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Production of a neutron-rich $^6\Lambda H$ hypernucleus in the $^6\text{Li}(\pi^-, K^+)$ reaction

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We study phenomenologically the production of the neutron-rich hypernucleus $^6\Lambda H$ in the $^6\text{Li}(\pi^-, K^+)$ reaction at 1.2 GeV/c, using a distorted-wave impulse approximation in a one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^- \rightarrow \Lambda n$ coupling. The production cross section of $^6\Lambda \text{H}(1_{\text{exc}}^-)$ is evaluated by a coupled ($^6\text{H}(-\Lambda)$ + ($^6\text{He}$-$\Sigma$)) model with a spreading potential, in comparison with the data of the missing mass spectrum at the J-PARC E10 experiment. The result indicates that the $\Sigma^-$ mixing probabilities in $^6\Lambda \text{H}(1_{\text{exc}}^-)$ are $P_{\Sigma^-} \sim 0.2\%$ both for $s_\Sigma$ state and for $p_{\Sigma}$ state in order to reproduce no significant peak in the $\Lambda$ production data, so that the cross section of $^6\Lambda H$ is less than on the order of 0.4 nb/sr. The sensitivity of the $\Sigma\Lambda$ coupling and $\Lambda$ potentials to the near-$\Lambda$-threshold spectrum is discussed. The shape and magnitude of the spectrum provide valuable information on the $\Sigma\Lambda$ coupling in the production mechanism and also the nuclear structure of $^6\Lambda \text{H}(1_{\text{exc}}^-)$.

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I. INTRODUCTION

Recently, the J-PARC E10 collaboration [1,2] performed experimental measurements of the double-charge-exchange (DCX) reaction ($\pi^-, K^+$) on a $^6\text{Li}$ target at $p_{\pi^-} = 1.2$ GeV/c in order to confirm a neutron-rich hypernucleus $^6\Lambda H$ in which an unbound $^5\text{H}$ nuclear core with neutron-proton excess ratio $(N - Z)/(N + Z) = 0.6$ is expected to be stable by $\Lambda$ stabilization or glue [3,4]. No significant peak structure below and near the $^6\Lambda H + 2n$ threshold was observed in missing mass spectra with $K^+$ forward-direction angles of $\theta_{\text{lab}} = 2^\circ$–14$^\circ$. This is inconsistent with the observation by the $^6\text{Li}(K_{\text{sup}}, \pi^+)$ reaction in FINUDA experiments [5] which indicated evidence of $^6\Lambda H$ with a binding energy of $B_{^6\Lambda H} = 4.5 \pm 1.2$ MeV with respect to the $^3\text{H} + \Lambda$ threshold.

Dalitz and Levi–Setti [3] first discussed the $\Lambda$ stabilization of the neutron-rich $^6\Lambda H$ hypernucleus with the particle-unstable $^5\text{H}$ nuclear core beyond the neutron drip line, Akashi and Myint [6] paid attention to $^6\Lambda H$ as a test ground for an attractive three-body $\Lambda NN$ force caused by the $\Lambda N$–$N\Lambda$ coupling which may be more coherently enhanced in such neutron-excess environments [7,8]. Thus the $^6\Lambda H$ was predicted to have a large binding energy of $B_{^6\Lambda H} = 5.8$ MeV with respect to the $^6\Lambda H + \Lambda$ threshold due to rather large contribution of 1.4 MeV by the coherent $\Lambda\Sigma$ mixing [6]. Gal and Millener [9] showed that recent shell-model calculations including the $\Lambda\Sigma$ coupling give $B_{^6\Lambda H} = 3.8 \pm 0.2$ MeV which seems to be in good agreement with $B_{^6\Lambda H} = 4.5 \pm 1.2$ MeV reported in the FINUDA experiments [5,9]. Hiyama et al. [10] suggested a less binding energy of $B_{^6\Lambda H} = 2.47$ MeV corresponding to an unbound state with respect to the $^6\Lambda H + 2n$ threshold in $nn\Lambda$ four-body cluster-model calculations. The value of $B_{^6\Lambda H}$ is often calculated by the $\Lambda$-nucleus potential which strongly depends on the structure of the nuclear core as well as $\Lambda NN$ interaction involving the $\Lambda\Sigma$ coupling. Therefore, it is very important to clarify the production and structure of the $^6\Lambda H$ which is strongly related to the structure of $^3\text{H}$ in nuclear physics.

The DCX ($\pi^-, K^+$) reaction is one of the most promising ways of searching for a bound state of the neutron-rich $\Lambda$ hypernuclei with stabilized effects by $\Lambda$ added. Indeed, Saha et al. [11] performed the first measurement of a significant yield for the $^6\Lambda \text{Li}$ hypernucleus in ($\pi^-, K^+$) reactions on a $^10\text{B}$ target, whereas no clear peak has been observed with the lack of the experimental statistics. The data show that the absolute cross section for $^6\Lambda \text{Li}$ at 1.20 GeV/c ($d\sigma /d\Omega \sim 11$ nb/sr) is twice larger than that at 1.05 GeV/c ($d\sigma /d\Omega \sim 6$ nb/sr). This incident-momentum dependence of $d\sigma /d\Omega$ exhibits a trend in the opposite direction for the theoretical prediction by Tretyakova and Lanskoy [12]. This might imply that the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^- \rightarrow \Lambda n$ coupling [13] is rather favored over the two-step mechanism, $\pi^- p \rightarrow \pi^-n$ followed by $\pi^0 p \rightarrow K^+\Lambda$ (or $\pi^- p \rightarrow K^0\Lambda$ followed by $K^0 p \rightarrow K^-n$) in the production of neutron-rich $\Lambda$ hypernuclear states, as pointed out in Ref. [11].

In this paper, we study phenomenologically the production of the neutron-rich $^6\Lambda H$ hypernuclear states in the $^6\text{Li}(\pi^-, K^+)$ reaction at 1.2 GeV/c. We demonstrate the calculated spectrum near the $\Lambda$ threshold within a distorted-wave impulse approximation (DWIA) by using a coupled ($^6\text{H}(-\Lambda)$ + ($^6\text{He}$-$\Sigma$)) model with a spreading potential [14]. Comparing the spectrum with the data of the J-PARC E10 experiment [1,2], we discuss the strengths of the $\Sigma\Lambda$ couplings related to the $\Sigma$-mixing probabilities and the strengths of the $\Lambda\Sigma$ potentials which depend on the structure of the $^6\Lambda H$ nuclear core in $^6\Lambda H$.

II. CALCULATIONS

A. Distorted wave impulse approximation

The inclusive $K^+$ double-differential laboratory cross section of $\Lambda$ production on a nuclear target in the DCX ($\pi^-, K^+$) reaction [15] is calculated by the Green’s function method.

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Thus the s.p. wave functions for the proton in 1p$_{3/2}$ and 1s$_{1/2}$ are calculated by the Woods–Saxon (WS) potential with $a = 0.67$ fm, $R = 1.27A^{1/3} = 2.31$ fm [20]. The strength parameter of the potential is adjusted to be $V_{0}^{N} = -55.5$ MeV (−58.0 MeV) for the proton in the p$_{3/2}$ (s$_{1/2}$) state, and $V_{0}^{p} = -0.44V_{0}^{N}$, in order to reproduce the data of proton s.p. energies in $^{6}\text{Li}(p, p'p)$ reactions [21,22]. Thus the s.p. energies for 1p$_{3/2}$ and 1s$_{1/2}$ amount to about 6.61 MeV and −21.48 MeV, respectively. The charge radius for $^{6}\text{Li}(1_{2}^{+})$ becomes 2.48 fm of which value is slightly smaller than that of 2.56 ± 0.05 fm in electron elastic scatterings [23] due to the s.p. description. If we replace the s.p. wave function for the 1p$_{3/2}$ (1s$_{1/2}$) state by a spectroscopic amplitude describing a p$_{3/2}$ (s$_{1/2}$) proton removal from $^{6}\text{Li}(1_{2}^{+})$ within the α + d cluster model [24], we recognize that the calculated cross sections decrease by about 5%, in comparison with the results which will be discussed in Sec. III B. Thus our conclusion obtained in the s.p. description would be reliable.

### B. Wave functions for $^8\text{He}$

To fully describe the one-step process, as shown in Fig. 1 and to estimate the production cross section of $^8\Lambda\text{H}$, we perform Λ−Σ coupled-channel calculations [14] which reproduce the shape and magnitude of the data of the J-PARC E10 experiment in the Λ and Σ−quasi-free (QF) regions [19]. Here we employ a multichannel coupled wave function of the Λ−Σ nuclear state for a total spin $J_B$ within a weak-coupling basis. It is written as

$$|\Psi_{J_{B}}(5H)\rangle = \sum_{J J' J_{3} J_{0}} \left[ \Phi_{J_{B}}(5H), \Phi_{J_{B}}(1) \right]_{J_{B}}$$

with

$$\Phi_{J_{B}}(5H) = A(\Phi_{J_{B}}(s^{3}p), \Phi_{J_{B}}(s^{0}p)), \Phi_{J_{B}}(\text{He}) = A(\Phi_{J_{B}}(s^{3}p), \Phi_{J_{B}}(s^{0}p))$$

where $\Phi_{J_{B}}(s^{3}p)$ is a wave function of the $s^{3}p$ configuration state, $A$ is the antisymmetrized operator for nucleons, and $\Phi_{J_{B}}(1)$, $\Phi_{J_{B}}(\text{He})$ describe the relative wave functions of shell model states (that occupy $j_{L}$, $j_{S}$, and $j_{n,p}$ orbits) for the Λ, Σ, and neutron (proton), respectively; $r_{n}(p_{r})$ denotes the relative coordinate between the $s^{3}p$ nucleus and the neutron or proton, and $r_{\Lambda}(\Sigma)$ denotes the relative coordinate between the center of mass of the $^3\text{He}$ ($^7\text{Be}$) subsystem and the Λ ($\Sigma$). We take the $^3\text{He}$ core-nucleus state with $J^p = 1/2^+$ (ground state ($g.s.$)) and the $^7\text{Be}$ core-nucleus states with $J^p = 3/2^-$, $1/2^+$, $3/2^+$, and $1/2^-$ that are in (1 + $\otimes$ $s_{1/2}$) configurations formed by a proton-hole state on $^{6}\text{Li}(1_{2}^{+})$. If the Λ component is dominant in a bound or resonant state, we can identify it as a state of the hypernucleus $^8\Lambda\text{H}$, in which the $\Sigma^{-}$-mixing probability can be estimated by

$$P_{\Sigma^{-}} = \sum_{j_{L}} \int_{0}^{\infty} \rho_{j_{L}}(r)r^{2}dr,$$
we assume that the structure of 5H is still uncertain experimentally [10,22,28], with respect to 5H ground state with \( Jπ \) = 0 + . Reaction, we consider positive-parity (negative-parity) states where \( ρj/Σ1 \) and \( ρj/\Lambda1 \) are selective with the 5Li targets; the 0 + ground state of 5H(0⁺gs) is forbidden.

Several theoretical calculations [22,25] described the 5H ground state with \( J^P = 1/2^+ \) (g.s.), \( T = 3/2 \) as a continuum or unbound state, \( E_r ≈ 1.6–4.0 \) MeV with respect to the 5H + 2n threshold in 5H + n + n three-body calculations [25] and \( E_r ≈ 3.0–4.5 \) MeV in standard shell-model calculation with \( sπd \) space [26,27]. Since the structure of 5H is still uncertain experimentally [10,22,25], we assume that the 5H(1/2⁺gs) nuclear core is a resonant state with \( E_r = 4.0 \) MeV [25] in the viewpoint of shell-model calculations, rather than that with \( E_r = 1.7 \) MeV in Ref. [29].

Thus the energy difference between 5He + \( Σ− \) and 5H + \( Λ \) channels is \( ΔM = M(5^eHe) + M(5^oHe) − M(5^oHe) − M(5^eHe) + M(5He) + M(5^oHe) − M(5^oHe) + M(5He) \), and \( M(5He) \), \( M(5^oHe) \), \( M(5^oHe) \), and \( M(5He) \) are masses of 5He, 5oHe, 5He, and 5He, respectively. Figure 2 illustrates the energy spectrum and decay threshold for the 5H hypernucleus, where \( BΣ(5H) \) and \( BΛ(5H) \) denote the binding energies with respect to 5H + \( Λ \) and 5H + \( Λ \) (1 + 2n) thresholds, respectively.

**C. Multichannel Green’s functions**

The Green’s function method is one of the most powerful treatments in calculations for the spectrum [16]. The complete Green’s function \( G(E) \) describes all information concerning (5H \( ⊗ \) Λ + (5He \( ⊗ \) Σ)) coupled-channel dynamics. We obtain it by solving the following equation with the hyperon-nucleus potential \( U \) numerically:

\[
G(E) = G^{(0)}(E) + G^{(0)}(E)U G(E),
\]

where

\[
G(E) = \left( \begin{array}{cc} G_Λ(E) & G_Σ(E) \\ G_Σ(E) & G_Σ(E) \end{array} \right), \quad U = \left( \begin{array}{ccc} U_Λ & U_Σ & 0 \\ U_Σ & U_Λ & 0 \\ 0 & 0 & U_Σ \end{array} \right).
\]

and the free Green’s function \( G^{(0)}(E) \). The diagonal parts \( U_Λ \) (\( U_Σ \)) for \( U \) are the \( Λ \)-nucleus (\( Σ \)-nucleus) potentials, and the off-diagonal parts \( U_Χ \) are the \( Σ \)-\( Λ \) coupling potentials. Thus the inclusive \( Σ \)-\( Λ \) spectrum in Eq. (2) can be decomposed into different physical processes [14,16] by using the identity

\[
\begin{align*}
\text{Im}(F_Σ^1 G_Σ(E) F_Σ) & = F_Σ^1 \Omega^{−1}(\text{Im} G^{(0)}_Λ(E)) \Omega^{−1}(F_Σ) \\
& + F_Σ^1 G_Σ(E) \text{Im} U_Λ G_Σ(E) F_Σ \\
& + F_Σ^1 G_Σ(\text{Im} U_Σ) G_Σ(E) F_Σ,
\end{align*}
\]

where \( Ω^{−1} \) is the Möller wave operator and \( F_Σ \) is the production amplitude for \( Σ− \). The remarkable production of 5H arises from the term \( F_Σ^1 G_Σ(\text{Im} U_Σ) G_Σ(E) F_Σ \).

The \( Y \)-nucleus (optical) potentials for \( Y = Λ \) or \( Σ− \) are given by the Woods–Saxon (WS) form:

\[
U_Y(r) = [V_Y + i W_Y g(E_0)] f_Y(r),
\]

where \( f_Y(r) = \{1 + \exp[(r − R)/a]\}^{−1} \). For the 5H-\( Λ \) channel, we use \( a = 0.60 \) fm, \( R0 = 1.080 + 0.395 A^{−2/3} \) fm and \( R = R0 A^{1/3} \) = 2.05 fm [30]. Considering that the 5H nuclear core can be an unbound state or a broad resonant state [10], the strength parameters of \( V_Σ \) should be adjusted appropriately to reproduce the experimental data. The spreading imaginary potential, \( U_Y \), can represent complicated excited states for 5H; \( g(E_0) \) is assumed to be an energy-dependent function which linearly increases from 0 at \( E_0 = 0 \) MeV to 1 at \( E_0 = 60 \) MeV with respect to the 5H + 2n threshold, as often used in nuclear optical models. For the 5He-\( Σ− \) potential, we use the WS potential with \( R = 1.1A^{1/3} \) = 1.88 fm and \( a = 0.67 \) fm, in comparison with the data of the J-PARC E10 experiment [2]. We take the strengths of \( (V_Σ, W_Σ) = (+20 \text{ MeV}, \text{–}20 \text{ MeV}) \) which can freely reproduce the data in \( Σ− \) region, leading to the reduced \( χ^2 \) value of \( χ^2/N ≃ 0.97 \) [19].

The spreading potential \( W_Σ \) expresses nuclear core breakup processes caused by the \( Σ− p → Λ n \) conversion in the 5He nucleus, and its effect is not involved in \( U_X \) which we will mention below.

**D. \( Σ \)-\( Λ \) coupling potentials**

The \( Σ \)-\( Λ \) coupling potential \( U_X \) in off-diagonal parts of \( U \) can be obtained by a two-body \( Σ N − Λ N \) potential \( v_{ΣN,ΛN}^S(r′, r′) \) with the spin \( S = 1 \), 0 isospin \( I = 1/2 \) state. Here we use a zero-range interaction \( v_{ΣN,ΛN}^S(r′, r′) = v_{ΣN,ΛN}^S δ(r′ − r) \) in a real potential for simplicity, where \( v_{ΣN,ΛN}^S \) is the strength parameter that should be connected with volume integral \( \int v_{ΣN,ΛN}^S(r) dr = v_{ΣN,ΛN}^S \). Thus the matrix elements can be easily estimated by use of Racah
where \( \tau_j \) denotes the \( j \)th nucleon isospin operator and \( \phi \) is defined as \( \langle \Sigma | \phi | \Lambda \rangle \) in isospin space [33], and \( \mathcal{Y}_{jfs} = \{ Y_j \otimes X_{fs} \} \), is a spin-orbit function and \( c_{LSK}(J'J'') \) is a purely geometrical factor [31]; \( F_{LSK}(r) \) is the nuclear form factor including a one-body transition density for the \( A = 5 \) shell model [26] and the center-of-mass correction of a factor \( \sqrt{A/(A-1)} \) [34].

Three parameters, \( \tilde{v}_{\Sigma N,\Lambda N}, \tilde{v}_{\Sigma N,\Lambda N}^0, \) and \( V_\Lambda \), are very important for determining the \( \Sigma^- \) mixing probability in \( \frac{6}{5}H \) and the production cross section of \( \frac{6}{5}H \) within the one-step mechanism [13]. These parameters are strongly connected for each other for the shape of the spectrum and its magnitude. The effects of the \( \Sigma N-\Lambda N \) coupling can be evaluated by the volume integrals for \( \Sigma N-\Lambda N \) g matrices based on Nijmegen potentials [35–38], in which these values are model dependent; for example, \(-216.3, -351.0, -478.3, \) and \(-826.6 \text{ MeV fm}^3 \) for \( S = 1 \) in ESC08c, ESC08a, ESC08b, and ESC08a potentials with \( k_F = 1.0 \text{ fm}^{-1} \), respectively [38]. Here we use the volume integrals calculated by the \( g \) matrix based on the D2' potential (D2'g) which can reproduce the binding energies of \( \frac{4}{1}H, \frac{4}{1}He, \frac{4}{1}He^*, \) and \( \frac{6}{1}He \) [39], and we assume \( \tilde{v}_{\Sigma N,\Lambda N} = -900 \text{ MeV} (\tilde{v}_{\Sigma N,\Lambda N}^0 = 500 \text{ MeV}) \) as a standard, which corresponds to \( \tilde{v}_{\Sigma N,\Lambda N} = -941.2 \text{ MeV} (\tilde{v}_{\Sigma N,\Lambda N}^0 = 513.6 \text{ MeV}) \) obtained by D2'g with \( k_F = 1.05 \text{ fm}^{-1} \). To see the dependence of the spectrum on the \( \Sigma N-\Lambda N \) coupling strength, we choose typical values of \( \tilde{v}_{\Sigma N,\Lambda N} = -450, -900, -1350, \) and \(-1800 \text{ MeV} (\tilde{v}_{\Sigma N,\Lambda N}^0 = 250, 500, 750, \) and \( 1000 \text{ MeV}) \). Thus we attempt to determine important parameters of \( \tilde{v}_{\Sigma N,\Lambda N} \) and \( V_\Lambda \), demonstrating the calculated spectrum in comparison with the shape and magnitude of the experimental data, whereas no significant peak structure was observed near the \( \frac{6}{5}H + 2n \) threshold in the J-PARC E10 experiment.

III. RESULTS

Now let us examine the dependence of the shape and magnitude of the spectrum on \( \tilde{v}_{\Sigma N,\Lambda N} \) and \( V_\Lambda \), comparing the calculated inclusive \( \Lambda \) spectrum for \( \frac{6}{5}H \) with the data of the \( ^6\text{Li}(\pi^-, K^+) \) reaction at the J-PARC E10 experiment. In our calculation, we also took the energy-dependent Fermi-averaged \( t \) matrix for the \( \pi^-p \rightarrow K^+\Sigma^- \) reaction which is essential to explain the \( \Sigma^- \) QF spectra of the \( (\pi^-, K^+) \) data on nuclear targets [17]. Therefore, it should be noticed that the following calculated spectra have reproduced the data in the \( \Sigma^- \) and \( \Lambda \) QF regions [19].

The nuclear \( (\pi^-, K^+) \) reaction can predominantly populate spin-stretched states of \( ^6\text{He} \otimes \Sigma^- \) doorways with \( T = 3/2 \) because the momentum transfer is very large (\( q \approx 359 \text{ MeV/c} \) around the \( \Lambda \) threshold) in the \( \pi^-p \rightarrow K^+\Sigma^- \) reaction at \( 1.20 \text{ GeV/c} \) [40]. It is also considered that non-spin-flip processes are dominant near the forward direction in this reaction [41]. Thus the \( 0^+ \) ground state of \( ^6\text{H}(0^+) \) that is expected to have a large contribution by the coherent \( \Lambda \Sigma \) mixing [6] is forbidden by spin-parity conservation when choosing \( ^6\text{Li}(1^+) \) as a target, whereas the \( 1^+ \) excited state of \( ^6\text{H}(1^+) \) can be produced in the reaction. For \( ^6\text{H}(1^+) \) in the one-step mechanism via \( \Sigma^- \) doorways, we have

\[ ^6\text{Li}(1^+);T = 0 \]

\[ \frac{p_{pNS}^-p_{NS}^-}{\Delta L=0.2} [^6\text{He}(3/2^-),1/2^+;T_c = 1/2] \otimes (s_{1/2})\Sigma^-;1^+] \]

\[ = [^6\text{H}(1^+);T_c = 3/2] \otimes (s_{1/2})\Lambda;1^+] \]  

in the \( s_{1/2}^-s\Sigma_1 \) configuration formed by the \( \pi^-p \rightarrow K^+\Sigma^- \) reaction, and

\[ ^6\text{Li}(1^+);T = 0 \]

\[ \frac{p_{pNS}^-p_{NS}^-}{\Delta L=0.2} [^6\text{He}(3/2^-),1/2^+;T_c = 1/2] \otimes (p_{3/2,1/2})\Sigma^-;1^+] \]

\[ = [^6\text{H}(1^+);T_c = 3/2] \otimes (s_{1/2})\Lambda;1^+] \]  

in the \( p_{pNS}^-p_{NS}^-i \) configuration. Figure 3 illustrates these shell-model configurations in \( ^6\text{H}(1^+) \) schematically. The former process indicates the coherent \( \Lambda \Sigma \) coupling with the \( p_{pNS}^-p_{NS}^- \) transition [7]. The latter process also contributes to \( ^6\text{H}(1^+) \) due to the \( s_{1/2}^-s_{NS} \rightarrow p_{NS}^-s_{NS} \) transition which inducesucleon-hole states with nuclear core-excitation in the \( \Lambda \) hypernucleus, as discussed in \textit{ab initio} calculations for \( ^6\text{He}(1^+) \) by Nemura et al. [42]. The type of this coupling is called as “incoherent” \( \Lambda \Sigma \) coupling. We used single-particle wave functions for a proton in \( ^6\text{Li}(1^+) \), reproducing the s-hole and p-hole energies in \( ^6\text{Li}(p, 2p) \) reactions [21].
cross section for $^6\Lambda H(1_{\text{exc}}^+)$ in one-step mechanism. In Table I, we show configurations of the $(J^c_\Lambda \otimes (\ell j)_\Lambda)$ state in $^6\Lambda H(1_{\text{exc}}^+)$ composed by the $A = 5$ core nucleus with $J^c_\Lambda$ and $(\ell j)_\Lambda$-shell hyperon. In Fig. 4, we display the calculated $\Sigma\Lambda$ coupling potentials $U_\Lambda(r)$ between $[^5\text{He}(J^c_\Lambda \otimes (\ell j)_\Sigma \otimes (s_1/2)_\Lambda)]$ and $[^3\text{H}(1/2^+_\Lambda) \otimes (s_1/2)_\Lambda]$ states in $^6\Lambda H(1_{\text{exc}}^+)$ as a function of a relative distance between $^3\text{He}$ and $\Lambda = \Lambda$ ($\Sigma^-$), using the $\Sigma\Lambda$ coupling strengths of $v_{\Sigma\Lambda,N,N} = -900$ MeV and $v_{\Sigma\Lambda,N,N}^0 = 500$ MeV in Eq. (11); these coupling potentials are classified by the orbital angular momentum transfers $\Delta \ell$ to the hyperon in $^6\Lambda H(1_{\text{exc}}^+)$, where $\Delta \ell = |\ell_{\Sigma} - \ell_{\Lambda}|$. We find that the following coupling potentials are dominant:

<table>
<thead>
<tr>
<th>$J^c_\Lambda$</th>
<th>$(\ell j)_\Lambda$</th>
<th>$J^c_\Sigma$</th>
<th>$(\ell j)_\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>$s_1/2$</td>
<td>$3/2^-$</td>
<td>$p_1/2, p_1/2, f_5/2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1/2^+$</td>
<td>$3/2^-$</td>
</tr>
</tbody>
</table>

This nature may originate from the fact that a significant $\pi + \rho$ meson exchange related with the SU(3) coupling constant generates a $(\sigma_N - \sigma_f)(\gamma \cdot \Phi_f)$ coupling constant in $\Lambda - \Sigma$ potentials, and that the nuclear form factors $F_{\Sigma,K}^2(r)$ in Eq. (13) have the collectivity of nuclear core excitations in microscopic $\Lambda = 5$ shell-model calculations. We recognize that the $p_{\pi\Sigma}^- \rightarrow p_{\rho\Sigma}$ transitions are significant to describe $\Lambda - \Sigma$ dynamics in $^6\Lambda H(1_{\text{exc}}^+)$ as well as the $p_{\rho\Sigma}^- \rightarrow p_{\rho\Sigma}$ transitions caused by coherent $\Lambda \Sigma$ couplings, as discussed by Akaishi et al. [7].

To see the dependence of the spectrum on $v_{\Sigma\Lambda,N,N}^0$, here we take $V_\Lambda = -19$ MeV for $^6\Lambda H(1_{\text{exc}}^+)$, whose potential gives the binding energy of $B_\Lambda(\Lambda H) = 1.492$ MeV when omitting $v_{\Sigma\Lambda,N,N}^0$. This value of $B_\Lambda$ is moderately larger than that of $B_\Lambda(\Lambda H) = 0.96 \pm 0.04$ MeV for the $^5\Lambda H(1^+) \subset ^6\Lambda H$. We consider single-particle wave functions for $\Lambda, n$ in $^6\Lambda H(1_{\text{exc}}^+)$ as well as those for $^6\Lambda H(0_{\text{exc}}^+)$ in which the $s_\Lambda$ state has the root-mean-square radius of $(r_{\Sigma}^{1/2})_{\text{rms}} = 3.35$ fm, in comparison with $(r_{\Sigma}^{1/2})_{\text{rms}} = 4.01$ fm for valence neutrons in $^6\Lambda H$. Thus the $\Lambda, n$ distributions in $^6\Lambda H$ simulate a similar structure to the layer distributions of single-particle $t, \Lambda$, and $n$ densities obtained by the $\text{inn}\Lambda$ four-body calculations [10].

### 1. Binding energies and $\Sigma^-$-mixing probabilities

In Table II, we show the results of the binding energies and $\Sigma^-$-mixing probabilities in $^6\Lambda H(1_{\text{exc}}^+)$. When we take $v_{\Sigma\Lambda,N,N}^0 = -450$, $-900$, $-1350$, and $-1800$ MeV ($v_{\Sigma\Lambda,N,N}^0 = 250, 500, 750$, and $1000$ MeV), we find the $\Sigma^-$-mixing probabilities of $P_{\Sigma} = 0.07\%$, $0.32\%$, $0.79\%$, and $1.58\%$, respectively. We stress that there appear not only $s_{\Sigma}$ components but also $p_{\Sigma}$ components in the $\Sigma^-$-mixing probabilities; the value of $P_{\Sigma} (s_{\Sigma}) = 0.04\% - 0.82\%$ is larger than that of $P_{\Sigma} (s_{\Sigma}) = 0.03\% - 0.68\%$. The $d_{\Sigma}$ components are relatively small. The corresponding energy positions of $^6\Lambda H(1_{\text{exc}}^+)$ are shifted downward by the $\Sigma\Lambda$ coupling. We obtain the energy-level shift $\Delta E_\Lambda$ caused by the $p_{\rho\Sigma}^- \leftrightarrow p_{\rho\Sigma}$ coupling in Eq. (12), e.g., $\Delta E_\Lambda \approx -148$ keV when $v_{\Sigma\Lambda,N,N}^0 = -900$ MeV and $v_{\Sigma\Lambda,N,N}^0 = 500$ MeV. This value is slightly smaller than that of $^{10}\text{Li}$ in several microscopic shell-model calculations [43,44]. For $\Delta E_\Lambda$ caused by the $s_{\rho\Sigma}^- \leftrightarrow p_{\rho\Sigma}$ coupling in Eq. (13), we estimate $\Delta E_\Sigma \approx -201$ keV. This effect may be often eliminated in the model space by g-matrix description, and it is not taken into account explicitly in standard calculations [43,44].

In Fig. 5, we display the density distribution of $\rho_{\Sigma} (r)$ for $Y = \Lambda, \Sigma^-$ with $\alpha = (n\ell j)$ in $^6\Lambda H(1_{\text{exc}}^+)$ when we use the $\Sigma\Lambda$ coupling potential given in Fig. 4. Thus we have $P_{\Sigma} (s_{\Sigma}) = 0.13\%$ and $P_{\Sigma} (p_{\Sigma}) = 0.17\%$, as seen in Table II. We find that the $\Sigma^-$ components are located near the center of $^6\Lambda H(1_{\text{exc}}^+)$, e.g., the renormalized root-mean-square radius of $(r_{\Sigma}^{1/2})_{\text{rms}} = 1.47 (1.70)$ fm for $s_{\Sigma}$ $(p_{\Sigma})$ states, respectively, in comparison with those of $(r_{\Sigma}^{1/2})_{\text{rms}} = 1.98 (3.03)$ fm for $s_{\Sigma}$ $(p_{\Sigma})$ states in $^6\text{Li}(1^+_{\text{g.s.}})$. This compactness of these $\Sigma^-$ distributions
TABLE II. Calculated production cross sections of $d\sigma/d\Omega$ for $^6$H($^{1}\text{exc}$) by one-step mechanism in the $^6$Li($\pi^-, K^+$) reaction at 1.2 GeV/$c$ ($^7$), depending on the $\Sigma\Lambda$ coupling parameters of $\tilde{v}_{\Sigma\Lambda}$. $P_{\Sigma^-}$ is the $\Sigma^-$-mixing probability, and $B_1(6^1H)$ and $B_{2\text{sh}}(6^1H)$ are binding energies of $\Lambda$ and $2n$, respectively. $V_{\Lambda} = -19$ MeV is used.

<table>
<thead>
<tr>
<th>$\tilde{v}_{\Sigma\Lambda,N}$ (MeV)</th>
<th>$B_1(6^1H)$</th>
<th>$B_{2\text{sh}}(6^1H)$</th>
<th>$P_{\Sigma^-}$ (%)</th>
<th>$d\sigma/d\Omega$ (nb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 1$</td>
<td>$S = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-$450</td>
<td>250</td>
<td>1.576</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
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<td>500</td>
<td>1.841</td>
<td>0.13</td>
<td>0.02</td>
</tr>
<tr>
<td>$-$1350</td>
<td>750</td>
<td>2.328</td>
<td>0.34</td>
<td>0.04</td>
</tr>
<tr>
<td>$-$1800</td>
<td>1000</td>
<td>3.100</td>
<td>0.68</td>
<td>0.08</td>
</tr>
</tbody>
</table>

may originate from the short-range nature of the $\Sigma\Lambda$ coupling potentials obtained in Eq. (11), and this nature is already seen in the $ab$ initio calculation by Ref. [42].

2. Inclusive $\Lambda$ spectra and cross sections

In Fig. 6, we show the calculated inclusive $\Lambda$ spectrum of the $^6$Li($\pi^-, K^+$) reaction at $p_{\pi^-} = 1.20$ GeV/$c$ and $\theta_{lab} = 7^\circ$, together with the data for the average cross section $\tilde{\sigma}_{2\pi^-}$, taken into account a detector resolution of 3.2 MeV FWHM. We find that the calculated spectrum below the $^4\Lambda + 2n$ threshold is rather sensitive to $\tilde{v}_{\Sigma\Lambda,N}$ in the one-step mechanism, whereas $\tilde{v}_{\Sigma\Lambda,N}$ is particle unstable above the $^4\Lambda + 2n$ threshold. The integrated cross sections for $^6$H($^{1}\text{exc}$) account for $d\sigma/d\Omega = 0.04–1.32$ nb/sr for $\tilde{v}_{\Sigma\Lambda,N} = (-450)–(-1800)$ MeV and $\tilde{v}_{\Sigma\Lambda,N} = 250–1000$ MeV, as listed in Table II. We display the values of $d\sigma/d\Omega$ for $^6$H($^{1}\text{exc}$) as a bin with a finite width of 1 MeV for particle decay channels at $M_\chi \approx 5800.16–5804.63$ MeV/$c^2$, as also shown in Fig. 6. It is remarkable that the $\Lambda$ production spectra are composed of proton-hole states, $s_p$, and $p_p$, populated by the $(\pi^-, K^+)$ reactions. The value of $d\sigma(p_p^-)/d\Omega = 0.01–0.32$ nb/sr is considerably smaller than that of $d\sigma(s_p^-)/d\Omega = 0.03–1.00$ nb/sr, whereas $P_{\Sigma^-}(s_p^-) = 0.04%–0.82%$ is larger than $P_{\Sigma^-}(s_p^-) = 0.03%–0.68%$, as mentioned above.

IV. DISCUSSION

A. $s$-hole proton vs $p$-hole proton

To see the feasibility of producing the neutron-rich $\Lambda$ hypernucleus in the one-step mechanism, we consider the contribution of the inclusive spectra via $\Sigma^-$ doorways from the proton $p_p^−$ ($s_s^-$) state on the $^6$Li target. The integrated laboratory cross section may be roughly written as

$$
\frac{d\sigma(J_p^-)}{d\Omega} \approx |b_p^-|^2 S_p(p_p^-) F_{\Sigma\Lambda}^{(1/2-j_s)}(q)^2 P_{\Sigma^-}(J_p^-) \approx \frac{1}{2} \left[ 1 - \frac{1}{3} (b\bar{q})^2 + \frac{7}{180} (b\bar{q})^4 \right] \approx 0.20
$$

for $q \approx 360$ MeV/$c$ corresponding to the $\Lambda$ threshold at 1.2 GeV/$c$. Here we adopted $S_p(p_p^-)/S_p(s_p^-) \approx 1/2$ for $^6$Li and the oscillator radius parameter $\bar{b} = 1.38$ fm. This $\bar{b}$ value indicates that the $\Sigma^-$ components are distributed near the center of $^6$H($^{1}\text{exc}$). As a result, we confirm that the value of $d\sigma(p_p^-)/d\Omega$ is considerably smaller than that of $d\sigma(s_p^-)/d\Omega$, whereas $P_{\Sigma^-}(s_p^-)$ and $P_{\Sigma^-}(s_p^-)$ have almost the same value.

B. $\tilde{v}_{\Sigma\Lambda,N}$ strengths

As far as $\tilde{v}_{\Sigma\Lambda,N}^0 = (0.0)–(-900)$ MeV and $\tilde{v}_{\Sigma\Lambda,N}^0 = 0.0–500$ MeV leading to $P_{\Sigma^-}(s_s^-) = 0.0%–0.13%$ and $P_{\Sigma^-}(p_p^-) = 0.0%–0.17%$, the calculated spectra can fairly explain the data of the J-PARC E10 experiment. No peak structure of $^6$H originates from the small $\Sigma\Lambda$ coupling.
FIG. 6. Calculated missing mass spectra of the $^6\text{Li}(\pi^-, K^+)\Lambda$ reactions near the $\Lambda$ threshold at 1.2 GeV/c and $\theta_{lab} = 7^\circ$, with a detector resolution of 3.2 MeV FWHM. The $\Sigma\Lambda$ coupling strengths of $\tilde{v}_{\Sigma N,\Lambda}^1 = (a) -1800, (b) -1350, (c) -900$, and (d) -450 MeV [$\tilde{v}_{\Sigma N,\Lambda}^0 = (a) 1000, (b) 750, (c) 500$, and (d) 250 MeV] are used, together with $V_{\Lambda} = -19$ MeV for the $\Lambda-5\text{H}$ potential. Solid, dashed and dot-dashed curves denote contribution of total, $p$-hole, and $s$-hole spectra, respectively. The data are taken from Ref. [1]. The bins with a finite width of 1 MeV denote the cross sections for $^6\Lambda\text{H}(1^+_{\text{exc}})$ which is located in particle decay channels.

C. $V_{\Lambda}$ strengths

On the other hand, another important parameter $V_{\Lambda}$ for the $^5\text{H}-\Lambda$ potential also affects the binding energies and the production cross sections for $^6\Lambda\text{H}(1^+_{\text{exc}})$. The energy position of $^6\Lambda\text{H}(1^+_{\text{exc}})$ is shifted downward by the attraction of $V_{\Lambda}$. We find that, when $\tilde{v}_{\Sigma N,\Lambda}^1 = -900$ MeV and $\tilde{v}_{\Sigma N,\Lambda}^0 = 500$ MeV, the binding energies are $B_{\Lambda}(^6\text{H}) = 0.050, 1.841, 3.726$ and 5.493 MeV for $V_{\Lambda} = -11, -19, -24$, and $-28$ MeV, respectively, so that the $\Sigma$-mixing probabilities amount to $P_{\Sigma^-} = 0.07\%, 0.32\%, 0.38\%$, and 0.40%. In Fig. 7, we show the dependence of the inclusive $\Lambda$ spectrum for the $^6\Lambda\text{H}(1^+_{\text{exc}})$ production on these values of $V_{\Lambda}$ when $\tilde{v}_{\Sigma N,\Lambda}^1 = -900$ MeV and $\tilde{v}_{\Sigma N,\Lambda}^0 = 500$ MeV. We show that the calculated spectrum for $^6\Lambda\text{H}(1^+_{\text{exc}})$ is considerably changed by the value of $V_{\Lambda}$, where the integrated cross sections of $^6\Lambda\text{H}(1^+_{\text{exc}})$ become $d\sigma/d\Omega = 0.04, 0.22, 0.34$.
and 0.41 nb/str for $V_{\Lambda} = -11$, -19, -24, and -28 MeV, respectively. The calculated spectra with $V_{\Lambda} = -(24) - (28)$ MeV seem to disagree with the data of no peak structure below the $^5\text{H} + \Lambda$ threshold. This fact may indicate that the $^5\text{H}$-\Lambda potential is quite shallow in comparison with the $\Lambda$-nucleus potentials which are well known as $V_{\Lambda} \simeq -28$ MeV in ordinary nuclei [30], and the neutron-rich nuclear core $^3\text{H}$ should be an unbound or broad resonant state.

D. $V_{\Sigma}$ and $W_{\Sigma}$ strengths

As discussed above, we recognize that the calculated spectrum is in good agreement with that of the $^6\text{Li}(\pi^-, \Lambda^+)$ data when we use the $\Sigma\Lambda$ coupling strengths of $\delta_{\Sigma,N,\Lambda} \simeq -900$ MeV and $\tilde{v}_{\Sigma,N,\Lambda}^0 \simeq 500$ MeV, together with $V_{\Sigma} \simeq (20) - (30)$ MeV and $W_{\Sigma} \simeq -20$ MeV for the $^3\text{He}$-$\Sigma\Lambda$ potential [19]. The nature of the repulsive component in this potential is consistent with that in the $\Sigma$-nucleus potential obtained on heavier targets [17]. The calculated spectrum fully explains the data in the $\Sigma^-$ and $\Lambda$ QF regions by the one-step mechanism, $\pi^- + p \rightarrow K^+\Sigma^-$ via $\Sigma^-$ doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling.

E. $^3\text{H}(1/2^+_{g.s.})$ resonant state

Current experiments have reported that the $^3\text{H}$ ground state is located at $E_r = 1.7 \pm 0.3$ MeV with $\Gamma = 1.9 \pm 0.4$ MeV above the $^3\text{H} + 2n$ threshold [29], or at $E_r = 5.5 \pm 0.2$ MeV with $\Gamma = 5.4 \pm 0.6$ MeV [45]. The problem of whether the $^5\text{H}(1/2^+_{g.s.})$ ground state exists as a narrow resonant state with $E_r = 1.7$ MeV and $\Gamma = 1.9$ MeV may still be unsettled [22,28]. Several theoretical investigations [22,25] suggest the energy of the $^3\text{H}$ ground state with $E_r \simeq 1.6$–3.0 MeV, $\Gamma \simeq 1.5$–4.0 MeV in $\text{tnn}$ three-body calculations [25] and $E_r \simeq 3.0$–4.5 MeV in the shell-model calculations with $\text{spds}$ space [26,27]. It is expected that the $\Sigma\Lambda$ coupling matrix elements work reasonably within the shell-model description. In the viewpoint of shell-model calculations, we assume that the $^5\text{H}(1/2^+_{g.s.})$ core nucleus is a resonant state with $E_r = 4.0$ MeV, rather than that with $E_r = 1.7$ MeV if we have $E_r = 1.7$ MeV in the shell models, we would need to artificially add an extreme attraction to the $^3\text{H}$ system, e.g., by three-nucleon forces [10]. To see effects of the energy of the $^5\text{H} + \Lambda$ threshold on $^3\text{H}(1^+_\text{exc})$, we calculate the inclusive spectrum near the $\Lambda$ threshold, changing the energy of the $^3\text{H}(1/2^+_{g.s.})$ resonant state. In Fig. 8, we show the dependence of the inclusive spectrum for $^3\text{H}(1^+_\text{exc})$ near the $\Lambda$ threshold, using $E_r = 4.0$ MeV and 1.7 MeV which determine the position of the $^3\text{H} + \Lambda$ threshold. We recognize that the shape of the calculated spectrum for $^3\text{H}(1^+_\text{exc})$ is considerably changed by the value of $E_r$, which depends on whether $^3\text{H}$ is a narrow resonant state. The structure of $^3\text{H}$ may influence the scenario of production of $^5\text{H}$ at FINUDA [5]. Thus the spectrum near the $\Lambda$ threshold provides the ability to study the structure of the $^3\text{H}$ core nucleus in detailed comparison with the precise data, as well as the structure of $^3\text{H}(1^+_\text{exc})$.

F. Finite range

To clarify the one-step mechanism for production of the neutron-\Lambda hypernucleus, we obtained the $\Sigma\Lambda$ coupling potential constructed by the zero-range two-body interaction for simplicity, using the WS form for diagonal potentials in $^1\text{H} + \Lambda$ and $^3\text{He} + \Sigma^-$ channels. On the other hand, it is known that a finite range of the two-body interaction provides modified nuclear potentials [31]. To see effects of the finite range of the interaction, we have a Gaussian shape, $v_{\Sigma,N,\Lambda}(r') = v_{\Sigma,N,\Lambda}(\text{FR}) \exp(-|r' - r|^2/\beta^2)$, where $\beta$ is a range parameter. Here we choose $v_{\Sigma,N,\Lambda}(\text{FR}) = -369.4$ MeV and $v_{\Sigma,N,\Lambda}(\text{FR}) = 205.2$ MeV for $\beta = 0.8$ fm; these strength parameters correspond to a spin-averaged $\Lambda N$ strength of $v_{\Sigma,N}(\text{FR}) = -105.9$ MeV with $\beta = 0.8$ fm, which reproduce the $\Lambda$ binding energies for light $p$-shell nuclei. In the folding potential model, we realize that the radial shape of the $\Sigma\Lambda$ coupling potential $U_{\Sigma}(r)$ is more smoothly behaved and the range of $U_{\Sigma}(r)$ becomes slightly extended. Thus we find that the $\Sigma^-$-mixing probabilities for $^5\text{H}(1^+_\text{exc})$ is $\sigma_{\Sigma^-}/\Omega = 0.13\%$ and $P_{\Sigma^-}(p_{\Sigma^-}) = 0.11\%$ in comparison with 0.13% and 0.17% shown in Table II. The integrated cross section for $^5\text{H}(1^+_\text{exc})$ is $d\sigma/d\Omega = 0.17$ nb/str and the ($\pi^-, K^+$) spectrum is not so modified. It seems that a value of $P_{\Sigma^-}(p_{\Sigma^-})$ is relatively reduced whereas $P_{\Sigma^-}(s_{\Sigma^-})$ is not changed. This modification may depend on nuclear structures of the $^3\text{H}$ and $^3\text{He}$ core states as well as properties of the two-body $\Lambda N$, $\Sigma N$ and $\Lambda N - \Sigma N$ effective interactions. Therefore, more investigation is needed to qualitatively clarify nuclear dynamics by sophisticated microscopic calculations.

G. Two-step processes of $\pi^- p \rightarrow K^0\Lambda$ followed by $K^0 p \rightarrow K^+ n$

Finally we discuss the integrated laboratory cross sections of $d\sigma/d\Omega$ for $^4\text{H}(1^+_\text{exc})$ by the two-step mechanism, $\pi^- p \rightarrow K^0\Lambda$ followed by $K^0 p \rightarrow K^+ n$ or $\pi^- p \rightarrow \pi^0 n$ followed by $\pi^0 p \rightarrow K^+ \Lambda$ in the DCX $^6\text{Li}(\pi^-, K^+)$ reaction for production of the neutron-\Lambda hypernuclei [13]. We
roughly estimate the contribution of the two-step processes for \( \pi^- p \rightarrow K^+n \) followed by \( K^0 p \rightarrow K^+\Lambda \), which are expected to be a main component, rather than those for \( \pi^- p \rightarrow \pi^0n \) followed by \( \pi^0 p \rightarrow K^+\Lambda \). The sum of the cross sections by a harmonic oscillator model [46] for \(^6\text{Li}\) targets at \( p_\pi = 1.2 \text{ GeV/c} \) (\( 0^0 \)) is given as

\[
\sum_f \left( \frac{d\sigma_f}{d\Omega_f} \right)_{0^0} \approx \frac{2\pi\xi}{\rho K} \left( \frac{1}{r^2} \right) \left( \alpha \frac{d\sigma}{d\Omega} \right)_{0^0} \times \left( \frac{\alpha}{d\sigma/d\Omega} \right)_{0^0} \frac{K_{\text{eff}}^{pp}}{N_{\text{eff}} \Lambda},
\]

where \( \xi = 0.0370 \text{ mb}^{-1} \) is the factor integrated over angle \( \theta_{\text{lab}}^{(K^0)} \) for \( \pi^- p \rightarrow K^0\Lambda \) with \( -\theta_{\text{lab}}^{(K^+)} \) for \( K^0 p \rightarrow K^+n \) to restore \( \theta_{\text{lab}} = 0^0 \) in the angular distributions of the two elementary processes, \( p_K \approx 0.842 \text{ GeV/c} \) is the intermediate kaon momentum, and \( (1/r^2) \simeq 0.0280 \text{ mb}^{-1} \) is the mean inverse-square radial separation of the proton pair. \( K_{\text{eff}}^{pp} \approx 1 \) is the effective number of proton pairs including the nuclear distortion effects. The elementary laboratory cross section \( (\alpha d\sigma/d\Omega)_{0^0} \) is estimated to be \( \sim 0.55 \text{ mb/sr} \) for \( \pi^- p \rightarrow K^0\Lambda \) or \( \sim 1.96 \text{ mb/sr} \) for \( K^0 p \rightarrow K^+n \), depending on the nuclear medium corrections. The results show \( \sum_f (d\sigma_f/d\Omega_f)_{0^0} \approx 1.4 - 1.9 \text{ nb/sr} \) for \( \pi^- p \rightarrow K^0\Lambda \) followed by \( K^0 p \rightarrow K^+\Lambda \), and also \( 0.20 - 0.34 \text{ nb/sr} \) for \( \pi^- p \rightarrow \pi^0n \) followed by \( \pi^0 p \rightarrow K^+\Lambda \). Considering the large momentum transfer \( q \simeq 360 \text{ MeV/c} \) in the \( (\pi^- K^+) \) reactions, we expect that the production probabilities for loosely bound or resonant \( \Lambda \) states do not exceed \( 10^{-3}\% \) in the quasielastic \( \Lambda n \) production, so that the cross section of \( \Lambda \) in the two-step mechanism may be on the order of \( 10^{-2} \text{ nb/sr} \) at \( \theta_{\text{lab}} = 7^\circ \). This result suggests that the one-step mechanism, \( \pi^- p \rightarrow K^+\Sigma^- \) via \( \Sigma^- \) doorways caused by the \( \Sigma^- p \leftrightarrow \Lambda n \) coupling is rather favored than the two-step mechanism.

V. SUMMARY AND CONCLUSION

We studied phenomenologically the production of a neutron-rich hypernucleus \(^6\Lambda\) in the \(^6\text{Li}(\pi^- K^+)\) reaction at 1.2 GeV/c, considering the DWIA in the one-step mechanism, \( \pi^- p \rightarrow K^+\Sigma^- \) via \( \Sigma^- \) doorways caused by the \( \Sigma^- p \leftrightarrow \Lambda n \) coupling. We evaluated the production cross section of \( ^6\Lambda\text{H}(1_{\text{exc}}^+) \) by using the coupled \((^5\text{He}-\Lambda)+(^5\text{He}-\Sigma^-)\) model with a spreading potential and compared it with the data of the missing mass spectrum at the J-PARC E10 experiment. The results are summarized as follows:

(i) The \( \Sigma^- \)-mixing probabilities in \(^6\Lambda\text{H}(1_{\text{exc}}^+) \) are \( P_{\Sigma^-} \lesssim 0.2\% \) both for \( \Sigma^- \) state and for \( P_{\Sigma^+} \) state in order to reproduce no significant peak in the \( \Lambda \) production data, so that the cross section of \(^6\Lambda\text{H} \) is less than on the order of 0.4 nb/sr.

(ii) The shape and magnitude of the near-\( \Lambda \)-threshold spectrum significantly depend on the \( \Sigma \) \( \Lambda \) coupling and \( \Lambda \) potentials.

(iii) The cross section of \(^6\Lambda\text{H}(1^+_{\text{exc}}) \) is also sensitive to the structure of the \(^5\Lambda \) core nucleus independent of whether the \(^5\Lambda(1/2^+_{\text{exc}}) \) ground state exists as a resonant state bound with a narrow width.

(iv) The one-step mechanism via \( \Sigma^- \) doorways seems to be rather favored over the two-step mechanism because the cross section of \(^6\Lambda\text{H} \) in the two-step mechanism may be on the order of \( 10^{-2} \text{ nb/sr} \) at \( \theta_{\text{lab}} = 7^\circ \) by the harmonic-oscillator model.

In conclusion, the calculated spectrum of the \(^6\Lambda\text{H} \) hypernucleus by the one-step mechanism via \( \Sigma^- \) doorways can evaluate the near-\( \Lambda \)-threshold data of the DCX \(^6\text{Li}(\pi^- K^+)\) reaction at 1.2 GeV/c. The result shows that the \( \Sigma^- \)-mixing probabilities in \(^6\Lambda\text{H}(1^+_{\text{exc}}) \) are \( P_{\Sigma^-} \lesssim 0.2\% \) both for \( \Sigma^- \) state and for \( P_{\Sigma^+} \) state in order to explain no significant peak in the \( \Lambda \) production spectrum obtained at the J-PARC E10 experiment. The sensitivity to the potential parameters implies that the nuclear \((\pi^- K^+)\) reactions with much less background experimentally provide the high ability to study precise wave functions for \( \Lambda \), \( \Sigma^- \) and the \(^5\Lambda \) nuclear core in the neutron-rich \( \Lambda \) hypernucleus. Systematic analysis based on microscopic calculations is required for the extended J-PARC E10 experiment [47]. This investigation is in progress.

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