学 位 論 文 内 容 の 要 旨

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学 位 論 文 題 名

Study on quantum mechanical drift motion and expansion of variance of a charged particle in non-uniform electromagnetic fields

(非一様電磁場中の荷電粒子の量子力学的ドリフト運動と分散の膨張に関する研究)

In the field of plasma fusion, grad-B drift and $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity are a well-known topic. In recent years, motion of charged particles is getting attention not only in classical approach but also quantum approach. In this study, the quantum mechanical effects of a non-relativistic spinless charged particle in the presence of variation inhomogeneity of electromagnetic field are shown.

In chapter 2, the two-dimensional time-dependent Schrödinger equation, for a magnetized proton in the presence of a fixed field particle and of a homogeneous magnetic field is numerically solved. In the relatively high-speed case, the fast-speed proton has the similar behaviors to those of classical ones. However, in the extension of time, the relatively high-speed case shows similar behavior to the low-speed case: the cyclotron radii both in mechanical momentum and position are appreciably decreasing with time. However, the kinetic energy and the potential energy do not show appreciable changes. This is because of the increasing variances, i.e. uncertainty, both in momentum and position. The increment in variance of momentum corresponds to the decrement in the magnitude of mechanical momentum in a classical sense: Part of energy is transferred from the directional (classical kinetic) energy to the uncertainty (quantum mechanical zero-point) energy.

In chapter 3, by solving the Heisenberg equation of motion operators for a charged particle in the presence of an inhomogeneous magnetic field, the analytical solution for quantum mechanical grad-B drift velocity operator is shown. Using the time dependent operators, it is shown the analytical solution of the position. It is also numerically shown that the grad-B drift velocity operator agrees with the classical counterpart. Using the time dependent operators, it is shown the variance in position and momenta grow with time. The expressions of quantum mechanical expansion rate for position and momenta are also obtain analytically.

In chapter 4, the Heisenberg equation of motion for the time evolution of the position and momentum operators for a charged particle in the presence of an inhomogeneous electric and magnetic field is solved. It is shown that the analytical $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity obtained in this study agrees with the classical counterpart, and that, using the time dependent operators, the variances in position and momentum grow with time. It is also shown that the theoretical expansion rates of variance expansion are in good agreement with the numerical analysis. The expansion rate of variance in position and momentum are dependent on the magnetic gradient scale length, however, independent of the electric gradient scale length. Therefore, a higher order of inhomogeneous electric is introduced in next chapter.

In chapter 5, a charged particle in a higher order of electric field inhomogeneity is introduced and the quantum mechanical drift velocity is solved analytically. The analytical solution of the time dependent momenta operators and position operators is shown. With further combination the operators, the quantum mechanical expansion rate of variance is shown and the results agree with the numerical results. Finally, it is analytically shown the analytical result of quantum mechanical drift velocity, which coincides with the classical drift velocity. The result implies that light particles with low energy would drift faster than classical drift theory predicts. The drift velocity and the expansion rate of variance are dependent on the both electric and magnetic gradient scale length.

In summary, it is analytically shown that the variance in position reaches the square of the interparticle separation, which is the characteristic time much shorter than the proton collision time of plasma fusion. After this time the wavefunctions of the neighboring particles would overlap, as a result the conventional classical analysis may lose its validity. The expansion time in position implies that the probability density function of such energetic charged particles expand fast in the plane perpendicular to the magnetic field and their Coulomb interaction with other particles becomes weaker than that expected in classical mechanics.