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Abstract. We study an axion inflation model recently proposed within the framework of type IIB superstring theory, where we pay a particular attention to a sub-Planckian axion decay constant. Our axion potential can lead to the small field inflation with a small tensor-to-scalar ratio, and a typical reheating temperature can be as low as GeV.

Keywords: inflation, axions, string theory and cosmology

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Contents

1 Introduction

The moduli generically appear in superstring theory with compactification and their vacuum expectation values correspond to the size and shape of the compact space. The moduli fields hence can offer characteristic features in superstring theory on the six-dimensional compact space, and they can play important roles in particle phenomenology and cosmology of the four-dimensional low-energy effective field theory.

The moduli have perturbatively flat potential and their imaginary parts, axions, posses the shift symmetries. The moduli, in particular axions, hence are good candidates for the inflaton field driving the cosmological inflation. A well-known example includes the natural inflation [1] where the non-perturbative effects break the shift symmetry into the discrete one and induce the non-flat potential for the axion. A notable requirement for the successful natural inflation model is the super-Planckian axion decay constant, i.e. $f \sim 5M_{\rm p}$ ($M_{\rm p}$ denotes the reduced Planck scale $M_{\rm p} = 2.4 \times 10^{18}$ GeV) while a typical decay constant in superstring theory is sub-Planckian, $f \lesssim M_{\rm p}$ [2]. The possibilities for realizing a super-Planckian decay constant hence have been explored such as the studies on the alignment mechanism [3] and the one-loop effects [4, 5]. Another interesting axion inflation scenario in superstring theory is the axion monodromy inflation [6–9].[1]

These axion inflation models in the string theory discussed in the literature typically involve the super-Planckian inflaton amplitudes and a potentially large tensor-to-scalar ratio r is featured for the large field excursion $\Delta\phi$ as [12]

$$\frac{\Delta\phi}{M_{\rm p}} \simeq \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2}. \tag{1.1}$$

On the contrary, in this paper, we study the small field axion inflation where the field excursion of the axion inflaton is small compared with $M_{\rm p}$.[2] The tensor-to-scalar ratio can be consequently small and, in our string axion models with a sub-Planckian axion decay constant, the reheating temperature can be as low as GeV.

For the illustrative purpose, we study in details the concrete axion inflation model which was recently derived within the framework of type IIB superstring theory [17]. It is the extension of the work [18] to the compactification with generic fluxes, and the inflation potential consists of the mixture of polynomial functions and sinusoidal functions of the axion.[3]

[1] See also refs. [10, 11].
[2] See, e.g. refs. [13–16].
[3] See also refs. [19, 20].

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The paper is organized as follows. In section 2, we study the inflation dynamics for our axion inflation scenarios with a sub-Planckian axion decay constant and demonstrate that the axion inflation energy scale can be quite low compared to the conventional axion inflation scenarios with a super-Planckian axion decay constant. In section 3 we study the reheating temperature in our model and discuss the thermal history after the inflation, followed by the conclusion in section 4.

2 Axion inflation with a small axion decay constant

We, in this section, present the axion inflation model based on type IIB superstring theory [17]. In particular, we consider the inflation model with a sub-Planckian axion decay constant which can lead to a small tensor-to-scalar ratio r. We give the quantitative discussions for our axion inflation scenarios in terms of the slow-roll parameters $\epsilon \equiv \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2, \eta \equiv \frac{V_{\phi\phi}}{V}$ in view of the Planck constraints [21, 22]

$$P_\xi = \left(\frac{H^2}{2\pi|\dot{\phi}|}\right)^2 = \frac{V}{24\pi^2\epsilon} = 2.20 \pm 0.10 \times 10^{-9}, \tag{2.1}$$

$$n_s = 1 + 2\eta - 6\epsilon = 0.9655 \pm 0.0062, \tag{2.2}$$

$$r = 16\epsilon < 0.12. \tag{2.3}$$

2.1 Axion inflation potential in type IIB string theory

Recently, within the framework of type IIB superstring theory, the following form of axion potential was derived [17],

$$V(\phi) = \Lambda_1\phi^2 + \Lambda_2\phi\sin\left(\frac{\phi}{f}\right) + \Lambda_3\left(1 - \cos\left(\frac{\phi}{f}\right)\right), \tag{2.4}$$

where $\Lambda_{1,2,3}$ are constant, and f is the axion decay constant.

We consider the flux compactification of type IIB superstring theory. We can, in general, stabilize all of the complex structure moduli and the dilaton by choosing proper 3-form fluxes [23, 24]. We here choose the 3-form fluxes such that only one of the complex structure moduli, Φ, does not appear in the tree-level superpotential, while the other complex structure moduli as well as the dilaton are stabilized by the 3-form fluxes.[4] However, the geometrical corrections induce the superpotential,

$$W = w_0 + (c + c'\Phi)e^{-\Phi/f}, \tag{2.5}$$

where w_0, c, c' are constants determined by fluxes and vacuum expectation values of other moduli. The Kahler potential of Φ also receives the correction,

$$\Delta K = \left(k + k'\mathrm{Re}(\Phi)\right)\cos(\mathrm{Im}(\Phi)/f)e^{-\mathrm{Re}(\Phi)/f}, \tag{2.6}$$

in addition to the tree-level Kähler potential $K = -\ln i\int_{\mathcal{M}} \Omega \wedge \bar{\Omega}$ with the holomorphic three-form Ω of the CY manifold $\mathcal{M}$, where k and k' are constants determined by fluxes and other moduli vacuum expectation values. We assume that the real part of Φ, $\mathrm{Re}(\Phi)$, is

[4] We also assume that all of the Kähler moduli are stabilized by non-perturbative effects [25] and a proper uplifting scenario is available such as [26–29].

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heavy, and integrating out $\mathrm{Re}(\Phi)$ leads to the above scalar potential eq. (2.4) for the axion $\phi = \mathrm{Im}(\Phi)$. Such a situation is realized by the scenario where Φ is stabilized at the minimum satisfying $\partial_\Phi K = 0$ where K is the ϕ-independent Kähler potential given at the tree-level.[5]

We further assume that the Kähler moduli T^i $(i = 1, 2, \cdots, h^{1,1})$ with the hodge number $h^{1,1}$ are stabilized at the minimum realized by the LARGE Volume Scenario (LVS) [31] where the Kähler potential is described by $K = -2\ln(\mathcal{V} + \Delta\mathcal{V})$ with the volume of "Swiss-Cheese" CY manifold $\mathcal{V}$ and loop-correction $\Delta\mathcal{V}$, whereas the superpotential is the sum of contributions from the flux-induced superpotential $W_{\rm flux}$ and non-perturbatively generated superpotential, $W_{\rm non} \simeq \sum_i A_i e^{-a_i T^i}$ with the constants A_i and a_i. Although the energy density of scalar potential changes during and after the inflation, the superpotential can be regarded as the constant in the inflationary era, i.e., $W \simeq w_0$ where w_0 involves both $W_{\rm flux}$ and $W_{\rm non}$. This is because the first term in the superpotential (2.5) can be taken parametrically larger than the second term in eq. (2.5) which induces the inflaton potential. It is then possible that the stabilization of Kähler moduli is achieved at the scale above the inflation scale through the LVS mechanism, since the mass scale of lightest Kähler modulus (volume modulus) $\frac{w_0}{\mathcal{V}^{3/2}}$ can be larger than the Hubble scale for the mild volume of CY manifold $\mathcal{V} \sim 10^2$ in string units. As discussed in refs. [17, 18], the backreaction from the Kähler moduli are also suppressed, since the energy scale of scalar potential determined by the LVS is larger than that of inflaton potential.

Note that the superpotential as well as the Kähler potential includes the linear term, exponential term and their products. This is the origin of the mixture between polynomial functions and sinusoidal functions in the scalar potential. See, for details, ref. [17]. The natural scale for the decay constant would be of order $f \sim 1/2\pi$, even though one can expect a wider range depending on the vacuum expectation values of the real parts of moduli corresponding to the sizes of cycles. For concreteness, in the following discussions, we mainly consider the range[6]

$$0.01 \leq f \leq 1.0. \tag{2.7}$$

Note that we focus on a small axion decay constant which does not exceed the Planck scale, while a large axion decay constant has usually been explored in the previous literature on the axion inflation scenarios [17]. The magnitudes and ratios of $\Lambda_{1,2,3}$ can vary depending on the flux magnitudes and vacuum expectation values of moduli [17], and we here treat $\Lambda_{1,2,3}$ as free parameters to make our discussions as general as possible.

This potential consists of a mixture of polynomial functions and sinusoidal functions. It reduces to the simple ϕ^2 chaotic inflation when $\Lambda_2 = \Lambda_3 = 0$, which is in a tight tension with the observations due to a large r [21]. For the non-vanishing Λ_2 and Λ_3, the potential consists of many bumps and plateaus, as shown in figure 1, as well as several local minima. The form of the potential eq. (2.4) heavily depends on the oscillation parameter f which determines the width size of the flat plateau regime. A small f leads to a high frequency potential with a small interval between each plateau, and our main focus is on a smaller value of f making each flat plateau closer to each other. The potential is shown in figure 1 for $f = 0.1$ and $f = 0.01$, where, for concreteness, we chose $\Lambda_2/\Lambda_3 = 1, \Lambda_1/\Lambda_3 = 7.3$ for $f = 0.1$ and $\Lambda_2/\Lambda_3 = 1, \Lambda_1/\Lambda_3 = 97$ for $f = 0.01$. The inflation can occur on a flat plateau and we, in the following, study the inflation dynamics for our axion inflation scenarios with a sub-Planckian inflaton field excursion.

[5]Recently, the authors of ref. [30] pointed out that the light complex structure moduli appear in the explicit Calabi-Yau (CY) manifolds.

[6]Throughout this paper, we use the units where the reduced Planck scale $M_{\rm p} = 2.4 \times 10^{18}$ GeV $= 1$.

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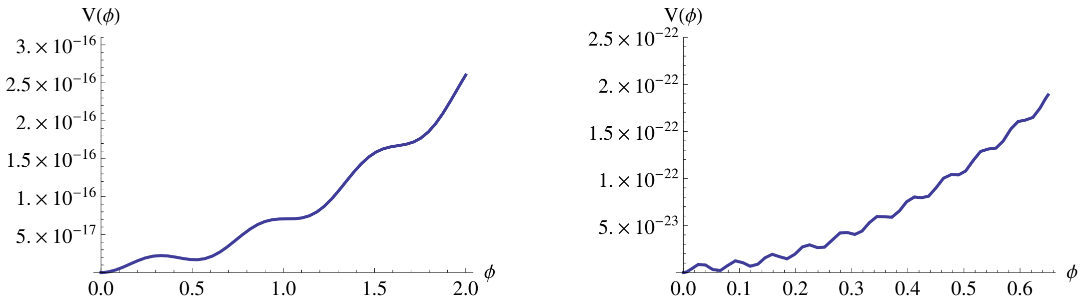

$f = 0.1.$ $\qquad$ $f = 0.01.$

Figure 1. The axion inflation potentials with a sub-Planckian axion decay constant $f = 0.1$(left) and $f = 0.01$(right) for the small field inflation (the field excursion $\Delta\phi < 1$ during the inflation).

2.2 Small field axion inflation

The inflation can occur when an axion inflaton field slowly rolls over a flat plateau region in our axion potential. We shall demonstrate that the small field inflation can be realized for a small axion decay constant f when an enough number of e-folds are induced for a sufficiently flat potential. The first derivative of the potential is written by

$$V_\phi = \left(2\Lambda_1 + \frac{\Lambda_2}{f}\cos\left(\frac{\phi}{f}\right)\right)\phi + \left(\Lambda_2 + \frac{\Lambda_3}{f}\right)\sin\left(\frac{\phi}{f}\right). \tag{2.8}$$

For our potential to become flat enough for a sufficient number of e-folds, we require $(V_\phi)^2 \ll V^2$, which is satisfied for $\phi \sim 1$ and $f \ll 1$ (as well as $\cos(\phi/f)$, $\sin(\phi/f) \sim \mathcal{O}(1)$) when

$$\Lambda_1 f \sim \Lambda_2 \sim \Lambda_3, \tag{2.9}$$

with proper signs of $\cos(\phi/f)$ and $\sin(\phi/f)$. Another condition $V_{\phi\phi} \ll V$ can also be satisfied in the same parameter region. The consequent small inflaton field variation results in a small tensor-to-scalar ratio r as estimated in the following.

For the inflaton variation $\Delta\phi$ around $|V_\phi| \approx 0$ and $|V_{\phi\phi}| \approx 0$, the second derivative can be estimated as

$$V_{\phi\phi} \sim V_{\phi\phi\phi}\Delta\phi \sim \left(-\frac{\Lambda_3}{f^3}\sin\left(\frac{\phi}{f}\right) - \frac{\Lambda_2}{f^3}\cos\left(\frac{\phi}{f}\right)\right)\Delta\phi. \tag{2.10}$$

Note, for a small f, the terms with f^{-3} can be dominant in the third derivative $V_{\phi\phi\phi}$. For $V \sim \Lambda_1\phi^2 \sim \Lambda_3/f$, with the relation (2.9) and $\phi = \mathcal{O}(1)$, we estimate

$$\eta \sim \frac{\Delta\phi}{f^2}. \tag{2.11}$$

Demanding $\eta \ll 1$ results in $\Delta\phi \ll f^2$, which leads to $r \ll 0.01 \times f^4$ from eq. (1.1). Explicitly, we can write

$$r \sim 10^{-6} \times f^4 \times \left(\frac{\eta}{0.01}\right)^2. \tag{2.12}$$

In addition, we can estimate $\eta \sim 10^{-2}$ because $r = 16\epsilon \ll 0.01$ and $2\eta \approx n_s - 1 \approx -0.03$. With this approximation, we estimate $r \sim 10^{-6} \times f^4$ and tensor-to-scalar ratio r can be suppressed greatly as f becomes small.

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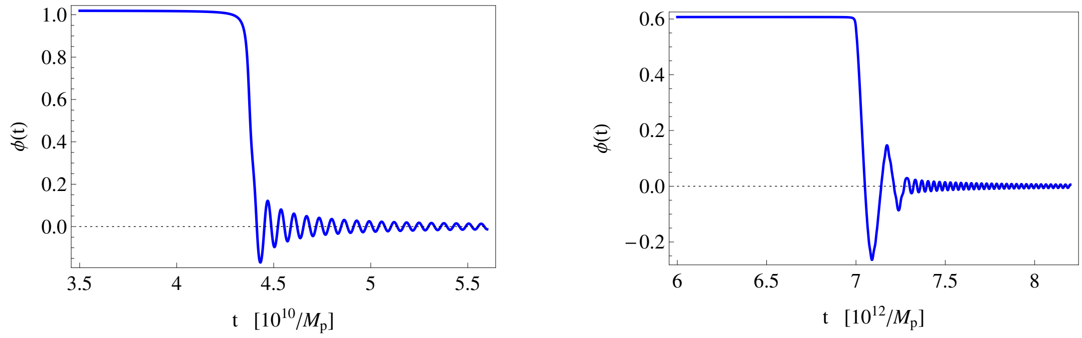

$\phi(0) = 1.0$ for $f = 0.1$, $\Lambda_1/\Lambda_3 = 7.3$, $\Lambda_2/\Lambda_3 = 1$.

$\phi(0) = 0.6$ for $f = 0.01$, $\Lambda_1/\Lambda_3 = 97$. $\Lambda_2/\Lambda_3 = 1$.

Figure 2. Inflaton trajectories.

Figure 2 shows examples of inflaton trajectories. For the illustrative purpose, the initial values of the inflaton field are chosen such that a big enough e-folding number is realized at the second and tenth plateaus, respectively, for $f = 0.1$ and 0.01. The inflaton rolls down through lower plateaus to finally reach the global minimum $\phi = 0$. The e-folding numbers, which are obtained from the other plateaus, are negligible for these examples. We concentrate on such parameter regions for concreteness where the total number of e-folds originates from a single plateau in the following discussions. We then aim to illustrate the characteristic features of our small field axion inflation scenarios which can be applicable for a wider range of the parameters.

For $f = 0.1$, figure 3 shows how the inflaton field evolves as a function of the number of e-folds (counted from the end of inflation), and the corresponding tensor-to-scalar ratio r and n_s are shown. In figure 3, we consider the scenario where a sufficient number of e-folds are induced while the inflaton axion rolls over the second lowest plateau in the potential shown in figure 1. As reference values to indicate the energy scale of inflation, the Hubble parameter and the potential energy at $N = 55$ in this example are $H_{\rm inf}(N = 55) = 2.2 \times 10^{-9}$ and $V_{\rm inf}^{1/4}(N = 55) = 6.1 \times 10^{-5}$. The inflation on another plateau also can lead to a similar result, so that it can induce an enough number of e-folds from a single plateau with a small tensor-to-scalar ratio.

The same story applies for a smaller $f = 0.01$ as shown in figure 4 (the scenario where a sufficient number of e-folds are induced on the tenth lowest plateau in figure 1) corresponding to $V_{\rm inf}^{1/4}(N = 55) = 4.0 \times 10^{-6}$ and $H_{\rm inf}(N = 55) = 9.0 \times 10^{-12}$.

For completeness, we also show the potential for $f = 1.0$ in figure 5 and the evolution of ϕ along with (n_s, r) in figure 6 which corresponds to $V_{\rm inf}^{1/4}(N = 55) = 9.0 \times 10^{-4}$ and $H_{\rm inf}(N = 55) = 4.7 \times 10^{-7}$. The inflaton field excursion during the inflation is sub-Planckian $\Delta\phi < 1$ (we hence call it the small field inflation), even though the amplitude itself can be larger than the Planck scale.

The above numerical analysis demonstrates that our axion potential with a sub-Planckian axion decay constant as well as $f = 1$ can lead to the inflation with a sub-Planckian inflaton field excursion. One notable feature compared with the conventional axion inflation scenarios with the Planckian f and inflaton amplitude is a small tensor to scalar ratio $r \ll 1$. As discussed by eq. (2.12), r is suppressed as the fourth power of f. A rough estimation eq. (2.12) fits with our numerical results by taking $\eta \sim 10^{-2}$ as mentioned above, and we

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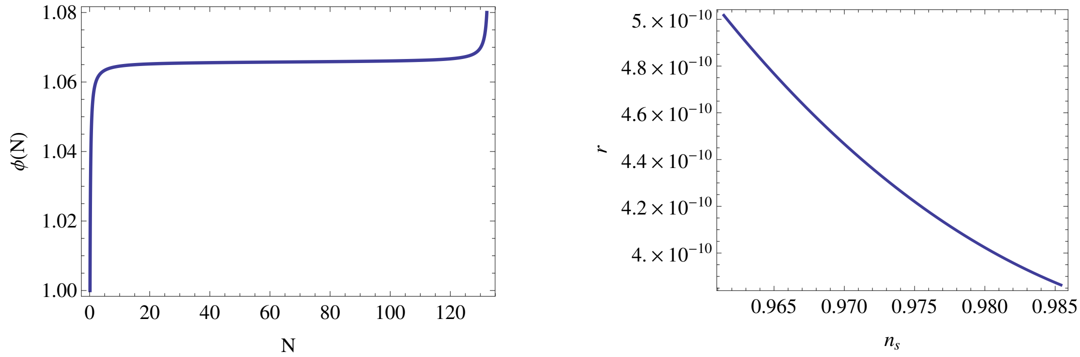
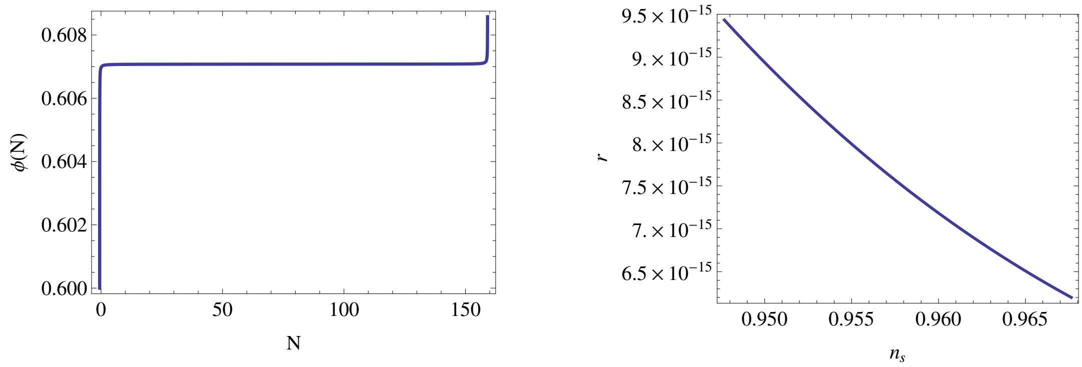


(N, ϕ). (n_s, r).

Figure 3. The inflaton amplitude as a function of the number of e-folds $\phi(N)$ (Left) and (n_s, r) for $N = [50, 60]$ (Right) for $f = 0.1$, $\Lambda_1/\Lambda_3 = 4.9$ and $\Lambda_2/\Lambda_3 = 0.25$ (corresponding to $V_{\rm inf}^{1/4}(N = 55) = 6.1 \times 10^{-5}$).



(N, ϕ). (n_s, r).

Figure 4. The inflaton amplitude as a function of the number of e-folds $\phi(N)$ (Left) and (n_s, r) for $N = [50, 60]$ (Right) for $f = 0.01$, $\Lambda_1/\Lambda_3 = 97$, and $\Lambda_2/\Lambda_3 = 1$ (corresponding to $V_{\rm inf}^{1/4}(N = 55) = 4.0 \times 10^{-6}$).

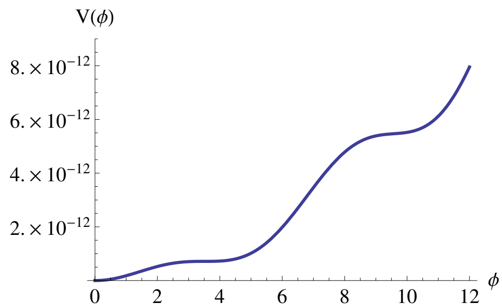
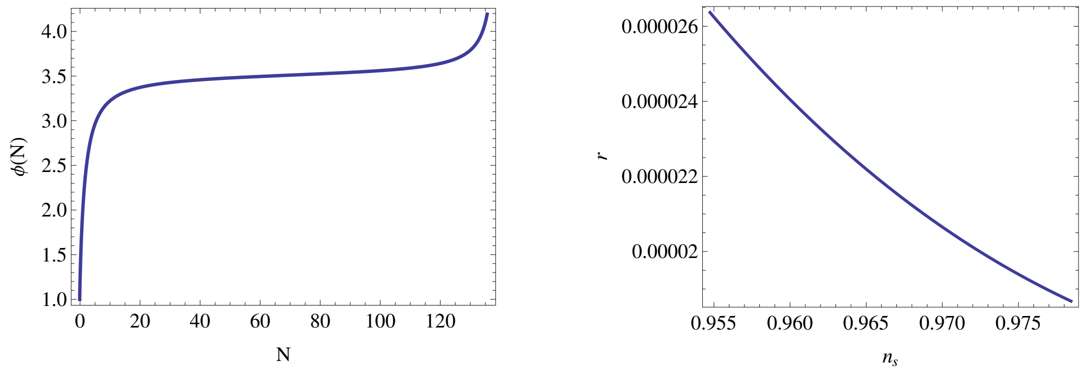
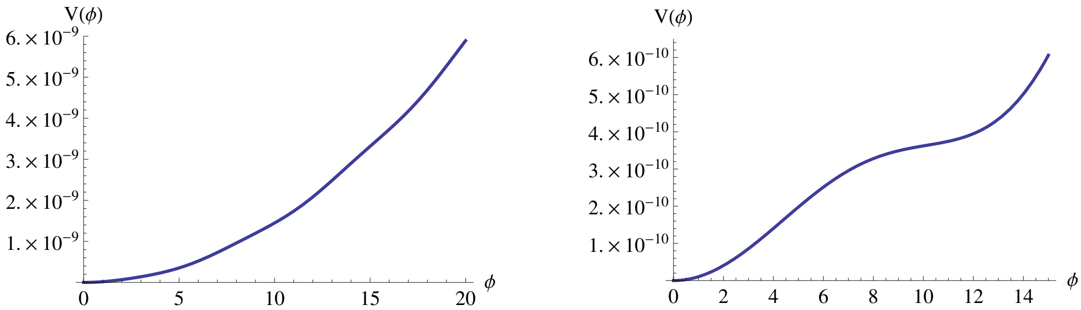


Figure 5. The axion inflation potential with $f = 1.0$ for the small field inflation (the field excursion $\Delta\phi < 1$ during the inflation).



N vs $\phi(N)$. (n_s, r).

Figure 6. (The inflaton amplitude as a function of the number of e-folds $\phi(N)$ (Left) and (n_s, r) for $N = [50, 60]$ (Right) for $f = 1.0$, $\Lambda_1/\Lambda_3 = 1.0$ and $\Lambda_2/\Lambda_3 = 1.9$ (corresponding to $V_{\text{inf}}^{1/4}(N = 55) = 9.0 \times 10^{-4}$).



$f = 1.$ $f = 3.$

Figure 7. The axion inflation potential with a large axion decay constant for the large field inflation.

estimate the typical parameter values of our axion inflation scenarios as

$$r \sim 10^{-6} \times f^4, \qquad V_{\text{inf}}^{1/4} \sim 5 \times 10^{-4} \times f, \qquad H_{\text{inf}} \sim 10^{-7} \times f^2, \qquad \Lambda_3 \sim 6 \times 10^{-14} \times f^5, \quad (2.13)$$

because of $V_{\text{inf}} \sim \Lambda_3/f$. The energy scale of our axion inflation scenarios can be quite low compared with the conventional axion inflation with the Planckian decay constant, and we expect the consequent low reheating temperature as discussed in the next section.

Before concluding this section focusing a small f, let us briefly discuss the scenarios for a larger $f \gtrsim 1$ commonly discussed in the literature for comparison. For a Planckian value of the axion decay constant, the large field inflation can be induced. The typical potentials are shown in figure 7 for $f = 1$ and $f = 3$. Compared with our axion potential with a sub-Planckian f, the tensor-to-scalar ratio r, along with the other parameters, can become large. For instance, with $f = 3.0$ for concreteness, the first term $\Lambda_1\phi^2$ can become dominant in both the potential (2.4) and the first derivative V_ϕ when $\phi \gg 1$ and $\Lambda_1 \sim \Lambda_2 \sim \Lambda_3$. The tensor-to-scalar-ratio ratio r can be estimated as $r \sim 10/\phi^2$, e.g. $r \sim 0.1$ for $\phi \sim 10$. The representative examples for a Planckian f are given, for illustration, in figure 7 and table 1 showing the observables including the inflaton potential energy scale V_{inf} at the horizon exit $N = 55$.

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	N	n_s	r	$V_{\rm inf}^{1/4}$	Λ_1/Λ_3	Λ_2/Λ_3
$f=1$	55	0.95	0.13	8.0×10^{-3}	5.0	1.0
$f=3$	55	0.97	0.011	4.3×10^{-3}	1.0	4.9

Table 1. The typical parameters for $f=1,3$ for the large field inflation.

f	m_ϕ^2	$T_{\rm reh}$
3.0	1.0×10^{-11}	4.3 PeV
1.0	1.9×10^{-13}	220 TeV
0.1	1.2×10^{-16}	860 GeV
0.01	3.4×10^{-20}	1.9 GeV

Table 2. Typical reheating temperature for the cases $f=3.0,\ 1.0,\ 0.1,\ 0.01$ with $c=1$.

3 Phenomenology after inflation

We now discuss the phenomenology after the inflation including the reheating temperature and the dark matter abundance in our small field axion scenarios. The inflaton field is the axionic part of the complex structure modulus, and, in type IIB superstring theory, the complex structure moduli appear in one-loop corrections on gauge kinetic functions [32, 33]. The modulus thus couples with the gauge bosons through one-loop effects,

$$-\frac{1}{4g_a^2}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{4}\frac{\Delta(\Phi)}{16\pi^2}F^a_{\mu\nu}F^{a\mu\nu}, \tag{3.1}$$

where $a=1,2,3$ correspond to the gauge groups of the standard model, $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)$ and $\mathrm{SU}(3)$, respectively, and $\Delta(\Phi)$ is a function of Φ. Through these couplings, the inflation decays into the gauge bosons $g^{(a)}$, and its decay width is estimated as [17]

$$\begin{aligned}\Gamma_\phi &= \sum_{a=1}^{3}\Gamma(\phi\to g^{(a)}+g^{(a)})\\ &= \sum_{a=1}^{3}\frac{N_G^a}{128\pi}\left(\frac{\partial_\phi(\Delta(\Phi))g_a^2}{16\pi^2 d}\right)^2\frac{m_\phi^3}{M_{\rm p}^2}\\ &\simeq 5.8\times10^{-5}c^2\left(\frac{m_\phi}{10^{13}\mathrm{GeV}}\right)^3\mathrm{GeV},\end{aligned} \tag{3.2}$$

where $\sum_{a=1}^3 N_G^a=12$, $d=\mathcal{O}(\sqrt{K_{\Phi\bar\Phi}})=\mathcal{O}(1)$, $g_a^2\simeq0.53$, and for concreteness, we assumed the form $\Delta(\Phi)=c\,\Phi$. When such a decay into the gauge bosons is the dominant decay channel, the reheating temperature can be estimated as

$$T_{\rm reh}=\left(\frac{\pi^2 g_*}{90}\right)^{-1/4}\sqrt{\Gamma_\phi M_{\rm p}}\simeq 6.4\times10^6 c\left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^{3/2}\mathrm{GeV}, \tag{3.3}$$

where the effective degrees of freedom $g_*=106.75$. Table 2 lists the reheating temperature along with the inflaton mass for the concrete examples of $f=3.0,\ 1.0,\ 0.1,\ 0.01$ illustrated in the last section. A smaller f corresponds to a smaller inflation energy scale, which hence

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leads to a smaller $T_{\rm reh}$. The order of magnitude for the inflaton mass can be estimated as follows. For $f \ll 1$ with the relation (2.9), the dominant term of second derivative, $V_{\phi\phi}$, at $\phi = 0$ is evaluated by $V_{\phi\phi} \sim \Lambda_3/f^2$, i.e. $m_\phi^2 \sim \Lambda_3/f^2$. Then, using eq. (2.13), we can estimate the inflation mass by

$$m_\phi^2 \sim 5 \times 10^{-14} \times f^3. \tag{3.4}$$

The complex structure moduli may appear in Yukawa couplings and higher dimensional couplings of matter fields within the framework of type IIB superstring theory (see for concrete computations, e.g. ref. [34, 35]). The inflaton hence can also decay into the matter fields, and, when such a decay channel dominates, the reheating temperature can be estimated as [17]

$$T_{\rm reh} \simeq 8.8 \times 10^7 (\partial_\Phi Y_{ijk}) \left(\frac{m_\phi}{10^{13}\text{GeV}}\right)^{3/2} \text{GeV}, \tag{3.5}$$

where $\partial_\Phi Y_{ijk}$ denotes the first derivative of moduli-dependent Yukawa couplings Y_{ijk}. $T_{\rm reh}$ estimated assuming the dominant decay via the Yukawa couplings is hence comparable or smaller than that estimated assuming the dominant decay into the gauge bosons.

Our models hence lead to the low reheating temperature (as low as GeV). Such a low reheating temperature has important effects on the thermal history following the inflation. Dark matter relic abundance for instance could be affected significantly. For example, if the reheating temperature is smaller than the freeze-out temperature of dark matter, $T_{\rm reh} < T_{\rm f}$, the dark matter yield can be estimated by considering the non-thermal abundance from the inflaton decay

$$\frac{n_{\rm dm}}{s} \simeq \frac{n_{\rm inf}}{s}\text{Br}_{\rm dm} \simeq \frac{\rho}{m_\phi s} \simeq \frac{3T_{\rm reh}}{4m_\phi}\text{Br}_{\rm dm} \simeq 1.5 \times 10^{-12} \left(\frac{c}{10}\right) \left(\frac{m_\phi}{10^8 \text{ GeV}}\right)^{1/2} \left(\frac{\text{Br}_{\rm dm}}{10^{-4}}\right), \tag{3.6}$$

where $n_{\rm dm}(n_{\rm inf})$ is the number density of dark matter (inflaton), s is the entropy density of the Universe, and $\text{Br}_{\rm dm}$ is the inflaton decay branching ratio to dark matter. The current dark matter abundance reads

$$\Omega_{\rm dm} h^2 \simeq m_{\rm dm} \frac{n_{\rm dm}}{s} \frac{s_0}{\rho_{\rm cr}} \simeq 0.04 \left(\frac{c}{10}\right) \left(\frac{m_{\rm dm}}{100 \text{ GeV}}\right) \left(\frac{m_\phi}{10^8 \text{ GeV}}\right)^{1/2} \left(\frac{\text{Br}_{\rm dm}}{10^{-4}}\right), \tag{3.7}$$

where h denotes the dimensionless Hubble parameter and the ratio of critical density to the current entropy densities of the Universe is given by $\rho_{\rm cr}/s_0 \simeq 3.6h^2 \times 10^{-9}$.

Our low energy scale axion inflation scenarios hence can be distinguished from the conventional large field axion inflation scenarios with a high reheating temperature $T_{\rm reh} > T_{\rm f}$ where the dark matter abundance can be estimated as the thermal relic abundance. Another notable feature in our axion inflation scenarios with a small decay constant is the suppressed thermal production of the unwanted relics such as the gravitinos due to the low reheating temperature [36, 37]. In general, supersymmetric models have the gravitino problem, and the low-energy effective field theory derived from superstring theory has the moduli problem. The non-thermally produced gravitinos from the moduli decay could be still a problem, and a light moduli, which does not contribute to supersymmetry breaking, can help in diluting the relic abundance of unwanted particles [38]. The baryogenesis at a low temperature can be also a concern, and the low-energy scale Affleck-Dine mechanism can be a possibility in our scenarios to realize the desired baryon asymmetry of the Universe [39, 40].

In addition to the inflaton axion we have been discussing so far, there can be other axion fields sourcing the isocurvature perturbations which give the tight bounds on the inflation

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parameters. For example, the isocurvature perturbations due to the QCD axion requires

$$H_{\rm inf} < 0.87 \times 10^7 {\rm GeV} \left(\frac{f_a}{10^{11} {\rm GeV}} \right)^{0.408}, \tag{3.8}$$

where f_a is the QCD axion decay constant (different from f), to be consistent with the present observations [41]. Such a low scale inflation can be realized in our model with a sub-Planckian decay constant f. For instance, the models with $f = 0.1$ and 0.01 can lead to $H_{\rm inf} \sim 10^9$ GeV and 10^7 GeV, respectively, while the model with $f = 1.0$ leads to $H_{\rm inf} \sim 10^{12}$ GeV. It would be interesting to increase f_a, although there is an upper bound $f_a \lesssim 10^{12}$ to avoid the over-abundant axion while its precise upper bounds depend on the model details such as the initial displacement angles and the possible entropy dilution [38, 42, 43].

We so far limited our discussions to the case $f \gtrsim 0.01$ as expected in the framework of type IIB superstring theory [17]. We could in principle study an even lower f, and compute the reheating temperature with eqs. (3.4) and (3.3). However, lower f can, depending on c, lead to the reheating temperature of order MeV or below, and $f \sim \mathcal{O}(0.01)$ would be the lower parameter range of our interest for the successful Big-Bang nucleosynthesis (BBN).

4 Conclusion

We have studied the axion inflation model proposed recently within the framework of type IIB superstring theory with a particular emphasis on the sub-Planckian axion decay constant, $0.01 \lesssim f \lesssim 1.0$. The axion potential with such a sub-Planckian decay constant possesses many flat plateaus and the small field inflation can be realized with a sufficient number of e-folds.

A notable feature of our scenario with a small decay constant f is the low inflation energy scale $V_{inf} \propto f^4$ (eq. (2.13)). The implications of the consequent low reheating temperature in our string axion inflation scenarios were discussed including the dark matter abundance, gravitino/moduli problem and the isocurvature fluctuations of the QCD axion. More detailed studies would be of great interest where we combine concrete mechanism for the moduli stabilization/uplifting, fix the mass scale of light moduli, choose a candidate for dark matter, and embed the QCD axion in superstring theory. We leave such detailed studies through the concrete models and their generalization for our future work.

We have studied one concrete potential which is derived from superstring theory. The shift symmetry of axion is violated by quantum effects inducing the axion potential. Such an axion potential consists of the mixture of polynomial functions and sinusoidal functions with the periodicity $\phi \sim \phi + 2\pi/f$, represented as $V(\phi^m, \cos(\phi/f), \sin(\phi/f))$. For a small decay constant $f \ll 1$, such a potential can have many bumps and plateaus with the size of the flat regime $f/(2\pi)$, and the small field inflation can be realized on one of the plateaus.

We expect our concrete examples discussed in our paper can capture the generic features for a wider class of axion inflation consisting of the sinusoidal and polynomial terms with a sub-Planckian axion decay constant. For instance, let us assume that the sinusoidal parts are dominant in some derivatives of the potential. We then would find $V^{(n+1)} \sim V^{(n)}/f$ with $n \geq n_0$ for a certain value n_0, where $V^{(n)}$ denotes the n-th derivative ($V^{(n+1)} \sim V^{(n)}/f$ can well happen for a higher derivative of the potential including the sinusoidal terms because a polynomial term vanishes at a sufficiently large n). Analogous to eq. (2.11), we can then make a similar Ansatz, $\eta \sim \Delta\phi f^{-p}$. Here, p would depend on the form of the potential, e.g. n_0, while $p = 2$ in our model presented in this paper. This would lead to $r \sim 10^{-6} \times f^{2p}$ when the tensor-to-scalar ratio is small $r < \mathcal{O}(10^{-2})$ and we can estimate $2\eta \approx n_s - 1 \approx 0.03$.

In such a model analogous to ours discussed in this paper, the inflation energy scale and Hubble parameter could have the power law dependence on f and hence become rapidly small as f becomes small. As a consequence, the reheating temperature would become small too although its precise value depends on the detailed reheating processes such as couplings between the inflaton and light modes. We would also be able to put the tight lower bound on f from the BBN so that $T_{\rm reh} > \mathcal{O}(1)\,\text{MeV}$. Confirming such a generalization of our study is beyond the scope of current work, and we plan to present the analysis extending our studies here for a wider class of axion inflation models which can be explicitly derived from superstring theory in our future work.

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