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# Dust formation and mass loss around intermediate-mass AGB stars with initial metallicity $\boldsymbol{Z}_{\text {ini }} \leq 1 \mathbf{1 0}^{\mathbf{- 4}}$ in the early Universe - I. Effect of surface opacity on stellar evolution and the dust-driven wind 

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#### Abstract

Dust formation and the resulting mass loss around asymptotic giant branch (AGB) stars with initial metallicity in the range $0 \leq Z_{\text {ini }} \leq 10^{-4}$ and initial mass $2 \leq M_{\text {ini }} / M_{\odot} \leq 5$ are explored by hydrodynamical calculations of the dust-driven wind (DDW) along the AGB evolutionary tracks. We employ the MESA code to simulate the evolution of stars, assuming an empirical mass-loss rate in the post-main-sequence phase and considering three types of low-temperature opacity (scaled-solar, CO-enhanced and CNO-enhanced opacity) to elucidate the effect on stellar evolution and the DDW. We find that the treatment of low-temperature opacity strongly affects dust formation and the resulting DDW; in the carbon-rich AGB phase, the maximum $\dot{M}$ of $M_{\mathrm{ini}} \geq 3 \mathrm{M}_{\odot}$ stars with the CO-enhanced opacity is at least one order of magnitude smaller than that with the CNO-enhanced opacity. A wide range of stellar parameters being covered, the necessary condition for driving efficient DDW with $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ is expressed as effective temperature $T_{\text {eff }} \lesssim 3850 \mathrm{~K}$ and $\log \left(\delta_{\mathrm{C}} L / \kappa_{\mathrm{R}} M\right) \gtrsim 10.43 \log T_{\text {eff }}-32.33$, with the carbon excess $\delta_{\mathrm{C}}$ defined as $\epsilon_{\mathrm{C}}-\epsilon_{\mathrm{O}}$, the Rosseland mean opacity $\kappa_{\mathrm{R}}$ in units of $\mathrm{cm}^{2} \mathrm{~g}^{-1}$ in the surface layer and the stellar mass (luminosity) $M(L)$ in solar units. The fitting formulae derived for gas and dust mass-loss rates in terms of input stellar parameters could be useful for investigating the dust yield from AGB stars in the early Universe being consistent with stellar evolution calculations.


Key words: stars: abundances - stars: AGB and post-AGB - ISM: abundances - dust, extinction.

## 1 INTRODUCTION

While the major source of interstellar dust in the early Universe at redshift $z \gtrsim 5$ is believed to be core-collapsed supernovae (e.g. Todini \& Ferrara 2001; Nozawa et al. 2003), the possibility that asymptotic giant branch (AGB) stars are an important source of dust has been suggested and investigated. Dwek, Galliano \& Jones (2007) claimed that core-collapse supernovae (CCSNe) cannot reproduce the dust mass of about $4 \times 10^{8} \mathrm{M}_{\odot}$ in the highredshift ( $z=6.4$ ) quasar $\mathrm{J} 1148+5251$, unless the dust mass produced is much greater than that evaluated from the observations of CCSNe in nearby galaxies or the dust destruction efficiency is much lower than that inferred from theoretical calculations. Valiante et al. $(2009,2011)$ have shown that AGB stars can contribute to dust en-

[^0]richment even at redshifts $z<8-10$, based on the dust yield of AGB stars with initial metallicity $Z_{\text {ini }}=10^{-3}$ calculated by Zhukovska, Gail \& Trieloff (2008).

So far, investigations of dust formation around AGB stars have suggested that AGB stars with initial metallicity $Z_{\text {ini }} \lesssim 10^{-3}$ cannot be assigned as the source of Si-bearing dust such as silicate, since the abundance of silicon, scaled by the initial metallicity, is so small as to prevent the formation of Si-bearing dust in the winds (Di Criscienzo et al. 2013). Thus, only carbon dust is expected to form around AGB stars with initial metallicity $Z_{\text {ini }} \lesssim 10^{-3}$, owing to the progressive enrichment of carbon in the surface regions, favoured by repeated third dredge-up (TDU) events.

The upper limit on the initial mass of the star required to have production of carbon dust during the AGB phases decreases with decreasing $Z_{\text {ini }}$. This is because the core mass of the star is higher when the metallicity is lower and when the core mass is above a given threshold $\left(\sim 0.8 \mathrm{M}_{\odot}\right)$ the stars experience hot bottom burning (HBB: Renzini \& Voli 1981), with the destruction of surface carbon. Di Criscienzo et al. (2013) have inferred that only low-mass
stars of initial mass $M_{\text {ini }} \lesssim 1 \mathrm{M}_{\odot}$ with $Z_{\text {ini }} \leq 10^{-4}$ can produce carbon dust significantly and that AGB stars cannot be considered as important dust manufacturers at $Z_{\text {ini }}<10^{-4}$ : this conclusion was based on their calculation for $Z_{\text {ini }}=3.0 \times 10^{-4}$ and on stellar evolution calculations with $Z_{\text {ini }} \lesssim 2 \times 10^{-5}$ by Campbell \& Lattanzio (2008). On the other hand, Constantino et al. (2014) confirmed that the mass threshold for HBB is different between models with and without composition-dependent low-temperature opacity. Also, even if stars experience HBB, these stars could become carbon-rich after the cease of HBB, depending on not only the treatment of low-temperature opacity but also the initial mass, as well as the mass-loss rate during evolution (e.g. Ventura \& Marigo 2010; Nanni et al. 2013). Thus, the pros and cons of formation of carbon dust in AGB stars with $Z_{\text {ini }} \leq 10^{-4}$ have yet to be explored by investigating how the treatment of low-temperature opacity affects the stellar evolution and dust formation.
The formation of dust and the resulting mass loss around AGB stars not only are determined by the abundances of dust-forming elements in the surface layer, but also depend sensitively on the effective temperature (see Gail \& Sedlmayr 2013 and references therein). In this context, the most relevant input for stellar evolution and dust formation is the low-temperature opacity. During this last decade, it has been emphasized that the low-temperature opacity varying with the change of surface elemental composition due to TDU and HBB during the thermally pulsing AGB (TP-AGB) phase strongly affects the evolution of stars with $Z_{\text {ini }} \geq 10^{-4}$ (e.g. Marigo 2002; Cristallo et al. 2007; Ventura \& Marigo 2010; Constantino et al. 2014; Fishlock, Karakas \& Stancliffe 2014). In particular, these authors have demonstrated that a composition-dependent low-temperature opacity makes the effective temperature decrease drastically in carbon-rich (C-rich) stars, in comparison with the scaled-solar opacity. Thus, it can be expected that the treatment of low-temperature opacity directly influences the formation of dust and the resulting dust-driven wind (DDW).
Dust formation around AGB stars is a complicated process, associated with the dynamical as well as the thermal behaviour of gas above the photosphere; dust condenses in the high-density gas induced by the shock originating from stellar pulsation, then mass loss is driven by the radiation pressure force acting on the newly formed dust (so-called pulsation-enhanced DDW: e.g. Fleischer, Gauger \& Sedlmayr 1992; Winters et al. 2000). Thus, dust formation has to be treated self-consistently with the consequent gas outflow from AGB stars, considering the periodic change of stellar properties and the corresponding wind structure simultaneously. However, most previous studies on the dust yields of low-metallicity AGB stars (e.g. Ventura et al. 2012a, b, 2014; Di Criscienzo et al. 2013; Nanni et al. 2013) have followed the scheme developed by Ferrarotti \& Gail (2006), without solving the formation processes of dust grains and the resulting density structure of outflowing gas selfconsistently; the dust yield has been evaluated from calculations of dust growth in a stationary wind, given the number density of dust seed particles and the mass-loss rate. Thus, the derived properties of newly formed dust, such as the amount and the size, may suffer ambiguities inherent in the treatment. Although self-consistent hydrodynamical calculation of the DDW for C-rich AGB stars with subsolar metallicities has been carried out (Wachter et al. 2008), to our knowledge so far no attempt has been made for AGB stars with $Z_{\text {ini }} \leq 10^{-4}$ in the early Universe.

In order to explore whether AGB stars can produce and supply carbon dust in the early Universe, first we simulate the evolution of stars with initial mass ranging from 2-5 $\mathrm{M}_{\odot}$ with initial metallicity $Z_{\text {ini }} \leq 10^{-4}$. In the simulations, three types of low-temperature
opacity (scaled-solar, CO-enhanced and CNO-enhanced opacity) are considered, to clarify how the treatment of low-temperature opacity influences the stellar parameters related to dust formation during the AGB phase. Then, applying the calculated stellar parameters along the evolutionary track of TP-AGB to the hydrodynamical model of pulsation-enhanced DDW, we investigate the dependence of the properties of dust and the DDW produced in the TP-AGB phase on the treatment of low-temperature (surface) opacity, as well as on the initial mass and metallicity. In addition, we evaluate a necessary condition for realizing the DDW and derive the fitting formulae for gas and dust mass-loss rates caused by the DDW in terms of the input stellar parameters.

This article is organized as follows. In Section 2, we address the stellar evolution model briefly, focusing on the tools implemented in the Modules for Experiments in Stellar Astrophysics (MESA) code (Paxton et al. 2011, Paxton et al. 2013), and introduce the hydrodynamical model of pulsation-enhanced DDW used in this study. Section 3 provides the results of stellar evolution calculations and shows how the stellar parameters (e.g. effective temperature and elemental composition in the surface layer) controlling the dust formation and mass loss during the TP-AGB phase are affected by the treatment of low-temperature opacity. Then, the dependence of the dust formation and resulting mass loss during the C-rich AGB phase on the low-temperature opacity, as well as on the initial mass and metallicity, is presented in Section 4. In Section 5, a necessary condition for producing an efficient DDW with mass-loss rate $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ and formulae for the gas and dust mass-loss rates are derived, and the implication on the evolution of AGB stars and dust formation in the early Universe is discussed. A summary is presented in Section 6. The input stellar parameters of the hydrodynamical calculations, as well as the derived properties of the DDW with $\dot{M} \geq 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$, are tabulated for models with CO-enhanced and CNO-enhanced opacities in Appendix A.

## 2 THE MODELS

The formation of carbon dust and the resulting mass loss around AGB stars with $Z_{\text {ini }} \leq 10^{-4}$ after the C-rich star stage is reached are investigated through two separate steps: first, given the initial mass and metallicity, the stellar evolution is simulated from the pre-main-sequence to the end of the AGB phase. Secondly, the stellar parameters roughly every $0.05 \mathrm{M}_{\odot}$ along the evolutionary track on TP-AGB are applied to the hydrodynamical model of a pulsationenhanced DDW to evaluate the formation of carbon dust and the resulting DDW in the C-rich AGB phase. Here, we describe briefly the models used in these two steps.

### 2.1 Stellar evolution

We employ the MESA code for the calculation of stellar evolution models, evolved from the pre-main-sequence up to the end of the AGB phase of stars with initial masses $M_{\text {ini }}=2,3,4$ and $5 \mathrm{M}_{\odot}$ and metallicities $Z_{\text {ini }}=0,10^{-7}, 10^{-6}, 10^{-5}$ and $10^{-4}$. In addition to the treatment of convection, we describe the low-temperature opacities and the mass-loss formula implemented in the mESA code for the purpose of the present study in the following subsections.

### 2.1.1 Convection

In the calculations, the standard mixing-length theory (MLT: Cox \& Giuli 1968) is applied to treat convection as a diffusive process within convective regions, defined according to the

Schwarzschild criterion, $\nabla_{\text {ad }}<\nabla_{\text {rad }}$, where $\nabla_{\text {ad }}$ and $\nabla_{\text {rad }}$ are the adiabatic and radiative temperature gradient, respectively. In the convective region involving nuclear burning, MESA solves the coupled structure, burning and mixing equations, as detailed in Paxton et al. (2011, Paxton et al. 2013). The mixing-length parameter $\alpha_{\text {MLT }}$ is set to be 2.0 as a standard value to reproduce the evolution of the Sun. Overshooting expresses the physical concept of an exponentially decaying velocity field beyond the convective boundary and overshoot mixing is treated as a time-dependent, diffusive process, with the overshoot-mixing diffusion coefficient defined as $D_{\mathrm{OV}}=D_{\text {conv }, 0} \exp \left(-2 z / f H_{\mathrm{P}}\right)$, where $D_{\text {conv, } 0}$ is the MLT diffusion coefficient at the boundary, $z$ is the distance from the boundary, $f$ is a free parameter called the overshooting parameter and $H_{\mathrm{P}}$ is the pressure scaleheight (Herwig 2000). For overshooting parameters, we adopt $f=0.014$ at all convective boundaries, except for the bottom of the He-shell flash region, at which $f$ is set to be 0.008 throughout the evolution after the first thermal pulse (TP), referring to Paxton et al. (2011); note that we adpot $f=0.014$ at the bottom of the convective envelope, instead of $f=0.126$, since we consider that the formation of a ${ }^{13} \mathrm{C}$ pocket is not relevant to the purpose of this article. We note that the MLT scheme leads to less efficient HBB than the full spectrum of turbulence (FST) scheme (Canuto \& Mazzitelli 1991), as discussed by Ventura \& D'Antona (2005). Thus, if the FST scheme were applied, less carbon dust would be formed, since carbon burning by HBB would be much stronger.

### 2.1.2 Low-temperature opacity

In order to clarify the role played by low-temperature opacities, not only regarding stellar evolution but also in the formation of carbon dust and resulting mass loss during the AGB phase, in the MESA code we implement three types of low-temperature opacity: (1) the scaled-solar opacity, with the elemental composition of metals scaled by the solar composition (Grevesse \& Noels 1993) according to the initial metallicity; (2) the CO-enhanced opacity, in which the opacity is calculated according to the enhancement of C and O abundances with respect to the scaled-solar values; (3) the CNO-enhanced opacity, which also includes the variation of N with respect to the scaled-solar value, besides the variation of C and O abundances. Since the CN molecule dominates the Rosseland mean opacity at low temperature, $\log T \lesssim 3.6$ (Marigo \& Aringer 2009), the CNO-enhanced opacity is, among the three possibilities, the most suitable one to describe the evolution of AGB stars.

The opacity tables are constructed using the ÆSOPUS tool (Marigo \& Aringer 2009) and are incorporated into the MESA code; the tables consist of five grids of metallicity $\left(Z=10^{-12}, 10^{-7}\right.$, $10^{-6}, 10^{-5}$ and $10^{-4}$ ), three grids of the mass fraction of hydrogen ( $X_{\mathrm{H}}=0.50,0.65$ and 0.80 ) and 16 grids of the increment of the mass fraction $\left(0 \leq \mathrm{d} X_{i} \leq 9.72 \times 10^{-2}\right)$ for element $i(i=\mathrm{C}, \mathrm{N}$ and O ). The grids of temperature $T$ (in units of K ) and the parameter $R=\rho /\left(T / 10^{6}\right)^{3}$, with gas density $\rho$ in c.g.s. units, cover the ranges $3.20 \leq \log T \leq 4.50$ and $-8.0 \leq \log R \leq 1.0$, respectively. Note that we adopt the opacity tables for $Z=10^{-12}$ as representative of $Z=0$. For the high-temperature opacity ( $\log T \geq 4.0$ ), we adopt the Opacity Project at Livermore (OPAL) type 2 opacities (Iglesias \& Rogers 1993, 1996) implemented in the MESA code, accounting for the enhancement of carbon and oxygen during the evolution. The opacity in the range $3.8 \leq \log T \leq 4.0$ is calculated by interpolating linearly between the low- and high-temperature opacity tables at a fixed $\log R$.

### 2.1.3 Mass loss

Mass loss during the evolution of low- and intermediate-mass stars plays a decisive role in determining the efficiency of TDU and HBB (e.g. Weiss \& Ferguson 2009). In previous studies, empirical and/or theoretical mass-loss formulae have been applied: for example, Weiss \& Ferguson (2009) applied the Reimers formula (Reimers 1975) on the red giant branch (RGB) and the AGB with pulsation periods $P \leq 400 \mathrm{~d}$ and then the formulae proposed by van Loon et al. (2005) for oxygen-rich (O-rich) AGB and Wachter et al. (2002) for C-rich AGB; the formulae by Vassiliadis \& Wood (1993) and Blöcker (1995) were adopted for the whole AGB phase in Ferrarotti \& Gail (2006) and Ventura et al. (2012a,b), respectively. Although the DDW mechanism has been believed to be plausible for C-rich AGB stars in galaxies with solar and subsolar metallicities at the present time, we have no convincing knowledge of whether intermediate-mass stars in the early Universe, with $Z_{\text {ini }} \leq$ $10^{-4}$, can form carbon dust efficiently enough to drive mass loss on the AGB.

The aim of this article is to reveal whether and in what conditions the formation of carbon dust and the consequent DDW onset on the AGB, as the first step to exploring the role of AGB stars as the source of dust in the early Universe. Thus, we apply an empirical mass-loss formula in the post-main-sequence phase for simplicity. In this article, we adopt the formula by Schröder \& Cuntz (2005, hereinafter SC05), since the formula reproduces the mass-loss rate on the RGB reasonably; although the recent population synthesis model of AGB stars in metal-poor galaxies prefers a modified SC05 as the mass-loss formula on the AGB before the onset of DDW, its application on the RGB is questionable (Rosenfield et al. 2014). The formula of SC05 is given by
$\dot{M}_{\mathrm{SC} 05}=\eta \frac{L R}{M}\left(\frac{T_{\text {eff }}}{4000 \mathrm{~K}}\right)^{3.5}\left(1+\frac{1}{4300 g}\right)$,
where the mass-loss rate is in units of $\mathrm{M}_{\odot} \mathrm{yr}^{-1}$, the effective temperature $T_{\text {eff }}$ is in units of K and the stellar mass $M$, luminosity $L$ and surface gravity $g$ are in solar units. In the calculations, we adopt the fitting parameter $\eta=8 \times 10^{-14}$; the value of $\eta$ is adjusted by fitting to the observed mass-loss rates of red giant stars in globular clusters with different metallicities (SC05), with which the formula reproduces the observed mass-loss rates of Galactic giants and supergiants well (Schröder \& Cuntz 2007).

### 2.2 Dust-driven wind

Here, we describe the hydrodynamical model of a pulsationenhanced DDW by Yasuda \& Kozasa (2012) employed in this article. The hydrodynamical model treats the nucleation and growth processes of carbon dust in the gas lifted up by the pulsation shock and the consequent DDW self-consistently. The model adopts the scheme for the formation of carbon grains proposed by Gauger, Gail \& Sedlmayr (1990) and includes the decay process of dust by heating due to backward radiation. Given the stellar parameters at the photosphere (see below) at a given epoch during the AGB phase, the model allows evaluation of the physical quantities related to dust formation: the time evolution of the mass-loss rate, gas velocity and condensation efficiency of carbon dust (defined as the ratio of carbon locked into dust to carbon available for dust formation), as well as the amount and size distribution of dust particles in the wind, together with their time-averaged values at the outer boundary of the hydrodynamical model.

The stellar parameters necessary for hydrodynamical model calculations are the current stellar mass $M$, luminosity $L$, effective temperature $T_{\text {eff }}$, abundances of $\mathrm{H}, \mathrm{He}, \mathrm{C}, \mathrm{N}$ and O , Rosseland mean opacity $\kappa_{\mathrm{R}}$ in the photosphere, period $P_{0}$ and velocity amplitude $\Delta u_{\mathrm{p}}$ of pulsation. The temporal evolution of the stellar parameters, except for the period and amplitude of pulsation, can be obtained from the stellar evolution calculation. For the pulsation period, we apply the formula for the fundamental radial mode of pulsation by Ostlie \& Cox (1986), which is given by

$$
\begin{equation*}
\log P_{0}=-1.92-0.73 \log M+1.86 \log R \tag{2}
\end{equation*}
$$

where $P_{0}$ is in units of days and the stellar mass $M$ and stellar radius $R$ are in solar units. As for the velocity amplitude of pulsation, little is known, as mentioned in Gail \& Sedlmayr (2013), though Wood (1986) have estimated it to be a few $\mathrm{km} \mathrm{s}^{-1}$, by adjusting the variable pressure at the inner boundary to a certain photospheric density deduced from observations. Thus, in this article, $\Delta u_{\mathrm{p}}$ is set to be $2 \mathrm{~km} \mathrm{~s}^{-1}$ as a reference value. We address the reader to Yasuda \& Kozasa (2012) for details of the numerical schemes for the formation process of carbon grains and the dust-driven wind.
In the calculations, we use the optical constants of astronomical graphite (Draine 1985) for carbon dust. The physical quantities characterizing the dust formation and resulting DDW presented in the following sections are specified by the values averaged over the final 60 pulsation cycles at the outer boundary placed at 25 times the initial stellar radius of the hydrodynamical model. Note that we avoid calculations at the current stellar mass in the short time intervals of TPs during the evolution, since convergence problems often occur just after the onset of TP in the stellar evolution calculations, as presented in the following subsection.

### 2.3 Convergence problem

During the stellar evolution calculation, we often come across convergence problems just after the onset of TP, when the effective temperature decreases down to $\sim 3200 \mathrm{~K}$. The failed convergency is not caused by the dominance of radiation pressure in the convective envelope, i.e. the small ratio of gas pressure to total pressure, $\beta$, but may be associated with the opacity, as argued by Karakas \& Lattanzio (2007). While investigation of the precise cause of the convergence problems is postponed to a future work, in the present calculations we avoid convergence difficulties as follows: we set a minimum temperature ( $T_{\text {min,op }}$ ) from just before the onset of TP to the beginning of TDU and the low-temperature opacity is replaced with that calculated using $T_{\text {min,op }}$ in a region where the local temperature is lower than $T_{\text {min,op }}$.

This method might affect the TDU efficiency parameter $\lambda$, defined as the ratio between the mass dredged up after a thermal pulse, $\Delta M_{\text {DUP }}$, and the increment of core mass during the preceding interpulse phase, $\Delta M_{\mathrm{c}}$ (e.g. Herwig 2005); in the calculations of model stars with CNO-enhanced opacity, for example, the largest difference of $\lambda$ between successive TDUs with and without the convergence problem is 0.24 for $M_{\text {ini }}=5 \mathrm{M}_{\odot}$, with $Z_{\text {ini }}=10^{-4}$. Such a degree of difference can be seen between successive TDUs with no convergence problem. By setting the different values of $\log T_{\text {min,op }}$ for the successive TDUs at $M=1.69 \mathrm{M}_{\odot}$ and $1.60 \mathrm{M}_{\odot}$ during the evolution of $M_{\mathrm{ini}}=3 \mathrm{M}_{\odot}$ with $Z_{\mathrm{ini}}=10^{-7}$, the largest difference of $\lambda$ reaches 0.404 . However, only two models ( $M_{\mathrm{ini}}=3 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=10^{-7}$ and $M_{\text {ini }}=4 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=0$ ) suffer such a large increment at a TDU among the last few TDUs before the evolution calculation finally stops. In addition, the effective temperature quickly changes in a short time interval of TP. Thus, we consider that
the prescription for dealing with the convergence problem cannot cause a serious issue against the aim of this article.

## 3 EVOLUTION OF LOW-METALLICITY AGB STARS

Table 1 summarizes the stellar models and the calculated quantities characterizing their TP-AGB phase: initial metallicity $Z_{\text {ini }}$; type of low-temperature opacity; final stellar (core) mass $M_{\text {tot,f }}\left(M_{\mathrm{c}, \mathrm{f}}\right)$; total number of TPs; total number of TDUs; threshold stellar mass $M_{\mathrm{C} / \mathrm{O}>1}$, defined as the mass of the star at the time after which the $\mathrm{C} / \mathrm{O}$ ratio in the surface layers keeps exceeding unity; maximum temperature at the bottom of the convective envelope during the TP-AGB phase $T_{\text {bce,max }}$; minimum effective temperature during the interpulse phases $T_{\text {eff,min }}$; final carbon excess (defined as $\delta_{\mathrm{C}} \equiv$ $\epsilon_{\mathrm{C}}-\epsilon_{\mathrm{O}}$ with $\epsilon_{\mathrm{C}}\left(\epsilon_{\mathrm{O}}\right)$ the abundance of $\mathrm{C}(\mathrm{O})$ by number relative to H ); and also the final mass fractions of $\mathrm{C}, \mathrm{N}$ and O in the surface layer. Note that 'final' does not always mean the end of the AGB phase. The models that fail to evolve to the end of the TP-AGB phase due to convergence problems are denoted by the superscript $f$. Also, models undergoing HBB (weak HBB) during evolution are specified by the superscript $\mathrm{H}(\mathrm{WH})$ attached to the value of $T_{\text {bce,max }}$.

How the difference in the treatment of low-temperature opacity affects the evolution and structure of low-metallicity stars has been analysed in the work by Constantino et al. (2014), aimed at clarifying whether use of composition-dependent low-temperature opacities is necessary for modelling the evolution of metal-poor AGB stars. Here, we briefly address the effects of the treatment of low-temperature opacity on the evolution of AGB stars in relation to dust formation, primarily referring to the result of calculations for $Z_{\text {ini }}=10^{-7}$. In the C-rich envelope, the contribution of $\mathrm{C}_{2}$ and CN molecules to the opacity is enhanced around $T=3500 \mathrm{~K}$ (see fig. 13 in Marigo \& Aringer 2009). Thus, for a given set of gas density and temperature, the value of low-temperature opacity increases in the order scaled-solar, CO-enhanced and finally CNO-enhanced. In what follows, a model star with scaled-solar (CO-enhanced, CNO-enhanced) opacity is referred to as the scaledsolar (CO-enhanced, CNO-enhanced) model.

### 3.1 CNO abundances and carbon excess $\delta_{\mathrm{C}}$ in the surface layer

Fig. 1 shows the effects of the treatment of low-temperature opacity on the evolution of the mass fractions of $\mathrm{C}, \mathrm{N}$ and O (left panel) and the carbon excess $\delta_{\mathrm{C}}$ and the $\mathrm{C} / \mathrm{O}$ ratio (right panel) in the surface regions of stars with $M_{\text {ini }}=2$ (top), 3 (middle) and $4 \mathrm{M}_{\odot}$ (bottom) with $Z_{\text {ini }}=10^{-7}$.
The star with $M_{\text {ini }}=2 \mathrm{M}_{\odot}$ and $Z_{\text {ini }}=10^{-7}$ becomes C-rich after the first TDU and the final carbon excess $\delta_{\mathrm{C}}$ exceeds 0.001 , regardless of the treatment of low-temperature opacity. This holds for all the $2-\mathrm{M}_{\odot}$ models with $Z_{\text {ini }} \leq 10^{-4}$ (see Table 1). $\delta_{\mathrm{C}}$ and the surface mass fractions of C and O increase with time and become larger in decreasing order of the values of the low-temperature opacities. On the other hand, without HBB the surface mass fraction of N during the TP-AGB phase is not affected by the treatment of low-temperature opacities. Also, the C/O ratio declines quickly after first TDU and then converges to a constant, larger than 10, almost independent of the type of low-temperature opacity.
The 4- $\mathrm{M}_{\odot}$ models with $Z_{\text {ini }}=10^{-7}$, regardless of the treatment of low-temperature opacity, undergo HBB after several TDU events (see the bottom left panel of Fig. 1). While $\delta_{\mathrm{C}}$ increases with time during the initial AGB phases following the first TDU, the mass fraction of $\mathrm{N}(\mathrm{C})$ in the surface regions increases (decreases) quickly

Table 1. Models and characteristic quantities during AGB phase: initial metallicity $Z_{\text {ini }}$, type of low-temperature opacity ( $\kappa_{\text {solar }}, \kappa_{\text {CO }}$ and $\kappa_{\mathrm{CNO}}$ denote scaled-solar, CO-enhanced and CNO-enhanced opacities, respectively), final stellar mass $M_{\text {tot,f }}$ in solar units, final core mass $M_{\mathrm{c}, \mathrm{f}}$ in solar units with the superscript ' $f$ ' for those models that fail to evolve to the end of AGB phase, total number of TPs $N_{\mathrm{TP}}$, total number of TDUs $N_{\text {TDU }}$, stellar mass below which $\mathrm{C} / \mathrm{O}>1 M_{\mathrm{C} / \mathrm{O}>1}$ in solar units, maximum temperature at the bottom of convective envelope during interpulse phases $T_{\text {bee, max }}$ in units of $10^{6} \mathrm{~K}$ with the superscript $\mathrm{H}(\mathrm{WH})$ for the models that experience HBB (weak HBB), minimum effective temperature during interpulse phases $T_{\text {eff,min }}$ in units of K, final carbon excess $\delta_{\mathrm{c}, \mathrm{f}} \times 10^{4}$ and the mass fractions of $\mathrm{C}, \mathrm{N}$ and O in the surface layer $X_{\mathrm{f}}(\mathrm{C}), X_{\mathrm{f}}(\mathrm{N})$ and $X_{\mathrm{f}}(\mathrm{O})$, respectively. Note that the models of $M_{\mathrm{ini}}=5 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=0$ and $10^{-7}$ are excluded, since the star does not form carbon dust without undergoing TDU on TP-AGB.

| $Z_{\text {ini }}$ | $\kappa$ | $M_{\text {tot,f }}$ | $M_{\text {c,f }}$ | $N_{\text {TP }}$ | $N_{\text {TDU }}$ | $M_{\text {C/O }>1}$ | $T_{\text {bce, max }}$ | $T_{\text {eff,min }}$ | $\delta_{\mathrm{c}, \mathrm{f}} \times 10^{4}$ | $X_{\mathrm{f}}(\mathrm{C})$ | $X_{\mathrm{f}}(\mathrm{N})$ | $X_{\mathrm{f}}(\mathrm{O})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{\text {ini }}=2.0 \mathrm{M}_{\odot}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $10^{-4}$ | $\kappa_{\text {solar }}$ | 0.695 | 0.695 | 20 | 19 | 1.88 | 10.0 | 3927 | 49.4 | $4.20 \times 10^{-2}$ | $1.68 \times 10^{-4}$ | $4.74 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\mathrm{CO}}$ | 0.692 | 0.692 | 17 | 16 | 1.88 | 7.24 | 3254 | 22.0 | $2.02 \times 10^{-2}$ | $5.22 \times 10^{-5}$ | $2.29 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\text {CNO }}$ | $0.690^{f}$ | 0.690 | 17 | 16 | 1.88 | 6.92 | 3136 | 25.5 | $2.32 \times 10^{-2}$ | $5.97 \times 10^{-5}$ | $2.63 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {solar }}$ | 0.698 | 0.698 | 17 | 16 | 1.84 | 10.0 | 3974 | 35.8 | $3.12 \times 10^{-2}$ | $3.47 \times 10^{-4}$ | $3.28 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\mathrm{CO}}$ | 0.695 | 0.695 | 15 | 14 | 1.84 | 7.60 | 3439 | 24.1 | $2.16 \times 10^{-2}$ | $3.15 \times 10^{-4}$ | $2.21 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {CNO }}$ | $0.689^{f}$ | 0.689 | 13 | 12 | 1.84 | 6.04 | 2939 | 22.0 | $2.00 \times 10^{-2}$ | $2.94 \times 10^{-4}$ | $2.09 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\text {solar }}$ | 0.707 | 0.707 | 17 | 17 | 1.83 | 10.0 | 3980 | 25.7 | $2.25 \times 10^{-2}$ | $3.59 \times 10^{-4}$ | $2.27 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\mathrm{CO}}$ | 0.703 | 0.703 | 16 | 16 | 1.83 | 8.07 | 3575 | 20.2 | $1.79 \times 10^{-2}$ | $3.96 \times 10^{-4}$ | $3.02 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\mathrm{CNO}}$ | $0.699^{f}$ | 0.699 | 14 | 14 | 1.83 | 6.17 | 3059 | 14.5 | $1.31 \times 10^{-2}$ | $3.26 \times 10^{-4}$ | $1.33 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\text {solar }}$ | 0.712 | 0.712 | 20 | 18 | 1.81 | 10.4 | 3984 | 23.5 | $2.04 \times 10^{-2}$ | $2.08 \times 10^{-4}$ | $2.11 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\text {CO }}$ | 0.711 | 0.711 | 17 | 16 | 1.81 | 8.45 | 3627 | 18.8 | $1.65 \times 10^{-2}$ | $2.11 \times 10^{-4}$ | $1.72 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\text {CNO }}$ | $0.707{ }^{\text {f }}$ | 0.707 | 17 | 14 | 1.81 | 6.79 | 3153 | 15.0 | $1.34 \times 10^{-2}$ | $1.98 \times 10^{-4}$ | $1.42 \times 10^{-3}$ |
| 0 | $\kappa_{\text {solar }}$ | 0.727 | 0.727 | 26 | 16 | 1.76 | 10.7 | 3990 | 21.4 | $1.79 \times 10^{-2}$ | $2.70 \times 10^{-4}$ | $1.72 \times 10^{-3}$ |
| 0 | $\kappa_{\mathrm{CO}}$ | 0.726 | 0.726 | 25 | 14 | 1.76 | 9.02 | 3695 | 14.5 | $1.24 \times 10^{-2}$ | $2.23 \times 10^{-4}$ | $1.18 \times 10^{-3}$ |
| 0 | $\kappa_{\mathrm{CNO}}$ | 0.723 | 0.723 | 23 | 13 | 1.76 | 7.43 | 3272 | 12.6 | $1.08 \times 10^{-2}$ | $2.21 \times 10^{-4}$ | $1.03 \times 10^{-3}$ |
| $M_{\text {ini }}=3.0 \mathrm{M}_{\odot}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $10^{-4}$ | $\kappa_{\text {solar }}$ | 0.823 | 0.823 | 29 | 27 | 1.64 | $74.8{ }^{\text {H }}$ | 3912 | 16.1 | $1.56 \times 10^{-2}$ | $1.84 \times 10^{-2}$ | $4.05 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\text {CO }}$ | 0.828 | 0.827 | 29 | 27 | 1.67 | $74.3{ }^{\text {H }}$ | 3521 | 13.7 | $1.35 \times 10^{-2}$ | $1.75 \times 10^{-2}$ | $3.50 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\mathrm{CNO}}$ | $1.121^{f}$ | 0.828 | 28 | 26 | 2.75 | $56.9{ }^{\text {WH }}$ | 2693 | 21.6 | $1.95 \times 10^{-2}$ | $1.49 \times 10^{-3}$ | $2.20 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {solar }}$ | 0.820 | 0.819 | 28 | 26 | 1.56 | $73.6{ }^{\text {H }}$ | 3960 | 19.9 | $1.87 \times 10^{-2}$ | $1.90 \times 10^{-2}$ | $4.44 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\mathrm{CO}}$ | $0.868^{f}$ | 0.818 | 29 | 26 | 1.58 | $73.2{ }^{\text {H }}$ | 3660 | 13.1 | $1.31 \times 10^{-2}$ | $1.82 \times 10^{-2}$ | $3.65 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {CNO }}$ | $1.122^{f}$ | 0.822 | 26 | 24 | 2.68 | $56.0{ }^{\text {WH }}$ | 2698 | 22.0 | $1.97 \times 10^{-2}$ | $9.89 \times 10^{-4}$ | $2.09 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\text {solar }}$ | 0.815 | 0.815 | 28 | 26 | 1.66 | $72.2{ }^{\text {H }}$ | 3969 | 19.3 | $1.78 \times 10^{-2}$ | $1.87 \times 10^{-2}$ | $4.15 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa^{\text {CO }}$ | 0.817 | 0.817 | 28 | 26 | 1.70 | $71.7{ }^{\mathrm{H}}$ | 3685 | 17.5 | $1.62 \times 10^{-2}$ | $1.77 \times 10^{-2}$ | $3.61 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\mathrm{CNO}}$ | $1.510^{f}$ | 0.820 | 24 | 22 | 2.60 | 53.9 WH | 2873 | 18.2 | $1.61 \times 10^{-2}$ | $5.62 \times 10^{-4}$ | $1.47 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\text {solar }}$ | 0.816 | 0.816 | 28 | 27 | 1.66 | $71.5{ }^{\mathrm{H}}$ | 3980 | 22.4 | $1.93 \times 10^{-2}$ | $1.90 \times 10^{-2}$ | $3.86 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\mathrm{CO}}$ | 0.820 | 0.819 | 28 | 27 | 1.79 | $71.1{ }^{\text {H }}$ | 3726 | 18.4 | $1.61 \times 10^{-2}$ | $1.80 \times 10^{-2}$ | $3.21 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\mathrm{CNO}}$ | $1.072^{f}$ | 0.828 | 27 | 26 | 2.57 | 47.8 WH | 2883 | 22.3 | $1.88 \times 10^{-2}$ | $3.55 \times 10^{-4}$ | $1.77 \times 10^{-3}$ |
| 0 | $\kappa_{\text {solar }}$ | 0.805 | 0.805 | 40 | 37 | 2.55 | 61.0 WH | 4011 | 16.2 | $1.30 \times 10^{-2}$ | $1.49 \times 10^{-2}$ | $2.35 \times 10^{-3}$ |
| 0 | $\kappa^{\text {CO }}$ | 0.807 | 0.807 | 40 | 37 | 2.55 | $50.3{ }^{\text {WH }}$ | 3543 | 27.0 | $2.04 \times 10^{-2}$ | $7.72 \times 10^{-4}$ | $1.88 \times 10^{-3}$ |
| 0 | $\kappa_{\mathrm{CNO}}$ | $1.195^{f}$ | 0.798 | 32 | 31 | 2.55 | 31.9 | 2997 | 21.2 | $1.62 \times 10^{-2}$ | $3.75 \times 10^{-4}$ | $1.55 \times 10^{-3}$ |
| $M_{\text {ini }}=4.0 \mathrm{M}_{\odot}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $10^{-4}$ | $\kappa_{\text {solar }}$ | $0.914^{f}$ | 0.880 | 36 | 34 | 1.50 | $87.7{ }^{\text {H }}$ | 3928 | 13.5 | $1.17 \times 10^{-2}$ | $1.71 \times 10^{-2}$ | $2.73 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\text {CO }}$ | $0.968^{f}$ | 0.882 | 36 | 34 | 1.57 | $87.7{ }^{\mathrm{H}}$ | 3659 | 10.4 | $9.31 \times 10^{-3}$ | $1.67 \times 10^{-2}$ | $2.38 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\mathrm{CNO}}$ | $1.237{ }^{\text {f }}$ | 0.882 | 36 | 35 | 1.90 | $87.0{ }^{\mathrm{H}}$ | 2868 | 7.62 | $7.04 \times 10^{-3}$ | $1.41 \times 10^{-2}$ | $1.83 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {solar }}$ | 0.878 | 0.878 | 36 | 33 | 1.56 | $87.0{ }^{\mathrm{H}}$ | 3968 | 15.4 | $1.30 \times 10^{-2}$ | $1.68 \times 10^{-2}$ | $2.66 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa^{\text {CO }}$ | $0.918^{f}$ | 0.878 | 36 | 33 | 1.64 | $86.9{ }^{\text {H }}$ | 3768 | 11.9 | $1.04 \times 10^{-2}$ | $1.65 \times 10^{-2}$ | $2.31 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {CNO }}$ | $1.270^{f}$ | 0.878 | 35 | 33 | 1.92 | $86.2{ }^{\text {H }}$ | 2915 | 6.75 | $6.31 \times 10^{-3}$ | $1.42 \times 10^{-2}$ | $1.70 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\text {solar }}$ | 0.862 | 0.861 | 36 | 33 | 1.59 | $84.3{ }^{\text {H }}$ | 3975 | 16.4 | $1.37 \times 10^{-2}$ | $1.77 \times 10^{-2}$ | $2.68 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\mathrm{CO}}$ | 0.862 | 0.862 | 36 | 33 | 1.62 | $84.2{ }^{\text {H }}$ | 3774 | 15.0 | $1.26 \times 10^{-2}$ | $1.72 \times 10^{-2}$ | $2.46 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\mathrm{CNO}}$ | $1.236{ }^{\text {f }}$ | 0.859 | 35 | 33 | 1.94 | $83.3{ }^{\mathrm{H}}$ | 2882 | 7.42 | $6.89 \times 10^{-3}$ | $1.53 \times 10^{-2}$ | $1.81 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\text {solar }}$ | 0.858 | 0.857 | 37 | 32 | 1.57 | $83.5{ }^{\text {H }}$ | 3982 | 16.4 | $1.35 \times 10^{-2}$ | $1.80 \times 10^{-2}$ | $2.68 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\mathrm{CO}}$ | $0.893{ }^{f}$ | 0.857 | 37 | 32 | 1.62 | $83.4{ }^{\text {H }}$ | 3785 | 13.0 | $1.09 \times 10^{-2}$ | $1.73 \times 10^{-2}$ | $2.24 \times 10^{-3}$ |
| $10^{-7}$ | $\kappa_{\mathrm{CNO}}$ | $1.196{ }^{f}$ | 0.855 | 35 | 33 | 1.98 | $82.5{ }^{\mathrm{H}}$ | 2860 | 8.50 | $7.64 \times 10^{-3}$ | $1.52 \times 10^{-2}$ | $1.92 \times 10^{-3}$ |
| 0 | $\kappa_{\text {solar }}$ | 0.832 | 0.832 | 44 | 41 | 1.64 | $79.4{ }^{\text {H }}$ | 4019 | 9.16 | $7.77 \times 10^{-3}$ | $2.26 \times 10^{-2}$ | $2.52 \times 10^{-3}$ |
| 0 | $\kappa^{\text {CO }}$ | 0.833 | 0.833 | 45 | 41 | 1.64 | $79.2{ }^{\text {H }}$ | 3834 | 8.79 | $7.46 \times 10^{-3}$ | $2.23 \times 10^{-2}$ | $2.40 \times 10^{-3}$ |
| 0 | $\kappa_{\mathrm{CNO}}$ | $1.193{ }^{f}$ | 0.835 | 44 | 41 | 2.12 | $77.9{ }^{\text {H }}$ | 2867 | 8.43 | $7.04 \times 10^{-3}$ | $1.80 \times 10^{-2}$ | $1.96 \times 10^{-3}$ |
| $M_{\text {ini }}=5.0 \mathrm{M}_{\odot}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $10^{-4}$ | $\kappa_{\text {solar }}$ | $1.133^{f}$ | 0.969 | 44 | 41 | 1.53 | $101^{\mathrm{H}}$ | 3984 | 6.59 | $6.55 \times 10^{-3}$ | $1.59 \times 10^{-2}$ | $2.58 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\text {CO }}$ | $1.066^{f}$ | 0.971 | 45 | 36 | 1.59 | $100^{\mathrm{H}}$ | 3777 | 7.88 | $7.48 \times 10^{-3}$ | $1.53 \times 10^{-2}$ | $2.64 \times 10^{-3}$ |
| $10^{-4}$ | $\kappa_{\mathrm{CNO}}$ | $1.329^{f}$ | 0.968 | 44 | 41 | 1.73 | $100^{\mathrm{H}}$ | 3096 | 4.49 | $4.96 \times 10^{-3}$ | $1.43 \times 10^{-2}$ | $2.35 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\text {solar }}$ | $1.054^{f}$ | 0.947 | 42 | 37 | 1.51 | $98.2{ }^{\text {H }}$ | 3985 | 9.15 | $8.31 \times 10^{-3}$ | $1.63 \times 10^{-2}$ | $2.64 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\mathrm{CO}}$ | $1.019{ }^{f}$ | 0.947 | 42 | 38 | 1.56 | $97.9^{\mathrm{H}}$ | 3837 | 10.0 | $8.97 \times 10^{-3}$ | $1.62 \times 10^{-2}$ | $2.76 \times 10^{-3}$ |
| $10^{-5}$ | $\kappa_{\mathrm{CNO}}$ | $1.242^{f}$ | 0.945 | 42 | 38 | 1.77 | $97.5{ }^{\text {H }}$ | 3073 | 5.65 | $5.77 \times 10^{-3}$ | $1.50 \times 10^{-2}$ | $2.40 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\text {solar }}$ | $1.040^{f}$ | 0.941 | 44 | 35 | 1.51 | $98.2{ }^{\text {H }}$ | 3985 | 9.60 | $8.63 \times 10^{-3}$ | $1.62 \times 10^{-2}$ | $2.60 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa^{\text {CO }}$ | $0.994^{f}$ | 0.942 | 45 | 36 | 1.56 | $98.3{ }^{\text {H }}$ | 3849 | 11.0 | $9.61 \times 10^{-3}$ | $1.59 \times 10^{-2}$ | $2.67 \times 10^{-3}$ |
| $10^{-6}$ | $\kappa_{\mathrm{CNO}}$ | $1.317^{f}$ | 0.941 | 44 | 35 | 1.77 | $98.0{ }^{\mathrm{H}}$ | 3076 | 4.97 | $5.25 \times 10^{-3}$ | $1.48 \times 10^{-2}$ | $2.22 \times 10^{-3}$ |



Figure 1. The effect of low-temperature opacity on the temporal evolution of mass fractions of $\mathrm{C}, \mathrm{N}$ and O (left panel) and carbon excess $\delta_{\mathrm{C}}$ and $\mathrm{C} / \mathrm{O}$ ratio (right panel) in the surface layer for $M_{\text {ini }}=2$ (top), 3 (middle) and $4 \mathrm{M}_{\odot}$ (bottom) stars with $Z_{\text {ini }}=10^{-7}$. The scaled-solar, CO-enhanced and CNO-enhanced models are coloured in black, blue, and red, respectively. The mass fraction of $\mathrm{C}(\mathrm{N}, \mathrm{O})$ is denoted by the solid (dashed, dotted) line in the left panel and $\delta_{\mathrm{C}}$ (C/O ratio) by the solid (dotted) line in the right panel.
after the onset of HBB. Thus, the combination of TDU and HBB makes the elemental composition in the surface regions C-rich and O-rich alternately. However, HBB ceases after the stellar mass reduces to $1.57,1.62$ and $1.98 \mathrm{M}_{\odot}$ for the scaled-solar, CO-enhanced and CNO-enhanced models, respectively. Consequently, the subsequent TDUs cause the stars to be C -rich again and $\delta_{\mathrm{C}}$ as well as the C/O ratio to increase monotonically with time. The carbon excess $\delta_{\mathrm{C}}$ in the surface region is more enhanced in the CNO-enhanced model with the largest threshold mass $M_{\mathrm{C} / \mathrm{O}>1}$ than in other opacity models at the same current stellar mass. Although the details depend on the initial metallicity, this behaviour holds for all the $4-\mathrm{M}_{\odot}$ models with $Z_{\text {ini }} \leq 10^{-4}$ and the $5-\mathrm{M}_{\odot}$ models with $Z_{\text {ini }}$ $\geq 10^{-6}$; TDU is not experienced during the TP-AGB phase in the
models with $M_{\mathrm{ini}}=5 \mathrm{M}_{\odot}$ with $Z_{\mathrm{ini}}=0$ and $10^{-7}$, thus both models are excluded from Table 1 and the following discussion.
The evolution of the $3-\mathrm{M}_{\odot}$ model with $Z_{\text {ini }}=10^{-7}$ is sensitive to the treatment of low-temperature opacity. The CNO-enhanced model is always C-rich in the TP-AGB phase, with very weak HBB in the early phase. On the other hand, the scaled-solar and CO-enhanced models experience stronger HBB until the stellar masses are reduced to 1.66 and $1.77 \mathrm{M}_{\odot}$, respectively, and then turn out to be C-rich. These behaviours are common to all the $M_{\text {ini }}=3 \mathrm{M}_{\odot}$ models with $Z_{\text {ini }} \geq 10^{-7}$; the $M_{\mathrm{ini}}=3 \mathrm{M}_{\odot}$ models with $Z_{\text {ini }}=0$ are always C-rich in their TP-AGB phases, since the scaled-solar, CO-enhanced and CNO-enhanced models experience weak, very weak and no HBB, respectively, as shown in Fig. 2.


Figure 2. Same as the left panel of Fig. 1, but for $M_{\text {ini }}=3 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=0$.


Figure 3. The effect of the low-temperature opacity on the time evolution of effective temperature for the same models with $Z_{\text {ini }}=10^{-7}$ in Fig. 1; $M_{\text {ini }}=2$ (left), 3 (middle), and $4 \mathrm{M}_{\odot}$ (right) models with scaled-solar (black line), CO-enhanced (blue line) and CNO-enhanced (red line) opacities.

We note that, in the case of $M_{\mathrm{ini}}=3 \mathrm{M}_{\odot}$, the value of $M_{\mathrm{C} / \mathrm{O}>1}$ is significantly larger in the CNO-enhanced model than in the other two models. Thus, it can be expected that the model of $M_{\text {ini }}=3 \mathrm{M}_{\odot}$ with the CNO-enhanced opacity could start to form carbon dust at a significantly larger stellar mass. In addition, it should be emphasized that the carbon excess $\delta_{\mathrm{C}}$ and the C/O ratio in C-rich AGB stars of $M_{\text {ini }}=2$ and $3 \mathrm{M}_{\odot}$ with $Z_{\text {ini }} \leq 10^{-4}$ are much larger than the values considered in the dust-driven wind models for C -rich AGB stars with solar and subsolar metallicities (e.g. Winters et al. 2000; Wachter et al. 2002, 2008; Mattsson, Wahlin \& Höfner 2010).

### 3.2 Effective temperature

Fig. 3 shows the time evolution of effective temperature for the same models as presented in Fig. 1; note that the huge spikes of effective temperature during the TPs in the CNO-enhanced models with smaller current stellar mass are artificial, being associated with convergence problems. Owing to the very short duration of TPs, in which the effective temperature changes quickly, we will focus on the behaviour of effective temperature in the interpulse phases in what follows.

The effective temperature does not decrease below 3900 K in the scaled-solar models, regardless of the initial mass and metallicity
(see Table 1); conversely, in the CO-enhanced and CNO-enhanced models, the time evolution of the effective temperature depends strongly on the initial mass, which determines the change in elemental composition in the surface regions during the TP-AGB phase. The effective temperature of the $M_{\mathrm{ini}}=2 \mathrm{M}_{\odot}$ star decreases with time more rapidly in the CNO-enhanced model than in the CO-enhanced model; the minimum values reached are $T_{\text {eff,min }}=3627$ and 3153 K in the CO-enhanced and CNO-enhanced models with $Z_{\text {ini }}=10^{-7}$, respectively. Generally speaking, the minimum effective temperature reached by a model of given mass is larger for smaller metallicity. The only exception to this trend is the $Z_{\text {ini }}=10^{-4}$ CNO-enhanced model. It should be remarked here that, even if the abundance of N in the surface layer is not enhanced without HBB, the CN molecule dominates the low-temperature opacity and decreases the effective temperature efficiently, since the first TDU and carbon ingestion (see Siess, Livio \& Lattanzio 2002; Lau, Stancliffe \& Tout 2009) increase the surface abundance of N in the AGB stars evolved from extremely metal-poor stars considered in this article. We note that the higher $T_{\text {eff,min }}$ of the $Z_{\text {ini }}=10^{-4}$ model comes from the fact that the model does not experience carbon ingestion.

In the CO-enhanced models with $M_{\mathrm{ini}}=3 \mathrm{M}_{\odot}$, we can see from Fig. 3 that the effective temperature decreases with time in the initial C-rich phase, but increases after the onset of HBB. In the O-rich phase, when stronger HBB operates together with TDU, the effective temperature in the CO-enhanced models with $M_{\mathrm{ini}}=3$ and $4 \mathrm{M}_{\odot}$ is almost the same as that in the scaled-solar models at the same current stellar mass. After HBB ceases, the effective temperature decreases with time in the C-rich phase, until the minimum value is reached. On the other hand, in the CNO-enhanced model, the effective temperature decreases with time efficiently even if HBB makes the surface layer O-rich, as can be seen from the time evolution of effective temperature for $M_{\mathrm{ini}}=4 \mathrm{M}_{\odot}$ with CNO-enhanced opacity. This is because, even in O-rich environments, the enhancement of N increases the opacity through the CN molecule as demonstrated by Lederer \& Aringer (2009) and thus the stellar radius.

Thus, even in the extremely metal-poor stars considered in this article, the employment of the low-temperature opacity, appropriately taking into account the change of elemental composition, such as the CNO-enhanced opacity, is inevitable to investigate the evolution of a star during the TP-AGB phase. Furthermore, as demonstrated in the next section, the treatment of the low-temperature opacity definitely influences the formation of carbon dust and the resulting gas outflow around AGB stars with $Z_{\text {ini }} \leq 10^{-4}$.

## 4 FORMATION OF CARBON DUST AND RESULTING MASS LOSS

As presented in the previous section, all the models other than $M_{\text {ini }}=5 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=0$ and $10^{-7}$ satisfy the minimum requirement for formation of carbon dust on the AGB after the stellar mass decreases below $M_{\mathrm{C} / \mathrm{O}>1}$. However, not only $\delta_{\mathrm{C}}$ but also the effective temperature during the TP-AGB phase strongly influences the formation of carbon dust and the consequent DDW (see Gail \& Sedlmayr 2013). In addition, Winters et al. (2000), based on hydrodynamical calculations of the DDW, showed that C-rich stars with stable gas outflows dominated by the effects of radiation pressure on dust with time-averaged radiative acceleration $\langle\alpha\rangle>1$ experience mass-loss rates $\dot{M} \gtrsim 3 \times 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$.

The present results, based on hydrodynamical calculations, show that CO-enhanced and CNO-enhanced models with $T_{\text {eff }} \lesssim 4000 \mathrm{~K}$


Figure 4. The effect of low-temperature opacity on dust formation and dust-driven wind and its dependence on the initial mass as a function of current stellar mass: from top to bottom, (a) mass-loss rate $\dot{M}$, (b) condensation efficiency of carbon $f_{\mathrm{c}}$, (c) dust-to-gas mass ratio $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$, (d) terminal wind velocity $v_{\infty}$, (e) mass-averaged radius of dust $\langle a\rangle$ during the C-rich TP-AGB phase calculated by the hydrodynamical model of DDW and (f) effective temperature $T_{\text {eff }}$ for the CO-enhanced (open circle - thin solid line) and CNO-enhanced (filled circle - thick solid line) models of $M_{\text {ini }}=2$ (red), 3 (blue) and $4 \mathrm{M}_{\odot}$ (green) with $Z_{\text {ini }}=10^{-7}$.
develop a DDW with $M>10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ and $\langle\alpha\rangle>1$ (hereinafter the stable DDW), except for the CO-enhanced models of $M_{\text {ini }}=4 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=0$ and $5 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=10^{-5}$. On the other hand, the mass-loss rate of almost all scaled-solar models with $T_{\text {eff }}>4000 \mathrm{~K}$, excluding a few model stars, is limited to less than $10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ though $\langle\alpha\rangle>1$. The scaled-solar opacity, not reflecting the change of elemental composition in the surface regions during the TP-AGB phase, could be inadequate in the low-temperature regime. Thus, in this section, focusing on the CO-enhanced and CNO-enhanced models with $\dot{M} \geq 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ (stable DDW), we shall show the dependence of the formation of carbon dust and the consequent DDW around AGB stars on the treatment of lowtemperature opacity as well as on the initial mass and metallicity. The input parameters used in the hydrodynamical calculations and the derived properties of DDW for the CO-enhanced and CNOenhanced models are summarized in Appendix A: Table A1 for $Z_{\text {ini }}=10^{-7}$ and Table A2 for the other initial metallicities.

### 4.1 Effect of the low-temperature opacity and its dependence on the initial mass

Fig. 4 displays the time-averaged physical quantities characterizing dust formation and the consequent DDW, together with the
effective temperature as a function of the current stellar mass for CO-enhanced and CNO-enhanced models of $M_{\text {ini }}=2,3$ and $4 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=10^{-7}$.
First, it should be pointed out that the formation of carbon dust and the resulting mass outflow do not operate in the C-rich phases alternating with O-rich phases associated with HBB, since the carbon excess $\delta_{\mathrm{C}}<10^{-4}$ is insufficient and the effective temperature is too high to form carbon dust in a dense gas region close to the photosphere; formation of carbon dust occurs in regions where the temperature is below $\sim 1500 \mathrm{~K}$ (e.g. Yasuda \& Kozasa 2012). Also, in the case of $M \gtrsim 2 \mathrm{M}_{\odot}$, the larger gravitational force could prevent the star from driving gas outflow stably through the radiation pressure force acting on dust. Thus, the effective formation of carbon dust to drive the mass loss is activated only after the stellar mass is significantly reduced from the threshold stellar mass $M_{\mathrm{C} / \mathrm{O}>1}$, as described below.
The general trend shown in Fig. 4 is that as the mass of the star decreases the mass-loss rate becomes larger, while the effective temperature decreases. This behaviour continues for a while even after the minimum effective temperature is reached. During the very final evolutionary phases, when the effective temperature increases rapidly, owing to peeling of the external layers, the mass-loss rate and dust condensation efficiency decline.


Figure 5. The dependence of the mass-loss rate (left panel) and the time-averaged mass-weighted radius of carbon dust (right) on the initial metallicity for the CNO-enhanced model with $M_{\text {ini }}=3 \mathrm{M}_{\odot}$. The symbol and colour denote the metallicity: filled red square, blue square, green circle, red circle and blue triangle for $Z_{\text {ini }}=10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}$ and 0 , respectively.

This trend of mass-loss rate holds irrespective of the treatment of low-temperature opacity. However, the value of the mass-loss rate, as well as the current stellar mass at which the stable DDW onsets, is heavily influenced by the treatment of low-temperature opacity, depending on the initial mass.

In the case of $M_{\mathrm{ini}}=3 \mathrm{M}_{\odot}$, the current stellar mass at the onset of a stable DDW and the maximum value of mass-loss rate are remarkably different between the CNO-enhanced and CO-enhanced models (see Fig. 4a); as shown in Section 3.2, the large value of surface opacity causes the photosphere to expand and suppresses the increase in temperature in the innermost layers of the convective envelope. Although HBB occurs in both models, the CNO-enhanced model undergoes much weaker HBB than the CO-enhanced model and HBB ceases at $\sim 2.0 \mathrm{M}_{\odot}$ in the CNO-enhanced model and at $\sim 1.4 \mathrm{M}_{\odot}$ in the CO-enhanced model. Then, the CNO-enhanced model evolving at smaller effective temperatures and reaching higher values of $\delta_{\mathrm{C}}$ activates the formation of carbon dust in denser regions, to drive the stable gas outflow at $M=1.8 \mathrm{M}_{\odot}$; in the CO-enhanced model, dust formation begins only after the mass of the stars decreases to $M \sim 1.2 \mathrm{M}_{\odot}$. The mass-loss rate at the maximum is almost one order of magnitude smaller in the CO-enhanced model than in the CNO-enhanced model. On the other hand, the average radius of carbon dust is slightly smaller for the CNO-enhanced model, since more seed nuclei are produced in the outflowing gas, which accelerated efficiently owing to larger values of $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$.

The behaviour of the $4-\mathrm{M}_{\odot}$ models is different in comparison with their 3-M $\odot_{\odot}$ counterparts: both models experience active HBB, since the core grows massive $\left(\sim 0.85 \mathrm{M}_{\odot}\right)$, and HBB ceases at $\sim 1.8$ (1.4) $\mathrm{M}_{\odot}$ for the CNO (CO)-enhanced model. In the following phases in the CNO-enhanced model, the decrease in effective temperature leads to the onset of stable DDW when the mass is $\sim 1.6 \mathrm{M}_{\odot}$ and the mass-loss rate increases sharply above $10^{-5} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$. On the other hand, in the CO-enhanced model evolving at higher effective temperatures, the onset of a stable DDW is delayed to $M \sim 1.1 \mathrm{M}_{\odot}$ and the largest mass-loss rate experienced is $\sim 5 \times 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$. It should be noted that the terminal gas velocity and dust-to-gas mass ratio in the CNO-enhanced model are considerably smaller in comparison with the CNO-enhanced model of $M_{\text {ini }}=3 \mathrm{M}_{\odot}$, despite the fact that the mass-loss rate is comparable in both models for $M \lesssim 1.5 \mathrm{M}_{\odot}$. This arises from the smaller value of $\delta_{\mathrm{C}}$, due to the delayed onset of effective dredge-up of carbon starting at $M \sim 1.8 \mathrm{M}_{\odot}$. Thus, the slower gas outflow velocity
allows dust grains to grow larger and results in a mass-weighted average radius more than a factor of 2 larger than that in the other models.

In the $M_{\mathrm{ini}}=2 \mathrm{M}_{\odot}$ models evolving without suffering HBB, the carbon excess $\delta_{\mathrm{C}}$ increases monotonically and gets larger in the CO-enhanced model than in the CNO-enhanced model as the stellar mass decreases (see Fig. 1). On the other hand, the effective temperature at the same current stellar mass is significantly lower in the CNO-enhanced model and the difference increases during the evolution. Thus, the gas density in the region of carbon dust formation as well as gas acceleration is higher in the CNO-enhanced model than in the CO-enhanced model. Although the condensation efficiency is a little smaller in the CO-enhanced model, the larger value of $\delta_{\mathrm{C}}$ makes the dust-to-gas mass ratio almost comparable in both models, as well as the gas terminal velocity being roughly proportional to $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$. The higher gas density of the dust-formation region results in a larger mass-loss rate in the CNO-enhanced model than in the CO-enhanced model (see Fig. 4a). However, being different from the cases of $M_{\mathrm{ini}}=3$ and $4 \mathrm{M}_{\odot}$, the difference in mass-loss rate between the CO-enhanced and CNO-enhanced models remains less than a factor of 3 at $M \lesssim 1 \mathrm{M}_{\odot}$.

### 4.2 Dependence on the initial metallicity

The properties of the DDW, as well as the newly-formed carbon dust around AGB stars with $Z_{\text {ini }} \leq 10^{-4}$, are expected not to depend directly on the initial metallicity, since the dredge-up carbon is of secondary origin and thus independent of $Z_{\text {ini }}$. Fig. 5 displays the dependence of the mass-loss rate (left panel) and mass-weighted average radius of dust (right panel) on the initial metallicity for CNO-enhanced models of $M_{\text {ini }}=3 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=10^{-4}$ (red square), $10^{-5}$ (blue square), $10^{-6}$ (green circle), $10^{-7}$ (red circle) and 0 (blue triangle).

We can see from Fig. 5 that, except for $Z_{\text {ini }}=0$, the mass-loss rates are almost the same at a given current stellar mass and any clear dependence on the initial metallicity is not recognized, apart from some fluctuations. This is also true for the mass-weighted radius of carbon dust; regardless of the initial metallicity, the radii, with a few exceptions, have almost the same value at a given current stellar mass and tend to increase slightly with decreasing current stellar mass. On the other hand, in the $Z_{\text {ini }}=0$ case, the mass-loss rate at
$M \gtrsim 1.2 \mathrm{M}_{\odot}$ as well as the radius at $M \gtrsim 1.4 \mathrm{M}_{\odot}$ is significantly smaller in comparison with the values for $10^{-7} \leq Z_{\text {ini }} \leq 10^{-4}$. This difference reflects the fact that the higher effective temperature of a $Z_{\text {ini }}=0$ star without enrichment of N due to HBB (see Tables A1, A2, Section 3.2.2 and Fig. 2) during the AGB phase prevents carbon dust from forming in a dense region close to the photosphere. Thus, although the initial metallicity may subtly influence the properties of DDW and newly-formed carbon dust through its effects on stellar evolution, the present results demonstrate that carbon dust formation and the DDW do not show any significant dependence on the initial metallicity, as long as $10^{-7} \leq Z_{\text {ini }} \leq 10^{-4}$.

In summary, the treatment of low-temperature opacity strongly affects dust formation and the consequent DDW on TP-AGB through its effect on the surface elemental composition and effective temperature, depending on the initial stellar mass. The current stellar mass at the onset of a stable DDW is considerably smaller ( $\sim 1 \mathrm{M}_{\odot}$ ) in the CO-enhanced model in comparison with that in the CNO-enhanced model. The largest mass-loss rate in the COenhanced model is at least one order of magnitude smaller than in the CNO-enhanced model, except for the case of $M_{\text {ini }}=2 \mathrm{M}_{\odot}$, which does not experience HBB. Thus, the adoption of a low-temperature opacity varying with the change of elemental composition at the surface during the TP-AGB phase is inevitable to investigate dust formation and mass loss around AGB stars with extremely low initial metallicity, as considered in this article.
The mass-weighted radius of carbon dust formed in the outflowing gas is of the order of $0.01 \mu \mathrm{~m}$, regardless of the treatment of low-temperature opacity as well as the initial mass and metallicity, except for the models of $M_{\mathrm{ini}}=4$ and $5 \mathrm{M}_{\odot}$ with the CNO-enhanced opacity developing slow and denser winds ( $\langle a\rangle \sim 0.03 \mu \mathrm{~m}$ ). The derived radius of carbon dust is significantly smaller than the typical radius of carbon dust necessary for reproducing the colours of obscured C-rich AGB stars observed in the Magellanic Clouds; based on the stellar evolution calculations and the dust formation calculations employing the scheme developed by Ferrarotti \& Gail (2006), typical radii of carbon dust are $0.06-0.2 \mu \mathrm{~m}$ in the Magellanic Clouds, assuming the number ratio of seed particles to hydrogen nuclei $n_{\mathrm{s}} / n_{\mathrm{H}}=10^{-13}$ (Dell'Agli et al. 2015a, b), and $0.035-0.06 \mu \mathrm{~m}$ in the Small Magellanic Cloud (SMC), by varying $n_{\mathrm{S}} / n_{\mathrm{H}}$ up to $10^{-11}$ (Nanni et al. 2016). The assumed/considered values of $n_{\mathrm{s}} / n_{\mathrm{H}}$ in their models are considerably smaller than the values calculated in the present DDW models ( $7 \times 10^{-12} \leq n_{\mathrm{s}} / n_{\mathrm{H}}$ $\leq 10^{-8}$, depending on the initial mass as well as the input stellar parameters). Accordingly, the derived size of carbon dust is smaller, being roughly proportional to $\left(n_{\mathrm{H}} / n_{\mathrm{s}}\right)^{1 / 3}$. Since the aim of this article is not to construct a self-consistent model with stellar evolution, comparison with observations is beyond the scope of this article.

## 5 DISCUSSION

The hydrodynamical calculations of the DDW in the previous section clearly demonstrate that the treatment of low-temperature opacity strongly affects dust formation and the resulting mass loss. Although the CNO-enhanced opacity is the most appropriate one among the three types of opacity considered, it should be remarked that the hydrodynamical model of the DDW employed in this article derives the properties of the DDW once a set of stellar parameters is given, as mentioned in Section 2.2 and presented in Section 3. Thus, irrespective of the mass-loss rate and the low-temperature opacity assumed in the stellar evolution calculations, the results of hydrodynamical calculation of the DDW
along the evolutionary track on the AGB enable us to investigate the dependence of the properties of the DDW on the input stellar parameters; the ranges covered by the CNO-enhanced and CO-enhanced models with $\dot{M} \geq 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ are 2693 $<T_{\text {eff }} / \mathrm{K}<4037,1.23<L / 10^{4} \mathrm{~L}_{\odot}<3.23,0.7 \leq M / \mathrm{M}_{\odot} \leq 2.04$, $3.28<\delta_{\mathrm{C}} \times 10^{4}<27.0$ and $1.50<\kappa_{\mathrm{R}} / \mathrm{cm}^{2} \mathrm{~g}^{-1} \times 10^{4}<90.0$.
In this section, based on the results of DDW calculation presented in the previous section, we shall derive and discuss a condition necessary for an efficient DDW with $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ and the analytic formulae for gas and dust mass-loss rates in terms of the input stellar parameters. Furthermore, the implication for the evolution of intermediate-mass stars with $Z_{\text {ini }} \leq 10^{-4}$ in the early Universe is discussed in connection with dust formation and mass loss.

### 5.1 A necessary condition for an efficient dust-driven wind

The thresholds of the stellar parameters for the stable DDW around C-rich AGB stars with solar metallicity have been investigated by means of hydrodynamical calculations. Based on the range of stellar parameters inferred from observations of galactic carbon stars, Winters et al. (2000) found critical values of the various parameters for producing a stable DDW, depending on a combination of all the other parameters used in the hydrodynamical model. However, no attempt has been made to express the dependence explicitly in terms of the other parameters. Also, the hydrodynamical calculations by Winter et al. (2000) are confined to a narrower range of stellar parameters, especially for $T_{\text {eff }}$ and $\delta_{\mathrm{C}}$, compared with the range covered by the present calculations. Thus, it is instructive to attempt at constraining of the conditions necessary for driving the DDW as a combination of stellar parameters, based on the present results.
First, it should be noted that the assumption of position coupling (drift velocity of dust is set to be 0 ) and the setting of velocity amplitude $\Delta u_{\mathrm{P}}=2 \mathrm{~km} \mathrm{~s}^{-1}$ in the hydrodynamical model may influence the calculated mass-loss rates. Although Winters et al. (2000) have shown that the dependence of $\Delta u_{\mathrm{P}}$ on the mass-loss rate is weak as long as $\dot{M} \gtrsim 3 \times 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$, at the present time little is known about the value of $\Delta u_{\mathrm{P}}$ allowed for C-rich AGB stars (Gail \& Sedlmayr 2013). As for the assumption of position coupling, the recent two-fluid hydrodynamic model of the DDW considering dust formation as well as the interaction between gas and dust has demonstrated that the properties of the DDW are well reproduced by assuming position coupling for $\dot{M} \gtrsim 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ (Yasuda et al., in preparation ). Thus, the calculated properties of a DDW with $\dot{M} \gtrsim$ $10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ would not suffer significantly from the uncertainties arising from the assumption of position coupling underlying the hydrodynamical model used in the present article. Here, referring to the DDW with $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ as the efficient DDW in the following, we shall constrain the condition for producing the efficient DDW.

Among the stellar parameters used in the hydrodynamical model, the effective temperature is the most relevant parameter for the DDW, as discussed in previous studies (e.g. Winters et al. 2000, Wachter et al. 2002). The efficiency of gas acceleration due to radiation pressure on dust grains is roughly proportional to $\delta_{\mathrm{C}} L / M$. Also, the Rosseland mean opacity $\kappa_{\mathrm{R}}$ at the photosphere, which controls the density structure of the surface regions, is considered to have a significant effect on the DDW, in connection with the density of gas levitated by the pulsation shock.

Fig. 6 shows $\Lambda=\delta_{\mathrm{C}} L / \kappa_{\mathrm{R}} M$ versus $T_{\text {eff }}$ for the CNO-enhanced and CO-enhanced models tabulated in Appendix A. The dotted lines indicate the boundaries on the $\log T_{\text {eff }}-\log \Lambda$ plane for the


Figure 6. The plot of $\Lambda=\delta_{\mathrm{C}} L / \kappa_{\mathrm{R}} M$ versus $T_{\text {eff }}$, with $M$ and $L$ in solar units and $\kappa_{\mathrm{R}}$ in units of $\mathrm{cm}^{2} \mathrm{~g}^{-1}$; the red (blue) filled circle is for the CNO (CO)-enhanced model with $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ and the red (blue) open triangle for the $\mathrm{CNO}(\mathrm{CO})$-enhanced model with $\dot{M}<10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$. The dotted lines represent the boundaries for the possible formation of a DDW with $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$.
possible formation of an efficient DDW. From this plot, we can see that efficient DDW is possible only if $T_{\text {eff }} \lesssim 3850 \mathrm{~K}$ and $\log \Lambda \gtrsim$ $10.34 \log T_{\text {eff }}-32.33$, though the boundary seems to reflect the TP-AGB tracks of the models somewhat. Although the derived constraint condition is only a necessary condition, the condition could be useful for judging when the efficient DDW resulting from the formation of carbon dust onsets in the course of evolution of C-rich AGB stars, by referring to the stellar parameters along the evolutionary track derived by the stellar evolution calculation.

### 5.2 Analytic formulae for gas and dust mass-loss rates

The amount of gas and dust that C-rich AGB stars supply to interstellar space is crucial not only to reveal the origin of dust but also to investigate the formation and evolution of stars in galaxies through chemical evolution models in the early Universe (e.g. Grieco et al. 2014). Formulae for the mass-loss rate for C-rich AGB stars with solar and subsolar metallicities have been proposed, based on hydrodynamical calculations of the DDW (Arndt, Fleischer \& Sedlmayr 1997; Wachter et al. 2002, 2008). Although Weiss \& Ferguson (2009) applied the formula by Wachter et al. (2002) to investigate the evolution of stars with $Z_{\text {ini }}=5 \times 10^{-4}-0.04$, it is questionable whether the same formula can be applied to C-rich AGB stars of metallicity $Z_{\text {ini }} \leq 10^{-4}$. Here, we shall derive the analytic formulae for gas and dust mass-loss rates in terms of the input stellar parameters employed in the hydrodynamical calculations for CNO-enhanced and CO-enhanced models with the efficient DDW.

For simplicity, we derive the formulae under the assumption that the mass-loss rate is simply approximated by a linear function of the logarithms of the input parameters ( $M, L, T_{\text {eff }}, \kappa_{\mathrm{R}}, \delta_{\mathrm{C}}$ and $P$ ). Also, we shall consider the initial metallicity as a parameter, since the mass-loss rates of the $Z_{\text {ini }}=0$ models deviate from the others (see the left panel of Fig. 5), though the mass-loss rates of stars with $10^{-7} \leq Z_{\text {ini }} \leq 10^{-4}$ do not show a clear dependence on the initial metallicity. Applying the least-squares method to the massloss rates and the dust-to-gas mass ratios tabulated in Appendix A,
the fitting formula is expressed as

$$
\begin{align*}
& \log \dot{M}_{\mathrm{fit}}=a+b \log \left(\frac{T_{\text {eff }}}{3000 \mathrm{~K}}\right)+c \log \left(\frac{L}{10^{4} \mathrm{~L}_{\odot}}\right) \\
& \quad+d \log \left(\frac{M}{\mathrm{M}_{\odot}}\right)+e \log \left(\frac{\delta_{\mathrm{C}}}{10^{-4}}\right)+f \log \left(\frac{\kappa_{\mathrm{R}}}{10^{-4} \mathrm{~cm}^{2} \mathrm{~g}^{-1}}\right) \\
& \quad+g \log \left(\frac{P}{650 \mathrm{~d}}\right)+h \log Z_{\mathrm{ini}} \tag{3}
\end{align*}
$$

Note that we adopt $Z_{\text {ini }}=10^{-12}$ as representative of $Z_{\text {ini }}=0$ when fitting. The numerical coefficients from $a$ to $h$ for the formulae for gas (dust) mass-loss rate $\dot{M}_{\text {fit }}^{\mathrm{g}}\left(\dot{M}_{\mathrm{fit}}^{\mathrm{d}}\right)$ with and without including the initial metallicity are provided in Table 2, with correlation coefficient $R$ and maximum deviation $D$ from the calculated values. The formula for the gas (dust) mass-loss rate not including metallicity fits the values calculated by the DDW model with correlation coefficient 0.87 ( 0.92 ) and maximum deviation 38 (50) per cent. The fittings are only slightly improved by including the metallicity, reflecting the fact that the mass-loss rate does not depend sensitively on the initial metallicity, with coefficient $h \sim 0.02-0.03$.

The power of the effective temperature in the gas mass-loss formula is huge (e.g. $b=-19.13$ for the case without $Z_{\text {ini }}$ ) in comparison with that in the formulae by Wachter et al. (2008, hereafter W08). The huge power arises from the inclusion of $\delta_{\mathrm{C}}$ and $\kappa_{\mathrm{R}}$ in the fitting formula. In fact, the gas mass-loss rate being fitted by using $M, L$ and $T_{\text {eff }}$ in the same manner as W08, the formula is given by $\dot{M}_{\mathrm{fit}}^{\mathrm{g}}=7.62 \times 10^{-6}\left(M / \mathrm{M}_{\odot}\right)^{-4.28}$ $\left(T_{\text {eff }} / 2600 \mathrm{~K}\right)^{-7.64}\left(L / 10^{4} \mathrm{~L} \odot\right)^{1.66}$ with correlation coefficient 0.80 and maximum deviation 42 per cent. Thus, the power of the effective temperature is reduced to -7.64 , comparable with the value in the formula by W08. Here it should be noted that the inclusion of $\delta_{\mathrm{C}}$ and $\kappa_{\mathrm{R}}$ is inevitable in the fitting formula, since $\delta_{\mathrm{C}}$ and $\kappa_{\mathrm{R}}$ control the amount of carbon available for dust formation and the density of gas levitated by the pulsation shock, respectively.

Fig. 7 shows the evolution of the gas mass-loss rate of the CNOenhanced models with $Z_{\text {ini }}=10^{-7} ; M_{\text {ini }}=2 \mathrm{M}_{\odot}$ (left) and $3 \mathrm{M}_{\odot}$ (right). The mass-loss rates calculated by the DDW model and the fitting formula without including metallicity are denoted by the filled circles and the solid line, respectively, with dotted and dashed lines indicating the results obtained by assuming, respectively, the mass-loss formulae by W08 for the SMC models and SC05. We can see that the fitting formula equation (3) reproduces the massloss rates derived from the DDW model reasonably, as long as $\dot{M} \gtrsim 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ in both models.

As shown in Fig. 7, the mass-loss rate assuming the formula for the SMC models by W08 is more than one order of magnitude larger than the rate calculated for C-rich AGB stars with $Z_{\text {ini }}=10^{-7}$. Although the difference in the stellar parameters used in the calculations makes it difficult to compare the results directly, the gap in the calculated mass-loss rate is caused by differences in the Rosseland mean opacity $\kappa_{\mathrm{R}}$ at the surface and the amplitude of pulsation $\Delta u_{\mathrm{P}}: \kappa_{\mathrm{R}}=5 \times 10^{-5} \mathrm{~cm}^{2} \mathrm{~g}^{-1}$ and $\Delta u_{\mathrm{P}}=5 \mathrm{~km} \mathrm{~s}^{-1}$ for the SMC models in W08; $\kappa_{\mathrm{R}}=\sim 10^{-3} \mathrm{~cm}^{2} \mathrm{~g}^{-1}$ as a typical value (see Table A1) and $\Delta u_{\mathrm{P}}=2 \mathrm{~km} \mathrm{~s}^{-1}$ in the CNO-enhanced models with $Z_{\text {ini }}=10^{-7}$. The difference in the value of $\kappa_{\mathrm{R}}$ implies that the gas density in the surface region, being roughly proportional to $\kappa_{R}$, is a factor of 20 larger in the SMC models than in the CNO-enhanced models. Also, the carbon excess $\delta_{\mathrm{C}}=8.57 \times 10^{-5}$ in the SMC models, assuming $\mathrm{C} / \mathrm{O}=1.8$ and taking the oxygen abundance from Russell \& Dopita (1992), while $\delta_{\mathrm{C}} \sim 10^{-3}$ as a typical value in the CNO-enhanced models investigated here. Although the amount of carbon in the surface layer is comparable, the higher gas density in

Table 2. The coefficients of the mass-loss formulae (equation 3) of gas $\left(\dot{M}_{\mathrm{fit}}^{\mathrm{g}}\right)$ and dust $\left(\dot{M}_{\mathrm{fit}}^{\mathrm{d}}\right)$ for cases with and without $Z_{\text {ini }}$ and their correlation coefficient $R$ and maximum deviation $D$ in per cent from the values calculated by the hydrodynamical model.

|  | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $d$ | $d$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\dot{M}_{\text {fit }}^{g}$ without $Z_{\text {ini }}$ | -5.733 | -19.13 | 3.164 | -5.254 | 0.7768 | -0.7089 | -0.8955 | 0 | 0.88 |  |
| $\dot{M}_{\text {fit }}^{\mathrm{g}}$ with $Z_{\text {ini }}$ | -5.590 | -20.07 | 3.221 | -5.182 | 0.8836 | -0.8476 | -0.8138 | 0.02220 | 0.90 |  |
| $\dot{M}_{\text {fit }}^{\mathrm{d}}$ without $Z_{\text {ini }}$ | -8.991 | -19.21 | 2.874 | -5.361 | 1.843 | -0.6417 | -0.8834 | 0 | 39 |  |
| $\dot{M}_{\text {fit }}^{\mathrm{d}}$ with $Z_{\text {ini }}$ | -8.822 | -19.41 | 2.696 | -5.075 | 1.977 | -0.8163 | -0.5075 | 0.02774 | 0.94 | 51 |



Figure 7. Time evolution of mass-loss rates for CNO-enhanced models with $M_{\text {ini }}=2$ (left panel) and 3 (right panel) $\mathrm{M}_{\odot}$, with $Z_{\text {ini }}=10^{-7}$. The mass-loss rate calculated by the DDW model and the fitting formula (equation 3) are denoted by a red circle and solid line, respectively. The dotted (dashed) line shows the mass-loss rate calculated by the formula for the SMC models of W08 (SC05 assumed in the stellar evolution calculation) as a reference.
the surface region as well as the enhanced density of gas levitated by the pulsation shock with larger $\Delta u_{\mathrm{P}}$ leads to a larger mass-loss rate in the SMC models. Here, it should be addressed that the values of $\kappa_{\mathrm{R}}$ and $\delta_{\mathrm{C}}$ used in W08 seem to be unrealistic in comparison with the values derived from stellar evolution calculations and the applicability of their formula for C-rich AGB stars with $Z_{\text {ini }} \leq 10^{-4}$ should be checked by using the appropriate values. Also, the dependence of the mass-loss rate on the velocity amplitude of pulsation should be investigated, since the dependence is considered to be more sensitive for stars with larger $\delta_{\mathrm{C}}$ and $\kappa_{\mathrm{R}}$.
The mass-loss rate derived from the DDW model is significantly smaller than the mass-loss rate assumed in stellar evolution calculations. If we use the derived mass-loss formula after the necessary condition for the efficient DDW is satisfied on the TP-AGB, the dredged-up carbon accumulates in the surface regions; accordingly, the effective temperature decreases and the mass-loss rate could increase. Although in the present calculations the derived mass-loss rate is inconsistent with the assumed mass-loss rate, it should be recalled here again that the hydrodynamical model can derive the properties of the DDW by specifying a set of input parameters, being independent of the stellar evolution model. Thus, the derived formulae, together with a necessary condition for the efficient DDW presented in Section 5.1, could enable us to evaluate the mass-loss rate and dust yield during the C-rich AGB phase of stars with $Z_{\text {ini }} \leq 10^{-4}$ in a manner consistent with stellar evolution, including whether the efficient DDW can operate on the C-rich AGB.

### 5.3 Implication for evolution of C-rich AGB stars and the dust-driven wind in the early Universe

Investigations on the formation of stars in low-metallicity environments have revealed that the critical metallicity $Z_{\text {cri }}$ for the transition
from Population III to Population II stars is as low as $\sim 10^{-9}-$ $10^{-7.5}$, depending on the depletion factor of metal into dust (e.g. Omukai et al. 2005, Schneider et al. 2006, Chiaki et al. 2015). Although there is no information available for the initial mass function at the present time, intermediate-mass AGB stars with $Z_{\text {ini }} \leq$ $10^{-4}$ can contribute to the enrichment of dust in the early Universe if the condition for the efficient DDW derived in the previous subsection is satisfied during the TP-AGB phase. However, the possibility of developing dust formation and a resulting DDW on the AGB is strongly influenced by the mass-loss history during the evolution, on which the time evolution of effective temperature as well as the elemental composition and opacity in the surface layer depend strongly through the number of TDU episodes and/or the occurrence of HBB. At present, we have no knowledge of the mass-loss mechanism and mass-loss rate before the onset of the DDW for the extremely low-metallicity stars considered in this article.
In recent investigations focused on low-metallicity $(-1.6<[\mathrm{Fe} / \mathrm{H}]<-0.5)$ AGB stars observed in distant galaxies, Rosenfield et al. $(2014,2016)$ showed that a mass-loss rate higher than SC 05 is required during the AGB phases previous to the onset of the DDW, to reproduce the observed TP-AGB luminosity function as well as the number ratio of TP-AGB to red giant stars. If this is true for the AGB phase of stars with $Z_{\text {ini }} \leq$ $10^{-4}$, the so-called pulsation enhanced DDW would not operate as the mass-loss mechanism; in stars losing mass efficiently on the AGB, the number of TDUs is reduced and the inefficient decrease of effective temperature as well as insufficient carbon excess ( $\delta_{\mathrm{C}}$ ) prohibits the onset of dust formation and the efficient DDW. On the other hand, when the mass-loss rate in the pre-dust phase on the AGB is depressed, it is possible for the DDW to dominate the mass loss after the stellar mass is substantially reduced below $M_{\mathrm{C} / \mathrm{O}>1}$,
as demonstrated in Section 4. In cases such as $M_{\mathrm{ini}}=2 \mathrm{M}_{\odot}$ not experiencing any HBB, the increase in carbon excess and decrease in effective temperature could make the DDW more efficient. For stars of mass $M_{\text {ini }} \geq 3 \mathrm{M}_{\odot}$ that experience HBB, a smaller mass-loss rate on the AGB results in more active HBB that seems to decrease the threshold mass $M_{\mathrm{C} / \mathrm{O}>1}$ and the carbon excess in the C-rich phase. However, the enrichment of N in the surface layer associated with HBB can counteract HBB itself, since the enhanced surface opacity depresses the increase in gas temperature in the innermost layer of the convective envelope and makes HBB weaker. Thus, it can be expected that, even in massive stars experiencing HBB, mass loss by the DDW could operate, although the details depend on the initial mass and mass-loss rate in the pre-dust phase. Anyway, the present results of DDW calculations demonstrate that the formation of carbon dust and the resulting DDW is possible even in low-metallicity environments with $Z_{\text {ini }}$ $\leq 10^{-4}$, as long as the mass-loss rate in the pre-dust phase on the AGB is reduced to some extent from the rate given by SC 05 .

Finally, it is useful to note the following in connection with the uncertainties inherent in the present DDW model. In section 4, the DDW with $\dot{M} \geq 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ is referred to as the stable DDW, according to Winters et al. (2000), since the time-averaged value $\langle\alpha\rangle \gtrsim 1$. However, in the present calculations, contrary to the results of Winters et al. (2000), we have not found any sustainable wind with $\langle\alpha\rangle<1$, since the carbon excess $\delta_{\mathrm{C}}$ of C-rich AGB stars with $Z_{\text {ini }} \leq$ $10^{-4}$ is significantly larger than the value inferred from observations of galactic C-rich AGB stars; $\delta_{\mathrm{C}} \sim 6.76 \times 10^{-4}$, corresponding to $\mathrm{C} / \mathrm{O}=2.0$ for solar metallicity. Also, recent investigation using the two-fluid hydrodynamic calculation for the DDW has shown that the assumption of position coupling will break down around $\dot{M} \sim \operatorname{several} \times 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ (Yasuda et al., in preparation ). Thus, application of the two-fluid hydrodynamic model is inevitable to explore the constraint conditions for the onset of a stable DDW. In addition, although the velocity amplitude of pulsation $\Delta u_{\mathrm{p}}$ is set to be $2 \mathrm{~km} \mathrm{~s}^{-1}$, the large $\delta_{\mathrm{C}}$ and $\kappa_{\mathrm{R}}$ in the surface region may make the dependence of the value of $\Delta u_{\mathrm{p}}$ on the formation of carbon dust and the resulting DDW more sensitive than the case for Galactic C-rich stars; the increase of $\Delta u_{\mathrm{p}}$ up to $8 \mathrm{~km} \mathrm{~s}^{-1}$ (Winters et al. 2000) may enhance the mass-loss rate from the DDW substantially. These aspects should be investigated systematically in future works to explore the properties of the DDW and the nature of carbon dust formed around AGB stars in the early Universe, consistent with stellar evolution calculations.

## 6 SUMMARY

In order to explore dust formation and the resulting mass loss around intermediate-mass AGB stars with initial metallicity $Z_{\text {ini }} \leq 10^{-4}$ in the early Universe, hydrodynamical calculations of the dustdriven wind (DDW) are carried out for stars with initial mass in the range $2 \leq M_{\text {ini }} / \mathrm{M}_{\odot} \leq 5$. The input stellar parameters necessary for the hydrodynamical calculation are calculated by the mesa code, assuming the mass-loss rate given by Schröder \& Cuntz (2005) in the post-main-sequence phase as a first step for this study. In addition, three types of low-temperature opacity (scaled-solar, CO-enhanced and CNO-enhanced) are considered to elucidate the effect of the treatment of low-temperature opacity on the time evolution of stellar parameters related to the dust formation and consequent DDW.

We confirm that all model stars, except for $M_{\text {ini }}=5 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=0$ and $10^{-7}$, finally turn out to be C-rich and satisfy the minimum condition for the formation of carbon dust, regardless
of the treatment of low-temperature opacity. However, the effective temperature, the quantity most sensitive to the dust formation process, is strongly affected by the treatment of low-temperature opacity; the minimum effective temperature $T_{\text {eff,min }}$ in the interpulse phases does not decrease below 3900 K for stars with scaled-solar opacity, while $T_{\text {eff,min }}$ decreases below 3100 K for stars of $M_{\text {ini }} \geq$ $3 \mathrm{M}_{\odot}$ with CNO-enhanced opacity.

Hydrodynamical calculations of the DDW along the evolutionary track of C-rich AGB stars simulated with CO-enhanced and CNO-enhanced opacities show the following. The stellar mass at which the stable DDW with $\dot{M} \geq 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ onsets is significantly smaller in the CO-enhanced model than in the CNO-enhanced models and the maximum mass-loss rate on C-rich AGB is more than one order of magnitude smaller in the CO-enhanced models than in the CNOenhanced models for $M_{\text {ini }} \geq 3 \mathrm{M}_{\odot}$. Thus, the employment of composition-dependent low-temperature opacity, such as CNO-enhanced opacity, is inevitable to investigate the formation of dust and resulting mass loss in low-metallicity AGB stars. Also, we find that, given the initial mass, the time evolution of the mass-loss rate, as well as the time-averaged mass-weighted radius of carbon dust, is almost independent of the initial metallicity, as long as $10^{-7}$ $\leq Z_{\text {ini }} \leq 10^{-4}$.

The results of the DDW calculation covering a wide range of stellar parameters, regardless of the treatment of low-temperature opacity and the mass-loss rate assumed in the stellar evolution calculations, enable us to derive a necessary condition for driving the efficient DDW with $\dot{M} \geq 10^{-6} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ as a combination of stellar parameters and the fitting formulae for gas and dust massloss rates in terms of input stellar parameters; the fitting formula for the gas mass-loss rate reproduces the mass-loss rate calculated by DDW model reasonably. The derived necessary condition and the fitting formulae would enable us to evaluate when the efficient DDW onsets and how much dust is produced in intermediate AGB stars with $Z_{\text {ini }} \leq 10^{-4}$, when coupled with the stellar evolution calculations.

The present results of calculations employing the mass-loss rate by SC05 in the post main-sequence phase suggest that the efficient DDW being consistent with the stellar evolution could be possible if the mass-loss rate during the evolution of a star were somewhat enhanced before entering into the AGB and depressed on the AGB before the onset of the DDW from the rate given by SC05. Also, it should be emphasized here that the assumption of position coupling is not valid for the case of a low mass-loss rate such as $\dot{M} \sim$ several $\times 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$; the assumption of position coupling results in overestimation of the mass-loss rate of C-rich AGB stars with larger $\delta_{\mathrm{C}}$ considered in this article, since $\dot{M} \propto \delta_{\mathrm{C}}$. Thus, a twofluid hydrodynamical model calculation of the DDW is necessary to clarify when and in what conditions the DDW actually onsets during the course of evolution of AGB stars. Also, large values of $\delta_{\mathrm{C}}$ and $\kappa_{\mathrm{R}}$ may result in a sensitive dependence of mass-loss rate on the velocity amplitude of pulsation. These subjects are left for future investigations.

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## APPENDIX A: INPUT STELLAR PARAMETERS FOR HYDRODYNAMICAL CALCULATIONS AND THE DERIVED PROPERTIES OF DUST-DRIVEN WINDS

The input stellar parameters of hydrodynamical calculations and the derived properties of dust-driven winds are tabulated for CO-enhanced and CNO-enhanced models with mass-loss rate $\dot{M} \geq 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ : Table A1 for $Z_{\text {ini }}=10^{-7}$ and Table A2 for $Z_{\text {ini }}=10^{-4}, 10^{-5}, 10^{-6}$ and 0 .
Table A1. Input parameters and derived properties of the dust-driven wind for $M_{\text {ini }}=2,3$ and $4 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=10^{-7}$ : current stellar mass $M\left(\mathrm{M}_{\odot}\right)$, effective temperature $T_{\text {eff }}(\mathrm{K})$, luminosity $L\left(\mathrm{~L}_{\odot}\right)$, pulsation period $P$ (days), opacity ( $\mathrm{cm}^{2} \mathrm{~g}^{-1}$ ), number ratio at the surface of $\mathrm{He}, \mathrm{C}, \mathrm{O}$ and N to H , current stellar mass $M\left(\mathrm{M}_{\odot}\right)$, time-averaged acceleration ratio $\langle\alpha\rangle$, mass-loss rate $\dot{M}\left(\mathrm{M}_{\odot} \mathrm{yr}^{-1}\right)$, terminal velocity $v_{\infty}$ $\left(\mathrm{km} \mathrm{s}^{-1}\right)$, dust-to-gas mass ratio $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$, condensation efficiency $f_{\mathrm{C}}$, mass-weighted average radius of dust $\langle a\rangle(\mu \mathrm{m})$.

Table A1 - continued

Table A2. Same as Table A1, but for all models except $M_{\text {ini }}=2,3$ and $4 \mathrm{M}_{\odot}$ with $Z_{\text {ini }}=10^{-7}$ models.

| M | $T_{\text {eff }}$ | $L$ | $P$ | $\kappa$ | He/H | C/H | O/H | N/H | $\delta_{\text {C }}$ | $\alpha$ | $\dot{M}$ | $v_{\infty}$ | $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$ | $f_{\text {c }}$ | $\langle a\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{\text {ini }}=2 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-4}$ |  |  |  | $\times 10^{4}$ | $\times 10^{2}$ | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{6}$ | $\times 10^{3}$ |  | $\times 10^{7}$ |  | $\times 10^{3}$ |  | $\times 10^{3}$ |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.250 | 3338.94 | 12916.9 | 522.374 | 5.13550 | 9.39632 | 1.56638 | 1.37701 | 3.28089 | 1.42868 | 18.1663 | 3.55304 | 49.0221 | 9.26062 | 0.756227 | 9.2684 |
| 1.190 | 3272.60 | 12602.6 | 570.602 | 6.21670 | 9.51406 | 1.78034 | 1.53969 | 3.50404 | 1.62637 | 22.1714 | 4.78770 | 55.8699 | 10.8788 | 0.780381 | 8.4654 |
| 1.150 | 3242.08 | 12965.7 | 621.543 | 6.42589 | 9.51406 | 1.78034 | 1.53969 | 3.50404 | 1.62637 | 25.2123 | 8.59955 | 57.1860 | 10.9897 | 0.788343 | 8.9031 |
| 1.100 | 3220.76 | 13112.8 | 665.690 | 6.53481 | 9.51406 | 1.78034 | 1.53969 | 3.50404 | 1.62637 | 27.5729 | 16.0594 | 59.0110 | 11.5646 | 0.829580 | 9.6414 |
| 1.070 | 3215.90 | 13091.2 | 681.205 | 6.53279 | 9.69428 | 2.10699 | 1.80110 | 4.14161 | 1.92688 | 35.0746 | 39.8511 | 64.4142 | 15.2790 | 0.925098 | 8.6247 |
| 1.000 | 3136.99 | 12925.0 | 775.883 | 9.36333 | 9.69428 | 2.10699 | 1.80110 | 4.14161 | 1.92688 | 39.4253 | 39.2466 | 64.3599 | 13.5719 | 0.821735 | 9.7659 |
| 0.950 | 3143.10 | 13158.5 | 813.308 | 8.86638 | 9.69428 | 2.10699 | 1.80110 | 4.14161 | 1.92688 | 44.2370 | 78.8438 | 65.3057 | 13.9913 | 0.847132 | 10.104 |
| 0.900 | 3179.10 | 13264.8 | 817.389 | 7.62180 | 9.69428 | 2.10699 | 1.80110 | 4.14161 | 1.92688 | 48.3443 | 51.9786 | 64.3025 | 14.0708 | 0.851943 | 8.9385 |
| 0.850 | 3254.64 | 13325.5 | 783.725 | 5.91049 | 9.69428 | 2.10699 | 1.80110 | 4.14161 | 1.92688 | 51.8245 | 63.4992 | 61.8357 | 14.0329 | 0.849648 | 8.7691 |
| 0.820 | 3322.48 | 13310.3 | 743.431 | 5.01595 | 9.69428 | 2.10699 | 1.80110 | 4.14161 | 1.92688 | 54.1433 | 49.3138 | 61.7336 | 14.1852 | 0.858873 | 12.918 |
| 0.750 | 3438.10 | 12642.1 | 668.964 | 5.56045 | 10.0821 | 2.79086 | 2.37194 | 5.75815 | 2.55367 | 73.7649 | 49.2142 | 67.6928 | 18.6802 | 0.853423 | 6.8885 |
| CO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.230 | 3403.20 | 12514.1 | 478.131 | 4.25313 | 9.50928 | 1.76904 | 1.53220 | 3.41822 | 1.61582 | 18.8180 | 1.75194 | 43.8845 | 10.0931 | 0.728753 | 7.57766 |
| 1.200 | 3372.04 | 12936.6 | 519.498 | 4.33107 | 9.50928 | 1.76904 | 1.53220 | 3.41822 | 1.61582 | 22.1539 | 5.21534 | 52.8957 | 10.0044 | 0.722342 | 8.70826 |
| 1.150 | 3344.92 | 13149.8 | 560.765 | 4.39201 | 9.50928 | 1.76904 | 1.53220 | 3.41822 | 1.61582 | 23.9172 | 5.37421 | 56.3700 | 10.0490 | 0.725567 | 8.37805 |
| 1.120 | 3337.66 | 13124.8 | 574.796 | 4.40725 | 9.50928 | 1.76904 | 1.53220 | 3.41822 | 1.61582 | 24.9763 | 10.2121 | 56.8877 | 12.0135 | 0.867411 | 9.77634 |
| 1.070 | 3270.97 | 12904.9 | 630.094 | 5.42814 | 9.66725 | 2.05172 | 1.75441 | 4.05309 | 1.87628 | 33.0183 | 15.3307 | 62.2296 | 12.4833 | 0.776208 | 8.22694 |
| 1.050 | 3261.07 | 13081.8 | 655.680 | 5.45861 | 9.66725 | 2.05172 | 1.75441 | 4.05309 | 1.87628 | 34.9095 | 28.4310 | 61.9283 | 13.6059 | 0.846014 | 9.37263 |
| 1.000 | 3254.46 | 13275.1 | 693.645 | 5.35957 | 9.66725 | 2.05172 | 1.75441 | 4.05309 | 1.87628 | 38.7163 | 24.3635 | 62.7492 | 12.3544 | 0.768195 | 9.08380 |
| 0.950 | 3267.01 | 13348.5 | 713.416 | 5.05744 | 9.66725 | 2.05172 | 1.75441 | 4.05309 | 1.87628 | 42.6281 | 30.3574 | 61.5102 | 12.9003 | 0.802139 | 8.92826 |
| 0.915 | 3290.12 | 13286.0 | 709.327 | 4.71961 | 9.66725 | 2.05172 | 1.75441 | 4.05309 | 1.87628 | 44.3248 | 31.7759 | 61.5741 | 13.1414 | 0.817128 | 8.98153 |
| 0.870 | 3282.86 | 12408.8 | 697.772 | 5.45099 | 9.86517 | 2.40456 | 2.04328 | 5.00541 | 2.20023 | 52.6909 | 38.6171 | 66.1579 | 16.0982 | 0.853604 | 7.88866 |
| 0.850 | 3303.32 | 12857.3 | 716.755 | 5.03197 | 9.86517 | 2.40456 | 2.04328 | 5.00541 | 2.20023 | 56.6553 | 59.9712 | 65.8274 | 15.6191 | 0.828196 | 7.83694 |
| 0.800 | 3421.51 | 13249.5 | 676.313 | 3.91966 | 9.86517 | 2.40456 | 2.04328 | 5.00541 | 2.20023 | 62.5362 | 45.6232 | 63.6694 | 15.9021 | 0.843203 | 8.09609 |
| 0.750 | 3628.09 | 13416.6 | 577.272 | 3.25685 | 9.86517 | 2.40456 | 2.04328 | 5.00541 | 2.20023 | 62.3623 | 28.5157 | 62.2468 | 14.4193 | 0.764581 | 7.49792 |
| $M_{\text {ini }}=2 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-5}$ |  |  |  | $\times 10^{4}$ | $\times 10^{2}$ | $\times 10^{3}$ | $\times 10^{5}$ | $\times 10^{5}$ | $\times 10^{3}$ |  | $\times 10^{7}$ |  | $\times 10^{3}$ |  | $\times 10^{3}$ |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.300 | 3265.46 | 12576.9 | 536.763 | 7.06323 | 9.56070 | 1.32209 | 9.95511 | 2.84940 | 1.22254 | 13.0549 | 2.71896 | 43.6010 | 8.39711 | 0.801335 | 10.676 |
| 1.240 | 3194.93 | 12274.7 | 589.369 | 8.80511 | 9.65633 | 1.50140 | 11.4535 | 2.88266 | 1.38687 | 17.3460 | 4.55029 | 50.9599 | 9.28072 | 0.780718 | 10.938 |
| 1.200 | 3155.75 | 12681.9 | 653.168 | 9.64457 | 9.65633 | 1.50140 | 11.4535 | 2.88266 | 1.38687 | 19.4534 | 5.88823 | 54.1910 | 9.50918 | 0.799937 | 10.001 |
| 1.150 | 3125.97 | 12833.0 | 705.774 | 10.40010 | 9.65633 | 1.50140 | 11.4535 | 2.88266 | 1.38687 | 21.0090 | 13.5384 | 51.3133 | 10.2159 | 0.859391 | 10.213 |
| 1.130 | 3119.70 | 12812.0 | 718.086 | 10.56800 | 9.65633 | 1.50140 | 11.4535 | 2.88266 | 1.38687 | 21.4855 | 12.6548 | 53.0611 | 9.64660 | 0.811497 | 10.422 |
| 1.090 | 3052.30 | 12312.5 | 770.083 | 15.69420 | 9.79182 | 1.76374 | 13.6145 | 2.92969 | 1.62760 | 25.5039 | 17.3597 | 57.7088 | 10.5748 | 0.758006 | 9.8957 |
| 1.050 | 3024.09 | 12770.0 | 849.425 | 16.94940 | 9.79182 | 1.76374 | 13.6145 | 2.92969 | 1.62760 | 29.7327 | 24.9733 | 60.8263 | 11.6148 | 0.832556 | 10.287 |
| 1.000 | 3014.68 | 12958.9 | 901.493 | 16.99590 | 9.79182 | 1.76374 | 13.6145 | 2.92969 | 1.62760 | 33.3294 | 35.2992 | 60.9806 | 11.9640 | 0.857583 | 10.642 |
| 0.950 | 3027.22 | 13047.0 | 926.287 | 15.78720 | 9.79182 | 1.76374 | 13.6145 | 2.92969 | 1.62760 | 36.3065 | 47.1479 | 60.9727 | 12.2716 | 0.879634 | 10.943 |
| 0.900 | 3064.84 | 13042.9 | 921.328 | 13.36980 | 9.79182 | 1.76374 | 13.6145 | 2.92969 | 1.62760 | 39.2270 | 64.1142 | 60.7041 | 10.9092 | 0.781979 | 11.003 |
| 0.850 | 2947.29 | 12333.5 | 1052.74 | 29.79860 | 10.12650 | 2.38802 | 18.8011 | 3.03918 | 2.20001 | 54.5560 | 64.8613 | 70.8512 | 16.6264 | 0.881700 | 8.2605 |
| 0.800 | 3064.84 | 12975.7 | 995.711 | 17.87920 | 10.12650 | 2.38802 | 18.8011 | 3.03918 | 2.20001 | 62.7349 | 64.2945 | 68.4908 | 16.7248 | 0.886920 | 8.9058 |
| 0.750 | 3314.34 | 13198.9 | 787.439 | 8.39542 | 10.12650 | 2.38802 | 18.8011 | 3.03918 | 2.20001 | 66.1491 | 62.5999 | 64.4525 | 16.4666 | 0.873227 | 7.6490 |
| 0.700 | 3990.15 | 13338.4 | 422.963 | 5.88501 | 10.12650 | 2.38802 | 18.8011 | 3.03918 | 2.20001 | 44.4785 | 1.27711 | 44.1531 | 12.8502 | 0.681447 | 5.2637 |

Table A2 - continued

Table A2 - continued

| M | $T_{\text {eff }}$ | $L$ | $P$ | $\kappa$ | He/H | C/H | O/H | N/H | $\delta_{\text {C }}$ | $\alpha$ | $\dot{M}$ | $v_{\infty}$ | $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$ | $f_{\text {C }}$ | $\langle a\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{\text {ini }}=2 \mathrm{M}_{\odot}, Z_{\text {ini }}=0$ |  |  | $\times 10^{4}$ |  |  | $\times 10^{3}$ | $\times 10^{5}$ | $\times 10^{5}$ | $\times 10^{3}$ | $\times 10^{7}$ |  | $\times 10^{3}$ |  | $\times 10^{3}$ |  |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.150 | 3339.49 | 14291.4 | 610.218 | 4.68494 | 0.120970 | 1.08508 | 7.56827 | 2.25733 | 1.00940 | 14.0328 | 6.88367 |  |  | 33.4370 | 6.39255 | 0.738854 | 12.389 |
| 1.100 | 3319.37 | 14631.5 | 657.922 | 4.66262 | 0.120970 | 1.08508 | 7.56827 | 2.25733 | 1.00940 | 17.2337 | 12.8259 | 40.2044 | 5.89020 | 0.680793 | 13.928 |
| 1.060 | 3319.37 | 14620.9 | 675.352 | 4.58452 | 0.120970 | 1.08508 | 7.56827 | 2.25733 | 1.00940 | 18.0478 | 11.4310 | 41.1719 | 5.82789 | 0.673591 | 13.414 |
| 1.000 | 3271.61 | 14509.3 | 737.734 | 5.29422 | 0.121855 | 1.25492 | 8.92204 | 2.28337 | 1.16570 | 21.5347 | 18.5453 | 46.2308 | 7.77300 | 0.777945 | 13.282 |
| 0.950 | 3294.24 | 14806.8 | 761.586 | 4.93485 | 0.121855 | 1.25492 | 8.92204 | 2.28337 | 1.16570 | 24.4100 | 19.2805 | 47.3174 | 7.70166 | 0.773968 | 12.913 |
| 0.900 | 3347.03 | 14907.8 | 751.495 | 4.45024 | 0.121855 | 1.25492 | 8.92204 | 2.28337 | 1.16570 | 25.6475 | 20.9610 | 45.7600 | 7.60962 | 0.761593 | 13.163 |
| 0.870 | 3392.28 | 14828.1 | 727.643 | 4.15621 | 0.121855 | 1.25492 | 8.92204 | 2.28337 | 1.16570 | 26.4785 | 26.4916 | 45.6186 | 8.07064 | 0.807734 | 13.305 |
| 0.820 | 3500.37 | 14158.6 | 649.665 | 3.92207 | 0.122519 | 1.35533 | 9.70306 | 2.38750 | 1.25830 | 28.2349 | 18.2165 | 44.8536 | 7.94338 | 0.736492 | 11.008 |
| 0.800 | 3573.27 | 14557.1 | 628.565 | 3.69338 | 0.122519 | 1.35533 | 9.70306 | 2.38750 | 1.25830 | 29.1972 | 15.9277 | 43.9736 | 7.19478 | 0.667084 | 11.448 |
| 0.750 | 3900.07 | 14923.7 | 488.206 | 3.75327 | 0.122519 | 1.35533 | 9.70306 | 2.38750 | 1.25830 | 19.8531 | 1.78389 | 24.2369 | 4.88276 | 0.452719 | 6.8978 |
| CO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.100 | 3718.51 | 15066.7 | 443.232 | 1.79454 | 0.122083 | 1.30129 | 9.24046 | 2.27491 | 1.20889 | 11.5672 | 1.11605 | 24.7375 | 7.14649 | 0.689691 | 11.5388 |
| 1.072 | 3718.03 | 15021.3 | 450.342 | 1.77842 | 0.122083 | 1.30129 | 9.24046 | 2.27491 | 1.20889 | 12.6125 | 1.49146 | 25.7992 | 5.97437 | 0.576572 | 9.91552 |
| 1.030 | 3697.04 | 14885.3 | 469.897 | 1.79859 | 0.122943 | 1.46127 | 10.4594 | 2.30974 | 1.35668 | 19.3967 | 2.38930 | 40.9011 | 7.95007 | 0.683662 | 9.24964 |
| 1.000 | 3695.35 | 15152.8 | 489.095 | 1.76245 | 0.122943 | 1.46127 | 10.4594 | 2.30974 | 1.35668 | 22.8812 | 4.50433 | 42.2656 | 7.48530 | 0.643695 | 9.09147 |
| 0.950 | 3703.56 | 15270.8 | 507.228 | 1.74662 | 0.122943 | 1.46127 | 10.4594 | 2.30974 | 1.35668 | 25.0306 | 7.73868 | 44.0875 | 7.76989 | 0.668168 | 9.54643 |
| 0.900 | 3718.75 | 14540.6 | 496.562 | 1.83962 | 0.123551 | 1.56243 | 11.1908 | 2.41422 | 1.45052 | 27.9964 | 8.94191 | 46.9987 | 8.22067 | 0.661195 | 9.03381 |
| 0.850 | 3751.32 | 15220.9 | 522.871 | 1.87171 | 0.123551 | 1.56243 | 11.1908 | 2.41422 | 1.45052 | 31.1536 | 9.01878 | 45.8575 | 7.83613 | 0.630266 | 8.86316 |
| 0.800 | 3829.09 | 15379.6 | 511.494 | 2.10146 | 0.123551 | 1.56243 | 11.1908 | 2.41422 | 1.45052 | 31.9521 | 6.18426 | 42.7639 | 8.46901 | 0.681169 | 8.66284 |
| $M_{\text {ini }}=3 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-4}$ |  |  | $\times 10^{4}$ |  |  | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{4}$ | $\times 10^{4}$ | $\times 10^{7}$ |  | $\times 10^{3}$ |  |  | $\times 10^{3}$ |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2.000 | 3015.63 | 22232.3 | 893.878 | 19.3012 | 0.101197 | 1.45730 | 1.28232 | 1.52832 | 13.2907 | 12.9080 | 1.33334 | 39.9491 | 8.32059 | 0.730388 | 6.5853 |
| 1.950 | 2980.03 | 21988.8 | 944.510 | 23.2151 | 0.101736 | 1.55750 | 1.34679 | 1.52873 | 14.2282 | 15.8151 | 1.30993 | 49.0924 | 8.74770 | 0.717283 | 6.8049 |
| 1.900 | 2961.70 | 22044.1 | 988.392 | 24.5965 | 0.101736 | 1.55750 | 1.34679 | 1.52873 | 14.2282 | 16.8991 | 1.93883 | 51.7274 | 8.63146 | 0.711262 | 7.1828 |
| 1.850 | 2909.93 | 22038.6 | 1072.78 | 32.1942 | 0.102569 | 1.70563 | 1.45269 | 1.54045 | 15.6036 | 18.4978 | 2.79164 | 53.7915 | 10.5294 | 0.787270 | 6.4778 |
| 1.800 | 2893.75 | 21994.3 | 1116.66 | 34.0360 | 0.102569 | 1.70563 | 1.45269 | 1.54045 | 15.6036 | 19.6371 | 6.21234 | 56.3819 | 10.9576 | 0.825301 | 7.2759 |
| 1.765 | 2884.04 | 21916.9 | 1143.66 | 35.1872 | 0.102569 | 1.70563 | 1.45269 | 1.54045 | 15.6036 | 20.0889 | 5.50874 | 56.6878 | 10.6411 | 0.795623 | 7.3496 |
| 1.740 | 2840.90 | 21861.5 | 1217.93 | 45.5476 | 0.103697 | 1.88861 | 1.60004 | 1.54620 | 17.2861 | 22.7527 | 12.1768 | 58.3655 | 11.4786 | 0.774708 | 6.9923 |
| 1.700 | 2827.96 | 21889.2 | 1265.18 | 47.3895 | 0.103697 | 1.88861 | 1.60004 | 1.54620 | 17.2861 | 23.8294 | 14.0703 | 60.0060 | 12.7515 | 0.860621 | 7.6718 |
| 1.650 | 2813.94 | 21861.5 | 1312.44 | 49.6918 | 0.103697 | 1.88861 | 1.60004 | 1.54620 | 17.2861 | 27.1918 | 24.3158 | 62.2434 | 12.7081 | 0.857690 | 9.3932 |
| 1.610 | 2804.23 | 21822.8 | 1352.95 | 51.0732 | 0.103697 | 1.88861 | 1.60004 | 1.54620 | 17.2861 | 28.3828 | 17.3372 | 62.5936 | 11.7366 | 0.800490 | 8.4547 |
| 1.550 | 2753.54 | 21789.6 | 1484.59 | 67.8801 | 0.105069 | 2.11080 | 1.78883 | 1.55134 | 19.3192 | 35.1209 | 26.7236 | 68.8731 | 12.5486 | 0.757796 | 8.0252 |
| 1.500 | 2742.75 | 21817.3 | 1545.35 | 69.7220 | 0.105069 | 2.11080 | 1.78883 | 1.55134 | 19.3192 | 38.7608 | 44.0972 | 70.9562 | 14.4273 | 0.871254 | 8.3376 |
| 1.450 | 2735.20 | 21822.8 | 1602.73 | 70.6429 | 0.105069 | 2.11080 | 1.78883 | 1.55134 | 19.3192 | 40.4044 | 53.3852 | 71.4569 | 14.1214 | 0.852780 | 8.8723 |
| 1.400 | 2730.89 | 21784.1 | 1649.99 | 70.6429 | 0.105069 | 2.11080 | 1.78883 | 1.55134 | 19.3192 | 42.1429 | 41.2658 | 71.1171 | 13.8080 | 0.833854 | 8.6252 |
| 1.350 | 2693.14 | 21684.4 | 1778.26 | 89.5219 | 0.106638 | 2.36348 | 2.00525 | 1.55257 | 21.6296 | 50.6735 | 53.2345 | 77.4225 | 14.8148 | 0.799091 | 7.9518 |
| 1.300 | 2695.30 | 21800.7 | 1828.89 | 86.5289 | 0.106638 | 2.36348 | 2.00525 | 1.55257 | 21.6296 | 53.6790 | 129.891 | 78.3726 | 17.0171 | 0.917879 | 8.7478 |
| 1.250 | 2706.08 | 21850.5 | 1862.65 | 80.7731 | 0.106638 | 2.36348 | 2.00525 | 1.55257 | 21.6296 | 57.1269 | 78.5636 | 78.2777 | 16.1485 | 0.871027 | 7.9453 |
| 1.200 | 2725.50 | 21883.7 | 1869.40 | 72.9452 | 0.106638 | 2.36348 | 2.00525 | 1.55257 | 21.6296 | 60.6020 | 91.1719 | 76.7541 | 15.5582 | 0.839189 | 7.7713 |
| 1.150 | 2755.70 | 21889.2 | 1852.52 | 62.5848 | 0.106638 | 2.36348 | 2.00525 | 1.55257 | 21.6296 | 63.8397 | 91.6873 | 78.1608 | 16.2348 | 0.875684 | 8.2444 |
| 1.120 | 2777.27 | 21856.0 | 1825.52 | 56.5988 | 0.106638 | 2.36348 | 2.00525 | 1.55257 | 21.6296 | 67.0162 | 84.7668 | 77.3281 | 15.7920 | 0.851798 | 8.1264 |

Table A2 - continued

Table A2 - continued

| M | $T_{\text {eff }}$ | $L$ | $P$ | $\kappa$ | He/H | C/H | O/H | N/H | $\delta_{\text {C }}$ | $\alpha$ | $\dot{M}$ | $v_{\infty}$ | $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$ | $f_{\text {c }}$ | $\langle a\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{\text {ini }}=3 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-6}$ |  |  | $\times 10^{4}$ |  |  | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{5}$ | $\times 10^{4}$ | $\times 10^{7}$ |  | $\times 10^{3}$ |  | $\times 10^{3}$ |  |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.900 | 3090.39 | 21640.1 | 826.329 | 14.0478 | 0.107099 | 1.53665 | 1.07087 | 5.73803 | 14.2956 | 14.0679 | 1.31479 | 43.5430 | 8.73270 | 0.712675 | 6.5167 |
| 1.840 | 3052.12 | 21404.2 | 880.362 | 16.7795 | 0.107600 | 1.62608 | 1.12975 | 5.74357 | 15.1311 | 16.2432 | 2.52741 | 49.1540 | 9.54671 | 0.736091 | 6.8871 |
| 1.800 | 3032.07 | 21557.2 | 922.388 | 17.7101 | 0.107600 | 1.62608 | 1.12975 | 5.74357 | 15.1311 | 18.4198 | 5.25820 | 51.4604 | 8.49704 | 0.655157 | 7.3182 |
| 1.750 | 2982.87 | 21493.5 | 998.435 | 22.7366 | 0.108351 | 1.76695 | 1.21963 | 5.78645 | 16.4499 | 20.3402 | 4.67849 | 55.5300 | 10.5377 | 0.747359 | 6.6427 |
| 1.700 | 2963.74 | 21544.5 | 1046.46 | 24.1113 | 0.108351 | 1.76695 | 1.21963 | 5.78645 | 16.4499 | 21.9032 | 8.33625 | 56.8996 | 10.2316 | 0.725653 | 7.0933 |
| 1.640 | 2909.06 | 21397.8 | 1142.52 | 32.4794 | 0.109424 | 1.95476 | 1.34670 | 5.85009 | 18.2009 | 25.6788 | 25.7539 | 61.6647 | 11.3057 | 0.724690 | 7.3059 |
| 1.600 | 2894.49 | 21.4935 | 1190.55 | 33.9737 | 0.109424 | 1.95476 | 1.34670 | 5.85009 | 18.2009 | 27.5721 | 12.3439 | 61.6702 | 11.6330 | 0.745668 | 7.1334 |
| 1.550 | 2881.73 | 21487.1 | 1240.58 | 35.4082 | 0.109424 | 1.95476 | 1.34670 | 5.85009 | 18.2009 | 30.7266 | 18.5587 | 63.2823 | 12.2792 | 0.787088 | 8.0537 |
| 1.510 | 2872.62 | 21423.3 | 1272.60 | 36.2450 | 0.109424 | 1.95476 | 1.34670 | 5.85009 | 18.2009 | 31.8780 | 26.2352 | 64.0918 | 12.3664 | 0.792677 | 8.4409 |
| CO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.200 | 3712.30 | 21914.9 | 593.181 | 1.54595 | 0.119582 | 1.229240 | 2.78029 | 197.206 | 9.51211 | 9.77604 | 1.02568 | 17.5339 | 3.02928 | 0.371543 | 9.72747 |
| 1.150 | 3716.93 | 21941.6 | 610.051 | 1.53126 | 0.119582 | 1.229240 | 2.78029 | 197.206 | 9.51211 | 13.6675 | 2.31529 | 25.9417 | 4.97348 | 0.610000 | 14.3414 |
| 1.090 | 3684.49 | 21415.9 | 640.079 | 1.62921 | 0.121382 | 1.552080 | 3.05415 | 197.206 | 12.4667 | 27.4791 | 30.9771 | 39.9155 | 8.89785 | 0.832687 | 10.2609 |
| 1.050 | 3697.01 | 21815.9 | 661.335 | 1.61942 | 0.121382 | 1.552080 | 3.05415 | 197.206 | 12.4667 | 29.1651 | 19.1169 | 40.7562 | 7.77809 | 0.727897 | 11.3047 |
| 1.000 | 3725.74 | 21903.5 | 668.082 | 1.65860 | 0.121382 | 1.552080 | 3.05415 | 197.206 | 12.4667 | 31.1523 | 37.0494 | 41.2446 | 7.37330 | 0.690016 | 14.1229 |
| 0.950 | 3772.55 | 21876.8 | 660.660 | 1.75655 | 0.121382 | 1.552080 | 3.05415 | 197.206 | 12.4667 | 32.8810 | 17.4306 | 38.5650 | 6.84473 | 0.640550 | 11.7188 |
| 0.900 | 3806.85 | 21739.7 | 661.672 | 2.07980 | 0.124399 | 2.097770 | 3.52555 | 197.206 | 17.4522 | 51.9964 | 17.7039 | 48.5809 | 8.88470 | 0.593937 | 8.70068 |
| 0.850 | 3994.43 | 22013.9 | 582.385 | 2.85362 | 0.124399 | 2.097770 | 3.52555 | 197.206 | 17.4522 | 49.1353 | 4.38012 | 42.9501 | 9.04255 | 0.604489 | 7.19564 |
| $M_{\text {ini }}=3 \mathrm{M}_{\odot}, Z_{\text {ini }}=0$ |  |  |  | $\times 10^{4}$ |  | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{5}$ | $\times 10^{3}$ |  | $\times 10^{7}$ |  | $\times 10^{3}$ |  | $\times 10^{3}$ |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.705 | 3193.12 | 20915.0 | 769.728 | 9.40080 | 0.162251 | 1.83376 | 1.32750 | 4.41197 | 1.70101 | 16.0610 | 1.10253 | 43.0910 | 9.72756 | 0.667181 | 5.7245 |
| 1.650 | 3158.45 | 20944.1 | 821.882 | 10.4698 | 0.162556 | 1.89158 | 1.36560 | 4.42104 | 1.75502 | 16.5100 | 2.00024 | 46.2026 | 8.83906 | 0.587585 | 5.4973 |
| 1.600 | 3133.16 | 20766.3 | 858.738 | 11.5145 | 0.162828 | 1.94259 | 1.40051 | 4.43918 | 1.80254 | 18.8256 | 2.19967 | 50.3418 | 8.40362 | 0.543912 | 5.8097 |
| 1.557 | 3115.36 | 20900.5 | 900.462 | 12.0854 | 0.162828 | 1.94259 | 1.40051 | 4.43918 | 1.80254 | 21.3379 | 2.64154 | 51.3439 | 8.70258 | 0.563261 | 6.0833 |
| 1.500 | 3085.38 | 20911.4 | 960.245 | 13.5405 | 0.163100 | 1.99361 | 1.43543 | 4.45731 | 1.85007 | 23.9164 | 3.95056 | 54.9920 | 10.0183 | 0.614844 | 6.1652 |
| 1.440 | 3065.70 | 20577.6 | 999.120 | 14.7368 | 0.163383 | 2.04689 | 1.45924 | 4.47545 | 1.90097 | 26.4285 | 7.25363 | 57.8056 | 10.8110 | 0.663498 | 6.6509 |
| 1.400 | 3047.90 | 20958.6 | 1059.34 | 15.3826 | 0.163383 | 2.04689 | 1.45924 | 4.47545 | 1.90097 | 28.0893 | 14.2387 | 59.0958 | 10.4794 | 0.643146 | 7.2935 |
| 1.370 | 3043.22 | 20933.2 | 1077.63 | 15.4779 | 0.163383 | 2.04689 | 1.45924 | 4.47545 | 1.90097 | 30.0380 | 13.1574 | 58.3090 | 9.90343 | 0.607796 | 7.3926 |
| 1.325 | 3029.16 | 20788.1 | 1120.32 | 16.5578 | 0.163688 | 2.10585 | 1.51162 | 4.50266 | 1.95469 | 33.2075 | 20.4616 | 60.1866 | 11.1212 | 0.663778 | 8.3752 |
| 1.300 | 3024.48 | 20969.5 | 1151.57 | 16.6319 | 0.163688 | 2.10585 | 1.51162 | 4.50266 | 1.95469 | 35.4404 | 20.3661 | 60.5248 | 11.0766 | 0.661111 | 7.9328 |
| 1.252 | 3024.48 | 20984.0 | 1182.06 | 16.3037 | 0.163688 | 2.10585 | 1.51162 | 4.50266 | 1.95469 | 37.9885 | 29.2257 | 63.4203 | 10.1911 | 0.608261 | 8.1991 |
| 1.200 | 2999.27 | 21084.0 | 1265.51 | 18.9499 | 0.164645 | 2.28597 | 1.63134 | 4.55163 | 2.12284 | 51.7773 | 72.8822 | 69.8954 | 13.5973 | 0.747278 | 8.2983 |
| 1.150 | 3014.83 | 21238.9 | 1289.34 | 17.4436 | 0.164645 | 2.28597 | 1.63134 | 4.55163 | 2.12284 | 51.3141 | 55.9495 | 69.3102 | 13.1216 | 0.721137 | 7.9170 |
| 1.100 | 3044.16 | 21182.0 | 1281.16 | 15.3349 | 0.164645 | 2.28597 | 1.63134 | 4.55163 | 2.12284 | 54.3185 | 59.0855 | 70.0126 | 12.8698 | 0.707299 | 8.0798 |

Table A2 - continued

Table A2 - continued

Table A2 - continued

| M | $T_{\text {eff }}$ | $L$ | $P$ | $\kappa$ | He/H | C/H | O/H | N/H | $\delta_{\text {C }}$ | $\alpha$ | $\dot{M}$ | $v_{\infty}$ | $\rho_{\mathrm{d}} / \rho_{\mathrm{g}}$ | $f_{\text {c }}$ | $\langle a\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{\text {ini }}=5 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-4}$ |  |  | $\times 10^{4}$ |  |  | $\times 10^{4}$ | $\times 10^{4}$ | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{7}$ |  | $\times 10^{4}$ |  | $\times 10^{2}$ |  |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.450 | 3156.83 | 31888.7 | 1338.92 | 5.06238 | 0.165217 | 5.78915 | 2.42856 | 1.73913 | 3.36059 | 1.58642 | 4.09535 | 5.31577 | 3.71425 | 0.128944 | 2.4288 |
| 1.400 | 3100.55 | 31820.7 | 1465.27 | 6.71101 | 0.165788 | 6.98985 | 2.50124 | 1.73913 | 4.48861 | 7.63765 | 19.3213 | 19.1998 | 18.7954 | 0.488523 | 3.6688 |
| 1.350 | 3126.53 | 31937.4 | 1462.93 | 6.31041 | 0.165788 | 6.98985 | 2.50124 | 1.73913 | 4.48861 | 8.14753 | 20.0081 | 15.9822 | 16.2788 | 0.423115 | 3.2800 |
| 1.330 | 3137.71 | 31888.7 | 1458.25 | 6.17174 | 0.165788 | 6.98985 | 2.50124 | 1.73913 | 4.48861 | 8.12276 | 21.9824 | 14.6053 | 14.1822 | 0.368621 | 3.0160 |
| CO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.100 | 3870.29 | 32226.1 | 774.846 | 1.91793 | 0.169942 | 10.7233 | 2.84210 | 1.88837 | 7.88120 | 10.7892 | 1.31327 | 12.3453 | 15.7767 | 0.233545 | 0.9440 |
| $M_{\text {ini }}=5 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-5}$ |  |  | $\times 10^{4}$ |  | $\times 10^{4}$ |  | $\times 10^{4}$ | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{7}$ |  | $\times 10^{4}$ |  | $\times 10^{2}$ |  |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.450 | 3189.53 | 30675.0 | 1239.85 | 4.53491 | 0.166764 | 5.68728 | 2.41216 | 1.83212 | 3.27512 | 1.06547 | 1.25136 | 1.54306 | 3.80944 | 0.135700 | 2.3862 |
| 1.400 | 3115.15 | 30664.3 | 1389.61 | 6.23243 | 0.167337 | 6.93248 | 2.49292 | 1.83221 | 4.43956 | 8.44758 | 19.9802 | 22.3217 | 21.8578 | 0.574398 | 4.3401 |
| 1.350 | 3136.66 | 30664.3 | 1391.63 | 5.89292 | 0.167337 | 6.93248 | 2.49292 | 1.83221 | 4.43956 | 8.30818 | 22.7109 | 20.8376 | 20.7950 | 0.546470 | 4.2711 |
| 1.300 | 3096.33 | 30707.3 | 1502.94 | 7.59044 | 0.168007 | 8.20920 | 2.56376 | 1.83230 | 5.64544 | 12.7565 | 50.7684 | 24.7468 | 25.9658 | 0.536599 | 3.2370 |
| 1.250 | 3138.45 | 30739.5 | 1472.58 | 6.85485 | 0.168007 | 8.20920 | 2.56376 | 1.83230 | 5.64544 | 13.8330 | 48.4231 | 24.5847 | 25.9371 | 0.536008 | 3.4423 |
| $M_{\text {ini }}=5 \mathrm{M}_{\odot}, Z_{\text {ini }}=10^{-6}$ |  |  | $\times 10^{4}$ |  |  | $\times 10^{4}$ | $\times 10^{4}$ | $\times 10^{3}$ | $\times 10^{4}$ | $\times 10^{7}$ |  | $\times 10^{4}$ |  | $\times 10^{2}$ |  |
| CNO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.450 | 3142.78 | 30215.0 | 1294.98 | 5.36104 | 0.163058 | 6.11749 | 2.27678 | 1.78645 | 3.84071 | 2.11325 | 4.72581 | 7.09111 | 4.76467 | 0.144733 | 2.2432 |
| 1.417 | 3150.60 | 30190.3 | 1301.72 | 5.24999 | 0.163058 | 6.11749 | 2.27678 | 1.78645 | 3.84071 | 2.42082 | 5.82691 | 7.44480 | 5.26745 | 0.160006 | 2.4163 |
| CO-enhanced |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.050 | 3917.13 | 30586.5 | 728.383 | 1.84213 | 0.170688 | 13.8482 | 2.88997 | 1.96628 | 10.9582 | 10.7141 | 1.50135 | 15.5986 | 15.9509 | 0.169821 | 0.5885 |

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