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Equivalent Circuit of Antennas Generated by Model Order Reduction

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Abstract. It is known that the proper orthogonal decomposition (POD), which is one of the effective approaches for model order reduction (MOR), can effectively reduce computational time for finite element method (FEM). This paper proposes a new method for generation of equivalent circuits based on POD-based FEM. In this method, the circuit parameters are determined by genetic algorithm so that the input impedance of the equivalent circuit is coincident with that computed by POD-based finite element method (FEM). For test of the proposed method, POD-based FEM is applied to analysis of a dipole antenna which is loaded with the Cockcroft Walton circuit. It is shown that this method can reduce the computational time to generate the equivalent circuit without deterioration of accuracy.

Keywords: Model order reduction, antenna analysis, proper orthogonal decomposition, equivalent circuit.

1. Introduction

Finite element method (FEM) has widely been used for analysis of the high-frequency devices. Using FEM, we can easily analyze electromagnetic fields coupled with the external circuit [1]. However, its analysis needs considerably long computational time when the time constant of the external circuit is much longer than that of electromagnetic fields.

Use of Thevenin's equivalent circuit, shown in Fig. 1, would make the analysis of such a system much faster [2]. In this method, one computes the impedance Z_a of a high-frequency system using FEM or FDTD method at a driving frequency and then Z_a is connected to the external circuit. However, in this method, we have to compute Z_a for different driving frequencies. Moreover, for transient analysis, one has to perform frequency sweep for Z_a which requires very long computational time.

The authors have proposed the generation method [3] of the equivalent circuits of electromagnetic devices using proper orthogonal decomposition (POD) which is one of the model order reduction (MOR) techniques [4]-[8]. In this method, the impedance characteristic is computed over a frequency range of interest using POD-based FEM and then it is fitted to that of the Cauer and Fosters circuits shown Figs. 2 and 3 using optimization technique. The impedance of the equivalent circuit of inductors has been shown to agree well with that computed by the conventional FEM over the frequency range. However, it has remained unclear if this method is also valid for analysis of high-frequency devices which can be either inductive or capacitive.

In this paper, we propose a method to generate the equivalent circuit for antennas. In particular, we consider here a dipole antenna connected to the 2-stage Cockcroft Walton (CW) Circuit [9] for test of the proposed method.

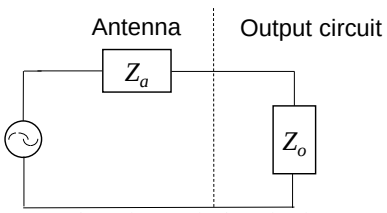

Fig. 1. Thevenin's equivalent circuit.

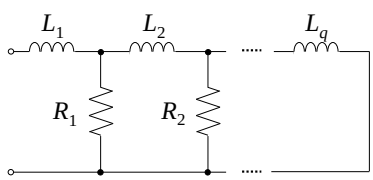

Fig.2 Cauer Circuit.

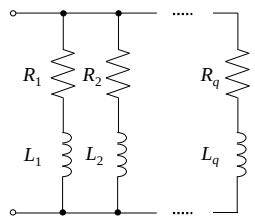

Fig. 3 Foster circuit (type 1)

*Corresponding author. E-mail: yukisato@em.ist.hokudai.ac.jp. Check if the checkbox in menu *Tools/Options/Compatibility/Lay out footnotes like Word 6.x/95/97* is selected if you make a footnote for the corresponding author.

2. Formulation

2.1 Antenna analysis with FEM

In our antenna analysis, the perfect matched layer [10][11] is employed to take reflections from the domain boundaries into account. The vector wave equation considering the PML in frequency domain is given by

$$\mathrm{rot}(\mu\Lambda(\omega))^{-1}\cdot\mathrm{rot}\boldsymbol{A}-\omega^2\varepsilon\Lambda(\omega)\boldsymbol{A}=0 \tag{1a}$$

where

$$\Lambda(\omega)=\frac{s_y s_z}{s_x}\hat{x}\hat{x}+\frac{s_y s_z}{s_x}\hat{y}\hat{y}+\frac{s_y s_z}{s_x}\hat{z}\hat{z} \tag{1b}$$

$$s_\varsigma=1+\frac{\sigma_\varsigma}{\mathrm{j}\omega}\ (\varsigma=x,y,z) \tag{1c}$$

and $\boldsymbol{A}$, μ, σ, ε and ω denote vector potential, magnetic permeability, conductivity, permittivity and angular frequency. Applying the weighted residual method and Galerkin method to (1), we obtain the FE equation

$$\sum_j A_j\int_V(\mathrm{rot}\boldsymbol{N}_i\cdot(\mu\Lambda(\omega))^{-1}\cdot\mathrm{rot}\boldsymbol{N}_j-\omega^2\boldsymbol{N}_i\cdot\varepsilon\Lambda(\omega)\cdot\boldsymbol{N}_j)\mathrm{d}V=0 \tag{2}$$

where $\boldsymbol{N}_i$ is the basis function of the edge element. The input impedance of the antenna can be obtained by solving (2). We have to solve (2) at different frequencies to obtain frequency characteristic of the input impedance. This computation needs heavy computational burden. To solve this difficulty, we apply the proper orthogonal decomposition described below to reduce (2)

2.2 Proper Orthogonal Decomposition

For simplicity, we rewrite (2) as

$$\mathrm{K}(\omega)\boldsymbol{a}(\omega)=\boldsymbol{b} \tag{3}$$

By solving (3) at s points over the frequency range of interest, we obtain the data matrix given by

$$\mathrm{X}=\begin{bmatrix}\boldsymbol{a}(\omega_1) & \boldsymbol{a}(\omega_2) & \cdots & \boldsymbol{a}(\omega_{s-1}) & \boldsymbol{a}(\omega_s)\end{bmatrix} \tag{4}$$

where $\mathrm{X}\in R^{n\times s}$ and s is chosen to be sufficiently smaller than n. The singular value decomposition is applied to X to have

$$\mathrm{X}=\mathrm{W}\Sigma\mathrm{V}^*=\sigma_1\boldsymbol{w}_1\boldsymbol{v}_1^*+\sigma_2\boldsymbol{w}_2\boldsymbol{v}_2^*+\cdots+\sigma_s\boldsymbol{w}_s\boldsymbol{v}_s^* \tag{5}$$

where σ_i i=1,2,…,s are singular values, $\boldsymbol{w}_i$ and $\boldsymbol{v}_i$ are the eigenvectors of XX^* and $\mathrm{X}^*\mathrm{X}$, respectively and * denotes Hermitian conjugate. The vector $\boldsymbol{w}_i$ is the orthonormal basis of the range space of X. We can reduce the number of $\boldsymbol{w}_i$ by neglecting the contributions from non-dominant singular values. The unknowns in the reduced system are expressed in the range space of W. That is, the original unknown vector $\boldsymbol{a}(\omega)$ is expressed in the form

$$\boldsymbol{a}(\omega)=\mathrm{W}\boldsymbol{y}(\omega) \tag{6}$$

Substituting (6) into (3), we obtain the reduced system given by

$$\mathrm{W}^{\mathrm{t}}\mathrm{K}(\omega)\mathrm{W}\boldsymbol{y}(\omega)=\mathrm{W}^{\mathrm{t}}\mathrm{B}\boldsymbol{i}(\omega)\ . \tag{7}$$

As the coefficient matrix of (7) is $s\times s$ where $n>>s$, (7) can be solved much faster than (3). Therefore, we can obtain the frequency characteristics by solving (7) effectively.

As written above, we employ POD in this study while there are other model order reduction techniques, for example Padé approximation via the Lanczos processes (PVL)[4], passive reduced interconnect macromodeling algorithm (PRIMA) [5] and so on. These methods have been successfully applied to high frequency problems [6] also they have never been applied to open boundary problems in which we

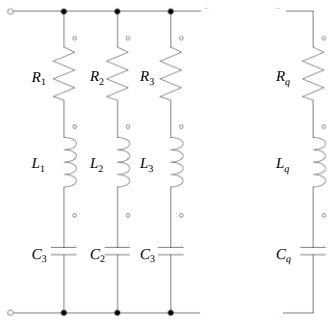

Fig.4 Foster circuit (Type 2)

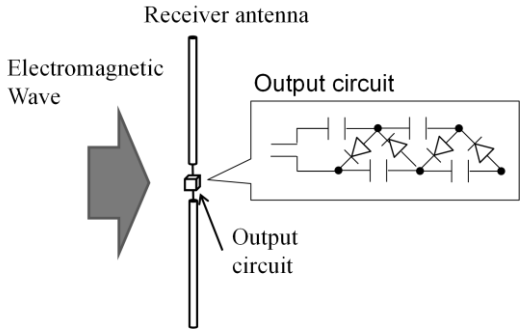

Fig. 5 Dipole antenna loaded with CW circuit

need to use the perfect matched layer (PML) on the open boundary. This is the reason why we choose POD-based MOR in this study.

2.3 Generation of Equivalent Circuit

We determine the circuit parameters in the equivalent circuit so that its frequency characteristic is fit to that computed by POD-based FEM. In this study, we employ the Foster circuit of type 2 composed of resistance, inductance as well as capacitance shown in Fig.4 as the equivalent circuit. The adaptive genetic algorithm (GA) is used to determine the circuit parameters, where that objective function is given by

$$f(\boldsymbol{R},\boldsymbol{L},\boldsymbol{C}) = \sum_{i=1}^{N} \left\| Z_i^{FEM} - Z_i^{Circuit}(\boldsymbol{R},\boldsymbol{L},\boldsymbol{C}) \right\|_2 \tag{8}$$

where $\boldsymbol{R}=[R_1, R_{2,\ldots,} R_q]$, $\boldsymbol{L}=[L_1, L_{2,\ldots,} L_q]$, $\boldsymbol{C}=[C_1, C_{2,\ldots,} C_q]$, Z_i^{FEM} and $Z_i^{Circuit}$ are obtained from POD-based FEM and equivalent circuit, respectively and N is the number of the sampling points in frequency domain. The impedance of the Foster circuit of type 2 with q stages is given by

$$Z_i^{Circuit}(\boldsymbol{R},\boldsymbol{L},\boldsymbol{C}) = \frac{1}{\displaystyle\sum_{j=1}^{q} \frac{1}{R_j + \mathrm{j}\omega_i L_j + \dfrac{1}{\mathrm{j}\omega_i C_j}}} \tag{9}$$

In the adaptive GA process to minimize (8), the crossover and mutation probabilities are adaptively changed depending on the population dispersion [12][13]. For example, if the dispersion is too high, the crossover probability is increased and the mutation probability is decreased and vice versa. This method can accelerate the convergence of GA.

3. Numerical Results

We consider a dipole antenna connected to the CW circuit shown in Fig. 5 in which the dipole antenna receives the plane wave and the receiving voltage at the antenna port is rectified by the CW circuit. The target frequency range is set to 1GHz $\leq f \leq$ 3GHz.

3.1 Reduced system for POD-based MOR

We compute the input impedance of the dipole antenna using FEM at 1.0, 1.5, 2.0, 2.5 and 3.0GHz to build the data matrix X in (4). Twenty-one points are sampled with 0.1 GHz intervals over the frequency range for generation of the equivalent circuit. This frequency sweep is performed by solving the reduced equation (7).

Figure 6 shows the impedance characteristics obtained by conventional FEM and POD-based FEM. They are found to be in good agreement. Note that POD-based FEM has been successfully applied to the practical antennas which have sharp resonance peaks in [7]. Also, the authors applied this method to spiral antenna and successfully obtained the accurate results [8]. Hence, the proposed method is valid not only for dipole antennas but also for practical antennas in principle.

The elapsed time for solving original equation (2) at 5 sampling points is about 13 minutes when using Intel(R) Core i7-4930K/3.4GHz (32GHz RAM). On the other hand, the elapsed time for solving reduced

equation (7) at 16 sampling points is about 11 minutes under the same computational environment. It takes about 41 sec. for one sampling where construction of the coefficient matrix K and $(WtKW)^{-1}$ need about 32 sec. and 5 sec., respectively. Elapsed time for singular value decomposition is about 1sec.

3.2 Equivalent Circuit of Dipole Antenna

The dipole antenna is represented by the Foster circuit of type 2 shown in Fig. 4 using the proposed method. For comparison, a Foster circuit of type 1, shown in Fig. 2, is also synthesized by optimizing the circuit parameters. The number of stages q of the circuit is set to 10 in both circuits.

The input impedances of the dipole antenna obtained from POD-based FEM, Foster circuits with and without capacitance are shown in Fig. 7 and the resultant circuit parameters are summarized in TABLE I. The impedances obtained by the Foster circuit of type 2 are almost the same as those obtained by the POD-based FEM while the Foster circuit of type 1 provides inaccurate results.

Figure 8 shows dependence of the impedance characteristics on q. Moreover, Fig. 9 shows the convergence history of the error of the elite individuals in the GA process, which is defined by

$$\text{error} = \frac{\sum_{i=1}^{21} \left\| Z_i^{FEM} - Z_i^{Circuit}(\boldsymbol{R},\boldsymbol{L},\boldsymbol{C}) \right\|_2}{\sum_{i=1}^{21} \left\| Z_i^{FEM} \right\|_2} \tag{10}$$

We can see from Figs. 8 and 9 that accuracy of the equivalent circuit is improved with the number of stages q.

3.3 Coupling Analysis of antennas and 2-stage CW circuit

Finally, we consider the dipole antenna coupled with the 2-stage CW circuit shown in Fig. 5. The dipole antenna is represented by the 10-stage Foster circuit of type 2 obtained by the proposed method. For comparison, we perform the FDTD analysis to consider the coupling between the circuit and electromagnetic waves. The reason why FDTD is employed is that its computational cost is smaller than FEM for time-domain analysis. The frequency of the input plane wave is set to 1GHz or 2GHz and its amplitude is set to 1V/m. In CW circuit, we set capacitance to 1pF and the load resistance R_L is set to $\infty\Omega$ or 5kΩ. The diode voltage $v_D(t)$ in CW circuit obeys

$$v_D(t) = \begin{cases} 0.026 \times \ln\left(\dfrac{10^{-7} + i_D(t)}{10^{-7}} \right) & i_D(t) \geq 0 \\ 10^{12} i_D(t) & i_D(t) < 0 \end{cases} \tag{11}$$

where $i_D(t)$ flows though the diode.

Figure 10 shows the output voltage of CW circuit. We can see that the output voltages obtained by FDTD are almost the same as those obtained from the equivalent circuit connected to the CW circuit when the load resistance is $\infty\Omega$. On the other hand, there are some differences in the amplitude when R_L=5kΩ. The time evolution of the output voltage when the antenna is illuminated by a complex wave composed of 1 and 2GHz sinusoidal waves is shown in Fig. 11. We observe the similar tendencies; both voltages computed from FDTD and the equivalent circuit are in good agreement with each other when R_L=$\infty\Omega$, whereas there are errors when R_L=5kΩ. These errors would be due to the fact that Thevenin's equivalent circuits are valid only for linear circuits. Nevertheless, the proposed method would be useful for design of the antennas because it gives fast evaluation of the output voltages. Using the proposed method, the circuit parameters can be determined. Then fine tuning of the circuit parameters might be made using time-consuming FDTD analysis or by experiments.

4. Conclusion

This paper has presented a novel generation method of the equivalent circuit for antennas using the POD-based FEM and genetic algorithm. The equivalent circuit of the dipole antenna has the input impedance which is in good agreement with that computed by FEM over the frequency range of interest.

TABLE I Circuit Parameters for Foster circuits of type 1 and 2

Resistance	R_1[Ω]	R_2[Ω]	R_3[Ω]	R_4[Ω]	R_5[Ω]	R_6[Ω]	R_7[Ω]	R_8[Ω]	R_9[Ω]	R_{10}[Ω]
Foster type 1	9690	2410	6440	218	9990	10600	1620	11800	17600	3670
Foster type 2	64.8	2850	3810	1060	41.4	2500	1610	2720	7.24	0.605
Inductance	L_1[H]	L_2[Ω]	L_3[Ω]	L_4[Ω]	L_5[Ω]	L_6[Ω]	L_7[Ω]	L_8[Ω]	L_9[Ω]	L_{10}[Ω]
Foster type 1	4.97e-10	3.78e-7	9.98e-7	2.15e-12	7.42e-10	7.58e-10	2.60e-7	2.39e-9	1.76e-9	6.14e-10
Foster type 2	2.64e-8	1.11e-11	1.21e-11	1.01e-7	9.67e-9	4.38e-11	1.55e-7	1.70e-11	1.56e-11	6.04e-10
Capacitance	C_1[Ω]	C_2[Ω]	C_3[Ω]	C_4[Ω]	C_5[Ω]	C_6[Ω]	C_7[Ω]	C_8[Ω]	C_9[Ω]	C_{10}[Ω]
Foster type 2	5.82e-13	2.69e-14	2.50e-14	7.78e-14	1.39e-13	4.54e-14	5.15e-14	4.35e-14	1.18e-14	2.13e-14

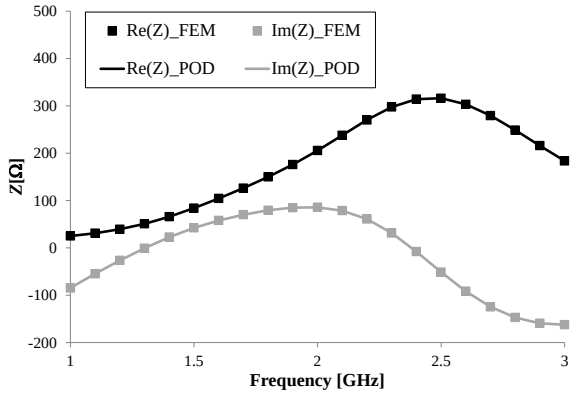

Fig. 6 Input impedance computed by FEN and POD-based FEM

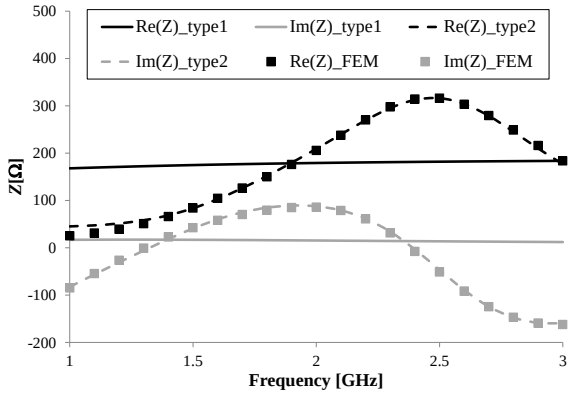

Fig. 7 Input impedance computed by equivalent circuits

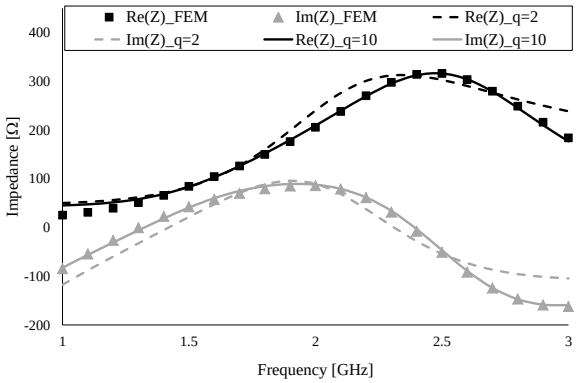

Fig. 8 Dependence of input impedance on *q*

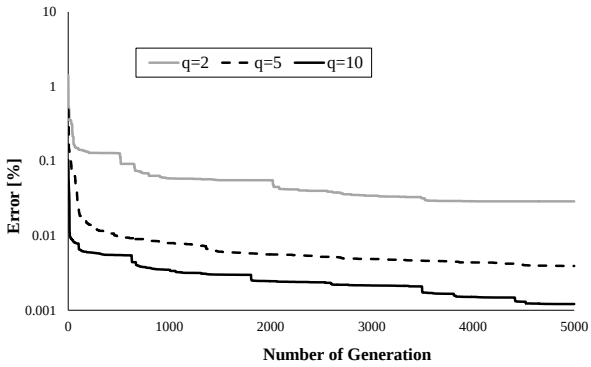

Fig. 9 Dependence of evolution of error on *q*

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When the equivalent circuit generated by the proposed method is connected to the CW circuit, accuracy of the output voltage depends on the load resistance. The equivalent circuits generated by the proposed method, which can be analyzed much faster than FEM and FDTD, would be useful for antenna and circuit design.

Acknowledgements

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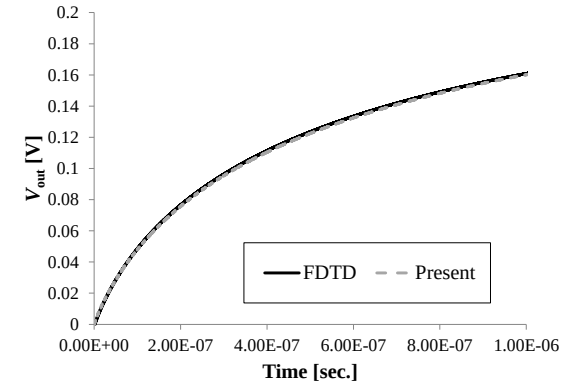
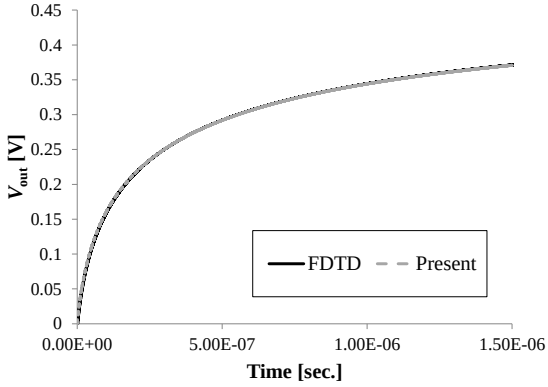
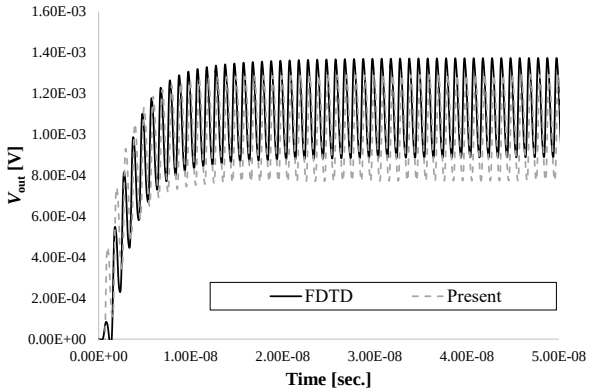


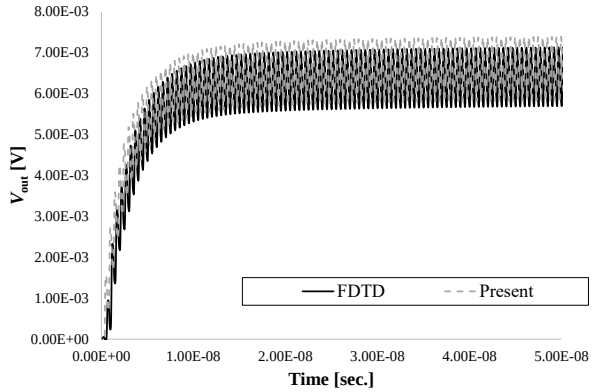
(a) f=1GHz and R_L=∞Ω



(b) f=2GHz and R_L=∞Ω

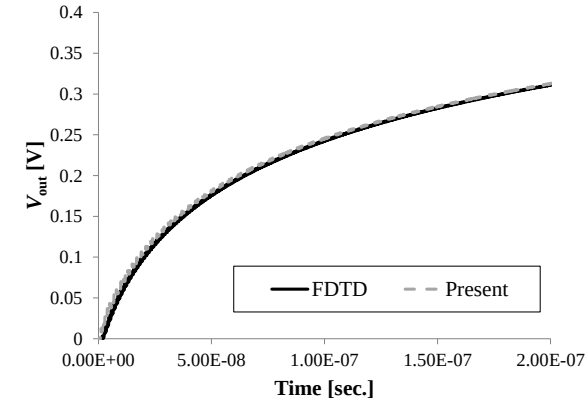


(c) f=1GHz and R_L=5kΩ

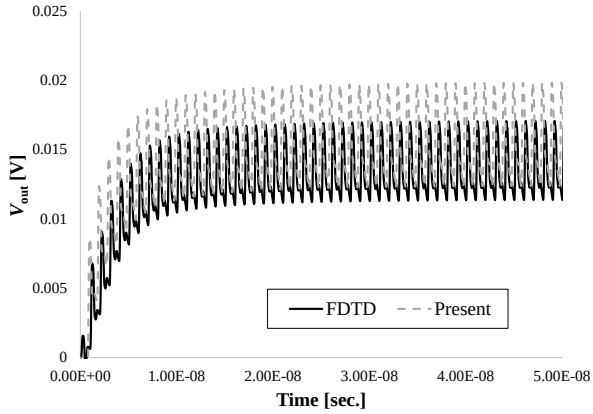


(d) f=2GHz and R_L=5kΩ

Fig. 10 Output voltage for CW circuit



(a) R_L=∞kΩ



(d) R_L=5kΩ

Fig. 11 Output voltage for CW circuit when plane wave is complex wave.

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