提要

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論文内容の要旨

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Abstract of Doctoral Dissertation

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Timelike Sabban curves in de Sitter space
(ド・ジッター空間内の時間的サバン曲線)

It is well known that Lorentzian space forms are classified into three types depending on the value of the scalar curvature. One of them is Lorentz-Minkowski space which has zero curvature. The Lorentz space form with negative curvature is Anti-de Sitter space. De Sitter space is a Lorentzian space form with positive curvature which has rich geometric properties. De sitter space is named after Willem de Sitter (1872-1934). In general relativity, a vacuum solution according to the Einstein field equation is a Lorentzian manifold whose Einstein tensor vanishes identically, and de Sitter space is one of the vacuum solutions of the Einstein field equation. In cosmology, the very early models of inflation are converging on a consistent model where our universe was best described as a de Sitter space, which is called de Sitter universe. In Mathematics, de Sitter space is defined to be the Lorentzian sphere in Minkowski space-time with a positive curvature, which is a good model for studying Lorentzian spherical geometry. This is the motivation for the investigation of submanifolds in de Sitter space from a mathematical viewpoint.

On the other hand, in physics, the locus of a moving particle is called a world line. In special relativity, world lines are timelike curves. World lines of particles/objects with constant speed are called geodesics which are straight lines in Minkowski space. The use of world lines in general relativity is basically the same as in special relativity, with the difference that space-time can be curved. Since the speed of a moving particle never exceed the speed of rays, in general relativity, world lines are timelike curves in space-time, where the tangent vectors of timelike curves fall within the lightcone.

In this thesis we investigate the singularities of hypersurfaces associate to timelike Sabban curves in de Sitter space as an application of the theory of singularities.

We first consider non-lightlike curves in de Sitter n-space and define the generalized Lorentzian Sabban curves. The generalized Lorentzian Sabban frame is well-defined under some conditions. We stick to timelike Sabban curves and investigate the de Sitter dual hypersurfaces of timelike Sabban curves in de Sitter space with a general dimension. By using singularity theory and Legendrian
duality theory, we give classifications and characterizations of the singularities of de Sitter dual hypersurfaces and reveal the relationship between the geometric properties of timelike Sabban curves and the singularities of dual hypersurfaces.

Secondly, we apply the singularity theory to the study of the geometric properties of focal sets of timelike Sabban curves as examples of special submanifolds in de Sitter space. These focal sets are de Sitter evolutes in the 2-dimensional de Sitter space, de Sitter focal surfaces in the 3-dimensional de Sitter space and de Sitter focal hypersurfaces in the 4-dimensional de Sitter space respectively. Some important model submanifolds along timelike Sabban curves are also defined, such as tangent de Sitter subspaces and osculating de Sitter subspaces. The notion of de Sitter height functions is introduced which is related to the timelike Sabban curve. It is quite useful to study the singularities of focal sets. We get some invariants of timelike Sabban curves which describe the contact between model submanifolds and timelike Sabban curves at singular points.

At last, from the view point of Legendrian duality theory, we discuss the focal sets of timelike Sabban curves. Izumiya introduced the mandala of Legendrian dualities between pseudo-spheres in Minkowski space-time, which are the fundamental tool for the study of spacelike submanifolds in pseudo-spheres in Minkowski space. Here, the regular parts of the focal sets of timelike Sabban curves are spacelike submanifolds. In this thesis the dual relationship between the tangent vector field of a timelike Sabban curve and the focal set is shown.