Elastocapillary Levelling of Thin Viscous Films on Soft Substrates – Supplementary Material –

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I. INTRODUCTION

As a reference state, we consider a thin viscous film of height h_0 sitting on an incompressible elastic layer of thickness s_0 (see Fig. S1). The elastic layer is itself placed atop a rigid substrate. We use a Cartesian coordinate system (x, y, z), with z being the vertical coordinate. We assume the system to be infinite in the x and y directions. The surface tension of the air-liquid interface is denoted γ, the viscosity of the fluid (assumed to be Newtonian) η, and the shear modulus of the elastic material (assumed incompressible, *i.e.* with a Poisson ratio of 1/2) μ. At initial time, $t = 0$, we perturb the air-liquid interface by adding a step function $h(x) = h_2 H(x)$ with $H(x < 0) = -1/2$ and $H(x > 0) = 1/2$. We assume invariance in the y direction, and that the step height h_2 is small compared with the reference height h_0.

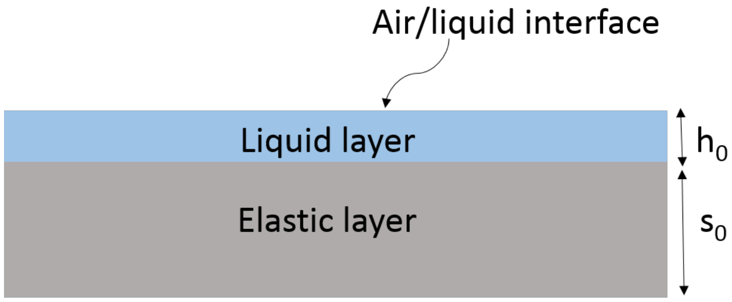

FIG. S1: Cross-sectional view of the reference equilibrium state, to which will be superimposed a deformation of the air-liquid interface at initial time ($t = 0$).

II. CONTROL EXPERIMENT WITH A STEPPED PERTURBATION

The previous $h_2 \ll h_0$ condition is not verified in our experiments (where $h_2 = h_1 = 2h_0/3$). However, we checked that this simplification in the model does not affect our general conclusions and is not the source of some discrepancy observed with the experiments. Indeed, in a test experiment with $h_2 \ll h_0$, we find that the profile width follows a $t^{1/6}$ power law with time t (see Fig. S2), consistently with the $h_1 = h_2$ experimental case (see Figs. 2 and 3), and in contrast to the theoretical prediction (see Fig. 6).

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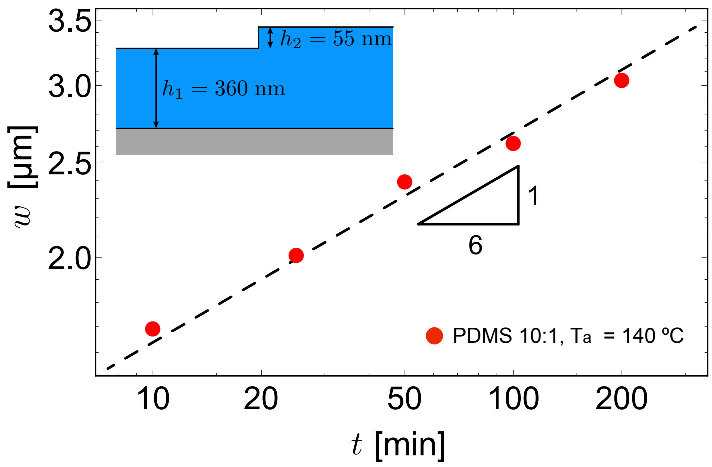

FIG. S2: Temporal evolution of the profile width w (defined in the inset of Fig. 2(a)), in log-log scale, for an experiment with $h_2 \ll h_1$ (see inset). We used the same annealing temperature T_a and PDMS substrate (in grey in the inset) as in the experiments reported in Figs. 1 and 2.

III. LUBRICATION-ELASTIC MODEL

A. Lubrication description of the liquid layer

As for the capillary levelling of a thin liquid film, of viscosity η, on a rigid substrate [39], we invoke the lubrication approximation which assumes that the typical horizontal length scale of the flow is much larger than the vertical one. As a result, at leading order, the vertical flow is neglected and the excess pressure field p (with respect to the atmospheric pressure) does not depend on z. The incompressible Stokes' equations thus reduce to:

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 v_x}{\partial z^2} \ , \tag{S1}$$

which can be integrated in z to get the horizontal velocity v_x. The main difference here with the previous model [39] is that the pressure acts on the elastic layer, giving rise to vertical and horizontal displacements of the liquid-elastic interface, $\delta(x,t)$ and $u_s(x,t)$ respectively. In addition, the no-slip condition at the liquid-elastic interface implies that a fluid particle in contact with the elastic surface will have a non-zero horizontal velocity $\partial u_s/\partial t$. Using this condition, the vanishing shear stress at the air-liquid interface, and invoking volume conservation, allow one to derive the following equation:

$$\frac{\partial \Delta}{\partial t} + \frac{\partial}{\partial x}\left[-\frac{(h_0+\Delta)^3}{3\eta}\frac{\partial p}{\partial x} + (h_0+\Delta)\frac{\partial u_s}{\partial t}\right] = 0 \ , \tag{S2}$$

where $\Delta(x,t) = h(x,t) - \delta(x,t) - h_0$ is the excess thickness of the liquid layer with respect to the equilibrium value h_0, and $h(x,t)$ is defined in Fig. 1(b). Since the pressure is independent of z, it is fixed by the proper boundary condition, *i.e.* the Laplace pressure at the air-liquid interface (we neglect the non-linear term of the curvature at small slopes):

$$p(x,t) = -\gamma \frac{\partial^2 h}{\partial x^2} = -\gamma \frac{\partial^2 (\Delta + \delta)}{\partial x^2} \ . \tag{S3}$$

Finally, as the perturbation is assumed to be small ($\Delta \ll h_0$), one can linearize Eq. (S2) and get the governing equation:

$$\frac{\partial \Delta}{\partial t} + \frac{\partial}{\partial x}\left[-\frac{h_0^3}{3\eta}\frac{\partial p}{\partial x} + h_0 \frac{\partial u_s}{\partial t}\right] = 0 \ . \tag{S4}$$

B. Coupling with the elastic layer

The surface displacements of the liquid-elastic interface are given by:

$$\delta(x,t) = -\frac{1}{\sqrt{2\pi}\mu}\int_{-\infty}^{\infty} \mathrm{d}x'\, k(x-x')p(x',t) \ , \tag{S5a}$$

$$u_{\mathrm{s}}(x,t) = -\frac{1}{\sqrt{2\pi}\mu}\int_{-\infty}^{\infty} \mathrm{d}x'\, k_{\mathrm{s}}(x-x')p(x',t) \ , \tag{S5b}$$

where k and k_{s} are the Green's functions of the elastic problem (see Section III C), corresponding to the vertical and horizontal displacements induced by a normal line load of magnitude $-\sqrt{2\pi}\mu$. We introduce the Fourier transform $\tilde{f}$ of a function f with respect to its variable x as:

$$\tilde{f}(\lambda) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{d}x\, f(x)e^{i\lambda x} \ , \tag{S6}$$

where λ is the Fourier variable (*i.e.* the angular wavenumber). Taking the Fourier transform of Eqs. (S3), (S4), and (S5), we obtain:

$$\tilde{\delta} = -\frac{\tilde{p}\tilde{k}}{\mu} = \frac{-\tilde{k}\gamma\lambda^2}{\mu(1+\tilde{k}\gamma\lambda^2/\mu)}\tilde{\Delta} \ , \tag{S7}$$

$$\tilde{u_{\mathrm{s}}} = -\frac{\tilde{p}\tilde{k_{\mathrm{s}}}}{\mu} = \frac{-\tilde{k_{\mathrm{s}}}\gamma\lambda^2}{\mu(1+\tilde{k}\gamma\lambda^2/\mu)}\tilde{\Delta} \ , \tag{S8}$$

$$\frac{\partial\tilde{\Delta}}{\partial t} = -\Omega(\lambda)\tilde{\Delta} \ , \tag{S9}$$

and:

$$\Omega(\lambda) = \frac{\gamma\lambda^4 h_0^3}{3\eta}\,\frac{1}{1+(\gamma\lambda^2/\mu)\left(\tilde{k}+i\lambda h_0\tilde{k_{\mathrm{s}}}\right)} \ . \tag{S10}$$

The solution of Eq. (S9) is:

$$\tilde{\Delta}(\lambda,t) = \tilde{\Delta}(\lambda,0)\exp[-\Omega(\lambda)t] = -\frac{h_2}{2i\lambda}\sqrt{\frac{2}{\pi}}\,\exp[-\Omega(\lambda)t] \ , \tag{S11}$$

where we have used the initial conditions $\Delta(x,0) = h_2 H(x)$ (see section I) and $\delta(x,0) = 0$. Finally, using Eq. (S7), one has:

$$\tilde{\Delta}(\lambda,t) + \tilde{\delta}(\lambda,t) = \frac{\tilde{\Delta}(\lambda,t)}{1+\tilde{k}\gamma\lambda^2/\mu} \ . \tag{S12}$$

Therefore, once the Green's functions $\tilde{k}$ and $\tilde{k_{\mathrm{s}}}$ are determined (see Section III C), the displacement $h(x,t) - h_0 = \Delta(x,t)+\delta(x,t)$ of the air-liquid interface with respect to its equilibrium position can be obtained by taking the inverse Fourier transform of Eq. (S12).

C. Green's functions for the elastic layer

We consider an incompressible and linear elastic layer of thickness s_0 supported on a rigid substrate (the latter is located at $z = -s_0$, see Fig. 1(a)). The deformation state of the elastic layer is that of plane strain, where the out-of-plane (*i.e.* along y, see Fig. 1(a))) displacement is identically zero. The horizontal and vertical displacement fields, $u_x(x,z,t)$ and $u_z(x,z,t)$ respectively, are both fixed to zero at the rigid substrate:

$$u_x(x,-s_0,t) = 0 \ , \tag{S13a}$$

$$u_z(x,-s_0,t)=0\ . \tag{S13b}$$

On the other side of the layer, the liquid-elastic interface (located at $z=0$ at zeroth order in the perturbation, see Fig. 1(a)) is subjected to the lubrication pressure field $p(x,t)$, but we assume no shear which is valid at leading lubrication order. Therefore, one has:

$$\sigma_{zz}(x,0,t)=-p(x,t)\ , \tag{S14a}$$

$$\sigma_{xz}(x,0,t)=0\ . \tag{S14b}$$

In plane strain, the stresses are given by the Airy stress function $\phi(x,z,t)$ which satisfies the spatial biharmonic equation. Specifically:

$$\sigma_{xx}=\frac{\partial^2\phi}{\partial z^2}\quad,\quad \sigma_{zz}=\frac{\partial^2\phi}{\partial x^2}\quad\text{and}\quad \sigma_{xz}=-\frac{\partial^2\phi}{\partial x\partial z}\ . \tag{S15}$$

The generalized Hooke's law for an incompressible material in plane strain reads:

$$2\mu\partial_z u_z=\sigma_{zz}-\Gamma\ , \tag{S16a}$$

$$2\mu\partial_x u_x=\sigma_{xx}-\Gamma\ , \tag{S16b}$$

$$\mu(\partial_x u_z+\partial_z u_x)=\sigma_{xz}\ , \tag{S16c}$$

where $\Gamma(x,z,t)$ is the pressure needed to enforce incompressibility, that can be found using the incompressibility condition:

$$\partial_x u_x+\partial_z u_z=0\quad\Rightarrow\quad \Gamma=\frac{\sigma_{xx}+\sigma_{zz}}{2}\ . \tag{S17}$$

Combining the above, and using the same Fourier-transform convention as in the previous section, we find the following relations:

$$\tilde\sigma_{xx}=\tilde\phi''\quad,\quad \tilde\sigma_{zz}=-\lambda^2\tilde\phi\quad\text{and}\quad \tilde\sigma_{xz}=i\lambda\tilde\phi'\ , \tag{S18}$$

$$2\mu\tilde u_z'=-\frac{\tilde\phi''+\lambda^2\tilde\phi}{2}\ , \tag{S19a}$$

$$-2i\lambda\mu\tilde u_x=\frac{\tilde\phi''+\lambda^2\tilde\phi}{2}\ , \tag{S19b}$$

$$\mu(-i\lambda\tilde u_z+\tilde u_x')=i\lambda\tilde\phi'\ , \tag{S19c}$$

where the prime denotes the partial derivative with respect to z. Taking the Fourier transform of the spatial biharmonic equation results in a fourth-order ordinary differential equation:

$$\lambda^4\tilde\phi-2\lambda^2\tilde\phi''+\tilde\phi''''=0\ , \tag{S20}$$

whose general solution is:

$$\tilde\phi(\lambda,z,t)=A(\lambda,t)\cosh(\lambda z)+B(\lambda,t)\sinh(\lambda z)+C(\lambda,t)z\cosh(\lambda z)+D(\lambda,t)z\sinh(\lambda z)\ . \tag{S21}$$

The parameters A, B, C, D are determined using the boundary conditions (Eqs. (S13a), (S13b), (S14a), and (S14b)) and the relations between the Airy stress function and the stresses/displacements (Eqs. (S18) and (S19)). After some algebra, we find:

$$A=\frac{\tilde p}{\lambda^2}\quad,\quad B=\frac{\tilde p}{\lambda^2}\,\frac{\sinh(\lambda s_0)\cosh(\lambda s_0)-\lambda s_0}{\cosh^2(\lambda s_0)+(\lambda s_0)^2}\quad,\quad C=-\lambda B\quad\text{and}\quad D=-\frac{\tilde p}{\lambda}\,\frac{\cosh^2(\lambda s_0)}{\cosh^2(\lambda s_0)+(\lambda s_0)^2}\ . \tag{S22}$$

Then, invoking Eqs. (S19b), (S19c), (S21) and (S22) the vertical displacement $\delta(x,t) = u_z(x,0,t)$ of the liquid-elastic interface reads in Fourier space:

$$\tilde{\delta}(\lambda,t) = \frac{1}{i\lambda}\left(\tilde{u}_x' - \frac{i\lambda\tilde{\phi}'}{\mu}\right)(\lambda,0,t) = -\frac{\tilde{p}}{2\mu\lambda}\,\frac{\sinh(\lambda s_0)\cosh(\lambda s_0) - \lambda s_0}{\cosh^2(\lambda s_0) + (\lambda s_0)^2}\ . \tag{S23}$$

Using Eqs. (S7) and (S23), we find:

$$\tilde{k}(\lambda) = \frac{1}{2\lambda}\,\frac{\sinh(\lambda s_0)\cosh(\lambda s_0) - \lambda s_0}{\cosh^2(\lambda s_0) + (\lambda s_0)^2}\ . \tag{S24}$$

In exactly the same way, the horizontal displacement $u_{\mathrm{s}}(x,t) = u_x(x,0,t)$ of the liquid-elastic interface reads in Fourier space:

$$\tilde{u_{\mathrm{s}}}(\lambda,t) = i\frac{\lambda^2\tilde{\phi} + \tilde{\phi}''}{4\mu\lambda}(\lambda,0,t) = \frac{i\tilde{p}}{2\mu}\,\frac{\lambda s_0^2}{\cosh^2(\lambda s_0) + (\lambda s_0)^2}\ , \tag{S25}$$

which gives:

$$\tilde{k_{\mathrm{s}}}(\lambda) = \frac{1}{2i}\,\frac{\lambda s_0^2}{\cosh^2(\lambda s_0) + (\lambda s_0)^2}\ . \tag{S26}$$

IV. STOKES-ELASTIC MODEL

The previous lubrication-elastic model assumes that the typical vertical length scale of the flow is much smaller than the horizontal one. However, the initial stepped interface and thus the early-time profiles are not compatible with this criterion. Therefore, we now instead solve the incompressible Stokes' equations for the liquid layer, in order to go beyond the lubrication approximation.

A. Hydrodynamic description of the liquid layer

We introduce the 2D stream function ψ that is related to the velocity field $\vec{v}$ via the relation $\vec{v} = \vec{\nabla} \times (\psi\vec{e_y})$ with $\vec{e_y}$ the out-of-plane unit vector and $\vec{\nabla}\times .$ the curl operator. Similarly to the Airy stress function, the stream function verifies a biharmonic equation. The kinematic and no-slip conditions at the liquid-elastic interface (located at $z = 0$ at zeroth order in the perturbation, see Fig. 1(a)) imply, respectively:

$$v_z(x,0,t) = \partial_x\psi(x,0,t) = \partial_t u_z(x,0,t) = \partial_t\delta(x,t)\ , \tag{S27a}$$

$$v_x(x,0,t) = -\partial_z\psi(x,0,t) = \partial_t u_x(x,0,t) = \partial_t u_{\mathrm{s}}(x,t)\ . \tag{S27b}$$

In addition, at the air-liquid interface (located at $z = h_0$ at zeroth order in the perturbation, see Figs. 1(a) and S1), we assume no shear and the pressure is set by the Laplace pressure. The continuity of stress thus gives:

$$\sigma_{xz}(x,h_0,t) = \eta(\partial_{xx}\psi - \partial_{zz}\psi)(x,h_0,t) = 0\ , \tag{S28a}$$

$$\sigma_{zz}(x,h_0,t) = -\mathcal{P}(x,h_0,t) + 2\eta\partial_z(\partial_x\psi)(x,h_0,t) = -p(x,t)\ . \tag{S28b}$$

with $\mathcal{P}(x,z,t)$ the excess pressure (with respect to the atmospheric pressure) in the liquid, and $p(x,t) = -\gamma\partial_{xx}h$ the Laplace pressure. Note that we neglect all nonlinear terms in $\partial_x h$ that come from the curvature in the Laplace pressure and the projection of the normal and tangential vectors onto the x and z axes. Now, we employ a similar method as the one developed in the previous lubrication-elastic model, and first take the Fourier transform of the biharmonic equation satisfied by the stream function:

$$\lambda^4\tilde{\psi} - 2\lambda^2\tilde{\psi}'' + \tilde{\psi}'''' = 0\ , \tag{S29}$$

whose general solution is:

$$\tilde{\psi}(\lambda, z, t) = A_2(\lambda, t)\cosh(\lambda z) + B_2(\lambda, t)\sinh(\lambda z) + C_2(\lambda, t) z\cosh(\lambda z) + D_2(\lambda, t) z\sinh(\lambda z) \ . \tag{S30}$$

Taking the Fourier transforms of the boundary conditions (Eqs. (S27) and (S28)), we find:

$$-i\lambda A_2 = \partial_t \tilde{\delta} \ . \tag{S31a}$$

$$\lambda B_2 = -\frac{i\tilde{p}}{2\eta\lambda}\frac{\sinh(\lambda h_0)\lambda h_0 + \cosh(\lambda h_0)}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} - i\partial_t\tilde{\delta}\frac{\sinh(\lambda h_0)\cosh(\lambda h_0) - \lambda h_0}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} - \partial_t \tilde{u_{\mathrm{s}}}\frac{(\lambda h_0)^2}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} \ , \tag{S31b}$$

$$C_2 = \frac{i\tilde{p}}{2\eta\lambda}\frac{\sinh(\lambda h_0)\lambda h_0 + \cosh(\lambda h_0)}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} + i\partial_t\tilde{\delta}\frac{\sinh(\lambda h_0)\cosh(\lambda h_0) - \lambda h_0}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} - \partial_t \tilde{u_{\mathrm{s}}}\frac{\cosh^2(\lambda h_0)}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} \ , \tag{S31c}$$

$$D_2 = \frac{-ih_0\tilde{p}}{2\eta}\frac{\cosh(\lambda h_0)}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} - i\partial_t\tilde{\delta}\frac{\cosh^2(\lambda h_0)}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} + \partial_t \tilde{u_{\mathrm{s}}}\frac{\lambda h_0 + \sinh(\lambda h_0)\cosh(\lambda h_0)}{\cosh^2(\lambda h_0) + (\lambda h_0)^2} \ . \tag{S31d}$$

Finally, we note that the pressure $\mathcal{P}(x, z, t)$ is entirely determined by the stream function. Indeed, in Fourier space, and invoking the stream function, the x-projection of the Stokes' equation reads:

$$i\lambda\tilde{\mathcal{P}} = \eta\left(\tilde{\psi}''' - \lambda^2\tilde{\psi}'\right) \ . \tag{S32}$$

B. Coupling with the elastic layer

As in the previous lubrication-elastic model, we solve the elastic part of the problem by introducing the Airy stress function ϕ given by Eq. (S21) in Fourier space. Assuming no displacement at the interface between the elastic layer and the rigid substrate (located at $z = -s_0$, see Fig. 1(a)), one has:

$$u_x(x, -s_0, t) = 0 \ , \tag{S33a}$$

$$u_z(x, -s_0, t) = 0 \ . \tag{S33b}$$

Equation (S19) can be used to relate the boundary conditions (Eq. (S33)) to the parameters A, B, C, D (Eq. (S21)). After some algebra, one finds:

$$\lambda A = 2\mu\,\frac{i\tilde{u_{\mathrm{s}}}(\lambda s_0)^2 - \tilde{\delta}[\cosh(\lambda s_0)\sinh(\lambda s_0) + \lambda s_0]}{\sinh^2(\lambda s_0) - (\lambda s_0)^2} \ , \tag{S34a}$$

$$\lambda B = -2\mu\tilde{\delta} \ , \tag{S34b}$$

$$C = 2\mu\,\frac{-i\tilde{u_{\mathrm{s}}}[\cosh(\lambda s_0)\sinh(\lambda s_0) - \lambda s_0] + \sinh^2(\lambda s_0)\tilde{\delta}}{\sinh^2(\lambda s_0) - (\lambda s_0)^2} \ , \tag{S34c}$$

$$D = 2\mu\,\frac{-i\tilde{u_{\mathrm{s}}}\sinh^2(\lambda s_0) + \tilde{\delta}[\cosh(\lambda s_0)\sinh(\lambda s_0) + \lambda s_0]}{\sinh^2(\lambda s_0) - (\lambda s_0)^2} \ . \tag{S34d}$$

At the liquid-elastic interface (located at $z = 0$ at zeroth order in the perturbation, see Fig. 1(a)), the normal-stress continuity reads:

$$-\mathcal{P}(x, 0, t) + 2\eta\partial_z(\partial_x\psi)(x, 0, t) = \partial_{xx}\phi(x, 0, t) \ , \tag{S35}$$

or, equivalently, in Fourier space:

$$\tilde{\mathcal{P}}(\lambda,0,t)+2i\lambda\eta\tilde{\psi}'(\lambda,0,t)=\lambda^2\tilde{\phi}(\lambda,0,t)\ . \tag{S36}$$

Then, by taking the $z\to 0$ limit of Eq. (S32) and by combining it with Eqs. (S21), (S30), and (S36), one obtains:

$$-2i\eta\lambda^2 B_2=-\lambda^2 A\ . \tag{S37}$$

Invoking Eqs. (S22) and (S31), Eq. (S37) becomes:

$$\begin{aligned}&-\tilde{p}\ \frac{\cosh(\lambda h_0)+\lambda h_0\sinh(\lambda h_0)}{\cosh^2(\lambda h_0)+(\lambda h_0)^2}-2\eta\lambda\partial_t\tilde{\delta}\frac{\cosh(\lambda h_0)\sinh(\lambda h_0)-\lambda h_0}{\cosh^2(\lambda h_0)+(\lambda h_0)^2}+2i\eta\lambda\partial_t\tilde{u_\mathrm{s}}\frac{(\lambda h_0)^2}{\cosh^2(\lambda h_0)+(\lambda h_0)^2}\\&=2\lambda\mu\frac{-i\tilde{u_\mathrm{s}}(\lambda s_0)^2+\tilde{\delta}[\cosh(\lambda s_0)\sinh(\lambda s_0)+\lambda s_0]}{\sinh^2(\lambda s_0)-(\lambda s_0)^2}\ .\end{aligned} \tag{S38}$$

For simplicity, we neglect the terms of order $T\partial_t\tilde{\delta}$ or $T\partial_t\tilde{u_\mathrm{s}}$ with respect to the terms of order $\tilde{\delta}$ or $\tilde{u_\mathrm{s}}$, where $T=\eta/\mu$ is a composite Maxwell-like viscoelastic time. This assumption essentially means that the elastic layer has an instantaneous response to the applied stress, or that we decouple the fast and slow dynamics and focus on the latter. This is relevant in our case since T is much smaller than the experimental time scale (see inset of Fig. 4). Doing so, we get in Fourier space:

$$-\tilde{p}\ \frac{\cosh(\lambda h_0)+\lambda h_0\sinh(\lambda h_0)}{\cosh^2(\lambda h_0)+(\lambda h_0)^2}=2\lambda\mu\frac{-i\tilde{u_\mathrm{s}}(\lambda s_0)^2+\tilde{\delta}[\cosh(\lambda s_0)\sinh(\lambda s_0)+\lambda s_0]}{\sinh^2(\lambda s_0)-(\lambda s_0)^2}\ . \tag{S39}$$

Besides, the tangential-stress continuity reads:

$$\eta(\partial_{xx}\psi-\partial_{zz}\psi)(x,0,t)=-\partial_{xz}\phi(x,0,t)\ , \tag{S40}$$

and thus, with a similar treatment, one gets in Fourier space:

$$\tilde{p}\ \frac{\lambda h_0\cosh(\lambda h_0)}{\cosh^2(\lambda h_0)+(\lambda h_0)^2}=2\lambda\mu\frac{-i\tilde{u_\mathrm{s}}(\cosh(\lambda s_0)\sinh(\lambda s_0)-\lambda s_0)+\tilde{\delta}(\lambda s_0)^2}{\sinh^2(\lambda s_0)-(\lambda s_0)^2}\ . \tag{S41}$$

By analogy with Eqs. (S5a) and (S5b) of the previous lubrication-elastic model, we introduce two new Green's functions $k_2(x)$ and $k_{\mathrm{s}2}(x)$. Equations (S39) and (S41) thus lead to:

$$\tilde{\delta}=-\frac{\tilde{p}\tilde{k_2}}{\mu}=\frac{-\tilde{p}}{2\mu\lambda}\ \frac{(\lambda s_0)^2(\lambda h_0)\cosh(\lambda h_0)+[\sinh(\lambda h_0)\lambda h_0+\cosh(\lambda h_0)][\cosh(\lambda s_0)\sinh(\lambda s_0)-\lambda s_0]}{[\cosh^2(\lambda h_0)+(\lambda h_0)^2][\cosh^2(\lambda s_0)+(\lambda s_0)^2]}\ , \tag{S42}$$

$$\tilde{u_\mathrm{s}}=-\frac{\tilde{p}\tilde{k_{\mathrm{s}2}}}{\mu}=\frac{i\tilde{p}}{2\mu\lambda}\ \frac{\lambda h_0\cosh(\lambda h_0)[\cosh(\lambda s_0)\sinh(\lambda s_0)+\lambda s_0]+[\cosh(\lambda h_0)+\sinh(\lambda h_0)\lambda h_0](\lambda s_0)^2}{[\cosh^2(\lambda h_0)+(\lambda h_0)^2][\cosh^2(\lambda s_0)+(\lambda s_0)^2]}\ , \tag{S43}$$

with:

$$\tilde{k_2}(\lambda)=\frac{1}{2\lambda}\ \frac{(\lambda s_0)^2(\lambda h_0)\cosh(\lambda h_0)+[\sinh(\lambda h_0)\lambda h_0+\cosh(\lambda h_0)][\cosh(\lambda s_0)\sinh(\lambda s_0)-\lambda s_0]}{[\cosh^2(\lambda h_0)+(\lambda h_0)^2][\cosh^2(\lambda s_0)+(\lambda s_0)^2]}\ , \tag{S44}$$

$$\tilde{k_{\mathrm{s}2}}(\lambda)=\frac{1}{2i\lambda}\ \frac{\lambda h_0\cosh(\lambda h_0)[\cosh(\lambda s_0)\sinh(\lambda s_0)+\lambda s_0]+[\cosh(\lambda h_0)+\sinh(\lambda h_0)\lambda h_0](\lambda s_0)^2}{[\cosh^2(\lambda h_0)+(\lambda h_0)^2][\cosh^2(\lambda s_0)+(\lambda s_0)^2]}\ . \tag{S45}$$

The two Green's functions k_2 and $k_{\mathrm{s}2}$ have forms that are quite similar to the ones of the previous lubrication-elastic model, k and k_s (see Eqs. (S24) and (S26)). Moreover, in the lubrication limit where $\lambda h_0\to 0$, k_2 and $k_{\mathrm{s}2}$ tend towards k and k_s, respectively.

C. Temporal evolution of the air-liquid interface

Let us write the mass conservation for the liquid layer:

$$\partial_t \Delta = -\partial_x \int_{\delta(x,t)}^{h(x,t)} \mathrm{d}z\, v_x(x,z,t) = \partial_x \int_{\delta(x,t)}^{h(x,t)} \mathrm{d}z\, \partial_z \psi(x,z,t) = \partial_x \psi[x,h(x,t),t] - \partial_x \psi[x,\delta(x,t),t] \ , \tag{S46}$$

with $\Delta(x,t) = h(x,t) - \delta(x,t) - h_0$ as in the previous lubrication-elastic model. At the lowest order in the perturbation, this general expression becomes:

$$\partial_t \Delta = \partial_x \psi(x,h_0,t) - \partial_x \psi(x,0,t) \ , \tag{S47}$$

or, equivalently, in Fourier space:

$$\partial_t \tilde{\Delta} + i\lambda[\tilde{\psi}(\lambda,h_0,t) - \tilde{\psi}(\lambda,0,t)] = 0 \ . \tag{S48}$$

Using Eqs. (S3), (S42), and (S43), one gets:

$$\tilde{\delta} = \frac{-\tilde{k_2}\gamma\lambda^2}{\mu(1+\tilde{k_2}\gamma\lambda^2/\mu)}\tilde{\Delta} \ , \tag{S49}$$

$$\tilde{u_{\mathrm{s}}} = \frac{-\tilde{k_{\mathrm{s}2}}\gamma\lambda^2}{\mu(1+\tilde{k_2}\gamma\lambda^2/\mu)}\tilde{\Delta} \ . \tag{S50}$$

By injecting Eqs. (S30) and (S31) in Eq. (S48), one obtains the ordinary differential equation:

$$\partial_t \tilde{\Delta} = -\Omega_2(\lambda)\tilde{\Delta} \ , \tag{S51}$$

with:

$$\Omega_2(\lambda) = \frac{\gamma\lambda}{2\eta}\,\frac{\mathcal{A}(\lambda h_0)}{\mathcal{B}(\lambda h_0) + \frac{\gamma\lambda^2}{\mu}\mathcal{C}(\lambda h_0)} \ , \tag{S52}$$

and:

$$\mathcal{A}(\lambda h_0) = \cosh(\lambda h_0)\sinh(\lambda h_0) - \lambda h_0 \ , \tag{S53a}$$

$$\mathcal{B}(\lambda h_0) = \cosh^2(\lambda h_0) + (\lambda h_0)^2 \ , \tag{S53b}$$

$$\mathcal{C}(\lambda h_0) = \tilde{k_2}\left[\cosh(\lambda h_0) + (\lambda h_0)\sinh(\lambda h_0)\right] + i\tilde{k_{\mathrm{s}2}}\lambda h_0 \cosh(\lambda h_0) \ . \tag{S53c}$$

This differential equation can be solved with the initial condition (step of height h_2, see Fig. 1(a)):

$$\tilde{\Delta}(\lambda,0) = -\frac{h_2}{2i\lambda}\sqrt{\frac{2}{\pi}} \ , \tag{S54}$$

thus leading to:

$$\tilde{\Delta}(\lambda,t) = -\frac{h_2}{2i\lambda}\sqrt{\frac{2}{\pi}}\exp\left[-\Omega_2(\lambda)t\right] \ . \tag{S55}$$

Then, using Eq. (S49), one has:

$$\tilde{\Delta} + \tilde{\delta} = \frac{\tilde{\Delta}}{1+\gamma\lambda^2\tilde{k_2}/\mu} \ . \tag{S56}$$

Finally, the displacement $h(x,t) - h_0 = \Delta(x,t) + \delta(x,t)$ of the air-liquid interface with respect to its equilibrium position can be obtained by taking the inverse Fourier transform of Eq. (S56). Figure S3 displays the temporal evolutions of the profile width ω (see definition in the inset of Fig.2(a)), as derived from the two models presented in this supplementary material. The Stokes-elastic model exhibits the same qualitative features as the lubrication-elastic one. In particular, the width of the profile depends on elasticity only at early times, and rapidly tends to a $1/4$ power law – characteristic of the rigid-substrate case. This result suggests that the lubrication approximation, which is not valid at early times, is not responsible for the discrepancy between the lubrication-elastic model and the experiments reported in the main text.

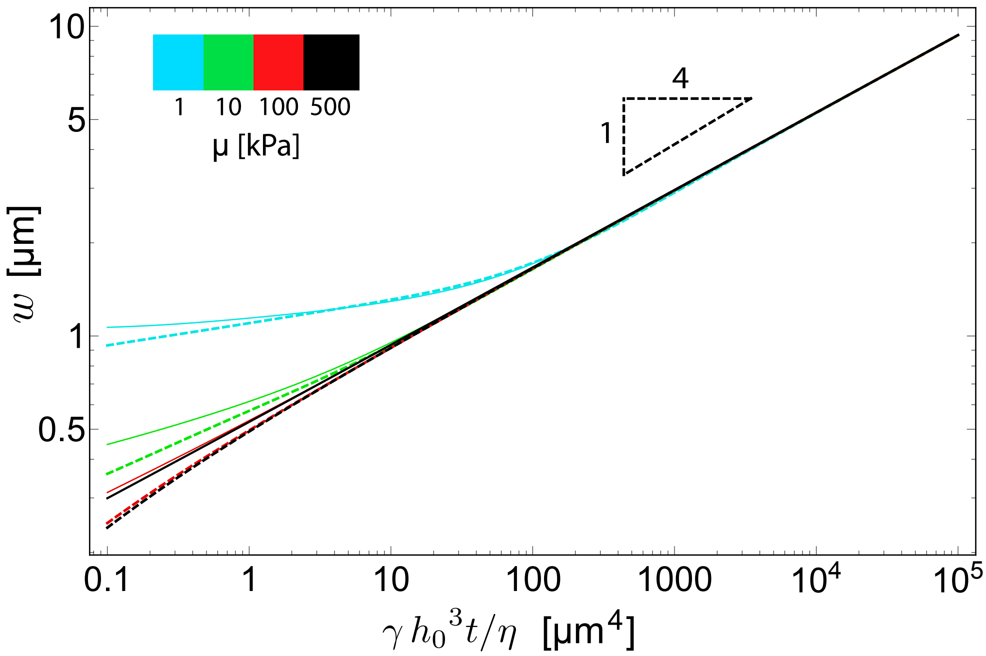

FIG. S3: Temporal evolution of the profile width (defined in the inset of Fig. 2(a)), in log-log scale, as predicted by both theoretical models, for different shear moduli, viscosities and liquid-film thicknesses. The 1/4 power law corresponding to a rigid substrate is indicated. The solid lines represent the lubrication-elastic model, and the dashed lines represent the Stokes-elastic model. The shear moduli are given by the color code, which is identical to the one in Fig. 6. All the other parameters are identical to the ones used in Fig. 6.