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LINEAR STABILITY ANALYSIS OF OPEN-CHANNEL SHEAR FLOW

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GENERATED BY VEGETATION

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4 ABSTRACT

A linear stability analysis of flow in an open-channel partially covered with vegetation was 5 performed. The differential drag between vegetated zones and adjacent non-vegetated zones is 6 known to induce a lateral gradient of the streamwise velocity. The velocity gradient may result in 7 flow instability in the shear layer around the edge of the vegetated zone, causing the generation 8 of discrete horizontal vortices. We assume that the base state flow field before the occurrence of 9 instability is characterized by turbulence, with a smaller length scale than the flow depth, which is 10 mainly generated by the bottom friction. By introducing perturbations to the flow depth as well as 11 the streamwise and transverse velocities in the base state, the conditions required for perturbations 12 grow in time were studied over a wide range of (1) Froude number, (2) normalized non-vegetated 13 zone width, and three other dimensionless parameters which represent the relative effect of (3) bed 14

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15	friction, (4) vegetation drag and (5) sub-depth eddy viscosity. All parameters were found to have
16	positive and negative growth rates of perturbations within their respective evaluated ranges. The
17	characteristic vortex shedding frequencies associated with the maximum growth rate was compared
18	with those observed in experiments. Although the analysis which employs a base state set without
19	the large scale lateral motions was shown to be capable of predicting the order of magnitude of
20	the frequencies, there is a systematic discrepancy between the predicted and observed frequencies
21	which may be due to the limitation of linear stability analysis.
22	Keywords: Linear stability analysis, shear flow, flow instability, kinematic eddy viscosity, lateral

vortices.

24 INTRODUCTION

The presence of vegetation is commonly observed in both natural and rectified watercourses. 25 Vegetation in watercourses is desirable in some cases as it prevents bank erosion and provides 26 habitat and food for numerous species. On the other hand, vegetation causes serious problems in 27 other cases as it increases channel resistance and reduces channel capacity for the draining of flood 28 water. Vegetation in a part of a channel produces transverse shear flow, which may lead to flow 29 instability and the generation of large-scale horizontal vortices. These horizontal vortices have a 30 strong influence on the velocity distribution and the amount of discharge conveyed by a channel 31 without overflow, and enhance the lateral mixing of not only the flow itself, but also the substances 32

33	transported by the flow both inside and outside the vegetated area. Therefore, it is important to
34	determine the conditions under which instability occurs, and the characteristics of the horizontal
35	vortices from both an engineering and an environmental points of view. Though various studies on
36	instability in channels with lateral velocity gradients have been performed, only theoretical studies
37	on instability in vegetated or compound channels are reviewed herein.
38	Tamai et al. (1986) observed the generation of large eddies on the water surface in a set of
39	experiments with compound channels consisting of a main channel and a flood plain. They con-
40	cluded that the shear layer in the lateral velocity profile around the interface between the main
41	channel and the flood plain is the predominant cause for the generation of large eddies. They
42	applied the stability analysis of Michalke (1964) with the use of the Rayleigh stability equation
43	to their experimental results, and found that their observations were able to be explained by the
44	analysis.
45	Chu et al. (1991) performed a linear stability analysis of shear flows in channels with varying
46	flow depths and varying bottom roughness. They employed the St. Venant shallow water equa-
47	tions with the free water surface approximated by a rigid lid, which is a valid simplification when
48	the Froude number is close to zero. The perturbation equations reduced to a modified Rayleigh
49	equation, which can be relatively easily solved. Because it is not possible to reproduce the lat-
50	eral gradient of the streamwise velocity without including the Reynolds stress in their formulation,

51	they adopted an assumption that the flow depth and the bottom roughness vary gradually across
52	the channel in order to approximate it. They found that flow stabilizes when the bottom roughness
53	is sufficiently large, and the lateral variation of flow is sufficiently small.
54	Ikeda et al. (1994) performed a temporal linear stability analysis of a partially vegetated chan-
55	nel. They obtained the base state flow with the use of the St. Venant shallow water equations
56	including the Reynolds stress expressed by the lateral kinematic eddy viscosity empirically deter-
57	mined in experiments. In the perturbed problem, however, they ignored the Reynolds stress and the
58	variation of the water surface elevation, in effect reducing their perturbed equations to a modified
59	Rayleigh equation again. They found that the dimensional angular frequency of maximum insta-
60	bility is uniquely correlated with the ratio of two velocities a sufficient distance from the boundary
61	between the non-vegetated and vegetated zones.
62	Ghidaoui and Kolyshkin (1999) performed a temporal linear stability analysis of a channel flow
63	with lateral velocity gradients without the rigid-lid assumption. The Reynolds stress was included
64	in their formulation by means of the eddy viscosity term of Chen and Jirka (1997). Semi-empirical
65	expressions were used to describe the base flow profile, containing regression parameters which
66	were not correlated to a specific flow field (i.e., the source of flow retardation in part of the channel
67	was not specified). Their computations showed that the influence of the Reynolds number, defined
68	using the eddy viscosity, on the stability domain is small when it surpasses 1000.

69	Prooijen and Uijttewaal (2002) have also included the turbulent viscosity as in Chen and Jirka
70	(1997) in their temporal and spatial linear stability analysis of a channel flow with a lateral velocity
71	gradient generated by two separate water supplies with different velocities. The mean flow field,
72	which varied along the streamwise direction, was assumed to be the base state and the rigid-lid
73	assumption was employed, which is a reasonable simplification, given that the Froude number in
74	their experimental runs did not exceed 0.5.
75	White and Nepf (2007) performed a complete set of experiments and a spatial stability anal-
76	ysis of a channel partially obstructed by an array of circular cylinders by the use of the modified
77	Rayleigh equations following Chu et al. (1991), with the Reynolds stress scaled with the width
78	of the shear layer around the edge of the array. They made use of the rigid-lid assumption, in
79	accordance with the condition of small Froude numbers met in their experiments (always below
80	0.25). They concluded that even though the drag differential due to the vegetation reinforces the
81	shear instability, the overall drag damps it if the background friction in the channel is sufficiently
82	large.
83	In this study, we perform a temporal linear stability analysis of flow in an open channel partially
84	covered with vegetation. By not employing the rigid-lid and the inviscid flow assumptions, we
85	could study the effects of the Froude number and the kinematic eddy viscosity, respectively, on the
86	growth rate of perturbations. We employ the St. Venant shallow water equations with the Reynolds

87	stress included to reproduce the velocity gradient due to the differential drag between the regions
88	with and without vegetation. The temporal and spatial variations of the flow vanish except for the
89	lateral variation of the streamwise velocity in the base state, which is used as a starting point of
90	the stability analysis. This base state flow field is not, however, simply a temporal average of the
91	flow affected by fully-developed horizontal vortices, but the flow undisturbed by the vortices. We
92	thus employ a kinematic eddy viscosity representing turbulence with a length scale smaller than
93	the flow depth. Differently from Chen and Jirka (1997), the eddy viscosity employed herein is
94	estimated for the flow unaffected by the large-scale horizontal vortices. We impose perturbations
95	on the base state flow velocities and flow depth, and study how various hydraulic parameters affect
96	the time development of the perturbations.

97 FORMULATION

⁹⁸ Suppose that water is flowing through a wide rectangular open-channel with lateral emergent ⁹⁹ rigid vegetation (trees) as shown in Fig. 1. The vegetation is modeled by an array of regularly ¹⁰⁰ spaced cylinders with a uniform diameter installed only on one side of the channel. The model of ¹⁰¹ cylinders as vegetation employed herein has been widely used in previous studies (e.g., Ikeda et al. ¹⁰² (1994), Tsujimoto and Kitamura (1992), White and Nepf (2007) and Xiaohui and Li (2002)). The ¹⁰³ region of the channel covered with vegetation is defined as the 'vegetated zone', the width of which ¹⁰⁴ is denoted by \tilde{B}_{v} . The region of the channel without vegetation is defined as the 'non-vegetated ¹⁰⁵ zone', the width of which is denoted by \tilde{B} .

106 Governing equations

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In this study, we focus on horizontal vortices generated in shallow flow in a wide rectangular 107 open channel. The horizontal length scale of the vortices is commonly large compared with the 108 length scale of flow depth. The generation of such thin vortices can be described by the depth-109 averaged shallow water formulation. In particular, the momentum equations employed in this 110 analysis need to include the Reynolds stress and the drag force due to vegetation in order to rep-111 resent the lateral velocity distribution due to the differential drag between the non-vegetated and 112 vegetated zones. The momentum equations in the streamwise and transverse directions (\tilde{x} and \tilde{y}) 113 and the continuity equation are 114

 $\frac{\partial \tilde{U}}{\partial \tilde{t}} + \tilde{U}\frac{\partial \tilde{U}}{\partial \tilde{x}} + \tilde{V}\frac{\partial \tilde{U}}{\partial \tilde{y}} = gS - g\frac{\partial \tilde{H}}{\partial \tilde{x}} - \frac{\tilde{T}_{bx} + \tilde{D}_x}{\rho \tilde{H}} + \frac{1}{\rho}\left(\frac{\partial \tilde{T}_{xx}}{\partial \tilde{x}} + \frac{\partial \tilde{T}_{xy}}{\partial \tilde{y}}\right),$ (1a)

$$\frac{\partial \tilde{V}}{\partial \tilde{t}} + \tilde{U}\frac{\partial \tilde{V}}{\partial \tilde{x}} + \tilde{V}\frac{\partial \tilde{V}}{\partial \tilde{y}} = -g\frac{\partial \tilde{H}}{\partial \tilde{y}} - \frac{\tilde{T}_{by} + \tilde{D}_y}{\rho \tilde{H}} + \frac{1}{\rho}\left(\frac{\partial \tilde{T}_{yx}}{\partial \tilde{x}} + \frac{\partial \tilde{T}_{yy}}{\partial \tilde{y}}\right),$$
(1b)

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$$\frac{\partial \tilde{H}}{\partial \tilde{t}} + \frac{\partial \tilde{U}\tilde{H}}{\partial \tilde{x}} + \frac{\partial \tilde{V}\tilde{H}}{\partial \tilde{y}} = 0, \qquad (1c)$$

where \tilde{t} is time, \tilde{x} is the streamwise coordinate, \tilde{y} is the lateral coordinate the origin of which is taken at the interface between the vegetated and non-vegetated zones, \tilde{U} and \tilde{V} are the \tilde{x} and \tilde{y} components of the flow velocity respectively, \tilde{H} is the flow depth, \tilde{T}_{bx} and \tilde{T}_{by} are the \tilde{x} and \tilde{y}

components of the bed shear stress respectively, \tilde{D}_x and \tilde{D}_y are the \tilde{x} and \tilde{y} components of the 121 drag force due to vegetation respectively, \tilde{T}_{ij} (i,j=x,y) is the Reynolds stress tensor, ho is the 122 density of water, q is the gravity acceleration, and S is the bed slope of the channel. The tilde 123 denotes dimensional variables, which is to be dropped after normalization. 124

The drag force vector $(\tilde{D}_x, \tilde{D}_y)$ is described by the expression 125

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$$(\tilde{D}_x, \tilde{D}_y) = \begin{cases} 0 & \text{in the non-vegetated zone,} \\ \frac{\rho C_D \tilde{a} \tilde{H}}{2} (\tilde{U}^2 + \tilde{V}^2)^{1/2} (\tilde{U}, \tilde{V}) & \text{in the vegetated zone,} \end{cases}$$
(2)

where C_D is the drag coefficient of vegetation, typically estimated to range from 1 to 2. In addition, 127 \tilde{a} is a parameter describing the density of vegetation, written by

129
$$\tilde{a} = \frac{d}{2\tilde{l}_x\tilde{l}_y},\tag{3}$$

where \tilde{d} is the diameter of cylinders and \tilde{l}_x and \tilde{l}_y are the distances between two adjacent cylinders 130 in the \tilde{x} and \tilde{y} directions respectively, as shown in Fig. 2. 131

The bed shear stress is related to the flow velocity by means of the bed friction coefficient C_f , 132 such that 133

134
$$(\tilde{T}_{bx}, \tilde{T}_{by}) = \rho C_f (\tilde{U}^2 + \tilde{V}^2)^{1/2} (\tilde{U}, \tilde{V}).$$
(4)

Though the bed friction coefficient C_f is a weak function of the flow depth relative to the roughness height, it is assumed to be constant and common in both vegetated and non-vegetated zones for simplicity.

¹³⁸ With the use of Boussinesq's kinematic eddy viscosity, the Reynolds stresses are expressed by

$$\tilde{T}_{xx} = 2\rho \tilde{\nu}_T \frac{\partial \tilde{U}}{\partial \tilde{x}}, \tag{5a}$$

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$$\tilde{T}_{xy} = \tilde{T}_{yx} = \rho \tilde{\nu}_T \left(\frac{\partial \tilde{U}}{\partial \tilde{y}} + \frac{\partial \tilde{V}}{\partial \tilde{x}} \right),$$
(5b)

$$\tilde{T}_{yy} = 2\rho\tilde{\nu}_T \frac{\partial\tilde{V}}{\partial\tilde{y}},\tag{5c}$$

where $\tilde{\nu}_T$ is the kinematic eddy viscosity. We assume that, in the base state before instability occurs, the flow is already affected by turbulence, the length scale of which is smaller than the flow depth (sub-depth scale turbulence). Where there is no influence of vegetation, the kinematic eddy viscosity $\tilde{\nu}_T$ should correspond to the sub-depth scale turbulence generated by the bottom friction. We employ the logarithmic velocity distribution as a sub-depth scale turbulent velocity distribution due to the bottom friction. The kinematic eddy viscosity then takes a parabolic form, which is depth-averaged from the bottom to the water surface, yielding

$$\tilde{\nu}_T = \frac{1}{6} \kappa \tilde{U}_{f\infty} \tilde{H}_{\infty},\tag{6}$$

150	where $\tilde{U}_{f\infty}$ and \tilde{H}_{∞} are the friction velocity and the flow depth in the region sufficiently far from
151	the vegetated zone, respectively, and κ is the Kármán constant (= 0.4). We assume that the sub-
152	depth scale turbulence is rather isotropic. Therefore, the above formulation is expected to describe
153	the Reynolds stresses in the streamwise and lateral directions a sufficient distance from the vege-
154	tated zone. Although the kinematic eddy viscosity in the horizontal direction is known to be larger
155	than in (6), as in Chen and Jirka (1997) where $\tilde{\nu}_T = 0.2 \tilde{U}_{f\infty} \tilde{H}_{\infty}$, we assumed that the increase in
156	the kinematic eddy viscosity is caused by large-scale horizontal vortices generated by instability.
157	In the shear layer formed around the boundary between the two zones, and inside the vegetated
158	zone, the velocity and the shear velocity are reduced because of the Reynolds stress and the drag
159	force due to vegetation. In addition, the length scale of sub-depth scale vortices may be affected by
160	a typical length scale of vegetation such as the vegetation spacing. According to the experimental
161	results of Ikeda et al. (1991), however, the depth-averaged kinematic eddy viscosity even in the
162	shear layer and the vegetated zone can be represented by (6). This may be attributed to the fact
163	that the sum of the resistant forces (the bed shear stress, the Reynolds stress and the vegetation
164	drag force) remains constant regardless of the reduction in the bed shear stress in the shear layer
165	and the vegetated zone. The kinematic eddy viscosity may be correlated to the total resistant force.
166	Furthermore, since the flow depth and the spacing of vegetation in Ikeda et al.'s experiments are
167	both in the same range, the kinematic eddy viscosity in the vegetated zone may not be strongly

affected by vegetation. These assumptions and (6) are employed in this study as well. Therefore,
 the Reynolds stresses in (1a-b) are expressed by the constant sub-depth kinematic eddy viscosity
 as in (6), for both the non-vegetated and vegetated zones.

At the side walls, the velocity vanishes in the directions both tangential and normal to the side walls. The following conditions therefore hold:

$$\tilde{U} = 0 \quad \text{at} \quad \tilde{y} = \tilde{B}, -\tilde{B}_v, \tag{7a}$$

$$\tilde{V} = 0 \quad \text{at} \quad \tilde{y} = \tilde{B}, -\tilde{B}_v. \tag{7b}$$

In the shallow water formulation, however, it is not easy to make use of the conditions of vanishing
 streamwise velocity (7a) (non-slip conditions). In place of these conditions, the following slip
 conditions are often used:

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$$\frac{\partial \tilde{U}}{\partial \tilde{y}} = 0 \quad \text{at} \quad \tilde{y} = \tilde{B}, -\tilde{B}_v.$$
(8)

At a sufficient distance from the boundary between the two zones, the streamwise velocity asymptotically approaches constant velocities in both the non-vegetated and vegetated zones in the base state. If \tilde{B} and \tilde{B}_v are sufficiently large, and the slip condition (8) holds, the streamwise velocity is constant at both side walls in the base state, i.e. for which

$$\tilde{U} = \tilde{U}_{\infty}$$
 at $\tilde{y} = \tilde{B};$ $\tilde{U} = \tilde{U}_{-\infty}$ at $\tilde{y} = -\tilde{B}_v,$ (9)

where \tilde{U}_{∞} and $\tilde{U}_{-\infty}$ are the velocities at a sufficient distance from the boundary in the nonvegetated and vegetated zones, respectively. We assume that both side walls are located at a sufficient distance from the boundary, and employ (7b) and (8) as the boundary conditions at the side walls.

Right at the boundary between the non-vegetated and vegetated zones, the velocities, flow depth and shear stresses are continuous, such that

¹⁹⁰
$$\lim_{\tilde{y}\to+0} \left(\tilde{U},\tilde{V},\tilde{H},\tilde{T}_{xx},\tilde{T}_{yy},\tilde{T}_{yy}\right) = \lim_{\tilde{y}\to-0} \left(\tilde{U},\tilde{V},\tilde{H},\tilde{T}_{xx},\tilde{T}_{xy},\tilde{T}_{yy}\right).$$
(10)

191 Normalization

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At a sufficient distance from the boundary between the two zones in the base state normal flow equilibrium condition, \tilde{U} and \tilde{H} are constant, and \tilde{V} vanishes. Thus, (1) allows the solutions

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$$\tilde{U}_{\infty} = \left(\frac{g\tilde{H}_{\infty}S}{C_f}\right)^{1/2}, \quad \tilde{U}_{-\infty} = \left(\frac{2g\tilde{H}_{\infty}S}{2C_f + C_D\tilde{a}\tilde{H}_{\infty}}\right)^{1/2}.$$
 (11)

¹⁹⁵ The velocity and flow depth at a sufficient distance from the vegetated zone, \tilde{U}_{∞} and \tilde{H}_{∞} are used ¹⁹⁶ for the normalization. The velocities and flow depth are then rendered dimensionless according to ¹⁹⁷ the following expressions:

$$(\tilde{U}, \tilde{V}) = \tilde{U}_{\infty}(U, V), \quad \tilde{H} = \tilde{H}_{\infty}H.$$
(12)

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The independent variables \tilde{x} , \tilde{y} and \tilde{t} are normalized with the use of the width of the non-vegetated zone \tilde{B} , such that

$$(\tilde{x}, \tilde{y}) = \tilde{B}(x, y), \quad \tilde{t} = \frac{B}{\tilde{U}_{\infty}}t.$$
 (13)

With the use of the above normalization, the governing equations (1) are rewritten in the form

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -F^{-2} \frac{\partial H}{\partial x} + \beta \left(1 - \frac{T_{bx} + D_x}{H}\right) + \epsilon \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right), \quad (14a)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -F^{-2} \frac{\partial H}{\partial y} - \beta \frac{T_{by} + D_y}{H} + \epsilon \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right), \quad (14b)$$

$$\frac{\partial H}{\partial t} + \frac{\partial UH}{\partial x} + \frac{\partial VH}{\partial y} = 0, \qquad (14c)$$

where (T_{bx}, T_{by}) and (D_x, D_y) are the normalized bed shear stress and vegetation drag vectors,

$$(T_{bx}, T_{by}) = (U^2 + V^2)^{1/2} (U, V), \qquad (15a)$$

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$$(D_x, D_y) = \begin{cases} \alpha \left(U^2 + V^2 \right)^{1/2} H(U, V) & \text{if } -B_v \le y \le 0, \\ 0 & \text{if } 0 \le y \le 1. \end{cases}$$
(15b)

The above normalized governing equations include the four non-dimensional parameters β , ϵ , F and α . The parameter β , which expresses the relative importance of the bed shear effect, is dependent on the aspect ratio of the non-vegetated zone $A_R = \tilde{B}/\tilde{H}_{\infty}$, and the bottom friction coefficient C_f , such that

$$\beta = \frac{C_f \tilde{B}}{\tilde{H}_{\infty}} = C_f A_R.$$
(16)

The parameter ϵ is associated with the eddy viscosity $\tilde{\nu}_T$, expressed by (6), in the form

$$\epsilon = \frac{\tilde{\nu}_T}{\tilde{U}_\infty \tilde{B}} = \frac{C_f^{1/2} \tilde{H}_\infty}{15\tilde{B}} = \frac{C_f^{1/2}}{15A_R}.$$
(17)

The Froude number \tilde{F} is given by

$$F = \frac{\tilde{U}_{\infty}}{\sqrt{\tilde{g}\tilde{H}_{\infty}}} = \left(\frac{S}{C_f}\right)^{1/2}.$$
(18)

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The parameter α is related to the vegetation drag and density, and is defined by

$$\alpha = \frac{C_D \tilde{a} \tilde{H}_{\infty}}{2C_f}.$$
(19)

221 BASE STATE NORMAL FLOW

The base state is set considering the fluid motion before the instabilities take place at the shear layer. Taking the average flow field as the base flow, or introducing a kinematic eddy viscosity including the effect of the vortices would result in a base state which is affected by the perturbations due to the transverse mixing.

In the base state, (14a) reduces to

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$$\beta \left(1 - U_0^2 \right) + \epsilon \frac{\mathrm{d}^2 U_0}{\mathrm{d} y^2} = 0 \quad \text{if} \quad 0 \le y \le 1,$$
 (20a)

$$\beta \left[1 - U_0^2 \left(1 + \alpha \right) \right] + \epsilon \frac{\mathrm{d}^2 U_0}{\mathrm{d} y^2} = 0 \qquad \text{if} \qquad -B_v \le y \le 0, \tag{20b}$$

where U_0 is the streamwise velocity in the base state, which is a function only of the transverse coordinate y.

The normalization of $(\tilde{U}_{\infty}, \tilde{U}_{-\infty})$ leads to $(1, \phi)$, where ϕ is the ratio between the undisturbed velocities in the vegetated and non-vegetated zones at a sufficient distance from their boundary, which is related to the non-dimensional parameter α , such that

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$$\phi = \frac{\tilde{U}_{-\infty}}{\tilde{U}_{\infty}} = \frac{1}{(1+\alpha)^{1/2}}.$$
(21)

The domain of ϕ is $0 < \phi \le 1$; ϕ approaches to 0 when vegetation obstructs the flow completely in the vegetated zone ($\alpha \to \infty$), and takes a value of unity when there is no vegetation ($\alpha = 0$). In the base state, the matching conditions (10) reduce to

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$$\lim_{y \to +0} U_0 = \lim_{y \to -0} U_0 = \psi, \quad \lim_{y \to +0} \frac{\mathrm{d}U_0}{\mathrm{d}y} = \lim_{y \to -0} \frac{\mathrm{d}U_0}{\mathrm{d}y}.$$
 (22)

There is a discontinuity in d^2U_0/dy^2 at y = 0. The velocity between the two zones ψ has a relation with ϕ as

$$\psi = \left(\frac{2\phi^2}{1+\phi}\right)^{1/3}.$$
(23)

The above expression was determined from the integration of (20a-b) with respect to y and the introduction of boundary conditions $U_0 = (\phi, 1)$ at $y = (-B_v, 1)$ and matching conditions (22). Solving (20a,b) under the above referred conditions, we obtain the explicit analytical solutions for U_0 in the form

$$U_{0}(y) = \begin{cases} 3 \tanh^{2} \left[\left(\frac{\beta}{2\epsilon} \right)^{1/2} y + \tanh^{-1} \left(\frac{\psi + 2}{3} \right)^{1/2} \right] - 2 & \text{if } 0 \le y \le 1, \\ 3\phi \coth^{2} \left[- \left(\frac{\beta}{2\epsilon\phi} \right)^{1/2} y + \coth^{-1} \left(\frac{\psi + 2\phi}{3\phi} \right)^{1/2} \right] - 2\phi & \text{if } -B_{v} \le y \le 0. \end{cases}$$
(24)

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The streamwise velocity in the base state U_0 is found to be expressed by hyperbolic-tangent and hyperbolic-cotangent functions which are invariant in time and in the streamwise direction and include four non-dimensional parameters, β , ϵ , ψ and ϕ , where the later two can be expressed as functions of only α .

Figs. 3(a), (b) and (c) depict the lateral distribution of U_0 as in (24) as functions of the parame-251 ters β , ϵ and α , respectively. Note that the velocities at the far right and left correspond to \tilde{U}_{∞} and 252 $\tilde{U}_{-\infty}$, and the value of U_0 at the far left is ϕ . It is found from Fig. 3(a) that, as β increases, the width 253 of the shear layer decreases. This is because an increase in β implies an increase in the relative 254 significance of the bed friction over the vegetation drag. In Fig. 3(b), on the other side, the increase 255 of ϵ results in the increase of the shear layer width following the increase of the relative importance 256 of the sub-depth kinematic eddy viscosity. The relative increase of the viscous effects will result 257 in a milder lateral gradient of the base state velocity. According to Fig. 3(c), ϕ decreases with 258 increasing α , as it is natural that the velocity difference between the two zones increases with the 259 vegetation drag parameter. In contrast to β and ϵ , the shear layer width does not strongly depend 260

261 On α .

262 LINEAR STABILITY ANALYSIS

A temporal linear stability analysis is performed herein. A disturbance undulating in the streamwise direction is introduced to the base state. The streamwise and lateral velocities U and V, and the flow depth H are then expanded in the form

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$$U(x, y, t) = U_0(y) + AU_1(y)e^{i(kx-\omega t)},$$
 (25a)

$$V(x, y, t) = AV_1(y)e^{i(kx-\omega t)}, \qquad (25b)$$

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$$H(x, y, t) = 1 + AH_1(y)e^{i(kx-\omega t)},$$
 (25c)

where A, k and ω are the amplitude, wavenumber and angular frequency of perturbation. In the scheme of temporal linear stability analysis, k is real while ω is complex such that $\omega = \omega_r + i\Omega$, where ω_r is the real angular frequency and Ω is the growth rate of perturbation.

Substituting (25) into the governing equations (14), we obtain the following perturbed equa-

tions in the non-vegetated zone:

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$$\left[i\left(kU_0 - \omega\right) + k^2\epsilon + 2\beta U_0 - \epsilon \frac{d^2}{dy^2}\right]U_1 + \frac{dU_0}{dy}V_1 + \left(ikF^{-2} - \beta U_0^2\right)H_1 = 0, \quad (26a)$$

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$$\left[i\left(kU_0 - \omega\right) + k^2\epsilon + \beta U_0 - \epsilon \frac{d^2}{dy^2}\right]V_1 + F^{-2}\frac{dH_1}{dy} = 0, \quad (26b)$$

ik
$$U_1 + \frac{\mathrm{d}V_1}{\mathrm{d}y} - \mathrm{i}(\omega - kU_0)H_1 = 0.$$
 (26c)

In the vegetated zone, (14) reduces to

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$$\left[i\left(kU_{0}-\omega\right)+k^{2}\epsilon+2\beta U_{0}\left(1+\alpha\right)-\epsilon\frac{d^{2}}{dy^{2}}\right]U_{1}+\frac{dU_{0}}{dy}V_{1}+\left(ikF^{-2}-\beta U_{0}^{2}\right)H_{1} = 0$$
(27a)

$$\begin{bmatrix} i \left(kU_0 - \omega \right) + k^2 \epsilon + \beta U_0 \left(1 + \alpha \right) - \epsilon \frac{\mathrm{d}^2}{\mathrm{d}y^2} \end{bmatrix} V_1 + F^{-2} \frac{\mathrm{d}H_1}{\mathrm{d}y} = 0$$
(27b)

280
$$ikU_1 + \frac{dV_1}{dy} - i(\omega - kU_0)H_1 = 0$$
(27c)

Since the amplitude of the perturbation A is assumed to be infinitesimally small, the terms containing A^2 have been dropped in the linear stability analysis, so that the results of the analysis are valid only in the range of small amplitudes.

The boundary conditions of vanishing lateral velocity (7b) and vanishing shear stress (8) at the

side walls take the following forms at O(A):

$$\frac{\partial U_1}{\partial y} = 0$$
 at $y = -B_v, 1,$ (28a)

286

 $V_1 = 0$ at $y = -B_v, 1.$ (28b)

At the interface between the vegetated and non-vegetated zones y = 0, the matching conditions reduce to

$$\lim_{y \to +0} \left(U_1, V_1, H_1, \frac{\partial U_1}{\partial y}, \frac{\partial V_1}{\partial y} \right) = \lim_{y \to -0} \left(U_1, V_1, H_1, \frac{\partial U_1}{\partial y}, \frac{\partial V_1}{\partial y} \right).$$
(29)

Although there are five matching conditions in (29), only four of them are independent since, if four of them are imposed, the other condition is automatically satisfied. Thus, one of these conditions can be dropped afterwards.

A numerical scheme is necessary to solve (26) and (27) under the boundary and matching conditions (28) and (29), as the equations obviously do not admit analytical solutions. We employ a spectral collocation method with the Chebyshev polynomials. In the non-vegetated zone ($0 \le y \le 1$), the variables U_1 , V_1 and H_1 are expanded in the form

²⁹⁸
$$U_1 = \sum_{j=0}^{N} a_j T_j(\xi), \quad V_1 = \sum_{j=0}^{N} a_{(N+1)+j} T_j(\xi), \quad H_1 = \sum_{j=0}^{N} a_{2(N+1)+j} T_j(\xi),$$
 (30)

and, in the vegetated zone ($-B_v \le y \le 0$), they are expanded in the form

$$U_{1} = \sum_{j=0}^{N} a_{3(N+1)+j} T_{j}(\zeta), \quad V_{1} = \sum_{j=0}^{N} a_{4(N+1)+j} T_{j}(\zeta), \quad H_{1} = \sum_{j=0}^{N} a_{5(N+1)+j} T_{j}(\zeta), \quad (31)$$

where a_j (j = 0, 1, 2, ..., 6N + 5) are the coefficients of the Chebyshev polynomials, and $T_j(\xi)$ and $T_j(\zeta)$ are the Chebyshev polynomials in ξ and ζ of degree j. The independent variables ξ and ζ both range from -1 to 1, and are related to y by the equations $\xi = 2y - 1$ $(0 \le y \le 1)$ and $\zeta = 2y/B_v + 1$ $(-B_v \le y \le 0)$, respectively. The expansions (30) and (31) are substituted into the governing equations (26) and (27) respectively, and the resulting six equations are evaluated at the Gauss-Lobatto points, defined by

$$\xi_m = \cos\frac{m\pi}{N}, \quad \zeta_m = \cos\frac{m\pi}{N}, \tag{32}$$

where m = 0, 1, ..., N. Therefore, the number of points where the governing equations are evaluated is N + 1. We obtain a system of 6(N + 1) algebraic equations with 6(N + 1) unknown coefficients $a_0, a_1, a_2, ..., a_{6N+5}$. Eight equations of the system are then replaced by the four boundary conditions (28) and four of the matching conditions (29). The resulting linear algebraic system can be written in the form

313
$$\mathcal{M}\left[\begin{array}{cc} a_0 & a_1 & \dots & a_{6N+5} \end{array}\right]^T = 0, \tag{33}$$

where \mathcal{M} is a $6(N+1) \times 6(N+1)$ matrix in which the elements consist of the coefficients of U_1 , V_1 and H_1 in the governing equations (26) and (27), and the boundary and matching conditions (28) and (29). The condition for (33) to have a non-trivial solution is that \mathcal{M} should be singular. Thus,

$$|\mathcal{M}| = 0. \tag{34}$$

³¹⁹ The solution of the above equation takes the functional form

$$\omega = \omega \left(k, \beta, \epsilon, \alpha, B_v, F \right). \tag{35}$$

321 RESULTS AND DISCUSSION

As seen in (35), there are six important non-dimensional parameters k, β , ϵ , α , B_v , and F 322 determining the growth rate Ω . The contours of the growth rate Ω on the plane of these parameters 323 are shown in Figs. 4–8. When Ω is positive, the perturbations grow with time, whereas the pertur-324 bations decay until they vanish when Ω is negative. The thick solid lines in the figures indicate the 325 neutral instability curve on which $\Omega = 0$ and the perturbations neither grow nor decay, and divide 326 the planes into stable ($\Omega < 0$) and unstable regions ($\Omega > 0$). In the figures, Ω typically becomes 327 negative in the range of sufficiently small and large values of k, and takes a maximum value be-328 tween them. It follows that Ω as a function of k commonly possesses a characteristic wavenumber 329

 k_m associated with the maximum growth rate Ω_m , implying the selection of a preferential wavelength at the linear level.

332	The parameters β , ϵ , α , B_v , and F were systematically varied from a base set of numerical
333	values consisting of $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, $\alpha = 10$, $B_v = 0.55$ and $F = 0.5$. These values were
334	defined based on typical values of the experiments of Ikeda et al. (1991). $N = 30$ was adopted in
335	the Chebyshev polynomials, with results independent of the numerical resolution.
336	In Fig. 4(a), the dependence of Ω on β is studied for the case $\epsilon = 6 \times 10^{-4}$, $\alpha = 10$, $B_v = 0.55$
337	and $F = 0.5$, following the base set of values for these parameters. The growth rate Ω is maximized
338	when β is around 10^{-1} . As β increases or decreases from this value, the flow becomes less stable,
339	and in the range $\beta \gtrsim 0.5,$ the region of positive Ω completely disappears and the flow becomes
340	stable. In the range of large β , the effect of the bottom friction inhibits the effect of the lateral
341	velocity gradient, as already shown by Chu et al. (1991) and White and Nepf (2007). On the other
342	side, in the range of small β , the relative effect of the small scale turbulences generated by the
343	bottom friction will be reduced, allowing the shear layer to expand further along the transverse
344	direction and leading to a milder lateral gradient of the base state velocity, as in Fig. 3(a), leading
345	to the reduction of the growth rate Ω . Although the present shallow water formulation may no
346	longer be valid in the range of $\beta \lesssim 5 \times 10^{-3}$, the decrease in the growth rate Ω is expected to
347	occur in this range. In the range of sufficiently small β , the shear layer may be affected by the zero

348	disturbance boundary condition of the walls, as pointed by Kolyshkin and Ghidaoui (2002). The
349	range of k for positive Ω and the characteristic wavenumber k_m increase with β . This is because
350	the wavenumber k is normalized by the width of the non-vegetated zone \tilde{B} .
351	The effect of the parameter ϵ on Ω is studied in Fig. 5. The parameter ϵ measures the relative
352	effect of the sub-depth kinematic eddy viscosity. Because the sub-depth kinematic eddy viscosity
353	is derived taking into consideration the small scale turbulences generated by the bottom friction
354	before instability takes place, ϵ does not contain the effect of the large-scale turbulences. It is
355	found from Fig. 5 that the flow becomes stable when $\epsilon \gtrsim 4.0 \times 10^{-3}$. In this range, the dissipation
356	of energy caused by the small scale turbulences will be sufficiently large to suppress the effect
357	of the transverse mixing. On the other side, as ϵ decreases, Ω increases. In the range of very
358	small ϵ , the flow approaches to the inviscid case. The characteristic wavenumber k_m increases
359	with the decrease of ϵ because of the normalization of the wavenumber k. If we had employed the
360	kinematic eddy viscosity as in Chen and Jirka (1997), the growth rate of perturbations would have
361	been underestimated, following that a larger eddy viscosity would correspond to a smaller growth
362	rate Ω .
363	The dependence of Ω on the vegetation drag parameter α is studied in Fig. 6. It is found that, in
364	the range of small α , Ω is negative and the flow is stable as already pointed out by Chu et al. (1991)
365	and White and Nepf (2007). As α increases, Ω increases with a slight increase in k_m , which peaks

366	around the point $(\alpha, k) \approx (10^2, 6)$. In the range of $\alpha \gtrsim 10^2$, Ω decreases with a slight decrease in
367	k_m , and Ω becomes negative and the flow becomes stable again when $\alpha \gtrsim 4 \times 10^3$. In the present
368	analysis, we assume that the kinematic eddy viscosity in the vegetated zone is represented by (6)
369	since the typical length scale of vegetation is not significantly smaller than the flow depth. When
370	the vegetation density reaches a certain density, this assumption may no longer be valid. However,
371	it is natural that the flow becomes stable with increasing α since the vegetated zone becomes like
372	a cavity region when α is sufficiently large, and the large-scale horizontal vortices are damped by
373	strong retardation effects. Therefore, the contours of Ω in the range of large α in Fig. 6 are at least
374	qualitatively correct.
375	The effect of the width of the vegetated zone B_v on Ω is shown in Fig. 7. It is found from the
376	figure that Ω is negative for any value of k when $B_v \leq 0.1$, and Ω is almost independent of B_v
377	when $B_v \gtrsim 0.5$. In this analysis, we assume that B_v is large enough that U_0 is almost constant at
378	$y = -B_v$ as described in (9). Therefore, Fig. 7 is not reliable in the range of small B_v . However,
379	Fig. 7 is at least qualitatively correct because the lateral displacement of water is suppressed and
380	the flow becomes stable when B_v is sufficiently small.
381	The dependence of Ω on the Froude number F is studied in Fig. 8. Because the rigid-lid
382	assumption was not employed in the present analysis, the growth rate of perturbations can also be
383	studied for values of F which are not close to zero. It is found from the figure that the flow is

unstable in the range of small F. As F increases, the instability weakens and a stable region is 384 observed in the range $F \approx 2.3 - 2.6$. When $F \gtrsim 2.6$ under the conditions of this figure, the flow is 385 again found to be unstable. It has been empirically known that rapid flow plays stabilizing effects 386 for lateral velocity gradients, which is theoretically explained in this analysis. On the other side, 387 as pointed by Kolyshkin and Ghidaoui (2002), the rapid flow becomes unstable to gravity waves 388 if F is sufficiently large. Therefore, although the flow is found to be stable to the lateral velocity 389 gradients when $F \gtrsim 2.3$ in Fig. 8, it is unstable to gravity when $F \gtrsim 2.6$. Fig. 9 depicts the neutral 390 instability curves ($\Omega = 0$) for the case $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, and $B_v = 0.55$ for varying α and 391 multiple Froude numbers F. While the flow is stable when $\alpha \lesssim 0.5$ for F = 0.5 and F = 1.5, 392 it becomes unstable to gravity in this range when F = 2.5. As F increases from 0.5 to 2.5, the 393 unstable region for $\alpha \gtrapprox 0.5$ diminishes because the gravity effects weaken the instability due to 394 the transverse mixing. The combined effect of rapid flow and lateral mixing stabilizes the flow in 395 the range of $\alpha \approx 10^1$ for F = 2.5. 396

Once the characteristic frequency of the generation of vortices ω_{rm} associated with the maximum growth rate Ω_m is determined, the corresponding time period \tilde{T} can be calculated by the following relation:

$$\tilde{T} = \frac{2\pi B}{\omega_{rm}\tilde{U}_n}.$$
(36)

Assuming that the perturbation with the maximum growth rate Ω_m is realized in experiments and

400

in the field, we compare \tilde{T} predicted in the analysis with the results of the laboratory experiments obtained by Ikeda et al. (1994) and Tsujimoto (1991). In their experiments, vortices were generated by an array of regularly spaced cylinders installed on one side of channels, which is the same setup as assumed in the present analysis. The major hydraulic parameters of the experiments are listed in Table 1.

Fig. 10 depicts a comparison between the predicted and measured values of the period of gen-407 eration of vortices \tilde{T} . In the figure, the crosses correspond to the results obtained with the use of 408 (6), and the closed circles correspond to the results obtained with the use of the eddy viscosity 409 proposed by Chen and Jirka (1997). For the sub-depth eddy viscosity $\tilde{\nu}_T$ from (6), the predic-410 tions are generally smaller than the observations by a factor of approximately two. Ikeda et al. 411 (1994) made a comparison between the vortex shedding periods observed in their experiments 412 and those predicted by their linear stability analysis, and showed good agreement between them. 413 Their analysis employed the lateral kinematic eddy viscosity observed in their experiments which 414 is approximately twice as large as in (6). Similarly, we performed a linear stability analysis of 415 the experimental cases of Table 1 employing the eddy viscosity of Chen and Jirka (1997) and the 416 agreement was better in comparison with the results corresponding to the eddy viscosity from (6), 417 as depicted in Fig. 10. 418

419

The vortex shedding periods observed in the experiments correspond to a fully developed stage,

27

420	while the results of the linear analysis can reliably reflect only an initial stage of the growth of
421	infinitesimally small disturbances. Although the use of an increased eddy viscosity in the linear
422	analysis led to a good estimation of the vortex shedding periods, this is because the nonlinear
423	interactions may result in the increase of the mixing efficiency, which is apparently equivalent to
424	the increase in the kinematic eddy viscosity.
425	The vortex shedding period may be longer due to the nonlinear interaction among vortices with
426	a variety of length scales and frequencies. Therefore, in order to gain a qualitative understanding of
427	the effect of nonlinear interaction, a weakly nonlinear stability analysis will no doubt prove useful.
428	CONCLUSIONS
429	In this paper, we propose a new linear stability analysis of flow with a lateral velocity gradient
430	due to the presence of vegetation on one side of an open channel. In the analysis, we employ the
431	St. Venant shallow water equations, and include the Reynolds stresses represented by the kine-
432	matic eddy viscosity, which characterizes the sub-depth scale turbulence generated by the bottom
433	friction. In the base state, the velocity distributions inside and outside vegetation are expressed by
434	hyperbolic cotangent and hyperbolic tangent functions squared respectively. These functions are
435	determined analytically, by considering as the base state not the average flow, but the flow without
436	the effects of the large-scale horizontal vortices.

We performed a temporal linear stability analysis by imposing small perturbations on the base

438	state flow velocities and depth. We obtained a set of instability diagrams with respect to six non-
439	dimensional parameters, including a kinematic eddy viscosity parameter, following that we did not
440	employ a formulation for inviscid flow. Our results indicate that, while the base state flow field is
441	unstable in the range of typical, moderate values of the hydraulic parameters, stability is retained
442	in the range of sufficiently small and large vegetation densities, small widths of the vegetated zone,
443	large bed shear effect, large sub-depth eddy viscosity effect, and moderate Froude numbers where
444	the flow is stable to both the transverse mixing and the gravity. The growth rate of perturbations
445	could be evaluated for Froude numbers far from zero because the rigid-lid assumption was not
446	used. The use of a theoretical sub-depth kinematic eddy viscosity unaffected by the lateral mo-
447	tions permitted a consistent estimation of the growth rate of perturbations, as these perturbations
448	were imposed to a base flow which is independent of the large-scale lateral motions. Assuming that
449	the characteristic wavenumber and frequency of perturbations associated with maximum perturba-
450	tion growth rate correspond to those of vortices realized in experiments, we compare predicted
451	and observed vortex shedding frequencies. There is a systematic discrepancy in the frequencies
452	predicted from employing a sub-depth eddy viscosity undisturbed by the vortices, when compared
453	to the observed frequencies. This discrepancy, typically in the range of a factor of approximately
454	two, may be caused by the limitation of linear stability analysis.

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482 NOTATION

⁴⁸³ The following symbols are used in this paper:

484 A = amplitude;

- A_{R} = aspect-ratio of the non-vegetated zone;
- a = vegetation density parameter;
- $_{487}$ B = non-vegetated zone width;

488 $B_v =$ vegetated zone width;

489 $C_d =$ vegetation drag coefficient;

490 $C_f =$ bed friction coefficient;

 $_{491}$ $D_x, D_y =$ streamwise and transverse vegetation drag components, respectively;

 $_{492}$ d = diameter of cylinders;

F = Froude number;

g = gravity acceleration;

H = flow depth;

k = wavenumber;

 $l_x, l_y =$ distances between two adjacent cylinders in x and y directions;

- S = streamwise bed slope of the channel;
- $T_{bx}, T_{by} =$ streamwise and transverse bed shear stress components, respectively;
- $T_{i,j}$ (i, j = x, y) = Reynolds stress tensor;
- $T_j(\xi), T_j(\zeta) =$ Chebyshev polynomials in ξ and ζ of degree j;

t = time;

- U, V = streamwise and transverse velocities, respectively;
- $U_0 = base state flow velocity;$
- $U_1, V_1, H_1 =$ eigenfunctions;

 $U_f =$ friction velocity;

- x, y = streamwise and transverse coordinates, respectively;
- $\alpha =$ vegetation drag parameter;
- $\beta = \text{ bed friction parameter;}$

 $\epsilon =$ sub-depth eddy viscosity parameter;

511 $\kappa = \text{Kármán constant};$

⁵¹² ν_T = kinematic eddy viscosity;

 $\rho = \text{water density};$

- $\phi =$ ratio between the undisturbed velocities in the vegetated zone and the non-vegetated zone;
- $\psi = base state flow velocity at the interface between the non-vegetated and vegetated zones;$
- 516 $\omega = \omega_r + i\Omega =$ angular frequency.

517 Subscripts:

 $_{518}$ $\infty, -\infty =$ far field in the non-vegetated and vegetated zones, respectively;

m = most unstable mode.

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522		IW1–IW3						

Run	eta	$\epsilon (\times 10^{-4})$	α	B_V	F	Measured $\tilde{T}(s)$
1	0.060	5.02	10.2	0.57	0.41	6.4
2	0.061	5.07	10.0	0.57	0.75	3.8
5	0.025	5.48	17.2	0.57	0.44	9.0
IW1	0.024	4.84	67.7	0.43	0.69	3.3

5.84 51.0 0.43 0.78

5.40 61.2 0.43 0.93

2.5

1.9

IW2 0.030

IW3 0.025

TABLE 1. Hydraulic parameters in Ikeda et al.'s (1994), run 1–5, and Tsujimoto's (1991), run IW1–IW3.

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FIG. 1. Conceptual diagram of the channel with vegetation. (a) The cross-sectional view, and (b) the plan view.



FIG. 2. The plan view of an array of regularly-spaced cylinders as a model of vegetation.



FIG. 3. The lateral distribution of the streamwise velocity in the base state U_0 as functions of (a) β for the case $\epsilon = 6 \times 10^{-4}$, $\alpha = 10$, (b) ϵ for the case $\beta = 0.05$, $\alpha = 10$, and (c) α for the case $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$.



FIG. 4. The contours of the perturbation growth rate Ω in the β -k plane for the case $\epsilon = 6 \times 10^{-4}$, $\alpha = 10$, $B_v = 0.55$, F = 0.5.



FIG. 5. The contours of the perturbation growth rate Ω in the ϵ -k plane for the case $\beta = 0.05$, $\alpha = 10$, $B_v = 0.55$, F = 0.5.



FIG. 6. The contours of the perturbation growth rate Ω in the α -k plane for the case $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, $B_v = 0.55$, F = 0.5.



FIG. 7. The contours of the perturbation growth rate Ω in the B_v -k plane for the case $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, $\alpha = 10$, F = 0.5.



FIG. 8. The contours of the perturbation growth rate Ω in the *F*-*k* plane for the case $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, $\alpha = 10$, $B_v = 0.55$.



FIG. 9. The contours of the neutral instability ($\Omega = 0$) in the α -k plane for the case $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, $B_v = 0.55$ and multiple Froude numbers F.



FIG. 10. Comparison between the predicted and measured periods of generation of vortices \tilde{T} (×: eddy viscosity from (6); •: eddy viscosity from Chen and Jirka (1997)).