<table>
<thead>
<tr>
<th>Title</th>
<th>Linear Quadratic Regulator with Decentralized Event-Triggering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Nakajima, Kyohei; Kobayashi, Koichi; Yamashita, Yuh</td>
</tr>
<tr>
<td>Citation</td>
<td>IEICE transactions on fundamentals of electronics communications and computer sciences, E100A(2): 414-420</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2017-02</td>
</tr>
<tr>
<td>Doc URL</td>
<td><a href="http://hdl.handle.net/2115/65212">http://hdl.handle.net/2115/65212</a></td>
</tr>
<tr>
<td>Rights</td>
<td>©2017 IEICE</td>
</tr>
<tr>
<td>Type</td>
<td>article</td>
</tr>
</tbody>
</table>

File Information: Linear quadratic regulator with decentralized event-triggering.pdf
Linear Quadratic Regulator with Decentralized Event-Triggering

Kyohei NAKAJIMA†, Nonmember, Koichi KOBAYASHI†a), and Yuh YAMASHITA†, Members

SUMMARY Event-triggered control is a control method that the measured signal is sent to the controller only when a certain triggering condition on the measured signal is satisfied. In this paper, we propose a linear quadratic regulator (LQR) with decentralized triggering conditions. First, a suboptimal solution to the design problem of LQRs with decentralized triggering conditions is derived. A state-feedback gain can be obtained by solving a convex optimization problem with LMI (linear matrix inequality) constraints. Next, the relation between centralized and decentralized triggering conditions is discussed. It is shown that control performance of an LQR with decentralized event-triggering is better than that with centralized event-triggering. Finally, a numerical example is illustrated.

key words: decentralized triggering conditions, event-triggered control, linear matrix inequality, linear quadratic regulator

1. Introduction

A networked control system (NCS) is a control system where components such as plants, sensors, controllers, and actuators are connected through communication networks. In each component, messages about the control input or the measured output are sent and received (see Fig. 1). During the last decade, there have been a lot of studies on analysis and synthesis of NCSs from several viewpoints such as packet losses, transmission delays, and communication constraints (see, e.g., [1], [4], [11]–[14], [17]).

In the first step of design of NCSs, it is important to select either periodic or aperiodic control methods. In order to decrease the number of sent and received messages, aperiodic control methods are useful. As an aperiodic control method, event-triggered control and self-triggered control methods are well known (see e.g., [2], [3], [7]–[10], [15], [16], [18]–[26]). The basic idea of event-triggered control is that transmissions of the measured signal and the control input are executed, only when a certain triggering condition on the measured signal is satisfied (i.e., the event occurs). The basic idea of self-triggered control is that the next sampling time at which the control input is recomputed is computed based on predictions.

In this paper, we propose a linear quadratic regulator (LQR) with decentralized triggering conditions. The LQR problem is one of the fundamental problems in control theory. A suboptimal solution to the design problem of LQRs with centralized event-triggering has been obtained in [26]. Under the assumption that a triggering condition is given, a state-feedback gain can be obtained by solving a convex optimization problem with LMI (linear matrix inequality) constraints. Decentralized triggering conditions studied here have been proposed in [21]. In this method, each sensor has a certain triggering condition. When a triggering condition of at least one sensor is satisfied, the plant state, which is stored in the controller, is updated (i.e., all sensors send the message about the measured signal to the controller). In the case where sensors are located in a decentralized way, this control method is useful. In [26], decentralized triggering conditions have not been studied.

In this paper, first, a suboptimal solution to the design problem of LQRs with decentralized triggering conditions is derived based on the result in [26]. Then, a state-feedback gain can be obtained by solving a convex optimization problem with LMI constraints. Decentralized triggering conditions studied here have been proposed in [21]. In this method, each sensor has a certain triggering condition. When a triggering condition of at least one sensor is satisfied, the plant state, which is stored in the controller, is updated (i.e., all sensors send the message about the measured signal to the controller). In the case where sensors are located in a decentralized way, this control method is useful. In [26], decentralized triggering conditions have not been studied.

In this paper, first, a suboptimal solution to the design problem of LQRs with decentralized triggering conditions is derived based on the result in [26]. Then, a state-feedback gain can be obtained by solving a convex optimization problem with LMI constraints. Decentralized triggering conditions studied here have been proposed in [21]. In this method, each sensor has a certain triggering condition. When a triggering condition of at least one sensor is satisfied, the plant state, which is stored in the controller, is updated (i.e., all sensors send the message about the measured signal to the controller). In the case where sensors are located in a decentralized way, this control method is useful. In [26], decentralized triggering conditions have not been studied.
**Notation:** Let $\mathcal{R}$ denote the set of real numbers. Let $I_n$, $0_{m \times n}$ denote the $n \times n$ identity matrix, the $m \times n$ zero matrix, respectively. For simplicity, we sometimes use the symbol 0 instead of $0_{m \times n}$, and the symbol 1 instead of $I_n$. For a vector $x$, let $||x||$ denote the Euclidean norm of $x$. For a vector $x$, let $x_i$ denote the $i$-th element of $x$. For a matrix $M$, let $M^T$ denote the transpose matrix of $M$. For a matrix $M$, let $\text{tr}(M)$ denote the trace of $M$. For matrices $M_1, M_2, \ldots, M_n$, let $\text{diag}(M_1, M_2, \ldots, M_n)$ denote the block-diagonal matrix.

The symmetric matrix $[A \ B^T \ C]$ is denoted by $[A \ B \ C]$.

2. Problem Formulation

As a plant, consider the following discrete-time linear system:

$$x(k+1) = Ax(k) + Bu(k),$$

(1)

where $x(k) \in \mathcal{R}^n$ is the state, $u(k) \in \mathcal{R}^m$ is the control input, and $k \in \{0, 1, 2, \ldots\}$ is the discrete time. For the system (1), consider the following event-triggered state-feedback controller:

$$u(k) = K\hat{x}(k),$$

(2)

where $\hat{x}(k)$ is defined by

$$\hat{x}(k) = \begin{cases} x(k) & \text{if } u(k) \text{ is updated}, \\ \hat{x}(k-1) & \text{if } u(k) \text{ is not updated}, \end{cases}$$

(3)

where $x(0) = \hat{x}(-1)$ is given in advance.

In event-triggered control, a triggering condition is given. If a triggering condition is satisfied, then the control input is updated. Here, we explain a centralized triggering condition and a decentralized triggering condition.

First, based on [26], the centralized triggering condition is defined as follows.

**Definition 1:** The centralized triggering condition is given by

$$||\hat{x}(k-1) - x(k)|| > \sigma||x(k)||,$$

(4)

where $\sigma > 0$ is a given scalar.

Several triggering conditions have been proposed (see, e.g., [9]), but we consider a simple triggering condition. Using the centralized triggering condition, (3) can be rewritten as

$$\hat{x}(k) = \begin{cases} x(k) & \text{if } ||\hat{x}(k-1) - x(k)|| > \sigma||x(k)||, \\ \hat{x}(k-1) & \text{if } ||\hat{x}(k-1) - x(k)|| \leq \sigma||x(k)||. \end{cases}$$

(5)

Then, the following condition

$$||\hat{x}(k) - x(k)|| \leq \sigma||x(k)||$$

(6)

is always satisfied.

Next, the decentralized triggering condition is defined as follows.

**Definition 2:** The decentralized triggering condition is given by

$$\exists i \in \{1, 2, \ldots, n\} \ |\hat{x}_i(k-1) - x_i(k)| > \sigma|x_i(k)|,$$

(7)

where $\sigma > 0$ is a given scalar.

The decentralized triggering condition is utilized in the case where sensors are located in a decentralized way. That is, the sensor $i \in \{1, 2, \ldots, n\}$ has the triggering condition $|\hat{x}_i(k-1) - x_i(k)| > \sigma|x_i(k)|$. If at least one of these triggering conditions is satisfied, then the control input is updated.

Using the decentralized triggering condition, (3) can be rewritten as

$$\hat{x}(k) = \begin{cases} x(k) & \text{if } (7) \text{ holds}, \\ \hat{x}(k-1) & \text{otherwise}. \end{cases}$$

(8)

Then, the following condition

$$||\hat{x}_i(k) - x_i(k)|| \leq \sigma|x_i(k)||,$$

(9)

is always satisfied. That is, the condition (6) is always satisfied. Comparing (6) with (9), we see that (9) is a sufficient condition that (6) holds. In other words, if (9) is satisfied, then (6) is satisfied. Conversely, even if (6) is satisfied, then (9) may not be satisfied.

Using the centralized and decentralized triggering conditions, the design problems of linear quadratic regulators (LQRs) are formulated. First, the design problem of LQRs with the centralized triggering condition is given as follows.

**Problem 1:** For the system (1), suppose that the initial state and the parameter $\sigma > 0$ in (4) are given. Then, find an event-triggered state-feedback controller (2) minimizing the following cost function

$$J = \sum_{k=0}^{\infty} \{x^T(k)Qx(k) + u^T(k)Ru(k)\},$$

(10)

under the centralized triggering condition (5), where $Q \succeq 0$ and $R > 0$.

A suboptimal solution to this problem can be obtained by solving a convex optimization problem with LMI constraints [26]. The outline will be explained in Sect. 4.

Next, the design problem of LQRs with the decentralized triggering condition is given as follows.

**Problem 2:** For the system (1), suppose that the initial state and the parameter $\sigma > 0$ in (7) are given. Then, find an event-triggered state-feedback controller (2) minimizing the cost function (10) under the decentralized triggering condition (8).

In this paper, this problem is reduced to a convex optimization problem with LMI constraints. By solving it, we can derive a suboptimal solution to Problem 2.
Remark 1: An LQR with the decentralized triggering condition (8) is a centralized control scheme. Only a triggering condition is decentralized.

3. Suboptimal Solution to Problem 2

In order to derive a suboptimal solution to Problem 2, first, Problem 2 is transformed into the problem of minimizing an upper bound of the cost function (10). Next, this problem is transformed into a convex optimization problem with LMI constraints.

3.1 Transformation of Problem 2

First, the error variable is defined by \( e(k) := \hat{x}(k) - x(k) \). Then, (9) is replaced with
\[
|e_i(k)| \leq \sigma|x_i(k)|, \quad i \in \{1, 2, \ldots, n\}. \tag{11}
\]
From \( u(k) = K \hat{x}(k) \) and \( \hat{x}(k) = x(k) + e(k) \), the closed-loop system can be obtained as
\[
x(k + 1) = \Phi x(k) + BK e(k), \tag{12}
\]
where \( \Phi = A + BK \).

Next, we introduce the following quadratic Lyapunov function:
\[
V(k) = x^\top(k)P x(k),
\]
where \( P = P^\top > 0 \). In this paper, according to the result in [26], we consider designing a controller satisfying
\[
x^\top(k + 1)Px(k + 1) - x^\top(k)Px(k) < -\left\{ x^\top(k)Q x(k) + u^\top(k)Ru(k) \right\}. \tag{13}
\]
When this condition is satisfied, the closed-loop system (12) is asymptotically stable, that is, \( \lim_{k \to \infty} x(k) = 0 \) holds. Noting this fact, from (13), we can obtain
\[
\lim_{k \to \infty} x^\top(k)Px(k) - x^\top(0)Px(0) = -x^\top(0)Px(0) < -J.
\]
Furthermore, we can obtain
\[
J < x^\top(0)Px(0) < \text{tr}(P)\|x(0)\|^2. \tag{14}
\]
Since the initial state \( x(0) \) is given in advance, the problem of minimizing an upper bound of the cost function (10) is reduced to that of minimizing \( \text{tr}(P) \).

From the above discussion, the problem of finding a suboptimal solution to Problem 2 is given by the following problem.

Problem 3: For the system (1), suppose that the initial state and the parameter \( \sigma > 0 \) in (11) are given. Then, find a state-feedback gain \( K \) minimizing \( \text{tr}(P) \) subject to the conditions (11), (12), and (13).

3.2 Reduction to a Convex Optimization Problem with LMI Constraints

Consider transforming Problem 3 into a convex optimization problem with LMI constraints.

First, note \( u(k) = K \hat{x}(k) = K(x(k) + e(k)) \). Consider substituting (12) and \( u(k) = K(x(k) + e(k)) \) into (13). Then, the left-hand side of (13) can be obtained as
\[
x^\top(k + 1)Px(k + 1) - x^\top(k)Px(k) = (\Phi x(k) + BK e(k))^\top P(\Phi x(k) + BK e(k))
\]
\[
- x^\top(k)Px(k)
\]
\[
= \left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right]^\top \left[ \begin{array}{cc} \Phi^T P \Phi - P & \Phi^T B^T PK \\ K^T B^T PK & K^T BK \end{array} \right] \left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right].
\]
The right-hand side can be obtained as
\[
- \left\{ x^\top(k)Q x(k) + u^\top(k)Ru(k) \right\}
\]
\[
- x^\top(k)Q x(k)
\]
\[
- (x(k) + e(k))^\top K^T RK (x(k) + e(k))
\]
\[
= - \left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right]^\top \left[ \begin{array}{cc} Q + K^T RK & K^T RK \\ K^T RK & K^T BK \end{array} \right] \left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right].
\]
Hence, from (13), we can obtain
\[
\left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right]^\top P_1 \left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right] > 0, \tag{15}
\]
where
\[
P_1 = \left[ \begin{array}{cc} P - \Phi^T P \Phi - Q - K^T RK & K^T RK \\ -K^T B^T PK & -K^T BK - K^T RK \end{array} \right].
\]
From (11), we can obtain
\[
\left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right]^\top P_{2,i} \left[ \begin{array}{c} x(k) \\ e(k) \end{array} \right] \geq 0, \quad i \in \{1, 2, \ldots, n\}, \tag{16}
\]
where
\[
P_{2,1} = \text{diag}(\sigma^2, 0, 0, \ldots, 0, 0, -1, 0, 0, 0, 0, 0),
\]
\[
P_{2,2} = \text{diag}(0, \sigma^2, 0, \ldots, 0, 0, -1, 0, 0, 0, 0, \ldots, 0),
\]
\[
:,
\]
\[
P_{2,n} = \text{diag}(0, 0, 0, 0, \ldots, 0, 0, \sigma^2, 0, 0, 0, 0, \ldots, 0, 0, -1).
\]
By applying the S-procedure [5] to (15) and (16), we can obtain
\[
P_1 - \sum_{i=1}^n \tau_i P_{2,i} > 0, \tag{17}
\]
where \( \tau_i > 0 \) is a scalar decision variable. By defining
\[
X := \text{diag}(\tau_1, \tau_2, \ldots, \tau_n),
\]
\[
P_1 - \sum_{i=1}^n \tau_i P_{2,i} \text{ is given by}
\]
\[
\left[ \begin{array}{cc} P - Q - K^T RK & -\sigma^2 X \\ -K^T RK & X - K^T RK \end{array} \right]
\]
\[
- \left[ \begin{array}{cc} \Phi^T \\ K^T B^T \end{array} \right] P \left[ \begin{array}{c} \Phi \\ BK \end{array} \right]. \tag{18}
\]
Next, by applying the Schur complement [5] to (17) with (18), we can obtain
\[
\begin{bmatrix}
P - Q - K^T R K - \sigma^2 X & * & * & * & * \\
-K^T R K & X - K^T R K & * & * & * \\
\Phi & BK & P^{-1} & * & * \\
I & 0 & 0 & 0 & 0 \\
\end{bmatrix} > 0.
\]

Focusing on \(\sigma^2 X\), we use the Schur complement for the above inequality. Then, we can obtain
\[
\begin{bmatrix}
P - Q - K^T R K & * & * & * & * \\
-K^T R K & X - K^T R K & * & * & * \\
\Phi & BK & P^{-1} & * & * \\
I & 0 & 0 & 0 & 0 \\
\end{bmatrix} > 0.
\]

Furthermore, noting that
\[
\begin{bmatrix}
Q + K^T R K & * \\
K^T R K & K^T R K \\
\end{bmatrix}
= \begin{bmatrix}
Q^{1/2} & 0 \\
R^{1/2} K & R^{1/2} K \\
\end{bmatrix}^T \begin{bmatrix}
Q^{1/2} & 0 \\
R^{1/2} K & R^{1/2} K \\
\end{bmatrix}
\]
holds, we can use the Schur complement again, and we can obtain
\[
\begin{bmatrix}
P & * & * & * & * \\
0 & X & * & * & * \\
\Phi & BK & P^{-1} & * & * \\
I & 0 & 0 & 0 & \frac{1}{\sigma^2} X^{-1} & * \\
Q^{1/2} & 0 & 0 & 0 & I & * \\
R^{1/2} K & 0 & 0 & 0 & I \\
\end{bmatrix} > 0. \tag{19}
\]

Finally, left-right-multiplying (19) by the matrix diag(\(P^{-1}, P^{-1}, I, I, I\)), we can obtain
\[
\begin{bmatrix}
P^{-1} & * & * & * & * \\
0 & P^{-1} X P^{-1} & * & * & * \\
\Phi P^{-1} & BK P^{-1} & P^{-1} & * & * \\
P^{-1} & 0 & 0 & \frac{1}{\sigma^2} X^{-1} & * \\
Q^{1/2} P^{-1} & 0 & 0 & 0 & I & * \\
R^{1/2} K P^{-1} & R^{1/2} K P^{-1} & 0 & 0 & 0 & I \\
\end{bmatrix} > 0. \tag{20}
\]

From \((X^{-1} - P^{-1})^T X (X^{-1} - P^{-1}) \geq 0\), the following relation
\[
P^{-1} X P^{-1} \geq 2P^{-1} - X^{-1}
\]
can be obtained. Hence, a sufficient condition for (20) is given by the inequality in which \(P^{-1} X P^{-1}\) in (20) is replaced with \(2P^{-1} - X^{-1}\).

We define \(S := P^{-1}\) and \(Y := X^{-1}\). Noting that \(\text{tr}(S^{-1})\) is convex [6], we can arrive at the following theorem.

**Theorem 1:** A suboptimal solution to Problem 3 is obtained by solving the following convex optimization problem with LMI constraints:

**Problem 4:** Find \(S, W,\) and \(Y\) minimizing \(\text{tr}(S^{-1})\) subject to the following LMI

\[
\begin{bmatrix}
S & * & * & * & * \\
0 & 2S - Y & * & * & * \\
AS + BW & BW S & * & * & * \\
S & 0 & 0 & \frac{1}{\sigma^2} Y & * \\
Q^{1/2} S & 0 & 0 & 0 & I & * \\
R^{1/2} W & R^{1/2} W & 0 & 0 & 0 & I \\
\end{bmatrix} > 0.
\]

Then, the state-feedback gain \(K\) is obtained as \(K = WS^{-1}\).

**4. Relation between Centralized and Decentralized Triggering Conditions**

In this section, first, the result [26] in the design problem of LQRs with the centralized triggering condition is explained. Next, the relation between centralized and decentralized triggering conditions is discussed.

**4.1 A Suboptimal Solution to Problem 1**

A suboptimal solution to Problem 1 is explained as a lemma, which has been obtained in [26].

**Lemma 1:** A suboptimal solution to Problem 1 is obtained by solving the following convex optimization problem with LMI constraints:

**Problem 5:** Find \(S, W,\) and \(\alpha > 0\) minimizing \(\text{tr}(S^{-1})\) subject to the following LMI

\[
\begin{bmatrix}
S & * & * & * & * \\
0 & 2S - \alpha I & * & * & * \\
AS + BW & BW S & * & * & * \\
S & 0 & 0 & \frac{\alpha}{\sigma^2} I & * \\
Q^{1/2} S & 0 & 0 & 0 & I & * \\
R^{1/2} W & R^{1/2} W & 0 & 0 & 0 & I \\
\end{bmatrix} > 0. \tag{21}
\]

Then, the state-feedback gain \(K\) is obtained as \(K = WS^{-1}\).

By replacing \(Y\) in Problem 4 with \(\alpha I\) in Problem 5, Lemma 1 can be obtained. In derivation of Lemma 1, instead of (11), we consider (6). Then, instead of (16), we can obtain

\[
\begin{bmatrix}
x(k) \\
e(k) \\
\end{bmatrix}^T P_2 \begin{bmatrix}
x(k) \\
e(k) \\
\end{bmatrix} \leq 0, \quad P_2 = \begin{bmatrix}
\sigma^2 I & 0 \\
0 & -I \\
\end{bmatrix} \tag{22}
\]

Applying \(S\)-procedure to (15) and (22), we can obtain

\[
P_1 - \tau P_2 > 0. \tag{23}
\]

where \(\tau > 0\) is a scalar decision variable. From (23), we can obtain the LMI (21).

**4.2 Discussion**

Even if the decentralized triggering condition (7) is used, (6) must be satisfied. Hence, instead of (11), we may consider (6) as a constraint condition. Then, in design of LQRs with decentralized triggering, the state-feedback gain obtained by
solving Problem 5 may be used. However, it is not appropriate to apply Problem 5 to design of LQRs with decentralized triggering. Hereafter, we explain it.

First, in the case of using the centralized triggering condition (4), (23) is equivalent to

$$\begin{bmatrix} x(k) \\ e(k) \end{bmatrix}^T (P_1 - \tau P_2) \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} > 0,$$

and we can obtain

$$\begin{bmatrix} x(k) \\ e(k) \end{bmatrix}^T P_1 \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} - \tau \left( \sum_{i=1}^{n} \tau_i x_i^2(k) - \sum_{i=1}^{n} \tau_i e_i^2(k) \right) > 0.$$

From this expression, we see that in (23), the condition (6) is directly included. In the case of using the decentralized triggering condition (7), (17) is equivalent to

$$\begin{bmatrix} x(k) \\ e(k) \end{bmatrix}^T \left( P_1 - \sum_{i=1}^{n} \tau_i P_{2,i} \right) \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} > 0,$$

and we can obtain

$$\begin{bmatrix} x(k) \\ e(k) \end{bmatrix}^T P_1 \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} - \tau \left( \sum_{i=1}^{n} \tau_i x_i^2(k) - \sum_{i=1}^{n} \tau_i e_i^2(k) \right) > 0,$$

where $T = \text{diag}(\tau_1^{1/2}, \tau_2^{1/2}, \ldots, \tau_n^{1/2})$. From this expression, we see that in (17), the condition (11) is not directly included. In other words, $x(k)$ and $e(k)$ are scaled by the matrix $T$.

Comparing (24) with (25), we see that (24) is the special case of (25). That is, by imposing the constraint $\tau_1 = \tau_2 = \cdots = \tau_n = \tau$, (25) can be rewritten as (24). Hence, in the case of using Theorem 1 and Lemma 1, control performance of an LQR with decentralized triggering is better than that with centralized triggering. Thus, in the case of using the decentralized triggering condition (7), applying Problem 4 is appropriate.

We remark here that the number of times that the decentralized triggering condition (7) is satisfied may be more than the number of times that the centralized triggering condition (4) is satisfied. However, in the case where sensors are located in a decentralized way, the decentralized triggering condition is useful for decreasing the number of communications. When the centralized triggering condition is applied to this case, a certain sensor (or the controller) must aggregate all elements of the state from other sensors, and communications occur at each time.

5. Numerical Example

As an example, consider the following discrete-time linear system:

$$x(k+1) = \begin{bmatrix} 1.1 & 0.6 \\ 0 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} 0.9 \\ -1.0 \end{bmatrix} u(k).$$

For this system, we suppose that $Q$, $R$, and $x(0)$ are given by $Q = 10I_2$, $R = 1$, and $[10\ 30]^T$, respectively. The parameter $\sigma$ in the decentralized triggering condition (7) is given by $\sigma = 0.2$.

By solving Problem 4, we can obtain

$$S = \begin{bmatrix} 0.0092 & -0.0060 \\ -0.0060 & 0.0048 \end{bmatrix},$$

$$W = \begin{bmatrix} -0.0035 & 0.0037 \\ 0.0015 & 0 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0.0004 \\ 0 \end{bmatrix}.$$

The state-feedback gain $K$ can be obtained as

$$K = WS^{-1} = \begin{bmatrix} 0.6972 & 1.6484 \end{bmatrix}.$$

Figure 2 and Fig. 3 show the state trajectory and the control input, respectively. From Fig. 2, we see that the state converges to the origin. Figure 4 shows the event, where “1” implies the event occurs (i.e., the triggering condition is satisfied) and “0” implies the event does not occur. From Fig. 4, we see that the event does not occur at time 2, 3, 4, 8, 9, and 10.

Finally, we compare the proposed method with existing methods. In this example, an LQR with centralized triggering cannot be obtained. That is, Problem 5 was infeasible, because the condition (24) is stricter than the condition (25). Hence, we compare the proposed method with the conventional LQR for discrete-time linear systems. First, we discuss the optimality. In this example, the optimal value of the cost function $\text{tr}(S^{-1})\|x(0)\|^2$ in Problem 4 was derived as $\text{tr}(S^{-1})\|x(0)\|^2 = 1.48 \times 10^8$. On the other hand, the optimal value of the cost function in the conventional LQR was derived as $x^T(0)P_dx(0) = 2.58 \times 10^5$, where $P_d$ is the positive-definite solution of discrete-time algebraic Riccati equation, and was obtained as

$$P_d = \begin{bmatrix} 114.3021 & 134.5739 \\ 134.5739 & 184.4115 \end{bmatrix}.$$
From these facts, the number of times of transmission at time $k$ is $20 \times \left(\frac{3}{4}\right)$, which is the number of transmissions that occur within time interval $[0, 20]$ when a triggering condition is satisfied. Hence, the number of times of transmissions in the initial interval $[0, 20]$ is $1 + 3 \times 20 = 61$. In the case of using the LQR with centralized triggering, when the event occurs, sensor 1 (or 2) transmits the message about the measured signal to the controller. In addition, the controller transmits the message about measured signal to the controller. Finally, the number of times of transmissions is $1 + 3 \times 14 = 43$. Thus, the number of times of transmissions can decrease by using LQRs with decentralized triggering.

6. Conclusion

In this paper, we considered the design problem of LQRs with decentralized triggering conditions. First, a suboptimal solution to this problem was derived. Next, the relation between centralized and decentralized triggering conditions was discussed from the viewpoint of control performance. Finally, a numerical example was presented. The result in this paper provides us a basic result for event-triggered control.

One of the future efforts is to reduce conservativeness. For example, in (14), $\text{tr}(P)$ may be replaced with the maximum eigenvalue of $P$. Further discussion is needed. Optimizing the parameter $\sigma$ in triggering conditions is also an open problem. Finally, in the decentralized triggering condition studied here, when the decentralized triggering condition (8) is satisfied, all elements of the state must be sent to the controller. However, it is desirable that only elements that a triggering condition is satisfied are sent to the controller, and the control input is updated by using only sent elements. This topic has been studied in [22], [24], but the case of LQRs has not been studied. This topic is also one of the future efforts.

This work was partly supported by JSPS KAKENHI Grant Numbers 26420412, 16H04380.

References

[12] H. Ishii, “Stabilization under shared communication with message...


Kyohei Nakajima received the B.E. degree in 2016 from Hokkaido University. Since 2016, he has been pursuing the M.E. degree with the Graduate School of Information Science and Technology, Hokkaido University. His research interests include networked control systems.

Koichi Kobayashi received the B.E. degree in 1998 and the M.E. degree in 2000 from Hosei University, and the D.E. degree in 2007 from Tokyo Institute of Technology. From 2000 to 2004, he worked at Nippon Steel Corporation. From 2007 to 2015, he was an Associate Professor at Japan Advanced Institute of Science and Technology. Since 2015, he has been an Associate Professor at the Graduate School of Information Science and Technology, Hokkaido University. His research interests include analysis and control of discrete event and hybrid systems. He is a member of the SICE, ISCIE, IEEJ, and IEEE.

Yuh Yamashita received his B.S., M.S., and Ph.D. degrees from Hokkaido University, Japan, in 1984, 1986, and 1993, respectively. In 1988, he joined the faculty of Hokkaido University. From 1996 to 2004, he was an Associate Professor at the Nara Institute of Science and Technology, Japan. Since 2004, he has been a Professor of the Graduate School of Information Science and Technology, Hokkaido University. His research interests include nonlinear control and nonlinear dynamical systems. He is a member of SICE, ISCIE, SCEJ, and IEEE.