significant differences in the accuracy of both method, while the speed-up ratio of the former method is found larger than that for the latter method for the same sampling points.
Originality/value – The equivalent circuits of electric machines and devices have been synthesized on the basis of physical insight of engineers. This paper proposes a novel method by which the equivalent circuits are automatically synthesized from finite element model of the electric machines and devices using POD.

Introduction

Finite element method (FEM) has been widely used to design electromagnetic devices. For FE analysis of these devices connected to control and deriving circuits, we have to solve FE equations coupled with the circuit equations for many times in their design processes. If the FE models are replaced by equivalent circuit models, computational time could be drastically reduced. For example, to analyze electric machines such as motors or reactors connected to external circuits, we solve the FE equation governing quasi-static electromagnetic fields coupled with the circuit equation. In the design process, we have to repeat this time-consuming analysis for many times. It is possible to reduce the computational time by replacing the FE model to an equivalent circuit model. It has been difficult, however, to express frequency response accurately in sufficiently wide frequency range using the conventional equivalent circuit which is synthesized on the basis of knowledge and experience of engineers. If we synthesize the equivalent circuit directly from the FE model, its accuracy would be much improved.

Equivalent circuits can be synthesized using rational polynomial approximation applied to the transfer function computed from FE analysis. In this method, we compute the frequency characteristics using FEM. Then, they are approximately represented by the rational polynomials whose coefficients can be determined by, for example, the vector fitting algorithm efficiently without solving non-linear optimization problem [1]. To obtain the frequency characteristics, however, we need frequency sweep based on FE analysis or transient FE analysis which are time consuming.

Model order reduction based on proper orthogonal decomposition (POD) [2, 3] in which the size of FE equation is effectively reduced so that the computational time for FE analysis is shortened has been proposed and applied to FE analysis of the electromagnetic devices [4-7]. The authors have employed POD to realize a fast frequency sweep for synthesis of equivalent circuits [8]. In this method, the circuit parameters are determined using real coded genetic algorithm (RGA) so that the frequency characteristics of the equivalent circuit are sufficiently fit to those computed from FEM. The equivalent circuit parameters would be, however, dependent on the setting of the RGA parameters. Moreover, if the equivalent circuits can be synthesized without the frequency sweep for fitting, the computational time could be further reduced.

In this paper, we propose a new fast synthesis method of equivalent circuits using POD which does not need time-consuming fitting processes. We compare the numerical performance of the proposed method with that of the fitting-based method [8].

Proper orthogonal decomposition

Let us consider three dimensional magneto-quasistatic fields governed by

$$\mathrm{rot}\,\nu(\mathrm{rot}\boldsymbol{A}) + j\omega\kappa(\boldsymbol{A} + \mathrm{grad}\varphi) = \boldsymbol{J} \tag{1a}$$

$$\mathrm{div}\,\kappa(\boldsymbol{A} + \mathrm{grad}\varphi) = 0 \tag{1b}$$

where $\boldsymbol{A}, \varphi, \boldsymbol{J}, \nu, \kappa, \omega$ denote vector potential, scalar potential, current density, reciprocal of permeability, electric conductivity and angular frequency, respectively. By applying the edge-based FEM to (1), we obtain

$$\sum_i a_i \int_\Omega \left(\nu\,\mathrm{rot}\boldsymbol{N}_j \cdot \mathrm{rot}\boldsymbol{N}_i + j\omega\kappa\,\boldsymbol{N}_i \cdot \boldsymbol{N}_j\right)\mathrm{d}V + \sum_k \varphi_k \int_\Omega j\omega\kappa\,\boldsymbol{N}_j \cdot \mathrm{grad}N_k \mathrm{d}V = I \int_\Omega \boldsymbol{N}_j \cdot \boldsymbol{j}\mathrm{d}V \tag{2a}$$

$$\sum_i a_i \int_\Omega j\omega\kappa\,\boldsymbol{N}_i \cdot \mathrm{grad}N_u \mathrm{d}V + \sum_k \varphi_k \int_\Omega j\omega\kappa\,\mathrm{grad}N_k \cdot \mathrm{grad}N_u \mathrm{d}V = 0 \tag{2b}$$

where a_i, $\boldsymbol{N}_i$, N_k, φ_k, denote the line integral of $\boldsymbol{A}$ along i-th edge, edge-basis function, node-basis function, the scalar potential value at k-th node, respectively, and I, $\boldsymbol{j}$ denote the current and unit vector parallel to $\boldsymbol{J}$. The electromagnetic field is assumed to be coupled with a circuit governed by

$$V = RI + L\frac{\mathrm{d}I}{\mathrm{d}t} + \frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{3}$$

where V, R, L and Φ denote input voltage, external resistance and inductance and the magnetic flux which is computed from $\Phi = \Sigma_j a_j \int_\Omega \boldsymbol{N}_j \cdot \boldsymbol{j}\, dV$ [9, 10]. We express (1), (2) and (3) in a matrix form as

$$\mathrm{K}\boldsymbol{x} + j\omega\mathrm{N}\boldsymbol{x} = \boldsymbol{b}V \tag{4}$$

where K, N$\in\mathbb{R}^{n\times n}$, $\boldsymbol{x}\in\mathbb{C}^n$, and $\boldsymbol{b}=[0,\ldots,0,1]^t \in \mathbb{R}^n$ are coefficient matrices, unknown and source vectors, respectively. The output current is expressed by

$$I = \boldsymbol{l}^{\mathrm{t}}\boldsymbol{x} \tag{5}$$

where $\boldsymbol{l}$=[0,…,0,1]$^{\mathrm{t}}\in\mathbb{R}^{n}$. We solve (4) at s different frequencies to construct the data matrix X which is composed of the real part of the snapshotted fields as follows:

$$\mathrm{X} = \begin{bmatrix}\boldsymbol{x}_1 & \boldsymbol{x}_2 & \cdots & \boldsymbol{x}_s\end{bmatrix} \tag{6}$$

Note here that s is set much smaller than n. We want to construct orthonormal basis vectors which span s-dimensional space to reduce the number of unknowns. Such basis vectors could be found by solving the optimization problem defined by [11]

$$\sum_{i=1}^{s}\left|\boldsymbol{x}_i^t\,\boldsymbol{u}_j\right|^2 \rightarrow \text{max.}\;\; \text{sub. to } \left|\boldsymbol{u}_j\right| = 1, \boldsymbol{u}_j^t\boldsymbol{u}_k = 0 \text{ for } k = 1,2,..,j-1 \tag{7}$$

Because the objective function in (7) can be written as $\boldsymbol{u}_j^t\mathrm{XX}^t\boldsymbol{u}_j$, the optimal solution is found to be the j-th eigenvector of $\mathrm{XX}^{\mathrm{t}} \in \mathbb{R}^{n\times n}$. However, the solution of this eigenvalue problem is computationally expensive. It can be shown that such eigenvalues can also be obtained by solving eigenvalue problem for $\mathrm{X}^{\mathrm{t}}\mathrm{X} \in \mathbb{R}^{s\times s}$ which is much smaller than XX^{t}. Equivalently, we apply here the singular value decomposition to X, that is

$$\mathrm{X} = \mathrm{W}\Sigma\mathrm{V}^{\mathrm{t}} = \sigma_1\boldsymbol{w}_1\boldsymbol{v}_1^{\mathrm{t}} + \sigma_2\boldsymbol{w}_2\boldsymbol{v}_2^{\mathrm{t}} + \cdots + \sigma_s\boldsymbol{w}_s\boldsymbol{v}_s^{\mathrm{t}} \tag{8}$$

where $\mathrm{W} \in \mathbb{R}^{n\times s}, \mathrm{V} \in \mathbb{R}^{s\times s}, \Sigma = diag[\sigma_1, \sigma_2, \ldots, \sigma_s]$ and W is composed of the eigenvectors of XX^t. As a result, the unknown vector is approximately expressed as a linear combination of the eigenvectors, that is, $\boldsymbol{x}$=W$\boldsymbol{x}_r$. If there are negligibly small singular values, (7) can be truncated after adequate number of terms. Using this transformation, we obtain the reduced system given by

$$\mathrm{W}^{\mathrm{t}}\mathrm{KW}\boldsymbol{x}_r + j\omega\mathrm{W}^{\mathrm{t}}\mathrm{NW}\boldsymbol{x}_r = \mathrm{W}^{\mathrm{t}}\boldsymbol{b}V \tag{9a}$$

$$I = \boldsymbol{l}^{\mathrm{t}}\mathrm{W}\boldsymbol{x}_r \tag{9b}$$

Because $s \ll n$, (9a) can be solved much faster than the original system (4).

Synthesis of equivalent circuits

We consider here two different approaches for synthesis of equivalent circuits on the basis of the reduced equations (8) and (9). To synthesize the equivalent circuit, we employ here the Foster realization of RL circuits [12]. The Foster equivalent circuit can be readily synthesized from a partial fraction expansion of the admittance function which can be obtained from (9a) as will be mentioned below. This is the reason why we choose the Foster realization. They are illustrated in Fig.1. In the first method proposed in [8], the circuit parameters in the Foster circuit are determined by optimization algorithm so that the frequency characteristics of the circuit are fitted to those obtained by solving (8) and (9). In contrast, the proposed method directly determines the Foster circuit from the transfer function of the reduced system.

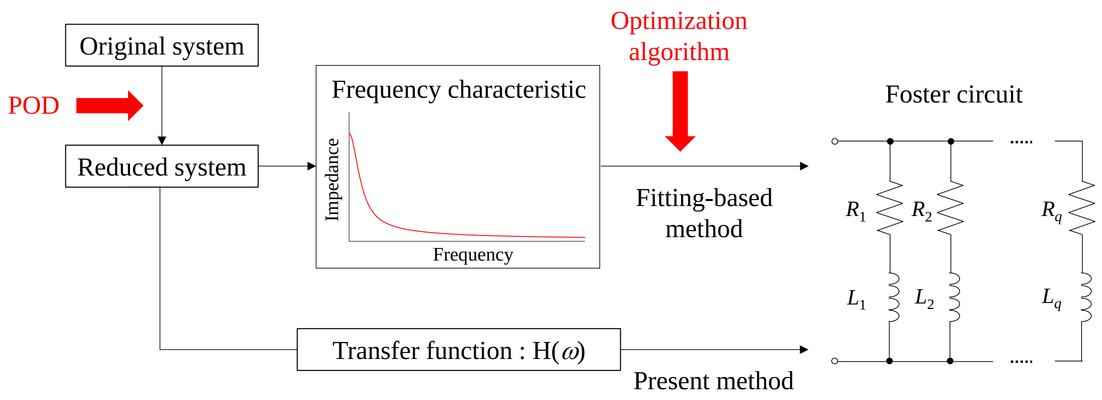


Fig.1 Two ways for equivalent-circuit synthesis

Fitting-based method

We synthesis the equivalent circuit from the frequency characteristics computed from the reduced system. In the Foster realization, the admittance $Y^{\text{fitting}}=I/V$ is expressed by

$$Y^{\text{fitting}} \approx \sum_{k=1}^{q} \frac{1}{R_k + j\omega L_k} \tag{10}$$

where R_k, L_k and q denote resistance, inductance and number of the stage of the ladder circuit in Fig.1, respectively. The circuit parameters $\boldsymbol{R} = [R_1, R_2, \ldots, R_q]$, $\boldsymbol{L} = [L_1, L_2, \ldots, L_q]$ in (10) are determined by solving the optimization problem defined by

$$\begin{aligned} & f(\boldsymbol{R}, \boldsymbol{L}) = \sqrt{\sum_{i}^{M} \left| Y^{\text{FEM}}(\omega_i) - Y^{\text{fitting}}(\omega_i, \boldsymbol{R}, \boldsymbol{L}) \right|^2} \rightarrow \min . \\ & \text{sub . to } R_k, L_k \geq 0 \end{aligned} \tag{11}$$

where $Y^{\text{FEM}}(\text{j}\omega_i)$, $Y^{\text{fitting}}(j\omega_i,\boldsymbol{R},\boldsymbol{L})$ represent the admittance obtained from the reduced system and the equivalent circuit, and M is the number of sampling points for fitting, where usually $M \geq s$. The above optimization problem is solved here using the RGA.

Proposed method

In proposed method, we pay attention to the transfer function of the system. The transfer function corresponds to the admittance $Y=I/V$ function when the input and output are set to the voltage and current. From (8) and (9) admittance is expressed by

$$\begin{aligned} Y &= l_r^{\text{t}} \left(\text{K}_r + j\omega \text{N}_r \right)^{-1} \boldsymbol{b}_r \\ &= l_r^{\text{t}} \left(\text{I} + j\omega \text{A}_r \right)^{-1} \boldsymbol{r}_r \end{aligned} \tag{12}$$

where $\boldsymbol{l}_r^{\mathrm{t}}=\boldsymbol{l}^{\mathrm{t}}\mathrm{W}$, $\mathrm{K}_r=\mathrm{W}^{\mathrm{t}}\mathrm{KW}$, $\mathrm{N}_r=\mathrm{W}^{\mathrm{t}}\mathrm{NW}$, $\boldsymbol{b}_r=\mathrm{W}^{\mathrm{t}}\boldsymbol{b}$, $\mathrm{A}_r=\mathrm{K}_r^{-1}\mathrm{N}_r$ and $\boldsymbol{r}_r=\mathrm{K}_r^{-1}\boldsymbol{b}_r$. We apply spectral decomposition to A_r in (12) which results in

$$\begin{aligned} Y &= \boldsymbol{l}_r^{\mathrm{t}}\left(\mathrm{I}+j\omega \mathrm{S}_r \Lambda_r \mathrm{S}_r^{-1}\right)^{-1}\boldsymbol{r}_r \\ &= \boldsymbol{f}_r^{\mathrm{t}}\left(\mathrm{I}+j\omega\Lambda_r\right)^{-1}\boldsymbol{g}_r \end{aligned} \tag{13}$$

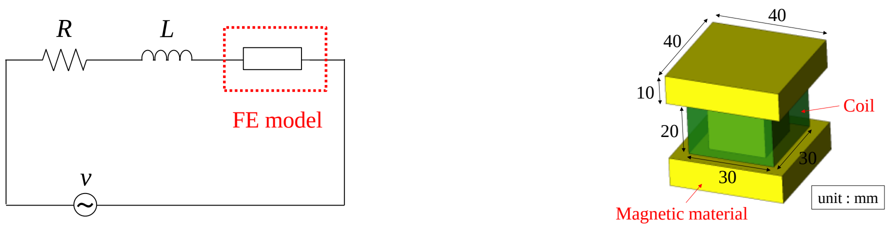
where matrices S_r and Λ_r are composed of the eigenvectors and eigenvalues of A_r, $\boldsymbol{f}_r=\mathrm{S}_r^{\mathrm{t}}\boldsymbol{l}_r$ and $\boldsymbol{g}_r=\mathrm{S}_r^{-1}\boldsymbol{r}_r$, respectively. Note that one does not have heavy computational burden for the spectral decomposition of A_r because A_r is the reduced matrix. Finally, Y is expressed in a form of the rational polynomial approximation as follows:

$$Y=\sum_{k=1}^{s}\frac{1}{\dfrac{1}{f_{rk}g_{rk}}+j\omega\dfrac{\lambda_{rk}}{f_{rk}g_{rk}}} \tag{14}$$

The real and imaginary parts of the denominator of (14) correspond to R_k and L_k for the Foster circuit shown in Fig. 1. In this method, the number of the stage q is set to the number of s. On the other hand, q can be chosen being independent from s in the fitting-based method. If R_k or L_k happens to be negative in the proposed method, we simply invert its sign to satisfy the passivity condition for R_k, L_k>0. Since this problem occurs for the higher circuit elements due to numerical errors, this rather empirical prescription give little effects to the system response. In the vector fitting [13, 14] which is widely used for the synthesis of rational function from the experimental or numerical data, this phenomena have been often obsered.

Numerical results

We apply both synthesis methods to FE analysis of the three-dimensional inductor model connected to the simple circuit shown in Fig. 2, which is a simplified model of a real inductor widely used in electric circuits whose cross-sectional picture can be found in [15]. The current is assumed to be uniform in the cross section of the coil. Because there are flux lines outside the inductor, the air region around the inductor is set more than five times wider than the inductor core. The analysis condition is summarized in Table1. In both methods, to construct X, we take s snapshots at equal intervals over the whole frequency range of the interest. In the fitting-based method, then solve (8) and (9) at 11frequency points, f = 1, 100, … , 10^3 Hz, for fitting of the frequency characteristics.



(a) Circuit model (c) FE model

Fig.2 Numerical model

Table 1. Analysis condition

Electrical conductivity κ [S/m]	5.0×10^{6}
Relative permeability μ_r	10
External resistance R [Ω]	1.0×10^{-5}
External inductance L [H]	1.0×10^{-15}
Amplitude of input voltage V [V]	1.0
Number of unknowns	373562

The frequency and time domain characteristics of the current obtained from equivalent circuits synthesized by fitting-based and proposed methods when s=5 and q=5 are shown in Fig. 3. The results obtained by the circuits are in good agreement with those obtained by the conventional FEM. We define the numerical errors E_f and E_t in frequency and time domain analysis as follows:

$$E_f = \sqrt{\frac{\sum_{i=1}^{N_f}\left|\dot{I}_i^{FEM}(\omega_i) - \dot{I}_i^{circuit}(\omega_i)\right|^2}{\sum_{i=1}^{N_f}\left|\dot{I}_i^{FEM}(\omega_i)\right|^2}}, \quad E_t = \sqrt{\frac{\sum_{i=1}^{N_t}\left(I_i^{FEM}(t_i) - I_i^{circuit}(t_i)\right)^2}{\sum_{i=1}^{N_t}\left(I_i^{FEM}(t_i)\right)^2}} \tag{15}$$

where, N_f, N_t, $\dot{I}_i^{FEM}(\omega_i)$, $\dot{I}_i^{circuit}(\omega_i)$, $I_i^{FEM}(t_i)$ and $I_i^{circuit}(t_i)$ denote sampling numbers in frequency and time domains, currents obtained by FE and circuits analysis in the frequency and time domain, respectively. It is found in Table.2 that in proposed method the errors are greatly improved as q increases. However, in the fitting-based method, accuracy may not be improved with q because increase of q leads to expansion of the search space for fitting which makes more difficult for RGA to find to the optimal solutions. The numerical errors and speedup ratios are plotted in Fig. 4, the latter of which is defined by $\tau^{FEM}/\tau^{circuit}$ where τ^{FEM} and $\tau^{circuit}$ are computational times to analyze frequency characteristics of the current using FE and circuit analysis. In Fig.4 “fitting” and “proposed” represent the results of the fitting-based and proposed methods. In the proposed method, as s increases, the speedup ratio and computational accuracy improve. The proposed method is superior over the fitting-based method in terms of the speed up ratio in each s.

The computational time to compute the frequency characteristics at m sampling points using FEM, fitting-based and proposed methods when s =5 are shown in Fig. 5. In the FE analysis, the computational time increases in proportion to m. However, in the equivalent-circuit approaches, the computational time depends little on m, because the computational burden for the circuit analysis is much smaller than that for circuit synthesis. The speed up ratio of fitting-based and proposed methods compared with the conventional FEM are about 8.6, 17 when m = 100. The values of R_k and L_k of the synthesized circuits are summarized in Table 3. In the fitting-based method, these values significantly vary as q changes. On the other hand, the proposed method provides the circuit parameters which are independent of q because they are uniquely determined from (14).

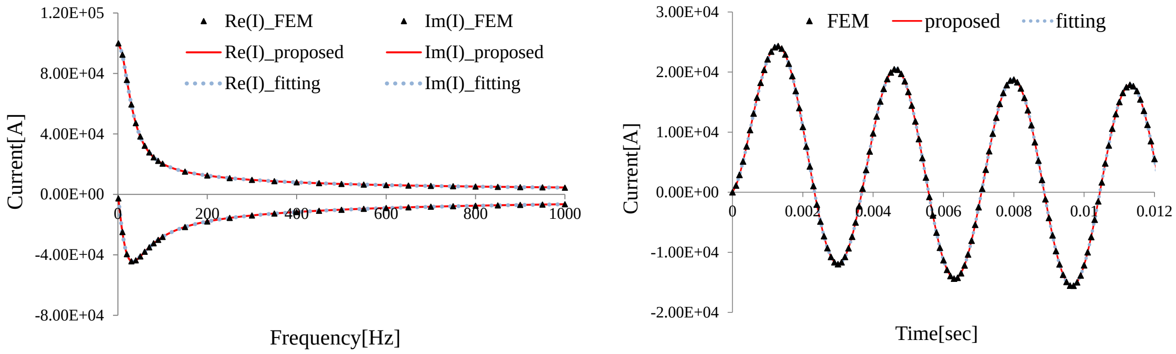
Conclusions

In this paper, we have compared the two fast synthesis methods of equivalent circuits using POD. These two methods have been applied to analysis for three-dimensional inductor models. The equivalent circuits obtained from the two methods give sufficiently accurate results when sufficient number of ladder stage is used. The fitting based method can always yield positive circuit parameters because the passivity condition can be easily imposed in the fitting process, whereas the proposed method can give negative values. On the other hand, the proposed method is superior to the fitting-based with respect to computational efficiency. Moreover, the proposed method can uniquely determine the circuit parameters.

We plan to synthesize equivalent circuits of more realistic inductor models which have multi-turn coils and core losses using the proposed method. Moreover, we plan to extend the proposed method to make it valid for nonlinear problems.

Table 2. Numerical errors

		E_f		E_t	
s	q	Fitting-based method	Proposed method	Fitting-based method	Proposed method
3	3	0.257%	1.92%	0.345%	2.34%
	5	5.78×10^{-2}%	----	6.93×10^{-2}%	----
	11	6.89×10^{-2}%	----	7.59×10^{-2}%	----
5	3	0.243%	----	0.259%	----
	5	4.94×10^{-2}%	0.276%	3.60×10^{-2}%	0.244%
	11	4.10×10^{-2}%	----	9.23×10^{-2}%	----
11	3	0.243%	----	0.259%	----
	5	5.10×10^{-2}%	----	9.22×10^{-2}%	----
	11	3.83×10^{-2}%	1.68×10^{-2}%	2.79×10^{-2}%	5.93×10^{-2}%



(a) Current-frequency response (b) Current-time response(300Hz)

Fig.3 Numerical results when s=5

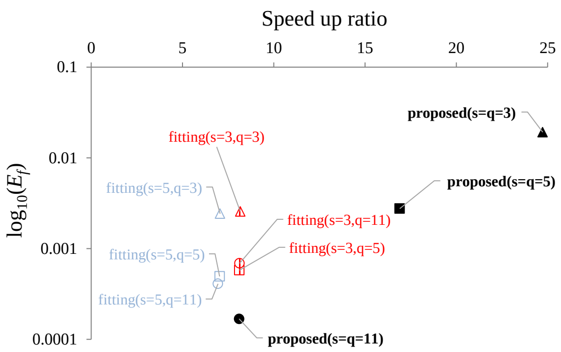


Fig. 4 Comparison of computational performances

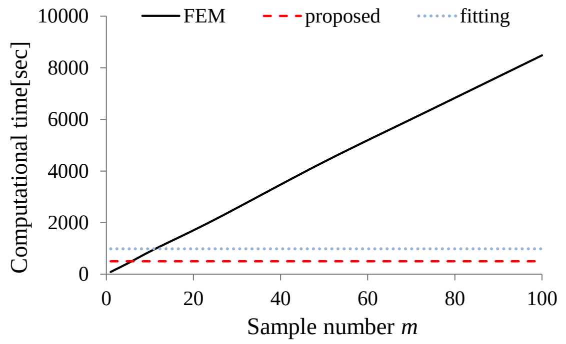


Fig. 5 Computational time (s=5)

Table 3. Circuit parameters

Fitting-based method (s=5)										
q	R_1[Ω]	R_2[Ω]	R_3[Ω]	R_4[Ω]	R_5[Ω]	L_1[H]	L_2[H]	L_3[H]	L_4[H]	L_5[H]
5	1.19×10^{-5}	3.58×10^{-4}	1.33×10^{-4}	3.46×10^{-4}	3.69×10^{-4}	5.82×10^{-8}	6.48×10^{-8}	7.76×10^{-8}	2.39×10^{-7}	1.05×10^{-8}
3	2.42×10^{-4}	1.16×10^{-4}	1.15×10^{-5}	----	----	1.36×10^{-8}	5.84×10^{-8}	5.64×10^{-8}	----	----
2	1.08×10^{-4}	1.10×10^{-5}	----	----	----	2.04×10^{-8}	5.04×10^{-8}	----	----	----
Proposed method										
q	R_1[Ω]	R_2[Ω]	R_3[Ω]	R_4[Ω]	R_5[Ω]	L_1[H]	L_2[H]	L_3[H]	L_4[H]	L_5[H]
5	1.15×10^{-5}	1.51×10^{-4}	5.78×10^{-4}	4.68×10^{-4}	3.82×10^{-4}	5.70×10^{-8}	9.39×10^{-8}	2.06×10^{-7}	6.47×10^{-8}	1.04×10^{-8}
3	1.15×10^{-5}	1.20×10^{-4}	2.00×10^{-4}	----	----	5.74×10^{-8}	6.30×10^{-8}	1.63×10^{-8}	----	----
2	1.14×10^{-5}	8.02×10^{-5}	----	----	----	5.64×10^{-8}	1.98×10^{-8}	----	----	----

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