Title	Proceedings of the 6th Pacific RIM Conference on Mathematics 2013
Author(s)	Giga, Y.; Jimbo, S.; Yamaguchi, K.
Citation	Hokkaido University technical report series in mathematics, 158, i, 1-iii, 146
Issue Date	2013-06-01
DOI	10.14943/81520
Doc URL	http://hdl.handle.net/2115/68082
Type	bulletin (article)
File Information	tech158.pdf

Proceedings of the 6th Pacific RIM Conference on Mathematics 2013

Organizers:
Y. Giga, S. Jimbo, H. Terao, K. Yamaguchi

Series #158. July, 2013

HOKKAIDO UNIVERSITY
TECHNICAL REPORT SERIES IN MATHEMATICS
http://eprints3.math.sci.hokudai.ac.jp/view/type/techreport.html

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#157 D. Suyama, T. ito, M. Kuroda, Y. goto, N. Teranishi, S. Futakuchi, T. Fuda and N. Miyagwa, 第 9 回数学総合若手研究集会, 344 pages. 2013.

Proceedings of the 6th Pacific RIM Conference on Mathematics 2013

Edited by
Y. Giga, S. Jimbo, H. Terao, K. Yamaguchi

Sapporo Convention Center

July 1 – July 5, 2013

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PREFACE

We welcome you to a great event for the mathematics: The 6th Pacific Rim Conference on Mathematics. This volume is intended as the proceeding of the 6th Pacific Rim Conference on Mathematics, held for the period of July 1 - 5, 2013 at Sapporo Convention Center.

The Pacific Rim Conference on Mathematics, organized and supported by major universities and research institutions in the Pacific Rim region, has been held triannually from 1998 to present the latest topics in various areas of mathematics. Past meetings were held in Hong Kong (1998), Taipei (2001), Shanghai (2005), Hong Kong (2007), Stanford (2010). This is the sixth meeting.

Importance of mathematics is significantly increasing in various areas of science and technology. This is because a key innovation often depends on innovation of mathematics. However, it is difficult, even for mathematicians, to keep pace with rapid development of mathematics. With several focusing sessions this conference emphasizes survey lectures by world-leading specialists on various areas of pure and applied mathematics to boost interactions and in-depth discussions among researchers working in mathematics and related fields.

We hope you enjoy the Pacific Rim Conference on Mathematics as well as your stay in Sapporo.

Organizers:
Yoshikazu Giga (University of Tokyo), Keizo Yamaguchi (President, Hokkaido University)
Local Organizers:
Shuichi Jimbo (Hokkaido University), Hiroaki Terao (Dean of Faculty of Sciences, Hokkaido University)

Acknowledgments

The 6th Pacific RIM Conference on Mathematics 2013 Organizing Committee would like to extend its sincere gratitude to the following associations and organizations for their valuable support and commitment to making this event successful.

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Session (I) “Spectral and Scattering Theory” is also supported by

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Session (L) “Topological Problems in Fluid Dynamics” is also supported by

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(14) Tohoku University
(15) Meiji University
(16) Hokkaido University
(17) University of Tokyo

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Tsuyoshi Yoneda (Hokkaido University)

Program (Plenary Lectures)

All plenary lectures will be held in Small Hall.

I Weixiao Shen (National University of Singapore) July 1 (Mon.), 10:05 – 10:50
On Stability of One-Dimensional Dynamics
Chairperson: Kuo-Chang Chen

II Hideo Tamura (Okayama University) July 1 (Mon.), 11:15 – 12:00
Aharonov-Bohm effect in resonances of magnetic Schrödinger operators in two dimensions
Chairperson: Claudio Fernandez

III Alan Carey (Australian National University) July 2 (Tue.), 10:05 – 10:50
Geometric Cycles and D-Branes
Chairperson: Bai-Ling Wang

IV Yoshio Sone (Kyoto University) July 2 (Tue.), 11:15 – 12:00
Fluid-dynamic-type equations derived from the Boltzmann equation for small Knudsen numbers and their boundary conditions
Chairperson: Shih-Hsien Yu

V Russel Caflisch (UC Los Angeles) July 3 (Wed.), 9:15 – 10:00
From Natural Science to Information Science and Back
Chairperson: Yoshikazu Giga

VI Izumi Takagi (Tohoku University) July 3 (Wed.), 10:05 – 10:50
Point-condensation phenomenon in a reaction-diffusion system: geometry of domain vs heterogeneity of media
Chairperson: Takayoshi Ogawa

VII Ben Andrews (Australian National University) July 3 (Wed.), 11:15 – 12:00
Minimal and Constant Mean Curvature Surfaces in the Three-Sphere: Brendle's Proof of the Lawson Conjecture
Chairperson: Sumio Yamada

VIII Jongil Park (Seoul National University) July 4 (Thu.), 9:15 – 10:00

The geography problems of 4-manifolds

Chairperson: Cheol-Hyun Cho

IX Doron Lubinsky (Georgia Institute of Technology) July 4 (Thu.), 10:05 – 10:50

Pushing Polynomial Reproducing Kernels to their Non-polynomial Limit

Chairperson: Roderick Wong

X Toshitake Kohno (University of Tokyo) July 4 (Thu.), 11:15 – 12:00

Quantum symmetry in homological representations of braid groups and hypergeometric integrals

Chairperson: Alejandro Adem

XI Bill Barton (The University of Auckland) July 5 (Fri.), 10:05 – 10:50

Being like Sakamoto Hayato: Lecturers as professionals

Chairperson: Ryosuke Nagaoka

XII Yujiro Kawamata (University of Tokyo) July 5 (Fri.), 11:15 – 12:00

Derived categories in algebraic geometry

Chairperson: Masa-Hiko Saito

Program (Sessions and Poster presentation)

July 1 (Mon.), 14:00 – 15:00, 15:30 – 16:30

July 2 (Tue.), 14:00 – 15:00, 15:30 – 16:30

(B) Dynamical Systems, Room 104＆105

Organizer: Kuo-Chang Chen (National Tsing Hua University)

Session local organizer: Zin Arai (Hokkaido University)

(E) Mathematical Physics, Room 206

Organizer: Bai-Ling Wang (Australian National University)

Session local organizer: Masao Jinzenji (Hokkaido University)

(F) Differential Geometry, Small Hall

Organizer: Richard Schoen (Stanford University)

Session local organizer: Yoshihiro Tonegawa (Hokkaido University)

(Note that the detailed time table is not as indicated above.)

(G) Topology and Related Topics, Room207

Organizers: Toshitake Kohno (University of Tokyo), Alejandro Adem (Pacific Institute for the Mathematical Sciences)

Session local organizer: Hiroaki Terao (Hokkaido University)

(I) Spectral and Scattering Theory, Room 107

Organizer: Claudio Fernandez (Pontificia Universidad Católica de Chile)

Session local organizer: Asao Arai (Hokkaido University)

(L) Topological Problems in Fluid Dynamics, Room 108

Organizer: Takashi Sakajo (Kyoto University)

Session local organizer: Tsuyoshi Yoneda (Hokkaido University)

(M) Mathematical Aspects of Crystal Growth and Image Analysis, Room 204

Organizers: Russel Caflisch (University of California, Los Angeles), Yoshikazu Giga (University of Tokyo)

Session local organizer: Shuichi Jimbo (Hokkaido University)

July 1 (Mon.), 16:30 – 17:00 Room 204 Poster presentation

July 2 (Tue.), 16:30 – 17:00 Room 204 Poster presentation

Program (Sessions and Poster presentation)

July 4 (Thu.), 14:00 – 15:00, 15:30 – 16:30

July 5 (Fri.), 14:00 – 15:00, 15:30 – 16:30

(A) Special Functions and Orthogonal Polynomials, Room 107

Organizer: Dan Dai (City University of Hong Kong)

Session local organizer: Hiroaki Terao (Hokkaido University)

(C) Symplectic Topology, Room 206

Organizer: Cheol-Hyun Cho (Seoul National University)

Session local organizer: Masao Jinzenji (Hokkaido University)

(D) Kinetic and Hyperbolic Equations, Small Hall

Organizer: Shih-Hsien Yu (National University of Singapore)

Session local organizer: Hideo Kubo (Hokkaido University)

(H) Undergraduate Mathematics Education, Room 108

Organizers: Judy Paterson, Mike Thomas, Bill Barton (The University of Auckland)

(This short session will be finished in July 4.)

(J) Algebraic Geometry, Room 207

Organizers: Masa-Hiko Saito (Kobe University), Yoshinori Namikawa (Kyoto University), Shigeru Mukai (Kyoto University)

Session local organizer: Iku Nakamura (Hokkaido University)

(Note that the detailed time table of this session is not as indicated above.)

(K) Elliptic and Parabolic Equations, Room 204

Organizer: Kazuhiro Ishige (Tohoku University), Takayoshi Ogawa (Tohoku University), Yoshihiro Tonegawa (Hokkaido University)

Session local organizer: Yoshihiro Tonegawa (Hokkaido University)

Program of Session (A)

(A) Special Functions and Orthogonal Polynomials, Room 107

Organizer: Dan Dai (City University of Hong Kong)

Session local organizer: Hiroaki Terao (Hokkaido University)

July 4 (Thu.), 14:00 – 14:25, Room 107

Roderick Wong (City University of Hong Kong)
Stieltjes-Wigert Polynomials and the q-Airy Function

July 4 (Thu.), 14:30 – 14:55, Room 107
Adri Olde Daalhuis (University Edinburgh, UK)

Exponentially-accurate uniform asymptotic approximations for integrals and Bleistein's method revisited

July 4 (Thu.), 15:30 – 15:55, Room 107
Walter Van Assche (Katholieke Universiteit Leuven, Belgium)
Ratio asymptotics and zero distribution for multiple orthogonal polynomials

July 4 (Thu.), 16:00 – 16:25, Room 107
Yoshitsugu Takei (Kyoto University, Japan)

On the turning point problem for Painlevé equations with a large parameter

July 5 (Fri.), 14:00 – 14:25, Room 107

Peter Clarkson (University of Kent, UK)

On the relationship between the Painlevé equations and semi-classical orthogonal polynomials

July 5 (Fri.), 14:30 – 14:55, Room 107
Arno Kuijlaars (Katholieke Universiteit Leuven, Belgium)
The tacnode Riemann-Hilbert problem

July 5 (Fri.), 15:30 – 15:55, Room 107
Yousuke Ohyama (Osaka University, Japan)

A connection problem for linear q-difference equations related to the q-Painlevé VI equation

July 5 (Fri.), 16:00 – 16:25, Room 107
Dan Dai (City University of Hong Kong)

Plancherel-Rotach asymptotic expansion for some polynomials from indeterminate moment problems

Program of Session (B)

(B) Dynamical Systems, Room 104＆105

Organizer: Kuo-Chang Chen (National Tsing Hua University)

Session local organizer: Zin Arai (Hokkaido University)

July 1 (Mon.), 14:00 – 14:25, Room 104＆105

Juan Eduardo Rivera Letelier (Pontificia Universidad Catolica de Chile, Chile)

Equilibrium states and large deviation principles for one-dimensional maps under a weak hyperbolicity assumption

July 1 (Mon.), 14:30 – 14:55, Room 104＆105

Yutaka Ishii (Kyushu University, Japan)

On parameter loci of the Hénon family

July 1 (Mon.), 15:30 – 15:55, Room 104＆105

Wen Huang (Chinese University of Science and Technology, China)

Stable Sets in Z^n-Systems with Positive Entropy

July 1 (Mon.), 16:00 – 16:25, Room 104＆105

Jung-Chao Ban (National Dong Hwa University, Taiwan)

On the Minkowski dimensions of multi-dimensional and coupled multiplicative systems

July 2 (Tue.), 14:00 – 14:2514:30, Room 104＆105

Hiroki Takahasi (Kyoto University, Japan)

Prevalence of non-uniform hyperbolicity at the first bifurcation of Hénon-like families

July 2 (Tue.), 14:30 – 14:55, Room 104＆105

Zin Arai (Hokkaido University, Japan)

On the monodromy and bifurcations of the Hénon map

July 2 (Tue.), 15:30 – 15:55, Room 104＆105

Kuo-Chang Chen (National Tsing Hua University, Taiwan)

On the barycenter set of some one-dimensional maps

Program of Session (C)

(C) Symplectic Topology, Room 206

Organizer: Cheol-Hyun Cho (Seoul National University)

Session local organizer: Masao Jinzenji (Hokkaido University)

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Yakov Eliashberg (Stanford University, USA)
Lagrangian non-intersection theory

July 4 (Thu.), 14:30 – 14:55, Room 206
Viktor Ginzburg (University of California, Santa Cruz, USA)
Periodic Orbits of Hamiltonian Systems: Beyond the Conley Conjecture

July 4 (Thu.), 15:30 – 15:55, Room 206
Urs Frauenfelder (Seoul National University, Korea)
A Gamma-structure on the Lagrangian Grassmannian

July 4 (Thu.), 16:00 – 16:25, Room 206
Kaoru Ono (Research Institute for Mathematical Sciences, Japan)
Non-displaceable Lagrangian submanifolds

July 5 (Fri.), 14:00 – 14:25, Room 206
Conan Leung (Chinese University of Hong Kong, China)
SYZ transformation for coisotropic A-branes

July 5 (Fri.), 14:30 – 14:55, Room 206
Sheel Ganatra (Stanford University, USA)
Symplectic cohomology and duality for the wrapped Fukaya category

July 5 (Fri.), 15:30 – 15:55, Room 206
Hansol Hong (Seoul National University, Korea)
Finite Group Actions and Lagrangian Floer Theory

Program of Session (D)

(D) Kinetic and Hyperbolic Equations, Small Hall

Organizer: Shih-Hsien Yu (National University of Singapore)

Session local organizer: Hideo Kubo (Hokkaido University)

July 4 (Thu.), 14:00 – 14:25, Small Hall
Shijin Deng (Shanghai Jiao Tong University, China)
Boltzmann equation and Green's function for shock profiles

July 4 (Thu.), 14:30 – 14:55, Small Hall
Jin-Cheng Jiang (National Tsing Hua University, Taiwan)
Collision operator of Boltzmann equation

July 4 (Thu.), 15:30 – 15:55, Small Hall
Tai-Ping Liu (Stanford University, USA / Academia Sinica, Taiwan)
Gas Dynamics and Kinetic Theory, Some Historical Perspectives

July 4 (Thu.), 16:00 – 16:25, Small Hall
Shigeru Takata (Kyoto University, Japan)
Some applications of symmetry relations for the steady/unsteady linearized Boltzmann equation

July 5 (Fri.), 14:00 – 14:25, Small Hall
Tong Yang (City University of Hong Kong, Hong Kong)
Smoothing effect for the homogeneous Boltzmann Equation

July 5 (Fri.), 14:30 – 14:55, Small Hall
Shih-Hsien Yu (National University of Singapore, Singapore)
Viscous wave propagation at interface

July 5 (Fri.), 15:30 – 15:55, Small Hall
Seung Yeal Ha (Seoul National University, Korea)
L^p-scattering and uniform stability of kinetic equations

Program of Session (E)

(E) Mathematical Physics, Room 206

Organizer: Bai-Ling Wang (Australian National University)

Session local organizer: Masao Jinzenji (Hokkaido University)

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Raphael Ponge (Seoul National University)

Vafa-Witten inequality and Poincare duality in noncommutative geometry

July 1 (Mon.), 14:30 – 14:55, Room 206

Bai-Ling Wang (Australian National University)

Virtual neighborhood technique for pseudo-holomorphic spheres

July 1 (Mon.), 15:30 – 15:55, Room 206

Kaoru Ono (Research Institute for Mathematical Sciences, Japan)

Lagrangian Floer theory and mirror symmetry on compact toric manifolds

July 1 (Mon.), 16:00 – 16:25, Room 206

Masao Jinzenji (Hokkaido University, Japan)

Mirror Map as Generating Function of Intersection Numbers

July 2 (Tue.), 14:00 – 14:25, Room 206

Kiyonori Gomi (Shinshu University, Japan)

Mickelsson's twisted K-theory invariant

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Siye Wu (Hong Kong University)

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Ruibin Zhang (University of Sydney, Australia)

Brauer category and fundamental theorems of classical invariant theory

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Toshitake Kohno (University of Tokyo, Japan)

Monodromy groups of conformal field theory

Program of Session (F)

(F) Differential Geometry, Small Hall

Organizer: Richard Schoen (Stanford University)

Session local organizer: Yoshihiro Tonegawa (Hokkaido University)

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Richard Schoen (Stanford University, USA)

A new method of constructing metrics on surfaces which maximize the first eigenvalue

July 1 (Mon.), 14:30 – 15:15, Small Hall

Haizhong Li (Tsinghua University, China)

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July 1 (Mon.), 15:30 – 16:15, Small Hall

Sumio Yamada (Gakushuin University, Japan)

On variational characterizations of exact solutions in general relativity

July 2 (Tue.), 14:00 – 14:45, Small Hall

Pin Yu (Tsinghua University, China)

Cauchy Data of Vacuum Einstein Equations Evolving to Black Holes

July 2 (Tue.), 15:00 – 15:45, Small Hall

Glen Wheeler (University of Wollongong, Australia)

The Willmore flow in Riemannian spaces

July 2 (Tue.), 16:00 – 16:45, Small Hall

Guoyi Xu (University of California, Irvine, USA)

An equation linking W-entropy with reduced volume

Program of Session (G)

(G) Topology and Related Topics, Room 207

Organizers: Toshitake Kohno (University of Tokyo), Alejandro Adem (Pacific Institute for the Mathematical Sciences)

Session local organizer: Hiroaki Terao (Hokkaido University)

July 1 (Mon.), 14:00 – 14:50, Room 207

Mario Salvetti (Universita di Pisa, Italy)

Some topological problems and computational methods in the theory of braids and related groups

July 1 (Mon.), 15:30 – 15:55, Room 207

Alejandro Adem (University of British Columbia, Canada)

Topology of Commuting Matrices

July 1 (Mon.), 16:00 – 16:25, Room 207

Masahiko Yoshinaga (Hokkaido University, Japan)

Milnor fibers of real line arrangements

July 2 (Tue.), 14:00 – 14:50, Room 207

Hiroaki Terao (Hokkaido University, Japan)

The freeness of ideal subarrangements of Weyl arrangements

July 2 (Tue.), 15:30 – 15:25, Room 207

Ryan Budney (University of Victoria, Canada)

Some maps out of spaces-of-embeddings

July 2 (Tue.), 16:00 – 16:25, Room 207

Tadayuki Watanabe (Shimane University, Japan)

Morse homotopy and invariants of manifolds

Program of Session (H)

(H) Undergraduate Mathematics Education, Room 108

Organizers: Judy Paterson, Mike Thomas, Bill Barton (The University of Auckland)

July 4 (Thu.), 14:00 – 14:25, Room 108

Ryosuke Nagaoka (Meiji University, Japan)

Mathematicians' Responsibility for Mathematics Education --- The Role of Lighthouse Keeping in the Dark

Program of Session (I)

(I) Spectral and Scattering Theory, Room 107

Organizer: Claudio Fernandez (Pontificia Universidad Católica de Chile)

Session local organizer: Asao Arai (Hokkaido University)

July 1 (Mon.), 14:00 – 14:25, Room 107

Asao Arai (Hokkaido University, Japan)

A New Asymptotic Perturbation Theory and Applications to Massless Quantum Fields

July 1 (Mon.), 14:30 – 14:55, Room 107

Carlos Villegas (Universidad Nacional Autonoma de Mexico, Mexico)

On a limiting resonance distribution theorem for the Stark effect in the semiclassical limit

July 1 (Mon.), 15:30 – 15:55, Room 107

Pablo Miranda (Pontificia Universidad Católica de Chile, Chile)

Eigenvalue Asymptotics for Dirichlet and Neumann Half-plane Magnetic Hamiltonians

July 1 (Mon.), 16:00 – 16:25, Room 107

Shu Nakamura (University of Tokyo, Japan)

Propagation of singularities for Schrodinger equations with long range perturbations

July 2 (Tue.), 14:00 – 14:25, Room 107

Kalyan Sinha (Jawaharlal Nehru Centre for Advanced Scientific Research, India)

Trace - Integral Formulae for functions of one and two operator variables

July 2 (Tue.), 14:30 – 14:55, Room 107

Rafael Tiedra (Pontificia Universidad Católica de Chile, Chile)

Commutator methods for the spectral analysis of time changes of horocycle flows

July 2 (Tue.), 15:30 – 15:55, Room 107

Takuya Mine (Kyoto Institute of Technology, Japan)

Two-solenoidal Aharonov-Bohm effect with quantized magnetic fluxes

July 2 (Tue.), 16:00 – 16:25, Room 107

Richard Froese (University of British Columbia, Canada)

Transversely periodic potentials on trees

Program of Session (J)

(J) Algebraic Geometry, Room 207

Organizers: Masa-Hiko Saito (Kobe University), Yoshinori Namikawa, (Kyoto University), Shigeru Mukai (Kyoto University)

Session local organizer: Iku Nakamura (Hokkaido University)

July 4 (Thu.), 14:00 – 14:40, Room 207
Baohua Fu (Academia Sinica, Beijing)
Sixty years of compactifications of C^n

July 4 (Thu.), 14:50 – 15:30, Room 207
Iku Nakamura (Hokkaido University)
Compactification of moduli of abelian varieties

July 4 (Thu.), 15:50 – 16:30, Room 207
Shigeyuki Kondo (Nagoya University)
On certain duality of Neron-Severi lattices of supersingular K3 surfaces

July 5 (Fri.), 14:00 – 14:40, Room 207
Young-Hoon Kiem (Seoul National University)
Categorification of Donaldson-Thomas invariants via Perverse Sheaves

July 5 (Fri.), 14:50 – 15:30, Room 207
Yukinobu Toda (IPMU, Tokyo University)
Gepner type stability conditions on graded matrix factorizations

July 5 (Fri.), 15:50 – 16:30, Room 207
Bumsig Kim (KIAS, Seoul)
ADE quiver bundles on a curve

Program of Session (K)

(K) Elliptic and Parabolic Equations, Room 204

Organizers: Kazuhiro Ishige (Tohoku University), Takayoshi Ogawa (Tohoku University), Yoshihiro Tonegawa (Hokkaido University)

Session local organizer: Yoshihiro Tonegawa (Hokkaido University)

July 4 (Thu.), 14:00 – 14:25, Room 204
Yoshiyuki Kagei (Kyushu University, Japan)
On time periodic problem of the compressible Navier-Stokes equation

July 4 (Thu.), 14:30 – 14:55, Room 204
Maria Schonbek (University of California, Santa Cruz, USA)
L^2 asymptotic stability of mild Navier-Stokes solutions

July 4 (Thu.), 15:30 – 15:55, Room 204
Senjo Shimizu (Shizuoka University, Japan)
Stability of equilibria for incompressible two-phase flows with phase transitions

July 4 (Thu.), 16:00 – 16:25, Room 204
Diogo Aguiar Gomes (Technical University of Lisbon, Portugal)
Classical solutions of mean-field games

July 5 (Fri.), 14:00 – 14:25, Room 204
Tai-Peng Tsai (University of British Columbia, Canada)
Forward Discretely Self-Similar Solutions of the Navier-Stokes Equations

July 5 (Fri.), 14:30 – 14:55, Room 204
Futoshi Takahashi (Osaka City University, Japan)
Convergence for a 2D elliptic problem with large exponent in nonlinearity

July 5 (Fri.), 15:30 – 15:55, Room 204
Ki-Ahm Lee (Seoul National University, Korea)
Homogenization of Nonlinear Equations in nondivergence type with Neumann data in Perforated Domains

July 5 (Fri.), 16:00 – 16:25, Room 204
Shigeru Sakaguchi (Tohoku University, Japan)
Fast diffusion and geometry of domain

Program of Session (L)

(L) Topological Problems in Fluid Dynamics, Room 108

Organizer: Takashi Sakajo (Kyoto University)

Session local organizer: Tsuyoshi Yoneda (Hokkaido University)

July 1 (Mon.), 14:00 – 14:25, Room 108
Darren G. Crowdy (Imperial College London, UK)
Applications of a new calculus for ideal hydrodynamics

July 1 (Mon.), 14:30 – 14:55, Room 108
Takashi Sakajo (Kyoto University, Japan)
Encoding of streamline topologies for incompressible vortex flows in 2D multiply connected domains

July 1 (Mon.), 15:30 – 15:55, Room 108
Yoshihiko Mitsumatsu (Chuo University, Japan)
Incompressible fluids on foliated manifolds

July 1 (Mon.), 16:00 – 16:25, Room 108
Alberto Enciso (Instituto de Ciencias Matematicas, Spain)
Knotted vortex tubes in steady solutions to the Euler equation

July 2 (Tue.), 14:00 – 14:25, Room 108
Yasuhide Fukumoto (Kyushu University, Japan)
Energy, pseudomomentum and Stokes drift of inertial waves and their application to stability of a columnar vortex

July 2 (Tue.), 14:30 – 14:55, Room 108
Jens Kasten (University of Leipzig, Germany)
Using persistence to quantify vortex significance

July 2 (Tue.), 15:30 – 15:55, Room 108
Kyo Yoshida (Tsukuba University, Japan)
Strong turbulence in nonlinear Schrödinger equation

July 2 (Tue.), 16:00 – 16:25, Room 108
Scott D. Kelly (University of North Carolina at Charlotte, USA)
Constrained Mechanics and Idealized Models for Aquatic Locomotion via Vortex Shedding

Program of Session (M)

(M) Mathematical Aspects of Crystal Growth and Image Analysis, Room 204

Organizers: Russel Caflisch (University of California, Los Angeles), Yoshikazu Giga (University of Tokyo)

Session local organizer: Shuichi Jimbo (Hokkaido University)

July 1 (Mon.), 14:00 – 14:25, Room 204
Harald Garcke (University of Regensburg, Germany)
New computational approaches to curvature driven interface evolution with applications to crystal growth and image analysis

July 1 (Mon.), 14:30 – 14:55, Room 204
Janko Gravner (University of California, Davis, USA)
A mesoscopic model for snow crystal growth

July 1 (Mon.), 15:30 – 15:55, Room 204
Piotr Mucha (Warsaw University, Poland)
Anisotropic models: analysis of at regions of solutions

July 1 (Mon.), 16:00 – 16:25, Room 204
Nao Hamamuki (University of Tokyo)
Asymptotically self-similar solutions to curvature flow equations with prescribed contact angle

July 2 (Tue.), 14:00 – 14:25, Room 204
Yasumasa Nishiura (Tohoku University, Japan)
Topological approach to pattern formation problems arising in materials science

July 2 (Tue.), 14:30 – 14:55, Room 204
Ji Hui (National University of Singapore, Singapore)
Non-stationary blind image de-deconvolution

July 2 (Tue.), 15:30 – 15:55, Room 204
Chiu-Yen Kao (Claremont McKenna College, USA)
Minimal Convex Combinations of Three Sequential Laplace-Dirichlet Eigenvalues

July 2 (Tue.), 16:00 – 16:25, Room 204
Hidekata Hontani (Nagoya Institute of Technology, Japan)
Multiscale contour shape analysis using the crystalline flow

Topics of Poster Presentation

Chairpersons: Tsuyoshi Yoneda (Hokkaido University), Masahiko Shimojo (Okayama University of Science)

Participants:

I-Kun Chen (National Chiao Tung University, Taiwan)
Boundary singularity of moments for the linearized Boltzmann equation
(related to: Session (D) Kinetic and Hyperbolic Equations)

Yi-Chiuan Chen (National Chiao Tung University, Taiwan)
Abrupt Bifurcations in Chaotic Scattering: view from the anti-integrable limit
(related to: Session (B) Dynamical Systems)

Noboru Chikami (Tohoku University, Japan)
The Cauchy problem for the compressible Navier-Stokes system with a potential term
(related to: Session (K) Elliptic and Parabolic equations)

Kenichi Fujishiro (University of Tokyo, Japan)
Approximate controllability of fractional diffusion equations by interior control
(related to: Session (K) Elliptic and Parabolic equations)

Mitsuhiro Imada (Keio University, Japan)
On Complex Contact Manifolds
(related to: Session (F) Differential Geometry)

Masaki Kasedou (Hokkaido University, Japan)
Differential geometry on submanifolds and singularity theory
(related to: Session (F) Differential Geometry)

Chua Seng Kee (National University of Singapore, Singapore)
Embedding and compact embedding for weighted and abstract Sobolev spaces
(related to: Session (K) Elliptic and Parabolic equations)

Hiroshi Matsuzoe (Nagoya Institute of Technology, Japan)

Hessian structures on deformed exponential families and their applications

(related to: Session (F) Differential Geometry)

Monika Muszkieta (Wroclaw University of Technology, Poland)

Two cases of squares evolving by anisotropic diffusion

(related to: Session (M) Mathematical Aspects of Crystal Growth and Image Analysis)

Kei Nishi (Hokkaido University, Japan)

Behaviors of a front-back pulse in some bistable reaction-diffusion system with heterogeneity

(related to: Session (B) Dynamical Systems)

Frantisek Stampach (Czech Technical University, Czech Republic)

One-parameter generalization of Charlier and Al-Salam-Carlitz polynomials

(related to : Session (A) Special Functions and Orthogonal Polynomials)

Li Zhiyuan (University of Tokyo, Japan)

Initial-boundary value problems for linear diffusion equation with multiple time-fractional derivatives and application to some inverse problems

(related to: Session (K) Elliptic and Parabolic equations)

Pacific Rim Conference
Lecture Timetable
(As of June 20, 2013)

	July 1 (Monday)	July 2 (Tuesday)	July 3 (Wednesday)	July 4 (Thursday)	July 5 (Friday)
09:00～	Registration				
09:15～10:00	Opening 9:30～ (Small Hall)		Plenary lecture Ⅴ Russel Caflisch	Plenary lecture Ⅷ Jongil Park	
10:00～10:05			Break	Break	
10:05～10:50	Plenary lecture Ⅰ Weixiao Shen	Plenary lecture Ⅲ Alan Carey	Plenary lecture Ⅵ Izumi Takagi	Plenary lecture Ⅸ Doron Lubinsky	Plenary lecture Ⅺ Bill Barton
10:50～11:15	Break	Break	Break	Break	Break
11:15～12:00	Plenary lecture Ⅱ Hideo Tamura	Plenary lecture Ⅳ Yoshio Sone	Plenary lecture Ⅶ Ben Andrews	Plenary lecture Ⅹ Toshitake Kohno	Plenary lecture Ⅻ Yujiro Kawamata
12:00～14:00	Lunch Break	Lunch Break	Excursion (Option) 13:30～	Lunch Break	Lunch Break
14:00～16:30	Session lectures (B) (E) (F) (G) (I) (L) (M)	Session lectures (B) (E) (F) (G) (I) (L) (M)		Session lectures (A) (C) (D) (H) (J) (K)	Session lectures (A) (C) (D) (J) (K)
16:30～17:00	Poster presentation Room 204	Poster presentation Room 204			

Sessions: (B) Dynamical Systems, Room 104&105
(E) Mathematical Physics, Room 206
(F) Differential Geometry, Small Hall
(G) Topology and Related Topics, Room 207
(I) Spectral and Scattering Theory, Room 107
(L) Topological Problems in Fluid Dynamics, Room108
(M) Mathematical Aspects of Crystal Growth and Image Analysis, Room 204

Sessions: (A) Special Functions and Orthogonal Polynomials, Room 107
(C) Symplectic Topology, Room 206
(D) Kinetic and Hyperbolic Equations, Small Hall
(H) Undergraduate Mathematics Education, Room 108
(J) Algebraic Geometry, Room 207
(K) Elliptic and Parabolic Equations, Room 204

Note: All plenary lectures will be held in Small Hall.

**Note: Light refreshments will be served around 10:50 and 15:00 (except Wednesday afternoon).
Lecture time includes Q&A discussion.**

Timetable of Monday's lectures (July 1)

09:30 Opening (Small Hall)

10:05〜10:50 Plenary lecture I (Small Hall), Weixiao Shen (National University of Singapore)
On Stability of One-Dimensional Dynamics
(Chairperson: Kuo-Chang Chen)

10:50〜11:15 Break

11:15〜12:00 Plenary lecture II (Small Hall), Hideo Tamura (Okayama University)
Aharonov-Bohm effect in resonances of magnetic Schrödinger operators in two dimensions
(Chairperson: Claudio Fernandez)

12:00〜14:00 Lunch Break

	Session (B) Room 104&105	**Session (E) Room 206**	**Session (F) Small Hall**	**Session (G) Room 207**	**Session (I) Room 107**	**Session (L) Room 108**	**Session (M) Room 204**
14:00〜14:25	Juan Eduardo Rivera Letelier	Raphael Ponge	14:00〜14:25 Richard Schoen	14:00〜14:50 Mario Salvetti	Asao Arai	Darren G. Crowdy	Harald Garcke
14:30〜14:55	Yutaka Ishii	Bai-Ling Wang	14:30〜15:15 Haizhong Li		Carlos Villegas	Alberto Enciso	Janko Gravner
15:00〜15:30	Break	Break	Break	Break	Break	Break	Break
15:30〜15:55	Wen Huang	Kaoru Ono	15:30〜16:15 Sumio Yamada	Alejandro Adem	Pablo Miranda	Yoshihiko Mitsumatsu	Piotr Mucha
16:00〜16:25	Jung-Chao Ban	Masao Jinzenji		Masahiko Yoshinaga	Shu Nakamura	Alberto Enciso	Nao Hamamuki
16:30〜17:00							Poster presentation

Timetable of Tuesday's lectures (July 2)

10:05～10:50 Plenary lecture Ⅲ (Small Hall), Alan Carey (Australian National University)
Geometric Cycles and D–Branes
(Chairperson: Bai–Ling Wang)

10:50～11:15 Break

11:15～12:00 Plenary lecture IV (Small Hall), Yoshio Sone (Kyoto University)
Fluid–dynamic–type equations derived from the Boltzmann equation for small Knudsen numbers and their boundary conditions
(Chairperson: Shih–Hsien Yu)

12:00～14:00 Lunch Break

	Session (B) Room 104&105	Session (E) Room 206	Session (F) Small Hall	Session (G) Room 207	Session (I) Room 107	Session (L) Room 108	Session (M) Room 204
14:00～14:25	Hiroki Takahasi	Kiyonori Gomi	14:00～14:45 Pin Yu	14:00～14:50 Hiroaki Terao	Kalyan Sinha	Yasuhide Fukumoto	Yasumasa Nishiura
14:30～14:55	Zin Arai	Siye Wu	Break		Rafael Tiedra	Jens Kasten	Ji Hui
15:00～15:30	Break	Break	15:00～15:45 Glen Wheeler	Break	Break	Break	Break
15:30～15:55	Kuo–Chang Chen	Ruibin Zhang	Break	Ryan Budney	Takuya Mine	Kyo Yoshida	Chiu–Yen Kao
16:00～16:25		Toshitake Kohno	16:00～16:45 Guoyi Xu	Tadayuki Watanabe	Richard Froese	Scott D. Kelly	Hidekata Hontani
16:30～17:00							Poster presentation

Timetable of Wednesday's lectures (July 3)

09:15～10:00 Plenary lecture V (Small Hall), Russel Caflisch (UC Los Angeles)
From Natural Science to Information Science and Back
(Chairperson: Yoshikazu Giga)

10:00～10:05 Break

10:05～10:50 Plenary lecture VI (Small Hall), Izumi Takagi (Tohoku University)
Point-condensation phenomenon in a reaction-diffusion system: geometry of domain vs heterogeneity of media
(Chairperson: Takayoshi Ogawa)

10:50～11:15 Break

11:15～12:00 Plenary lecture VII (Small Hall), Ben Andrews (Australian National University)
Minimal and Constant Mean Curvature Surfaces in the Three-Sphere: Brendle's Proof of the Lawson Conjecture
(Chairperson: Sumio Yamada)

13:30～ Excursion (Option)

Timetable of Thursday's lectures (July 4)

09:15〜10:00 Plenary lecture VIII (Small Hall), Jongil Park (Seoul National University)
The geography problems of 4-manifolds
(Chairperson: Cheol-Hyun Cho)

10:00〜10:05 Break

10:05〜10:50 Plenary lecture IX (Small Hall), Doron Lubinsky (Georgia Institute of Technology)
Pushing Polynomial Reproducing Kernels to their Non-polynomial Limit
(Chairperson: Roderick Wong)

10:50〜11:15 Break

11:15〜12:00 Plenary lecture X (Small Hall), Toshitake Kohno (University of Tokyo)
Quantum symmetry in homological representations of braid groups and hypergeometric integrals
(Chairperson: Alejandro Adem)

12:00〜14:00 Lunch Break

	Session (A) Room 107	Session (C) Room 206	Session (D) Small Hall	Session (H) Room 108	Session (J) Room 207	Session (K) Room 204
14:00〜14:25	Roderick Wong	Yakov Eliashberg	Shijin Deng	Ryosuke Nagaoka	14:00〜14:40 Baohua Fu	Yoshiyuki Kagei
14:30〜14:55	Adri Olde Daalhuis	Viktor Ginzburg	Jin-Cheng Jiang	Break	Break	Maria Schonbek
15:00〜15:30	Break	Break	Break		14:50〜15:30 Iku Nakamura	Break
15:30〜15:55	Walter Van Assche	Urs Frauenfelder	Tai-Ping Liu		Break	Senjo Shimizu
16:00〜16:25	Yoshitsugu Takei	Kaoru Ono	Shigeru Takata		15:50 – 16:30 Shigeyuki Kondo	Diogo Aguiar Gomes

18:30〜 **Banquet at Keio Plaza Hotel Sapporo**

Timetable of Friday's lectures (July 5)

10:05～10:50 Plenary lecture XI (Small Hall), Bill Barton (The University of Auckland)
Being like Sakamoto Hayato: Lecturers as professionals
(Chairperson: Ryosuke Nagaoka)

10:50～11:15 Break

11:15～12:00 Plenary lecture XII (Small Hall), Yujiro Kawamata (University of Tokyo)
Derived categories in algebraic geometry
(Chairperson: Masa-Hiko Saito)

12:00～14:00 Lunch Break

	Session (A) Room 107	**Session (C) Room 206**	**Session (D) Small Hall**	**Session (J) Room 207**	**Session (K) Room 204**
14:00～14:25	Peter Clarkson	Conan Leung	Tong Yang	14:00～14:40 Young-Hoon Kiem	Tai-Peng Tsai
14:30～14:55	Arno Kuijlaars	Sheel Ganatra	Shih-Hsien Yu	Break	Futoshi Takahashi
15:00～15:30	Break	Break	Break	14:50～15:30 Yukinobu Toda	Break
15:30～15:55	Yousuke Ohyama	Hansol Hong	Seung Yeal Ha	Break	Ki-Ahm Lee
16:00～16:25	Dan Dai			15:50 – 16:30 Bumsig Kim	Shigeru Sakaguchi

ON STABILITY OF ONE-DIMENSIONAL DYNAMICS

WEIXIAO SHEN

HOKKAIDO, JULY 2013

In this talk, we shall discuss stability of one-dimensional dynamical systems, from both topological and measure-theoretical point of view.

A time-discrete dynamical system is represented by a self map $f : M \to M$. For $n \geq 0$, let f^n denote the n-th iterate of f. For each $x \in M$, the orbit of x is defined to be the sequence $\mathrm{orb}(x) = \{f^n(x)\}_{n=0}^{\infty}$. The theory of dynamical system studies the orbit structure. Stability, roughly speaking, means that the global dynamical property remains unchanged under a small perturbation of the system. It is generally believed that "most" dynamical systems have certain kind of *hyperbolicity* which in turn implies stability in a suitable sense.

Structural stability. Let M denote a compact manifold (possibly with boundary). For each $r = 1, 2, \ldots$, let $C^r(M)$ denote the collection of C^r maps from M to itself, endowed with the C^r topology. A map $f \in C^r(M)$ is C^r-*structurally stable* if there is a neighborhood $\mathcal{U}$ of f in $C^r(M)$ such that each $g \in \mathcal{U}$ is topologically conjugate to f, i.e. there is a homeomorphism $h = h_g : M \to M$ such that $h \circ f = g \circ h$.

Conjecture. (Smale [12]) *If a map $f \in C^r(M)$ is C^r structurally stable, then it satisfies Axiom A.*

When $\dim(M) \geq 2$, the conjecture was proved in the case $r = 1$ (for diffeomorphisms and flows). In the case $\dim(M) = 1$, it is completely proved for all $r = 1, 2, \ldots$ by Kozlovski, Shen and van Strien ([7, 8]). Indeed, when $r \geq 2$, Axiom A maps are dense in $C^r([0,1])$ and $C^r(S^1)$. It should be noted that this result was proved by considering iteration of real polynomials on the complex plane. Recall that a one-dimensional map $f : M \to M$ satisfies Axiom A if f is uniformly expanding outside the attracting basin of periodic attractors.

The complex one-dimensional case (the *Fatou conjecture*) is still open.

Stochastic stability. Given a map $f : M \to M$, a probability Borel measure μ on M is called *invariant* if for any Borel subset $B \subset M$, we have $\mu(f^{-1}(B)) = B$. The basin of μ, denote by $B(\mu)$, is defined as the set of all $x \in M$ for which

$$\frac{1}{n}\sum_{i=0}^{n-1} \delta_{f^i(x)} \to \mu \text{ as } n \to \infty,$$

in the weak star topology, where δ_a denote the Dirac measure at the point a. An invariant probability measure μ is called *physical* if $B(\mu)$ has positive Lebesgue measure.

Let $M = [0,1]$ or S^1. We say that $f \in C^r(M)$ is *weakly stochastic*, if there exist finitely many physical measures μ_i, $i = 1, 2, \ldots, m$, such that

$$\text{Leb}\left(M \setminus \left(\bigcup_{i=1}^{m} B(\mu_i)\right)\right) = 0.$$

Furthermore, if for each i, μ_i is either supported on a periodic attractor or absolutely continuous, then we say that f is *strongly stochastic*. It was known (Jakobson [6]) that strongly stochastic but non-Axiom A maps are abundant in the measure-theoretical sense.

Conjecture.(Palis [10]) *"Most" dynamical systems are strongly stochastic and stochastically stable in a suitable sense.*

This conjecture was verified for the family of quadratic polynomials (and for more general unimodal maps with non-degenerate critical points) by Lyubich [9] and Avila-Moreira [3]. Recent works of Avila-Lyubich-Shen [2] and Avila-Lyubich [1] enable us to extend the result to unimodal maps with higher order criticality.

For multimodal interval maps, the Palis conjecture is still open. Nevertheless, it was shown by Bruin, Rivera-Letelier, Shen and van Strien [4] that a very weak *large derivatives* condition implies the strongly stochastic property and by Shen [11] that a summability condition implies strongly stochastic stability. In [5], a strengthened version of the Jakobson's theorem was proved.

References

[1] A. Avila, M. Lyubich. The full renormalization horseshoe for unimodal maps of higher degree: exponential contraction along hybrid classes. Publ. Math. IHES No. 114 (2011), 171-223.

[2] A. Avila, M. Lyubich, W. Shen. Parapuzzle of the Multibrot set and typical dynamics of unimodal maps. *J. Eur. Math. Soc.* 13 (2011), no. 1, 27-56.

[3] A. Avila, C. G. Moreira. Statistical properties of unimodal maps: the quadratic family. *Ann. Math.* 161, 831-881, 2005.

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Department of Mathematics, National University of Singapore

Aharonov–Bohm effect in resonances of magnetic Schrödinger operators in two dimensions

Hideo Tamura

In quantum mechanics, a vector potential is said to have a direct significance to particles moving in a magnetic field. This quantum effect is known as the Aharonov–Bohm effect (AB effect) ([1]). We study the AB effect in resonances through a simple scattering system in two dimensions. The system consists of three scatterers, one bounded obstacle and two scalar potentials with compact supports at large separation, where the obstacle is placed between two supports and shields completely the support of a magnetic field. The field does not influence particles from a classical mechanical point of view, but quantum particles are influenced by the corresponding vector potential which does not necessarily vanish outside the obstacle. The resonances are shown to be generated near the real axis by the trajectories trapped between two supports of the scalar potentials as the distances between the three scatterers go to infinity. The location is described in terms of the backward amplitudes for scattering by the two scalar potentials, and it depends heavily on the magnetic flux of the field. We also discuss what happens in the case of two obstacles. This system yields a two dimensional model of scattering by toroidal solenoids in three dimensions. The result depends on the location of the obstacles as well as on the fluxes.

We write

$$H(A,V) = (-i\nabla - A)^2 + V = \sum_{j=1}^{2} (-i\partial_j - a_j)^2 + V, \quad \partial_j = \partial/\partial x_j,$$

for the Schrödinger operator with the vector potential $A = (a_1, a_2) : \boldsymbol{R}^2 \to \boldsymbol{R}^2$ and the scalar potential $V : \boldsymbol{R}^2 \to \boldsymbol{R}$. The magnetic field $b : \boldsymbol{R}^2 \to \boldsymbol{R}$ associated with A is defined by

$$b(x) = \nabla \times A(x) = \partial_1 a_2 - \partial_2 a_1$$

and the quantity defined as the integral $\alpha = (2\pi)^{-1} \int b(x)\,dx$ is called the magnetic flux of b, where the integration with no domain attached is taken over the whole space.

Let $\mathcal{O} \subset \boldsymbol{R}^2$ be a simply connected bounded domain. We assume that $\mathcal{O}$ contains the origin as an interior point and its boundary is smooth. For $d \in \boldsymbol{R}^2$, we set

$$d_- = -\kappa_- d, \quad d_+ = \kappa_+ d, \quad \kappa_\pm > 0, \quad \kappa_+ + \kappa_- = 1,$$

so that $d_+ - d_- = d$. The distance $|d| \gg 1$ is treated as a large parameter, but the direction $\hat{d} = d/|d|$ is fixed. We consider the self–adjoint operator

$$H_d = H(A, V_d) = (-i\nabla - A)^2 + V_d, \quad \mathcal{D}(H_d) = H^2(\Omega) \cap H_0^1(\Omega),$$

over the exterior domain $\Omega = \boldsymbol{R}^2 \backslash \overline{\mathcal{O}}$ under the zero Dirichlet boundary conditions, where $V_d(x)$ takes the form

$$V_d(x) = V_{-d}(x) + V_{+d}(x) = V_-(x - d_-) + V_+(x - d_+)$$

with $V_\pm \in C_0^\infty(\boldsymbol{R}^2)$, and A is defined as the Aharonov–Bohm potential

$$A(x) = \alpha \left(-x_2/|x|^2, x_1/|x|^2 \right) = \alpha \left(-\partial_2 \log|x|, \partial_1 \log|x| \right)$$

over Ω. The potential A generates the solenoidal field

$$b = \nabla \times A = \alpha \left(\partial_1^2 + \partial_2^2 \right) \log|x| = 2\pi\alpha\delta(x),$$

which has the support only at the origin and α as a magnetic flux. Hence the field $b = \nabla \times A$ is entirely shielded by the obstacle $\mathcal{O}$, although the corresponding vector potential A does not necessarily vanish over Ω. The resolvent

$$R(\zeta; H_d) = (H_d - \zeta)^{-1} : L^2(\Omega) \to L^2(\Omega), \quad \operatorname{Re}\zeta > 0,\ \operatorname{Im}\zeta > 0,$$

is meromorphically continued from the upper half plane of the complex plane to the lower half plane across the positive real axis where the continuous spectrum of H_d is located. Then $R(\zeta; H_d)$ with $\operatorname{Im}\zeta \le 0$ is well defined as an operator from $L^2_{\mathrm{comp}}(\Omega)$ to $L^2_{\mathrm{loc}}(\Omega)$, where $L^2_{\mathrm{comp}}(W)$ denotes the space of square integrable functions with compact support in the closure $\overline{W}$ of a region $W \subset \boldsymbol{R}^2$ and $L^2_{\mathrm{loc}}(W)$ denotes the space of locally square integrable functions over $\overline{W}$. The resonances of H_d are defined as the poles of $R(\zeta; H_d)$ in the lower half plane. Roughly speaking, the resonances near the real axis are almost regarded as positive eigenvalues in some sense, although H_d has no positive eigenvalues.

One of the obtained results is stated as follows ([2]): The resonances are approximately determined as the solutions to the equation

$$\left(e^{2ik|d|}/|d| \right) \cos^2(\alpha\pi) f_-(-\hat{d} \to \hat{d}; \zeta) f_+(\hat{d} \to -\hat{d}; \zeta) = 1, \quad \operatorname{Im} k = \operatorname{Im} \zeta^{1/2} < 0,$$

for $|d| \gg 1$, where $f_\pm(\pm\hat{d} \to \mp\hat{d}; \zeta)$ is defined by analytic extension from the backward scattering amplitude $f_\pm(\pm\hat{d} \to \mp\hat{d}; E)$ at energy $E > 0$ for the Schrödinger operator

$$K_\pm = K_0 + V_\pm = -\Delta + V_\pm, \quad \mathcal{D}(K_\pm) = H^2(\boldsymbol{R}^2).$$

This relation makes sense only when the flux α is not a half–integer.

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[2] H. Tamura, Aharonov–Bohm effect in resonances of magnetic Schrödinger operators in two dimensions, *Kyoto J. Math.*, **52** (2012), 557–595.

GEOMETRIC CYCLES AND D-BRANES

ALAN L. CAREY

THIS IS JOINT WORK WITH BAI-LING WANG

Physical motivation. I will talk about a mathematical interpretation of D-branes in Type II string theory using B-L Wang's twisted geometric cycles [Wa].

In string theory D-branes were proposed as a mechanism for providing boundary conditions for the dynamics of open strings moving in space-time. Initially they were thought of as submanifolds. As D-branes themselves can evolve over time one needs to study equivalence relations on the set of D-branes. An invariant of the equivalence class is the topological charge of the D-brane which should be thought of as an analogue of the Dirac monopole charge as these D-brane charges are associated with gauge fields (connections) on vector bundles over the D-brane. These vector bundles are known as Chan-Paton bundles.

In [MM] Minasian and Moore made the proposal that D-brane charges should take values in K-groups and not in the cohomology of the space-time or the D-brane. In string theory there is an additional field on space-time known as the H-flux which may be thought of as a global closed three form. Locally it is given by a family of 'two-form potentials' known as the B-field. Mathematically we think of these B-fields as defining a degree three integral Čech class on the space-time, called a 'twist'. Witten [Wit], extending [MM], gave a physical argument for the idea that D-brane charges in the presence of a twist should take values in twisted K-theory (at least in the case where the twist is a torsion class). Subsequently Bouwknegt and Mathai [BouMat] extended Witten's proposal to the non-torsion case using ideas from operator algebra theory.

In this talk I will review the mathematical background. Then I will move to a discussion of the resolution of Witten's proposal.

Mathematical issues. From a mathematical perspective some immediate questions arise from the physical input summarised above. When there is no twist it is well known that K-theory provides the main topological tool for the index theory of elliptic operators. One version of the Atiyah-Singer index theorem establishes a relationship between the analytic viewpoint provided by elliptic differential operators and the geometric viewpoint provided by the notion of geometric cycle introduced in the fundamental paper of Baum and Douglas [BD2]. The viewpoint that geometric cycles in the sense of [BD2] are a model for D-branes in the untwisted case is surveyed in [Sz].

It is thus tempting to conjecture that there is an analogous picture of D-branes as a type of geometric cycle in the twisted case as well. More precisely we ask the question of whether there is a way to formulate the notion of 'twisted geometric cycle' (generalising[BD2]) so that it is adapted to the application to string theory. This question was answered in the affirmative by B-L. Wang [Wa]. It is important to emphasise that string geometry ideas from [FreWit] played a key role.

In Type II superstring theory on a manifold X, a string worldsheet is an oriented Riemann surface Σ, mapped into X with $\partial\Sigma$ mapped to an oriented submanifold M (called a D-brane world-volume). The theory also has a Neveu-Schwarz B-field, a system of local 2-forms on

X classified by a characteristic class $[\alpha] \in H^3(X, \mathbb{Z})$ (Čech cohomology) areferred to as a 'twist'. In physics, the D-brane world volume M carries a connection on a complex vector bundle (called the Chan-Paton bundle), and thus a D-brane is given by a submanifold M of X with a complex bundle E and a connection.

For a D-brane M to define a class in the K-theory of X, its normal bundle ν_M must be endowed with a $Spin^c$ structure. Equivalently, the embedding $\iota : M \longrightarrow X$ is such that the classical push-forward map in K-theory ([AH])

$$\iota_!^K : K^0(M) \longrightarrow K^{ev/odd}(X)$$

is well-defined, (it takes values in even or odd K-groups depending on the dimension of M). So the D-brane charge of $(\iota : M \to X, E)$ is given by pushing forward the K-theory class $[E]$ of E which we write as:

$$\iota_!^K([E]) \in K^{ev/odd}(X).$$

When the B-field is not topologically trivial, that is $[\alpha] \neq 0$, in order to have a well-defined worldsheet path integral, Freed and Witten in [FreWit] showed that the pull back under the imbedding ι of the twist class in $H^3(X, \mathbb{Z})$ should equal the third Stiefel-Whitney class of M, denoted W_3, which in general has to be allowed to be non-trivial so that M is not necessarily $Spin^c$.

This latter situation is summarised as

$$\iota^*[\alpha] + W_3(\nu_M) = 0. \tag{0.1}$$

When $\iota^*[\alpha] \neq 0$, then the push-forward map in K-theory ([AH])

$$\iota_!^K : K^0(M) \longrightarrow K^*(X)$$

is **not** well-defined.

In [Wa], the mathematical meaning of (0.1) was discovered. First of all we need to allow more general maps $\iota : M \to X$ than just imbeddings. He uses a generalisation of the notion of $Spin^c$ manifolds for a continuous map α into the classifying space for principal projective unitary group bundles that represent $[\alpha] \in H^3(X, \mathbb{Z})$.

In Wang's formulation we see that when M is $Spin^c$, the datum to describe a D-brane is exactly a Baum-Douglas geometric cycle. For a general manifold X with a twisting α, and a continuous map $\iota : M \to X$ with M having non-zero Stieffel-Whitney class, we need a new concept, which we call an α-twisted $Spin^c$ structure on M. Loosely speaking what Wang shows is that given a twisting α on a smooth manifold X, every twisted K-class is represented by a twisted geometric cycle supported on a twisted $Spin^c$ manifold M and an ordinary K-class $[E] \in K^0(M)$. It remains open as to whether this theory can be extended beyond targets X that are manifolds, a question of interest to topologists.

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Fluid-dynamic-type equations derived from the Boltzmann equation for small Knudsen numbers and their boundary conditions

Yoshio Sone*

The purpose of the study of the Boltzmann equation in gas dynamics is to clarify the behavior of a gas with non-small mean free path (or Knudsen number) and to investigate the theoretical background of the conventional gas-dynamic equations, which are used to describe the behavior of the gas in the vanishing Knudsen number. For the second purpose, the asymptotic behavior of solutions of initial or boundary-value problems of the Boltzmann equation for small Knudsen numbers has been studied. Their behavior is described by fluid-dynamic-type equations in overall region outside initial layer, Knudsen layer, or shock layer. Here, the fluid-dynamic-type systems derived from the Boltzmann system for small Knudsen numbers are reviewed with the interest of gas dynamics in mind. Various kinds of fluid-dynamic-type equations are derived depending on the physical situations under interest. Among them, there is a system that is different from the Euler or Navier-Stokes system, which is used in conventional gas dynamics [*incompleteness of the conventional fluid dynamics*, Case (ii) below].

The velocity distribution function describing the overall behavior of the gas approaches a Maxwell distribution f_e, whose parameters (density, flow velocity, and temperature) depend on time and position, in the limit $\mathrm{Kn} \to 0$ (Kn : Knudsen number). The fluid-dynamic-type equations that determine the macroscopic variables in the limit differ depending on the character of the Maxwellian. In time-independent problems, the systems are classified by the size of $|f_e - f_{e0}|/f_{e0}$, where f_{e0} is the reference stationary Maxwellian, as follows:
(i)$|f_e - f_{e0}|/f_{e0} = O(\mathrm{Kn})$: The limit $\mathrm{Kn} \to 0$ is a uniform state at rest. The nonuniform state of the first order of Knudsen number is described by the "incompressible Navier–Stokes set" with the energy equation modified owing to the *work done by (uniform) pressure*, which shows that the gas is *not incompressible.* (S system)
(i') $|f_e - f_{e0}|/f_{e0} = o(\mathrm{Kn})$: The fluid-dynamic-type equations are given by the Stokes set.
(ii)$|f_e - f_{e0}|/f_{e0} = O(1)$ with $|v_i|/(2RT)^{1/2} = O(\mathrm{Kn})$ (v_i and T : the flow velocity and temperature in f_e respectively): The temperature and density of the gas in the limit are determined together with the flow velocity of the first order of Kn amplified by $1/\mathrm{Kn}$ (*the ghost effect of an infinitesimal flow*), and the thermal stress of the order of $(\mathrm{Kn})^2$ must be retained in the momentum equations (the ghost effect of a non-Navier–Stokes stress). The thermal creep in the boundary condition must be taken into account (the ghost effect of a nonslip boundary condition). (SB system)

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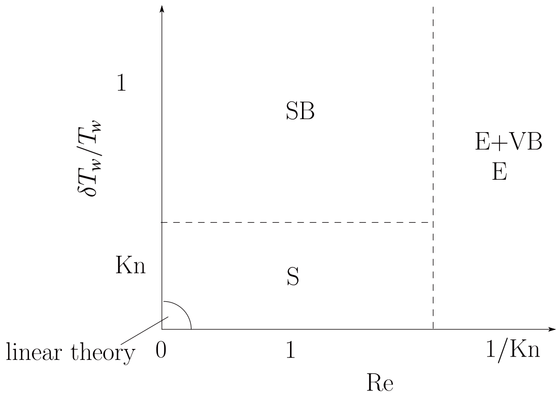

Figure 1: Fluid-dynamic-type equations in the plane $(\mathrm{Re}, \delta T_w/T_w)$. (Re : the Reynolds number, $\delta T_w/T_w$: the reference relative variation of the wall temperature)

(iii) $|f_e - f_{e0}|/f_{e0} = O(1)$ with $|v_i|/(2RT)^{1/2} = O(1)$:
(a) The behavior of the gas around a simple boundary, where $v_i n_i/(2RT)^{1/2} = 0$ (n_i: the normal to the boundary), is described by the combination of the Euler and viscous boundary-layer sets. (E+VB system)
(b) The behavior of the gas around the condensed phase of the gas, where evaporation or condensation with $v_i n_i/(2RT)^{1/2} = O(1)$, is taking place, is described by the Euler set. The Knudsen-layer correction is given by the nonlinear Boltzmann equation in contrast to the other cases, in which the Knudsen layer is governed by the linearized Boltzmann equation. (E system)

In time-dependent problems, there are two time scales of variation of variables, in addition to the mean free time: the time required for the sound wave to propagate over a gas-dynamic distance and a longer time given by the quotient of this time scale divided by the Knudsen number. The overall behavior is given by the Euler set for the first and by the set with the corresponding time-independent set being modified for the second.

The above discussion is for a general domain. A boundary of a ruled surface can be taken as the limit of a general surface with the generating straight lines being originally curves. The problem with such a boundary can be taken as various limiting cases where the Knudsen number and the curvature tend to zero simultaneously. The result depends on the relative speed of the parameters to the limit, and the effect of the curvature remains in the limiting equations (*ghost effect of infinitesimal curvature*). The *bifurcation of the plane Couette flow* at infinite Reynolds number and *nonexistence of the Poiseuille flow* with a parabolic profile through a circular straight pipe are shown in some cases.[1]

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From Natural Science to Information Science and Back

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ABSTRACT

The arrival of massive amounts of data from imaging, sensors, computation and the internet brought with it significant challenges for information science. New methods for analysis and manipulation of big data have come from many scientific disciplines. Most recently, the flow of ideas has reversed as new information theoretic methods are being harnessed for the natural sciences. The first example is variational principles, originally developed for mathematical modeling of physical phenomena but then used for image and data analysis, as in compressed sensing. The resulting ideas of soft-thresholding and sparsity are now being applied back to PDEs. A second example is neural nets, which derive from neurological models and form the basis for deep-learning and related machine learning techniques. These have been remarkably successful for applications such as voice, handwriting and facial recognition. Recently, machine learning algorithms have been used to mine datasets of structural and electronic properties for known materials, in order to predict the corresponding properties of hypothetical materials. The third, not so new example is entropy, which was developed in thermodynamics and statistical mechanics, then used by Shannon for analysis of communication channels. Now, information theoretic entropy is being used for closure of multiscale methods in plasma physics.

Point-condensation phenomenon in a reaction-diffusion system: geometry of domain vs. heterogeneity of media

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Abstract:
Turing regarded the Diffusion-Driven-Instability as the cause of pattern formation in developmental biology. The diffusion-driven instability is the destabilization of a spatially uniform steady state in a system consisting of chemicals with different diffusion rates. Following Turing's idea, Gierer and Meinhardt proposed an activator-inhibitor system in which the activator concentrates sharply around finitely many points in the closure of the domain. We say that such a solution exhibits a point-condensation phenomenon.

In the limit of infinite diffusion rate for the inhibitor, the activator-inhibitor system reduces to the so-called shadow system, which is essentially a single equation for the activator alone. In this talk we consider a (single) semilinear elliptic equation with power nonlinearity under homogeneous Neumann boundary conditions, which is a prototype problem for the activator-inhibitor system.

If the medium is homogeneous and the diffusion is isotropic, we have an equation with constant coefficients. It is known that when the diffusion coefficient is sufficiently small, the ground-state solution concentrates around a point on the boundary of the domain and decays exponentially away from the concentration point. By the ground-state solution, we mean the solution which has the least energy among positive solutions. As the diffusion coefficient tends to zero, the concentration point approaches the maximum point of the mean curvature function of the boundary. Here, the mean curvature is with respect to the inner normal, so that it is positive when the domain is convex. In addition to the ground state, solutions concentrating around multiple points in the interior of the domain or on the boundary of domain are found. The location of the concentration points are determined in terms of the mean curvature functions or solutions of the sphere-packing problem.

On the other hand, if the medium is heterogeneus, the coefficients in the equation depend on the spatical variable. Recently the effect of spatial heterogeneity on the patterns in reaction-diffusion systems has attracted much attention, but not so many results have appeared on the point-condensation phenomenon in heterogeneous media. In this talk, we introduce a locator function to find out concentration points and apply it to the ground state. Interestingly, the ground state can concentrate at an interior point but it occurs only when the global minimum of the locator function is significantly smaller than its minimum over the boundary.

This talk is based mainly on the joint work with Hiroko Yamamoto.

MINIMAL AND CONSTANT MEAN CURVATURE SURFACES IN THE THREE-SPHERE: BRENDLE'S PROOF OF THE LAWSON CONJECTURE

BEN ANDREWS

Minimal surfaces (surfaces of least area) and related objects such as constant mean curvature surfaces have formed a substantial part of the development of the field of geometric analysis. Not only are they beautiful objects with a rich theory spanning partial differential equations, calculus of variations and complex analysis, but they have been employed to great effect in efforts to understand the topology and geometry of manifolds, particularly in three dimensions.

Recently there has been some spectacular progress in this field, answering some old questions about the nature of these surfaces in the simplest possible three-dimensional manifold: The three-dimensional sphere. Simon Brendle gave a remarkable proof [B] of a conjecture of H. Blaine Lawson [L1] dating back to the 1970s, that the only *embedded* minimal torus in the three-sphere (up to congruence) is the Clifford torus, which is the product $S^1(1/\sqrt{2}) \times S^1(1/\sqrt{2}) \subset \mathbb{R}^2 \times \mathbb{R}^2$. Subsequently Haizhong Li and I used related ideas to give a classification of constant mean curvature embedded tori in the three-sphere, proving in particular a conjecture of Pinkall and Sterling [PS] that all such surfaces should be axially symmetric.

In this talk I will discuss briefly some of the background to these conjectures, including the beautiful construction of higher genus embedded minimal surfaces due to Lawson [L2], and the construction of large numbers of *immersed* minimal tori using integrable systems [PS]. Then I will describe one of the new ingredients employed by Brendle, which is a geometric estimate originating from some recent work of mine on mean curvature flow [A]. Finally, I will explain how this is used in the proof. If time allows I will also describe the situation for constant mean curvature tori and some more recent related work on Weingarten tori.

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The geography problems of 4-manifolds

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Abstract

Since M. Freedman classified simply connected topological 4-manifolds using intersection forms ([Fr]) and S. Donaldson introduced gauge theory ([DK]) to show that some of topological 4-manifolds do not admit a smooth structure in 1982, there has been a great progress in the study of 4-manifolds mainly due to Donaldson invariants, Seiberg-Witten invariants and Gromov-Witten invariants. But the complete understanding of 4-manifolds is far from reach.

One of the fundamental problems in 4-manifolds is to classify simply connected closed smooth (symplectic, complex) 4-manifolds. The classical invariants of a simply connected closed 4-manifold are encoded by its intersection form Q_X, a unimodular symmetric bilinear pairing on $H_2(X : \mathbf{Z})$. M. Freedman proved that a simply connected smooth 4-manifold is determined up to homeomorphism by Q_X ([Fr]). But it turned out that the situation is strikingly different in the smooth (symplectic, complex) category. That is, it has been known that only some unimodular symmetric bilinear integral forms are realized as the intersection form of a simply connected smooth (symplectic, complex) 4-manifold, and there are many examples of infinite classes of distinct simply connected smooth (symplectic, complex) 4-manifolds which are mutually homeomorphic. Hence it is a fundamental and important question in the study of 4-manifolds which unimodular symmetric bilinear integral forms are realized as the intersection form of a simply connected smooth (symplectic, complex) 4-manifold - called an *existence problem*, and how many distinct smooth (symplectic, complex) structures exist on it - called a *uniqueness problem*. Geometers and topologists call these *'geography problems of 4-manifolds'*.

The geography problem asks which lattice points in the $(\frac{b^+ +1}{2}, 3\sigma + 2e)$-plane are 'populated' by simply connected minimal smooth (symplectic, complex) 4-manifolds. These coordinates are chosen because of their relation to complex surfaces, where $\chi = \frac{b^+ +1}{2}$ and $c_1^2 = 3\sigma + 2e$. The geography problem for complex surfaces of general type has been studied extensively by algebraic surface theorists. For example, it is well-known that minimal complex surfaces of general type satisfy Noether inequality $(2\chi - 6 \leq c_1^2)$ and Bogomolov-Miyaoka-Yau inequality $(c_1^2 \leq 9\chi)$. Many remarkable results on the geography problems of smooth and symplectic 4-manifolds have also been obtained using Donaldson theory and Seiberg-Witten theory. For example, all known simply connected minimal symplectic 4-manifolds satisfy $0 \leq c_1^2 \leq 9\chi$ and most lattice points satisfying $0 \leq c_1^2 \leq 9\chi$ are populated by simply connected minimal smooth and symplectic 4-manifolds. Furthermore, it has also been proved that most known simply connected smooth 4-manifolds with $b_2^+ > 1$ and odd admit infinitely many distinct smooth structures.

One the other hand, in the case of $b_2^+ = 1$ (equivalently, $p_g = 0$ in complex category) and $c_1^2 > 0$, until 2003 the only previously known simply connected, minimal, symplectic 4-manifolds (or complex surfaces) are rational surfaces and Barlow surface ([BHPV]). Barlow surface has $c_1^2 = 1$ ([B]). So the natural question arises if there are other simply connected symplectic 4-manifolds with $b_2^+ = 1$ or complex surfaces of general type with $p_g = 0$ except Barlow surface.

Since I discovered a new simply connected symplectic 4-manifold with $b_2^+ = 1$ and $c_1^2 = 2$ ([P]) in 2004 by using a rational blow-down surgery, and Y. Lee and myself constructed a new family of simply connected, minimal, complex surfaces of general type with $p_g = 0$ and $c_1^2 = 2$ ([LP]) by modifying the symplectic 4-manifold in 2006, many new simply connected symplectic 4-manifolds with $b_2^+ = 1$ and complex surfaces of general type with $p_g = 0$ have been constructed ([FS, PPS, PSS, SS, SSW]) and now it is one of the most active research areas in 4-manifolds to find a new family of smooth (symplectic, complex) 4-manifolds with $b_2^+ = 1$ (equivalently, $p_g = 0$ in complex category).

The aim of this talk is to review briefly recent developments in this area. In particular, I'd like to survey the existence and the uniqueness problems of simply connected 4-manifolds with $b_2^+ = 1$ in three levels - smooth category, symplectic category and complex category.

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Speaker: Doron Lubinsky, Georgia Institute of Technology

Pushing Polynomial Reproducing Kernels to their Non-polynomial Limit

Let μ be a positive Borel measure on the real line, all of whose moments $\int x^j d\mu(x)$, $j = 0, 1, 2, 3, ...$ are finite. Then we may form orthonormal polynomials $p_n(x) = \gamma_n x^n$+lower powers, $\gamma_n > 0$, satisfying the orthonormality relation

$$\int p_n p_m d\mu = \delta_{mn}.$$

The nth reproducing kernel is

$$K_n(x,t) = \sum_{j=0}^{n-1} p_j(x) p_j(t).$$

Its reproducing property is

$$P(x) = \int K_n(x,t) P(t) d\mu(t),$$

for polynomials of degree $\leq n-1$. "Along the diagonal" it has an extremal property that is common to reproducing kernels in all inner product spaces:

$$K_n(x,x) = \sup\left\{ \frac{|P(x)|^2}{\int |P|^2 d\mu} : P \text{ is a polynomial of degree } \leq n-1 \right\}.$$

In the case of orthogonal polynomials, it is especially useful, as it shows that $K_n(x,x)$ decreases as μ increases. This allows us to compare $K_n(x,x)$ for different measures, and establish asymptotics.

Indeed, it played in a crucial role in a breakthrough 1991 result of Maté, Nevai, and Totik. They proved that if μ is a measure with support $[-1,1]$, and if, for example, $\int_{-1}^{1} \log \mu'(x) dx$ is finite, then for a.e. x in $(-1,1)$,

$$\lim_{n\to\infty} \mu'(x) \frac{K_n(x,x)}{n} = \frac{1}{\pi\sqrt{1-x^2}}.$$

In mathematical physics, this limit is loosely described as the *density of the states*. The asymptotic was subsequently generalized to measures μ with arbitrary compact support on the real line by Totik. In this case, $\frac{1}{\pi\sqrt{1-x^2}}$ is replaced by the (potential theoretic) equilibrium density for the support of the measure. There are analogues for measures with non-compact support, and also for sequences of measures, where we have an nth measure μ_n at the nth stage.

How do reproducing kernels connect to random matrices? It was the physicist Eugene Wigner who had the idea to model scattering theory for neutrons off heavy nuclei, using random Hermitian matrices. He placed a probability distribution $\mathcal{P}^{(n)}$ on the eigenvalues of $n \times n$ Hermitian matrices. A key statistic is the $m-$point correlation function

$$R_m(x_1, x_2, ..., x_m) = \frac{n!}{(n-m)!} \int ... \int P^{(n)}(x_1, x_2 ..., x_n)\, dx_{m+1}\, dx_{m+2} ... dx_n.$$

It can be used to measure the number of $m-$tuples of eigenvalues lying in a given set. Under appropriate assumptions on the underlying measure, there is the remarkable identity

$$R_m(x_1, x_{2,}..., x_m) = \det\left(\sqrt{\mu'(x_i)\,\mu'(x_j)}K_n(x_i, x_j)\right)_{1\le i,j\le m}.$$

The *universality limit in the bulk* asserts that for x in the interior of the support of μ, and real $a_1, a_2, ..., a_m$, we have

$$\begin{aligned}&\lim_{n\to\infty}\frac{1}{\mu'(x)\,K_n(x,x)^m}R_m\left(x+\frac{a_1}{\mu'(x)\,K_n(x,x)}, ..., x+\frac{a_m}{\mu'(x)\,K_n(x,x)}\right)\\ =\;&\det\left(\frac{\sin\pi(a_i-a_j)}{\pi(a_i-a_j)}\right)_{1\le i,j\le m}.\end{aligned}$$

Because R_m is the determinant of an $m\times m$ matrix, and m is fixed in this limit, this reduces to the limit for each entry, namely, for real a, b,

$$\lim_{n\to\infty}\frac{K_n\left(x+\frac{a}{\mu'(x)K_n(x,x)}, x+\frac{b}{\mu'(x)K_n(x,x)}\right)}{K_n(x,x)} = \frac{\sin\pi(a-b)}{\pi(a-b)}. \tag{1}$$

Thus, an assertion about the distribution of eigenvalues of random matrices has been reduced to a technical limit involving orthogonal polynomials. The right-hand side is independent of x, and the underlying measure μ, which well justifies the name *universality limit*.

This limit has been established for a very wide range of measures μ, with compact and non-compact support, and for varying measures, where μ changes as n does. In Wigner's original case, the so-called Gaussian unitary ensemble, $\mu'(x)$ was a scaled Hermite weight e^{-nx^2} at the nth stage. Techniques of proof include asymptotics for classical orthogonal polynomials, and the Christoffel-Darboux formula, Riemann-Hilbert methods, and more recently, classical tools from orthogonal polynomials. The limit itself has also given new insight into spacing of zeros of orthogonal polynomials. It is still unresolved how *universal is universality*, that is what is the full range of measures μ for which is is true? We'll discuss this.

Is it an accident that the famous sinc kernel

$$S(t) = \frac{\sin\pi t}{\pi t}$$

arises in (1)? It appears in so many contexts, most notably in the sampling theorem of signal processing. Of course it is scarcely surprising that the scaled limit of reproducing kernels for polynomials is a reproducing kernel for some space. We'll explain why it is $S(a-b)$, the reproducing kernel for *Paley-Wiener space*. We'll also discuss L_p analogues of $K_n(x,x)$, and their universality limits.

Quantum symmetry in homological representations of braid groups and hypergeometric integrals

Toshitake Kohno (The University of Tokyo)

The notion of braid groups was introduced by E. Artin in 1920's. Especially after the discovery of the Jones polynomial in the middle of 1980's braid groups have appeared in various areas of mathematics such as quantum groups, number theory and conformal field theory. In the beginning of 2000's S. Bigelow and D. Krammer investigated homological representations of braid groups and independently showed that these representations are faithful.

In this talk I will focus on the following developments concerning braid groups.

- correspondence between homological representations and monodromy of KZ connection
- quantum group symmetry and hypergeometric integrals
- description of KZ connection as Gauss-Manin connection
- the image and the kernel of the action of mapping class groups on the space of conformal blocks

For a space X we we define the configuration space of ordered distinct n points in X as

$$\mathcal{F}_n(X) = \{(x_1, \cdots, x_n) \in \mathcal{F}^n \mid x_i \neq x_j,\ i \neq j\}.$$

We define the configuration space of unordered distinct n points in X as $\mathcal{C}_n(X) = \mathcal{F}_n(X)/\mathfrak{S}_n$ where the symmetric group $\mathfrak{S}_n$ acts as the permutation of n points.

In the case X is the complex plane $\mathbf{C}$ the fundamental group $\pi_1(\mathcal{C}_n(\mathbf{C}))$ is by definition the braid group with n strands denoted by B_n. The fundamental group $\pi_1(\mathcal{F}_n(\mathbf{C}))$ is called the pure braid group with n strands and is denoted by P_n.

Let us first explain the construction of homological representations of braid groups. We fix a set of distinct n points in $\mathbf{C}$ and take a 2-dimensional disk D in $\mathbf{C}$ containing Q in the interior. We fix a positive integer m and consider the configuration space $\mathcal{F}_{n,m}(D) = \mathcal{F}_m(D \setminus Q)$. We set $\mathcal{C}_{n,m}(D) = \mathcal{F}_{n,m}(D)/\mathfrak{S}_m$. We have $H_1(\mathcal{C}_{n,m}(D); \mathbf{Z}) \cong \mathbf{Z}^{\oplus n} \oplus \mathbf{Z}$ and consider the abelian covering $\widetilde{\mathcal{C}}_{n,m}(D)$ corresponding to the map

$$\alpha : H_1(\mathcal{C}_{n,m}(D); \mathbf{Z}) \longrightarrow \mathbf{Z} \oplus \mathbf{Z}$$

defined by $\alpha(x_1, \cdots, x_n, y) = (x_1 + \cdots + x_n, y)$. The group of deck transformation is $\mathbf{Z} \oplus \mathbf{Z}$ and the homology group $H_{n,m} = H_m(\widetilde{\mathcal{C}}_{n,m}(D); \mathbf{Z})$ is considered as the module over the ring of Laurent polynomials $R = \mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$. It can be shown that $H_{m,n}$ is a free R-module. We obtain a representation of the braid group

$$\rho_{n,m} : B_n \longrightarrow \operatorname{Aut}_R H_{n,m}$$

which is called the homological representation of the braid group.

On the other hand, we have the following construction of flat connections on the configuration space $X_n = \mathcal{F}_n(\mathbf{C})$ associated with a complex semi-simple Lie algebra $\mathfrak{g}$ and its representations. We set $\Omega = \sum_\mu I_\mu \otimes I_\mu$. Let $r_i : \mathfrak{g} \to \mathrm{End}(V_i)$, $1 \le i \le n$, be representations of the Lie algebra $\mathfrak{g}$. We denote by Ω_{ij} the action of Ω on the i-th and j-th components of the tensor product $V_1 \otimes \cdots \otimes V_n$. We define the Knizhnik-Zamolodchikov (KZ) connection as the 1-form

$$\omega = \frac{1}{\kappa} \sum_{1 \le i < j \le n} \Omega_{ij} d \log(z_i - z_j)$$

with values in $\mathrm{End}(V_1 \otimes \cdots \otimes V_n)$ for a non-zero complex parameter κ. A horizontal section of the above flat bundle is a solution of the total differential equation $d\varphi = \omega\varphi$ for a function $\varphi(z_1, \cdots, z_n)$ with values in $V_1 \otimes \cdots \otimes V_n$. As the above holonomy of the connection ω we have a one-parameter family of linear representations of the pure braid group $\theta : P_n \to \mathrm{GL}(V_1 \otimes \cdots \otimes V_n)$.

For a complex number λ we denote by M_λ the Verma module of $sl_2(\mathbf{C})$ with highest weight λ. For $\Lambda = (\lambda_1, \cdots, \lambda_n) \in \mathbf{C}^n$ we put $|\Lambda| = \lambda_1 + \cdots + \lambda_n$ and consider the tensor product $M_{\lambda_1} \otimes \cdots \otimes M_{\lambda_n}$. For a non-negative integer m we define the space of weight vectors with weight $|\Lambda| - 2m$ by

$$W[|\Lambda| - 2m] = \{x \in M_{\lambda_1} \otimes \cdots \otimes M_{\lambda_n} \mid Hx = (|\Lambda| - 2m)x\}$$

and consider the space of null vectors defined by

$$N[|\Lambda| - 2m] = \{x \in W[|\Lambda| - 2m] \mid Ex = 0\}.$$

We have the following comparison theorem.

Theorem 0.1. *There exists an open dense subset U in $(\mathbf{C}^*)^2$ such that for $(\lambda, \kappa) \in U$ the homological representation $\rho_{n,m}$ with the specialization*

$$q = e^{-2\pi\sqrt{-1}\lambda/\kappa}, \quad t = e^{2\pi\sqrt{-1}/\kappa}$$

is equivalent to the monodromy representation of the KZ connection $\theta_{\lambda,\kappa}$ with values in the space of null vectors

$$N[n\lambda - 2m] \subset M_\lambda^{\otimes n}.$$

For the proof we use the description of the horizontal sections of the KZ connection by hypergeometric integrals due to V. Schechtman and A. Varchenko. By looking at the action of the quantum group on the homology of local systems we recover the quantum symmetry on the monodromy of the KZ connection due to V. G. Drinfel'd and myself. In the case of conformal field theory the parameters κ and λ are special. We can define the action of the braid group on the space of conformal blocks, which is a quotient space of the above space of null vectors and the KZ connection has a description as a Gauss-Mannin connection. This corresponds to representations of quantum groups at roots of unity. We study the structure of the image and the kernel of the action of braid groups and mapping class groups and get interesting series of finite index subgroups of mapping class groups. This part is a joint work with L. Funar.

Being like Sakamoto Hayato: Lecturers as professionals

Bill Barton, The University of Auckland

Abstract：
The best professionals, especially those at the top of their game, spend a lot of time improving. Just as people pay to watch baseball stars, so do students pay to come to university and be taught by us. They are paying for the best education they can buy. Are they getting it--or are they getting the same education we have given students for many years? What are we doing to ensure that the quality of our courses is continuing to improve? How can we go about improving undergraduate mathematics delivery to our students? What are the principles behind university teaching of mathematics? What practical steps can we take in our very full lives.

I will consider some innovative practices, and some professional development programmes for university mathematics staff. More than that, I will invite the audience to consider their practice as professionals so that we can all offer a better education than ever before.

Derived categories in algebraic geometry

Yujiro Kawamata (U. Tokyo)

Sapporo, June 28, 2013

A derived category is derived from an abelian category. An abelian category is closely attached to the original situation, but a derived category is not in the sense that derived categories of different origins are sometimes equivalent. This kind of rather unexpected equivalences of derived categories usually reveal deep mathematical facts. We review some of these phenomena in old and new examples.

(1) *Fourier-Mukai transform.*

Let X be an abelian variety and let $Y = \mathrm{Pic}^0(X)$ be its dual abelian variety. Y is the moduli space of line bundles on X which are algebraically equivalent to the trivial line bundle $\mathcal{O}_X$. Then their bounded derived categories of coherent sheaves $D^b(X)$ and $D^b(Y)$ are equivalent. The equivalence is given by the integration functor in the same way as the Fourier transform in analysis. The Fourier transform is given by $\Phi(h)(y) = \int_X e^{2\pi ixy}h(x)dx$, where $X = Y = \mathbf{R}/\mathbf{Z}$ and x, y are coordinates. The Fourie-Mukai transform is given by $\Phi(E) = Rp_{2*}(p_1^*E \otimes P)$, where p_1, p_2 are projections from the direct product $X \times Y$ and P is the universal line bundle.

(2) *Beilinson's decomposition.*

The bounded derived category of coherent sheaves on a projective space $D^b(\mathbf{P}^n)$ is generated by the line bundles $\mathcal{O}_X(-i)$ for $0 \le i \le n$. Let $A = \mathrm{End}(\bigoplus_{0\le i\le n} \mathcal{O}_X(-i))$. It is a non-commutative ring which is finite dimensional as a vector space over the base field k. Then $D^b(\mathbf{P}^n)$ is equivalent to the bounded derived category of finite right A-modules $D^b(\text{mod-}A)$.

$D^b(\mathbf{P}^n)$ is also generated by the locally free sheaves $\Omega^i_X(i)$ for $0 \le i \le n$, and is equivalent to the bounded derived category of finite right B-modules $D^b(\text{mod-}B)$ for $B = \mathrm{End}(\bigoplus_{0\le i\le n} \Omega^i_X(i))$.

(3) *BGG correspondence.*

Let V be a vector space of dimension n over a field k and W its dual space. Let x_i $(1 \leq i \leq n)$ be a basis of V and e_i $(1 \leq i \leq n)$ the dual basis. We put degree 1 on the x_i and -1 on the e_i.

Let $S = \mathrm{S}^*(V) \cong k[x_1, \ldots, x_n]$ be the symmetric algebra. It is a commutative ring and is infinite dimensional as a vector space over k. Let $E = \bigwedge^* W$ be the exterior algebra. It is a non-commutative ring and is finite dimensional as a vector space over k.

There is an equivalence $D(\text{Gr-Mod-}S) \cong D(\text{Gr-Mod-}E)$ of derived categories of graded modules. This equivalence is a refinement of the equivalence in the previous section.

(4) *McKay correspondence.*

Let G be a finite subgroup of $SL(n, \mathbf{C})$ which acts on $\mathbf{C}^n$ naturally. The quotient space $X = \mathbf{C}^n/G$ has normal singularities, but the quotient stack $\mathcal{X} = [\mathbf{C}^n/G]$ is a smooth Deligne-Mumford stack. A sheaf on $\mathcal{X}$ is by definition a sheaf on $\mathbf{C}^n$ with an equivariant action of G. A resolution of singularities $f : Y \to X$ is said to be *crepant* if $K_Y = f^*K_X$, i.e., the G-invariant holomorphic volume form on $\mathbf{C}^n$ lifts to that on Y. The McKay correspondence theorem says that, if $n \leq 3$, then there exists a crepant resolution $f : Y \to X$ and that there is an equivalence $D^b(Y) \cong D^b(\mathcal{X})$.

(5) *flops.*

This is a variant of the previous section. Let $f : Y \to X$ be a birational morphism from a smooth projective variety to a normal variety. Assume that $K_Y = f^*K_X$. Then in some cases we expect that there is a coherent sheaf of non-commutative $\mathcal{O}_X$-algebras $\mathcal{A}$ such that $D^b(Y) \cong D^b(\mathcal{A})$. For example, if $\dim Y = 3$ and f is small in the sense that f does not contract any divisor, then the assertion is true. This is generalized to the "K-equivalence and D-equivalence" conjecture.

(6) *Derived VGIT.*

I will also discuss more recent development on the relationship with the variation of GIT quotients.

Stieltjes-Wigert Polynomials and the q-Airy Function

Y. T. Li and R. Wong*

Abstract

Asymptotic formulas are derived for the Stieltjes-Wigert polynomials $S_n(z;q)$ in the complex plane, with the q-Airy function $A_q(z)$ being used as the approximant. One formula holds in any disc centered at the origin, and the other holds outside any smaller disc centered at the origin; the two regions together cover the whole plane. For $x > 1/4$, a limiting relation is also established between the q-Airy function $A_q(x)$ and the ordinary Airy function $\mathrm{Ai}(x)$ as $q \to 1$.

*The speaker.

Exponentially-accurate uniform asymptotic approximations for integrals and Bleistein's method revisited

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We obtain new uniform asymptotic approximations for integrals with a relatively exponentially small remainder via a surprisingly simple method. We illustrate how these results can be used to obtain remainder estimates in the so-called Bleistein method. Our new method is created to deal with integrals of the form

$$f_{a,b}(\lambda,\zeta)=\frac{1}{\Gamma(b)}\int_0^\infty \frac{t^{b-1}e^{-\lambda t}}{(1+t/\zeta)^a}G(t)\,dt,$$

where $\Re(\lambda)>0, \Re(b)>0$ and $|\mathrm{ph}\,(\zeta)|<\pi$. For these integrals the usual methods for remainder estimates fail.

As an application of our results we show that

$$\begin{aligned}\frac{\Gamma(\lambda)}{\Gamma(\lambda+b)}{}_2F_1\left({a,b \atop \lambda+b};-z\right) \sim & \zeta^b e^{-a\zeta}\left(\frac{e^\zeta-1}{\zeta}\right)^{a+b-1} U\left(b,b-a+1,\lambda\zeta\right)\\ & +\zeta^b\left(1-e^{-a\zeta}\left(\frac{e^\zeta-1}{\zeta}\right)^{a+b-1}\right)U\left(b,b-a+2,\lambda\zeta\right),\end{aligned}$$

as $\lambda\to\infty$ in $|\mathrm{ph}\,\lambda|\leqslant\pi/2$ uniformly for large $|z|$. In this result, a and b are fixed complex constants and $\zeta=\ln\left(1+z^{-1}\right)$. For the notation of the hypergeometric function and the Kummer-U function see chapters 13 and 15 in the DLMF. Note that this result is a generalisation of the well-known limit

$$\lim_{c\to\infty}{}_2F_1\left({a,b \atop c};-cx\right)=x^{-b}U\left(b,b-a+1,x^{-1}\right).$$

Ratio asymptotics and zero distribution for multiple orthogonal polynomials

Walter Van Assche
University of Leuven
Belgium

Abstract

The asymptotic behavior of the ratio of orthogonal polynomials can be obtained from the three term recurrence relation and involves a quadratic equation. We give the asymptotic behavior of the ratio of two neighboring multiple orthogonal polynomials under the condition that the recurrence coefficients in the nearest neighbor recurrence relations converge. This will involve an algebraic function of higher degree. The asymptotic distribution of the zeros can be obtained from the ratio asymptotics. We illustrate the result for some families of multiple orthogonal polynomials (multiple Hermite, multiple Laguerre, multiple Charlier, multiple Meixner, Jacobi-Piñeiro).

On the turning point problem for Painlevé equations with a large parameter

Yoshitsugu Takei
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In this talk we will discuss the turning point problem for Painlevé equations (P_J) $(J = \mathrm{I}, \ldots, \mathrm{VI})$ with a large parameter.

As is well known, a second order linear ordinary differential equation is transformed to the Airy equation near a simple turning point. In a similar manner, it is transformed to the Weber equation and to the Whittaker equation near a double turning point and near a simple pole, respectively. Our purpose is to generalize these results to Painlevé equations.

In [1] we proved that any Painlevé equation (P_J) is transformed to the first Painlevé equation (P_{I}) near a simple turning point. To be more specific, every formal instanton-type solution (or transseries solution) of (P_J) is transformed to that of the first Painlevé equation near its simple turning point. This result can be considered as a counterpart of the above result for a second order linear differential equation near a simple turning point. Then, what are the counterparts of the above results near a double turning point and a simple pole? Our answer to this question is given by the following

Theorem 1. *Near a double turning point every formal instanton-type solution of* (P_J) *is transformed to that of the* (*degenerate*) *second Painlevé equation.*

Theorem 2. *Near a simple pole every formal instanton-type solution of* (P_J) *is transformed to that of the third Painlevé equation of type* $(D8)$*, that is, the most degenerate third Painlevé equation.*

In the talk we will explain the precise meaning of these theorems and also some recent results related to them.

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On the relationship between the Painlevé equations and semi-classical orthogonal polynomials

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Abstract

In this talk I will be concerned with the relationship between the Painlevé equations and orthogonal polynomials with respect to semi-classical weights. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation and for some semi-classical weights, these coefficients in the recurrence relation can be expressed in terms of solutions of a Painlevé equation. I will show that the coefficients in these recurrence relations can be expressed in terms of Wronskians of special functions which arise in the description of classical solutions of Painlevé equations.

Specifically I shall discuss orthogonal polynomials with respect to a semi-classical Laguerre weight and variations of the Freud weight. For these orthogonal polynomials, the coefficients in the three-term recurrence relations can be expressed in terms of Wronskians of parabolic cylinder functions that arise in connection with special function solutions of the fourth Painlevé equation.

I shall also discuss semi-classical generalizations of the Charlier and Meixner polynomials, which are discrete orthogonal polynomials whose recurrence coefficients can be expressed in terms of Wronskians of modified Bessel functions and confluent hypergeometric functions (equivalently Kummer or Whittaker functions), respectively. These Wronskians arise in the description of special function solutions of the third and fifth Painlevé equations.

The tacnode Riemann-Hilbert problem

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Painlevé transcendents arise as special functions in an increasing number of models in mathematical physics and probability theory. A basic example is the appearance of Painlevé II in the description of the Tracy-Widom distribution for the largest eigenvalue of a random matrix.

The tacnode is a critical phenomenon for non-intersecting one-dimensional Brownian motions with prescribed starting and ending points. One can adjust parameters so that the Brownian motions fill out two tangent ellipses in the time-space plane. Local correlations at the tacnode were recently described in two different ways, namely on the one hand with Airy resolvent functions [1], [4], [5], and on the other hand with a Riemann Hilbert problem [3]. The connection between the two sets of formulas was found by Delvaux [2] and depends on an explicit solution of the tacnode Riemann-Hilbert problem in terms of functions related to Painlevé II, see [6].

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A connection problem for linear q-difference equations related to the q-Painlevé VI equation

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Abstract: For the continuous Painlevé equations, character varieties, which are cubic surfaces satisfied by monodromy invariants, play an important role to study nonlinear connection problems. In this talk, we show a q-analogue of a character variety for the q-Painlevé VI equation found by Jimbo and Sakai. We show a weak Riemann-Hilbert correspondence for q-linear equations, in the sense of G. D. Birkoff.

This is a joint work with Jean-Pierre Ramis and Jacques Sauloy in Toulouse.

Plancherel-Rotach asymptotic expansion for some polynomials from indeterminate moment problems

Dan Dai

May 22, 2013

Abstract

We study the Plancherel–Rotach asymptotics of four families of orthogonal polynomials, the Chen–Ismail polynomials, the Berg–Valent polynomials, the Conrad–Flajolet polynomials I and II. All these polynomials arise in indeterminate moment problems and three of them are birth and death process polynomials with cubic or quartic rates. We employ a difference equation asymptotic technique due to Wang and Wong. Our analysis leads to a conjecture about large degree behavior of polynomials orthogonal with respect to solutions of indeterminate moment problems.

This is joint work with Mourad E.H. Ismail and Xiang-Sheng Wang.

Equilibrium states and large deviation principles for one-dimensional maps under a weak hyperbolicity assumption

Juan Eduardo Rivera Letelier (Pontificia Universidad Catolica de Chile,Chile)

Abstract:

For a real or complex one-dimensional map satisfying a weak hyperbolicity assumption, we describe the thermodynamic formalism for Holder continuous potentials. We show that, rather unexpectedly, such a map has no phase transitions: The pressure function is real analytic on the space of Holder continuous potentials, and every Holder continuous potential has a unique equilibrium state. This last fact, together with various formulas to compute the pressure function, allows us to apply Kifer's method to obtain a full level-2 large deviation principle for periodic points, iterated preimages, and Birkhoff averages. This is a joint work with Huaibin Li.

On parameter loci of the Hénon family

Yutaka Ishii (Kyushu University, Japan)

Abstract:
In this talk I will report my ongoing project with Zin Arai (Hokkaido University). The purpose of our collaboration is to characterize the hyperbolic horseshoe locus and the maximal entropy locus of the Hénon family defined on R^2. Our basic strategy is to extend the Hénon map as well as the parameter space to C^2 and investigate its complex dynamical and complex analytic properties. We also employ interval arithmetic to verify certain numerical criteria which imply combinatorial and dynamical consequences.

Stable Sets in Z^n-Systems with Positive Entropy

Wen Huang (Chinese University of Science and Technology, China)

Abstract:
In this talk, the chaoticity appearing in the Z^n_+ -stable sets of a Z^n-dynamical system with positive entropy is investigated. It is shown that in any positive entropy Zn-system, there is a measure theoretically rather big set such that the Z^n-stable set of any point from the set contains a Mycielski Li-Yorke chaotical set under Z^n_-.

On the Minkowski dimensions of multi-dimensional and coupled multiplicative systems

Jung-Chao Ban (National Dong Hwa University, Taiwan)

Abstract:
This talk investigates a multiplicative integer system using a method that was developed for studying pattern generation problems. The entropy and the Minkowski dimensions of general multiplicative systems can thus be computed. A multi-dimensional decoupled system is investigated in three main steps. (I) identify the admissible lattices of the system; (II) compute the density of copies of admissible lattices of the same length, and (III) compute the number of admissible patterns on the admissible lattices. A coupled system can be decoupled by removing the multiplicative relation set and then performing procedures similar to those applied to a decoupled system. The admissible lattices are chosen to be the maximum graphs of different degrees which are mutually independent. The entropy can be obtained after the remaining error term is shown to approach zero as the degree of the admissible lattice tends to infinity.

Prevalence of non-uniform hyperbolicity at the first bifurcation of Hénon-like families

Hiroki Takahasi (Kyoto University, Japan)

Abstract:

We consider strongly dissipative Hénon-like maps in the plane, around the first bifurcation parameter a* at which the uniform hyperbolicity is destroyed by the formation of homoclinic or heteroclinic tangencies inside the limit set. In [Takahasi H.: Commun. Math. Phys. **312** 37-85 (2012)], it was proved that a* is a full Lebesgue density point of the set of parameters for which the non-wandering set of the corresponding map is transitive, and Lebesgue almost every initial point diverges to infinity under forward iteration. For these parameters, we show that all Lyapunov exponents of all invariant ergodic Borel probability measures are uniformly bounded away from zero, uniformly over all the parameters.

On the monodromy and bifurcations of the Hénon map

Zin Arai (Hokkaido University, Japan)

Abstract:
In this talk, we discuss the structure of the parameter space of the complex Hénon map. Our main tool is the monodromy representation that assigns an automorphism of the full shift to each loop in the hyperbolic parameter locus. Assuming that there exist infinitely many non-Wieferich prime numbers (it suffices to assume "the abc conjecture"), we show that automorphisms contained in the image of the monodromy representation must satisfy "Sign Gyration Compatibility Condition". This algebraic condition imposes some geometric restrictions on the structure of the parameter space.

On the barycenter set of some one-dimensional maps

Kuo-Chang Chen (National Tsing Hua University, Taiwan)

Abstract:
The standard tent map and baker map are often introduced as first examples of chaotic maps in standard textbooks on dynamical systems. By Birkhoff's ergodic theorem it is obvious that the average position or barycenter of generic orbits for these maps is 0.5. Periodic orbits are exceptional orbits in the sense that most of them have barycenters different from 0.5. In this talk we discuss some interesting properties about their barycenter set, provide some patterns of periodic orbits with the same barycenter, and discuss some related open questions.

Lagrangian non-intersection theory

Yakov Eliashberg (Stanford University, USA)

Abstract:
I will discuss recent flexibility results on constructions of Lagrangian embeddings and immersions with minimal number of double points. This work is joint with E. Murphy, and partially with T. Ekholm and I. Smith.

Periodic Orbits of Hamiltonian Systems: Beyond the Conley Conjecture

Viktor Ginzburg
University of California, Santa Cruz, USA

Abstract: How few periodic orbits can the Reeb flow have, when the contact form gives rise to the standard contact structure on a sphere? Can it have just one closed orbit? In this talk we discuss how methods from Hamiltonian dynamics, originally developed for the proof of the Conley conjecture, translate to the realm of Reeb flows to answer or at least to shed some light on these kinds of questions.

In particular, we prove, drawing from a joint work of the speaker with Hein, Hryniewicz and Macarini, that the existence of one simple closed Reeb orbit of a particular type (a symplectically degenerate maximum) forces the Reeb flow to have infinitely many periodic orbits. We use this result to give a different proof of a recent theorem of Cristofaro-Gardiner and Hutchings asserting that every Reeb flow on the standard contact three-sphere has at least two periodic orbits. (This approach together with several other ingredients leads to a more or less purely symplectic proof of the existence of infinitely many geodesics on the two-sphere.) We also discuss the effect of hyperbolic fixed points on the dynamics of Hamiltonian diffeomorphisms following a recent work of the speaker and Gürel.

A Γ-structure on the Lagrangian Grassmannian

Urs Frauenfelder *

March 1, 2013

This is joint work with Peter Albers and Jake Solomon. Given standard symplectic vector space $(\mathbb{R}^{2n}, \omega_0)$ the Lagrangian Grassmannian $\mathscr{L} = \mathscr{L}(n)$ is defined to be the manifold consisting of all Lagrangian subspaces of $(\mathbb{R}^{2n}, \omega_0)$. The Lagrangian Grassmannian can also be identified with the homogenous space $U(n)/O(n)$. Alternatively, we think of the Lagrangian Grassmannian as the space consisting of all linear, orthogonal antisymplectic involutions. Indeed, given an antisymplectic involution, the fixed point set is a Lagrangian subspace and this map identifies the two descriptions. Now thinking of points of the Lagrangian Grassmanian as linear, orthogonal, antisymplectic involutions we can define a product on the Lagrangian Grassmannian

$$\Theta : \mathscr{L} \times \mathscr{L} \to \mathscr{L}, \quad (R, S) \mapsto RSR.$$

Our main result is that for n odd, this product endows the Lagrangian Grassmannian with the structure of a Γ-manifold.

The notion of a Γ-manifold goes back to Hopf. Assume that M is a closed, orientable, connected manifold and $\Theta \colon M \times M \to M$ a smooth map which we think of as a product. If $x \in M$ plugging x into the left entry of Θ we get a map $\Theta_x := \Theta(x, \cdot) \colon M \to M$ which has a degree $\deg \Theta_x \in \mathbb{Z}$. Since M is connected, by invariance of the mapping degree under homotopies we see that $\deg \Theta_x$ is independent of the choice of x and we set $\ell(\Theta) := \deg(\Theta_x)$. Similarly, we can plug in x to the right entry of Θ and we get a degree $r(\Theta) \in \mathbb{Z}$ again independent of the choice of the point x. The left and right degree however do not need to agree. For example if Θ is a projection then one degree is zero while the other one is one. Now the tuple (M, Θ) is called a Γ-manifold if both degrees $\ell(\Theta$ and $r(\Theta)$ are different from zero. Note that this condition rules out trivial products like constant maps or projections to a factor. The interest in the existence of a Γ-structure comes from the discovery of Hopf that a Γ-structure endows the cohomology ring of the manifold with a Hopf algebra structure and therefore gives interesting topological information on the manifold. In particular, our main result allows us to recover some results by Fuks on the cohomology ring of the Lagrangian Grassmannian in the context of Hopf algebras.

*Seoul National University and University of Muenster

Non-displaceable Lagrangian submanifolds

Kaoru Ono
Research Institute for Mathematical Sciences, Kyoto University

Abstract:
I will review some criteria for non-displaceablity of Lagrangian submanifolds and give some examples, e.g., a certain Lagrangian torus in the one-point blow-up of a symplectically aspherical manifold.

SYZ transformation for coisotropic A-branes

Naichung Conan Leung, Chinese University of Hong Kong

Abstract:
Kapustin-Li observed that Lagrangian cycles alone is not enough for mirror symmetry for Calabi-Yau manifolds away from LCSL and they introduced the notion of coisotropic A-branes. In the semiflat case, Yi Zhang and I showed recently that the SYZ transformation of B-branes are precisely given by coisotropic A-branes. In this talk, I will explain this work.

Symplectic cohomology and duality for the wrapped Fukaya category

Sheel Ganatra, Stanford University, USA

Abstract:
Consider the wrapped Fukaya category W of a collection of exact Lagrangians in a Liouville manifold. Under a non-degeneracy condition implying the existence of enough Lagrangians, we show that natural geometric maps from the Hochschild homology of W to symplectic cohomology and from symplectic cohomology to the Hochschild cohomology of W are isomorphisms, in a manner compatible with ring and module structures. This is a consequence of a more general duality for the wrapped Fukaya category, which should be thought of as a non-compact version of a Calabi-Yau structure.

FINITE GROUP ACTIONS AND LAGRANGIAN FLOER THEORY

HANSOL HONG

This is joint work with Cheol-hyun Cho.

We construct a finite group action on the Lagrangian Floer theory when a symplectic manifold has a finite group action. For this, we first develop G-Novikov Morse theory. Then, we introduce a notion of a *spin profile* of a Lagrangian submanifold and define group actions on Floer cochain complexes for pairs of Lagrangian submanifolds with the same spin profiles. If time permits, we will also explain the case of Fukaya-Seidel category when the Lefschetz fibration is invariant under the group action on the total space of the fibration.

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Boltzmann equation and Green's function for shock profiles

Shijin Deng, Shanghai Jiao Tong University

The Abstract:
In this talk, we will discuss how to construct the Green's function for shock profiles of viscous conservation law in high dimension based on the Green's function for the Boltzmann equation.

Collision operator of Boltzmann equation

Jin-Cheng Jiang, National Tsing Hua University

Abstract: In this talk, we will discuss some fundamental issues of the collision operator of the Boltzmann equation. We should review the derivation of collision kernel first. Then we discuss the issues occurring in the classical derivation of collision kernel for the long range potential and some answers to these issues. If time allowed, we will also present some progress in understating of smoothing effect of the collision operator for the hard sphere model.

Gas Dynamics and Kinetic Theory, Some Historical Perspectives

Tai-Ping, Stanford University

Abstract:

We will focus on some key issues in the gas dynamics and the kinetic theory. Among the topics to be mentioned are the origin of modern theory of hyperbolic conservation laws, the renaissance of analytical study of multi-dimensional Euler equations, and recent connection to the kinetic theory. We will consider these issues from the historical point of view.

Some applications of symmetry relations for the steady/unsteady linearized Boltzmann equation

Shigeru Takata, Kyoto University

Abstract:
We will give a review of the symmetry relations for the steady/unsteady linearized Boltzmann equation for a rarefied gas that hold for arbitrary Knudsen numbers.

The talk will include a brief introduction of the concept of the relations, some specific relations that are predicted from the developed theory and their numerical demonstration, and a proposal of an approach to a hard problem by solving an easier adjoint problem by using the symmetry relations.

Smoothing effect for the homogeneous Boltzmann Equation

Tong Yang, City University of Hong Kong

Abstract:

In this talk, we will present our recent works about the smoothing effect on weak solutions to the homogeneous Boltzmann equation without angular cutoff.

Firstly, in a joint work with Alexandre-Morimoto-Ukai-Xu, we show that every L^1 weak solution with finite moments of all order acquires C^∞ regularity in any positive time. And then in a joint work with Morimoto, we show that Villani conjecture holds for the Maxwellian molecule type cross section. That is, any weak solution with measure initial datum except a single Dirac mass acquires C^∞ regularity in any positive time. Here, the coercivity estimate plays an important role. In particular, to prove Villani conjecture, a new time degenerate coercivity estimate is given.

The reseach was supported in part by the General Research Fund of Hong Kong, CityU # 104511.

Viscous wave propagation at interface

Shih-Hsien Yu (National University of Singapore, Singapore)

Abstract:
In this talk we implement the LY algorithm to derive a two-sided master relationship. This gives rise the operator to construct the surface wave at the interface and the construction of the Green's function of a simple variable coecient problem for viscous conservation laws.

L^p-scattering and uniform stability of kinetic equations

Seung-Yeal Ha, Dept. of Math., Seoul National University

Abstract

In this talk, we will review recent progress on the L^p-scattering and uniform stability of several kinetic equations with self-consistent forces. For the Vlasov equation with a self-consistent force, we will show that the Coulomb's potential in three dimensions is critical in the sense that if spatial dimension is larger than three, there exists a L^1-scattering, whereas for low dimensions less than equal to three, there is no L^1-scattering. We also present a framework for the L^p-stability of kinetic equations. This is a joint work with Sun-Ho Choi (NUS) and Qinghua Xiao (SNU).

Vafa-Witten inequality and Poincare duality in noncommutative geometry

Raphael Ponge, Seoul National University

Abstract:
The inequality of Vafa-Witten provides us with a bound on the first eigenvalue of a Dirac operator with coefficient in any Hermitian vector bundle.
The remarkable feature of this inequality is the fact that the bound depend only on the manifold, not on the datum of the Hermitian vector bundle. In the framework of noncommutative geometry the role of manifolds is played by spectral triples. In order to deal with some "type III" geometric situations (e.g., non-isometric group actions on manifolds) Connes-Moscovici introduced "twisted spectral triples".The main aim of this talk is to present a version of Vafa-Witten inequality for twisted spectral triples, including twisted spectral on the noncommutative torus associated to conformal weights. An important ingredient of the proof is a version of Poincar¥'e duality for twisted spectral triple. (Joint work with Hang Wang, Mathematical Science Center,Tsinghua University, Beijing).

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Virtual neighborhood technique for pseudo-holomorphic spheres

Bai-Ling Wang, Australian National University

Abstract:
The main analytical difficulty in defining the Gromov-Witten invariants for general symplectic manifolds is the failure of the transversality of the compactified moduli space of pseudo-holomorphic curves. The foundation to resolve this issue is to construct a virtual fundamental cycle for the compactified moduli space. For smooth projective varieties, the construction of this virtual fundamental cycle was carried out by Li-Tian. For general symplectic manifolds, the virtual fundamental cycle was constructed by Fukaya-Ono, Li-Tian, Liu-Tian. Ruan proposed a virtual neighborhood technique as a dual approach using the Euler class of a virtual neighborhood, in which the compactified moduli space is a zero set of a smooth section of a finite dimensional orbfiold vector bundles over an open orbifolds.

Further developments in Gromov-Witten theory and its applications require differential structures on these moduli spaces involved. Some of the analytical details have been provided by Ruan, Li-Ruan and Fukaya-Oh-Ohta-Ono. Other methods like the polyfold theory by Hofer-Wysocki-Zehnder are developed to deal with this issue. In a recent joint work with Bohui Chen and Anmin Li, we implement the full machinery of virtual neighborhood technique to the Gromov-Witten theory using virtual manifold/orbifolds developed by Chen-Tian. This provides an alternative approach to establish differentiable structure on moduli spaces arising from the Gromov-Witten invariants. In this short talk, I explain how this method can be applied to the genus 0 Gromov-Witten theory by resolving the analytical issue of the non-differentiable \$PSL(2, ¥C)\$-action.

Lagrangian Floer theory and mirror symmetry on compact toric manifolds

Kaoru Ono
Research Institute for Mathematical Sciences, Kyoto University

Abstract:
I will explain how Lagrangian Floer theory for torus fibers in a compact toric manifold is governed by the so-called potential function after joint works by Fukaya, Oh, Ohta and myself. I would also like to mention a generation criterion for Fukaya category and, in particular, the Fukaya category of a compact toric manifold is split-generated by objects corresponding to critical points of the potential function based on our (FOOO) joint work with Abouzaid.

Mirror Map as Generating Function of Intersection Numbers

Masao Jinzenji, Hokkaido University

Abstract:
In this talk, we discuss geometric construction of the mirror map used in the mirror computation of Gromov-Witten invariants. We reconstruct the mirror map as a generating function of intersection numbers of the moduli space of holomorphic maps compactified by chains of quasi maps. We also apply this formalism to compute some open Gromov-Witten invariants.

Mickelsson's twisted K-theory invariant

Kiyonori Gomi
Department of Mathematical Sciences, Shinshu University

Twisted K-theory, being a topological K-theory with certain local coefficients, was invented by Donovan-Karoubi and also by Rosenberg. This notion is originally applied to a generalization of the Thom isomorphism theorem, and recently to mathematical physics. For example, in the context of string theory, twisted K-theory is thought of as home of Ramond-Ramond charges of D-branes with background B-fields.

Mickelsson's invariant is an invariant of some odd twisted K-classes on 3-manifolds [1]. The point of this invariant is that it detects some torsion elements, in contrast to the Chern character, which detects all the non-torsion elements but no non-torsion elements.

The theme of my talk is a reformulation of Mickelsson's invariant: For its account, we denote by $K^1_P(M)$ the odd twisted K-group of a manifold M, where P is a principal bundle whose structure group is the projective unitary group $PU(H) = U(H)/U(1)$ of a separable infinite dimensional Hilbert space H. Such a principal bundle is classified by a cohomology class $h(P) \in H^3(M, \mathbb{Z})$. Then there is a simplest odd twisted K-theory invariant, namely, a natural homomorphism $\mu_1 : K^1_P(M) \to H^1(M, \mathbb{Z})$.

Theorem 1. *For any principal $PU(H)$-bundle P on a manifold M, there is a natural homomorphism*

$$\mu_3 : \ \mathrm{Ker}\mu_1 \longrightarrow H^3(M, \mathbb{Z})/(\mathrm{Tor} + h(P) \cup H^0(M, \mathbb{Z})),$$

where Tor *is the torsion subgroup. The homomorphism recovers original Mickelsson's invariant if M is compact, oriented, connected and* 3*-dimensional.*

The homomorphism μ_3, which is constructed by using a Čech-de Rham cocycle, factors through a homomorphism in computing the Atiyah-Hirzebruch spectral sequence. By the help of this factorization and a computation of μ_3, we can reproduce the known result that $K^1_P(SU(3)) \cong \mathbb{Z}/h$ in the case that $h = h(P) \in H^3(SU(3), \mathbb{Z}) \cong \mathbb{Z}$ is odd.

References

[1] J. Mickelsson, *Twisted K theory invariants.* Lett. Math. Phys. 71 (2005), no. 2, 109–121.

Index gerbe and differential K-theory

Siye Wu, University of Hong Kong

Abstract:
We discuss results on determinant line bundles of Bismut-Freed, Quillen and Witten, the notion of index gerbe due to Carey-Mickelsson-Murray and Lott, and the recent index theorem of Freed-Lott in differential K-theory. Then we show, by calculations in differential K-theory, that for a family of Riemannian manifolds of odd dimensions, the index in the odd differential K-group maps to the Deligne cohomology class of the index gerbe. This is a joint work with V. Mathai.

Brauer category and fundamental theorems of classical invariant theory

R. B. Zhang, University of Sydney

Abstract:
A strict monoidal category, referred to as the Brauer category, is introduced and applied to study the invariant theory of the orthogonal and symplectic groups. Full tensor functors are constructed from the Brauer category to the categories of tensor representations of these groups. This leads to a generalization of the first and second fundamental theorems of invariant theory to a category theoretical setting, that enables us to construct presentations for the endomorphism algebras of the tensor representations. This is joint work with Gus Lehrer.

References

1) G I Lehrer and R B Zhang, The second fundamental theorem of invariant theory for the orthogonal group, Annals of Mathematics **176** (2012) 2031-2054.
2) G I Lehrer and R B Zhang, The *Brauer Category* and Invariant Theory. arXiv:1207.5889.

Monodromy groups of conformal field theory

Toshitake Kohno (The University of Tokyo)

Abstract:
There is an action of the mapping class groups on the space of the conformal blocks for Riemann surfaces defined by monodromy. We give a qualitative estimate for the images of such representations of mapping class groups. In particular, we show that the image of any Johnson subgroup contains a non-abelian free group. In the case of braid groups we describe the monodromy group in relation with triangle groups. Based on the estimate of the monodromy groups we give an answer to conjectures by Squier on Burau representations of braid groups. This is a joint work with Louis Funar.

A new method of constructing metrics on surfaces which maximize the first eigenvalue

Richard Schoen, Stanford University

Abstract:
The problem of finding sharp upper bounds on the first eigenvalue for Riemannian surfaces has a long history. We will describe a new method for constructing smooth maximizing metrics both for compact surfaces and for surfaces with boundary.

Self-Shrinkers of the Mean Curvature Flow in Euclidean Space with Arbitrary Codimension

Haizhong Li (Tsinghua University, Beijing)

Abstarct: In this talk, we will report our results about self-shrinkers of the mean curvature flow in Euclidean space with arbitrary codimension, which include: lower volume growth estimates for self-shrinkers; classification and rigidity of self-shrinkers; the diameter estimate of compact shrinkers; gap theorems of self-shrinkers and F-stability for self-shrinkers.

On variational characterizations of exact solutions in general relativity

Sumio Yamada (Tohoku University, Japan)

Abstract: In this talk, I will report on the progress of a joint project with Marcus Khuri and Gilbert Weinstein, where we introduce a new characterization of the exact solutions to the Einstein Maxwell equation; namely the Reissner-Nordstrom metrics and the Majumdar-Papapetrou metrics. Those metrics are characterized as saturating a set of inequalities, often categorized as the Penrose-type inequalities, where several versions have been proved as the Positive Mass Theorem and the Riemannian Penrose Inequality.

Cauchy Data of Vacuum Einstein Equations Evolving to Black Holes

Pin Yu (Tsinghua University, China)

Abstract:
We show the existence of complete, asymptotically flat Cauchy initial data for the vacuum Einstein field equations, free of trapped surfaces, whose future development must admit a trapped surface. Moreover, the datum is exactly a constant time slice in Minkowski space-time inside and exactly a constant time slice in Kerr space-time outside. The proof makes use of the full strength of Christodoulou's work on the dynamical formation of black holes and Corvino-Schoen's work on the constructions of initial data set.

The Willmore flow in Riemannian spaces

Glen Wheeler (University of Wollongong, Australia)

Abstract:

In this talk I will detail recent developments on the Willmore flow of surfaces in 3-manifolds with a prescribed Riemannian structure. If this structure is flat then local parabolic regularity for data in terms of the local L^{2}-norm of the second fundamental form is contained in Kuwert and Schaetzle's previous work on the Willmore functional. If there is some ambient curvature then several problems arise: the evolution of key geometric quantities becomes more complex and tools such as the Sobolev inequality are either invalid or hold only in special circumstances. Using a new concentration of area method we are able to recover an analogue of Kuwert and Schaetzle's local parabolic regularity theorem. We present one application yielding a global existence result in certain ambient spaces. This is joint work with Jan Metzger and Valentina-Mira Wheeler.

An equation linking W-entropy with reduced volume

Guoyi Xu (Tsinghua University, China)

Abstract: W-entropy and reduced volume for Ricci flow were introduced by Perelman, which had proved their importance in the study of Ricci flow. Lei Ni studied the analogous concepts for heat equation on static manifolds, and proved an equation, which links the large timebehavior of these two. Due to the surprising similarity between those concepts in the Ricci flow and the linear heat equation, a natural question whether such equation holds for the Ricci flow ancient solution was asked by Lei Ni. In this talk, we show a proof of Lei Ni's equation based on a new method. And following the same philosophy of this method, we answer Lei Ni's question positively for type I k-solutions of Ricci flow.

Some topological problems and computational methods in the theory of braids and related groups

Mario Salvetti
Department of Mathematics
University of Pisa, Italy

The topological theory of Braid groups has been developed in connection with several important subjects, including homotopy theory and singularity theory. From the point of view of cohomological calculations, the theory can be developed in a very similar way for the whole class of groups usually called *Artin groups*. Each of these groups has some (finite or infinite) Coxeter group as quotient, which acts freely over a *configuration space* which is the complement to an *hyperplane arrangement*. The *orbit space* of this action has the original Artin group as its fundamental group.

Many calculations has been performed, starting from the 70's for the trivial cohomology of the braid group and of the Artin groups of finite type.

The explicit construction of CW-complexes over which the orbit space contracts give algebraic complexes computing the twisted cohomology of these groups. In particular, many calculations where produced for abelian local systems over the module of Laurent polynomial. This is a very important module, whose cohomology is strictly related to the (trivial) cohomology of the associated *Milnor fiber*.

We give here some new way to do computations, especially related to typical (even if not old) methods in Combinatorics. We make particular use of some variation of the so called Discrete (or Combinatorial) Morse Theory and we show how the twisted cohomology of the Artin groups can be unexpectedly related to the cohomology of certain graph complexes, which can be computed by using purely combinatorial methods.

The introduction of an interesting class of *weighted sheaves over posets*, being an interesting object by itself, constitutes the bridge between the cohomological theory of Artin groups and combinatorics.

We introduce a spectral sequence associated to such sheaves and we show how discrete (algebraic) Morse theory fits into this theory to allow explicit computations. In particular, besides the case of braid groups, we give explicit cohomology for other groups, including many affine type Artin groups.

Topology of Commuting Matrices

Alejandro Adem (University of British Columbia, Canada)

Abstract:

Let G denote a Lie group. In this talk we will present recent results on the homotopy and cohomology of certain classifying spaces built out of commuting elements in G. Models for these spaces will be given which involve homotopy colimits over topological posets. We will describe cohomology calculations for the case of the general linear groups over the complex numbers using multisymmetric polynomials. This is joint work with José Manuel Gómez.

Milnor fibers of real line arrangements

Masahiko Yoshinaga
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Let $Q(x, y, z) \in \mathbb{C}[x, y, z]$ be a homogeneous polynomial of degree $n + 1$. The variety $F = Q^{-1}(1) \subset \mathbb{C}^3$ is called the Milnor fiber of the hypersurface $X = Q^{-1}(0) \subset \mathbb{C}^3$. The Milnor fiber F is preserved by the scalar multiplication $(x, y, z) \longmapsto (\zeta x, \zeta y, \zeta z)$, where $\zeta = e^{2\pi i/(n+1)}$. It induces an automorphism $\rho : F \longrightarrow F$, so called the Monodromy automorphism. Obviously $\rho^{n+1} = id$. The homology $H_1(F, \mathbb{C})$ equipped with the automorphism induced by the monodromy ρ is an important object. Indeed it is related to several topological invariants (e.g., Local system homology groups, Alexander polynomial of the fundamental group, the number of certain plane curves passing through the prescribed points, and so on).

In this talk, we will discuss the case that $Q = \prod_{i=1}^{n+1} \alpha_i$ splits into the product of linear forms of real coefficients. Then $Q^{-1}(0) = \bigcup_{i=1}^{n+1} H_i$ is a line arrangement in the projective plane $\mathbb{RP}^2$. We will give a new algorithm which computes the monodromy eigen spaces of $H_1(F, \mathbb{C})$ in terms of real and combinatorial structures of chambers. We also give a new upper bound of the dimension of the eigen spaces and several conjectures.

This talk is based on the preprint *"Milnor fibers of real line arrangements"*. arXiv:1301.1430

The freeness of ideal subarrangements of Weyl arrangements

Hiroaki Terao, Hokkaido University

Abstract:
A Weyl arrangement is the arrangement defined by the root system of a finite Weyl group. When a set of positive roots is an ideal in the root poset, we call the corresponding arrangement an ideal subarrangement. Our main theorem asserts that any ideal subarrangement is a free arrangement and that its exponents are given by the dual partition of the height distribution. In particular, when an ideal subarrangement is equal to the entire Weyl arrangement, our main theorem yields the celebrated formula by Shapiro, Steinberg, Kostant and Macdonald. Our proof of the main theorem heavily depends on the theory of free arrangements and thus greatly differs from the earlier proofs of the formula. (This work was done with Takuro Abe, Mohamed Barakat, Michael Cuntz, Torsten Hoge.)

Some maps out of spaces-of-embeddings

Ryan Budney (University of Victoria, Canada)

Abstract:
This talk will be about operad actions on spaces of knots and associated bar constructions, the relation to the Goodwillie-Weiss calculus of functors and finite-type invariants.

Morse homotopy and invariants of manifolds

Tadayuki Watanabe

After Witten's discovery of path integral interpretation of quantum invariants of knots and 3-manifolds, several rigorous and powerful theories of universal invariant for homology 3-spheres appeared, e.g. perturbative Chern–Simons theory Z^{CS} of Axelrod–Singer and Kontsevich, and a combinatorial invariant Z^{LMO} of Le–Murakami–Ohtsuki. These invariants take values in a space of graphs called Jacobi diagrams or Feynman diagrams, and are known to be universal among Ohtsuki's finite type invariants for rational homology 3-spheres. Z^{CS} is defined by integration over spaces of configurations of points on a 3-manifold and hence can be considered as an "analytic" invariant. Z^{LMO} is constructed from Kontsevich's link invariant by ingenious combinatorial argument and can be considered as an "algebraic" invariant.

My talk is concerned with Fukaya's "topological" construction of invariant of 3-manifolds, obtained by using Morse theory. We give a generalization of Fukaya's invariant to graphs with arbitrary number of loops at least 2.

Mathematicians' Responsibility for Mathematics Education
— The Role of Lighthouse in the Dark —

Ryosuke Nagaoka
Meiji University

Without exaggeration, most depertments of mathematics of Japaneses Universities now cark serious disrelish for mathematial thinking among young students.

A short but intensive hearing of students in mathmatical departments of several universities proved that they have their firm conviction on their way to learn mathematics: Learning of mathematics is nothing but learning by heart how to "solve" problems and how to "prove" theorems in contrast eith their poor confidence in mathematics they learned in their university. Young students are delligent enough to learn mathematics by heart without any understanding of the mathematical theories underlying back in their background. Almost all of them agreed that they were taught to study mathematics in that style in high school days.

In Japan as in other counties, the standard style to teach mathematics up to secondary level is quite naturally to instruct basic techniques of calculation in the elementary level and to give wider chances to experience how to apply the basic techniques to apparently difficult problems.

In a sence, mathematics education has been carried out with a large emphasis the problem solving especially in Japan.

But problem solvings are not the goal of teaching mathematics, just the good style to inspire students' taste for mathematical thinking, But ironically, all "efforts" in high schools to give students the chances to learn the variety of mathematical problems are now to result in the firm belief among youth that learning mathematics is never more related to theorectical thinking than memoriging the correspondance of natural numbers with the historic events.

The deep discrepancy of the views over learning mathematics between students and professors is not easy to overcome. The real cause which has brought the discrepancy to so serious extent is not at all easy even to identity. multi-sided collaborative study to solve these difficult problems with prudent intelligence not bringing easy and quick conclusion is really needed.

However it is clear with few exception that young high school students cannnot have their happy encounters with mathematics without good mathematics high school teachers who love mathematics and who show students various interesting aspects of mathematics through "usual" class of "ordinary" mathematics with their confidence in understanding of basic modern mathematics.

Therefore we can have a hope that an innovative change to education system can be brought by mathematical department without appialing to political powers and without praying for big social changes, if we are more successful in bringing more and more graduates with mathematical confidence as well as mathematical competency. Ideas more in detail will be discussed.

A New Asymptotic Perturbation Theory and Applications to Massless Quantum Fields

Asao Arai
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Abstract

Let H_0 and H_{I} be a self-adjoint and a symmetric operator on a complex Hilbert space, respectively, and suppose that H_0 is bounded below and the infimum E_0 of the spectrum of H_0 is a simple eigenvalue of H_0 which is *not necessarily isolated*. In this paper, we present a new asymptotic perturbation theory for an eigenvalue $E(\lambda)$ of the operator $H(\lambda) := H_0 + \lambda H_{\mathrm{I}}$ ($\lambda \in \mathbf{R} \setminus \{0\}$) satisfying $\lim_{\lambda \to 0} E(\lambda) = E_0$. The point of the theory is in that it covers also the case where E_0 is a non-isolated eigenvalue of H_0. Under a suitable set of assumptions, we derive an asymptotic expansion of $E(\lambda)$ up to an arbitrary finite order of λ as $\lambda \to 0$. We apply the theory to a massless quantum field model.

On a limiting resonance distribution theorem for the Stark effect in the semiclassical limit

Carlos Villegas, Universidad Nacional Autonoma de Mexico

Abstract:
We study the distribution of resonances nearby the eigenvalues of the hydrogen atom hamiltonian under the influence of a constant electric field with a suitable intensity in the semiclassical limit. We show that such a limit involves averages of the position coordinate in the direction of the electric filed along the classical orbits of the Kepler problem.

Eigenvalue Asymptotics for Dirichlet and Neumann Half-plane Magnetic Hamiltonians

Pablo Miranda

Facultad de Física

Pontificia Universidad Católica de Chile

Abstract

In this talk we consider two Schrödinger operators with constant magnetic field on a h alf-plane, one defined with Dirichlet boundary conditions and another with Neumann boundary conditions. If V is a real, non-positive decaying electric potential, we study the discrete spectra of the original operators perturbed by V. In the Dirichlet case we show that, even under very weak perturbations V, infinitely many eigenvalues appear below the essential spectrum of the operator, while the Neumann case depends of the decaying rate of V. This is joint work with Vincent Bruneau and Georgi Raikov.

Propagation of singularities for Schrödinger equations with long range perturbations

Shu Nakamura (University of Tokyo)

Abstract: The singularities of solutions to Schrödinger equations (with short range perturbations) can be described using the scattering theory for the corresponding classical mechanics. If the perturbation is long range type, then we need to use long range scattering technologies, namely, we need to employ a solutions to the Hamilton-Jacobi equation in the momentum space. If the perturbation is modestly long-range, then we can use the Dollard type approximate solution, and we can describe the singularities rather explicitly. (Partly joint work with K. Horie)

Trace - Integral Formulae for functions of one and two operator variables

Kalyan B. Sinha
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Abstract:
Krein's classical theorem on the trace of the difference of a function of two self-adjoint operators, differing by a trace-class operator, is extended to the second and third-order with exact expressions. An associated question about functions of a pair of commuting self-adjoint bounded tuples will also be discussed.

Commutator methods for the spectral analysis of time changes of horocycle flows

Rafael Tiedra, Pontificia Universidad Católica de Chile

Abstract:
We show that all time changes of the horocycle flow on compact surfaces of constant negative curvature have purely absolutely continuous spectrum in the orthocomplement of the constant functions. This provides an answer to a question of A. Katok and J.-P. Thouvenot on the spectral nature of time changes of horocycle flows. Our proofs rely on positive commutator methods for self-adjoint operators and the unique ergodicity of the horocycle flow.

Two-solenoidal Aharonov-Bohm effect with quantized magnetic fluxes

Takuya Mine, Kyoto Institute of Technology

Abstract:
We consider the motion of a quantum particle confined in a plane under the influence of two infinitesimally thin magnetic solenoids perpendicular to the plane. Provided that two fluxes equal the quantum of magnetic flux, we give the generalized eigenfunctions of the corresponding Hamiltonian using various Mathieu functions. As a consequence, we give an explicit formula for the scattering amplitude using some special value of Mathieu functions.

Transversely periodic potentials on trees

Richard Froese, University of British Columbia

Abstract

We discuss a class of random Schrodinger operators on trees where a pair of potentials is chosen independently at random for every level in the tree and then repeating periodically across that level. This model exhibits a weak disorder transition between localization and delocalization as the joint distribution of the potentials is varied.

Sixty years of compactifications of $\mathbf{C}^n$

Baohua Fu
(Academia Sinica, Beijing)

Abstract:

In 1954, Hirzebruch posed the problem to classify analytic compactifications of $\mathbf{C}^n$ of $b_2 = 1$. I shall give a survey on results obtained for this problem over the sixty years and then discuss the new direction initiated by Hassett-Tschinkel in 1999 on equivariant compactifications of $\mathbf{C}^n$. This is based on joint works with Jun-Muk Hwang.

Compactification of moduli of abelian varieties

Iku Nakamura
(Hokkaido University)

Abstract:

The moduli space of abelian varieties is compactified geometrically by adding certain GIT-stable degenerate abelian varieties, which is similar to the Deligne-Mumford compactification of the moduli space of nonsingular curves of genus g by stable curves. We report on some relevant complete moduli spaces of degenerate abelian varieties and the connections between the moduli spaces.

On certain duality of Neron-Severi lattices of supersingular K3 surfaces (a joint work with Ichiro Shimada)

Shigeyuki Kondo
(Nagoya University)

Abstract:

A K3 surface defined over an algebraically closed field is said to be supersingular (in the sense of Shioda) if the rank of its Neron-Severi lattice is 22. Supersingular K3 surfaces exist only when the base field is of positive characteristic. In this talk, we present certain duality between Neron-Severi lattices of supersingular K3 surfaces.

Categorification of Donaldson-Thomas invariants via Perverse Sheaves

Young-Hoon Kiem
(Seoul National University)

Abstract:

I will talk about a joint work (arXiv: 1212.6444) with Jun Li, where we showed that there is a perverse sheaf on a fine moduli space of stable sheaves on a smooth projective Calabi-Yau three-fold, which is locally the perverse sheaf of vanishing cycles for a local holomorphic Chern-Simons functional. The Euler number of the hypercohomology of this perverse sheaf is the Donaldson-Thomas invariant. We further show that this perverse sheaf lifts to a mixed Hodge module and enables us to define the Gopakumar-Vafa invariants mathematically.

Gepner type stability conditions on graded matrix factorizations

Yukinobu, Toda
(IPMU, Tokyo University)

Abstract:

I will introduce Gepner type Bridgeland stability conditions on triangulated categories. I conjecture that an existence of such a stability condition on a Calabi-Yau 3 category implies a non-trivial relationship among Donaldson-Thomas invariants. The main result is to show the existence of Gepner type stability conditions on triangulated categories of graded matrix factorizations of weighted homogeneous polynomials with low degrees. I also discuss a conjectural construction of such a stability condition on a quintic 3-fold, and relate it to a conjectural stronger version of BG inequality for stable sheaves.

ADE quiver bundles on a curve

Bumsig Kim
(KIAS, Seoul)

Abstract:

This is work in progress with H. Lee and T. Logvinenko. According to Diaconescu's work, the moduli of stable ADHM quiver bundles on a smooth projective curve X is naturally isomorphic to the moduli of stable pairs on $\mathbf{C}^2 \times X$. We extend this with Γ-action on $\mathbf{C}^2$, where Γ is a finite subgroup of $SL_2(\mathbf{C})$. Namely, using McKay correspondence we relate stable ADE quiver bundles to "perverse stable pairs" on $\Gamma\text{-Hilb}(\mathbf{C}^2) \times X$.

Existence and stability of time-periodic solution of the compressible Navier-Stokes equation

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We consider a time periodic problem of the following compressible Navier-Stokes equation in $\mathbb{R}^n$ $(n \geq 3)$:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho v) = 0, \\ \rho(\partial_t v + v \cdot \nabla v) - \mu \triangle v - (\mu + \mu')\nabla \operatorname{div} v + \nabla P(\rho) = \rho g. \end{cases} \tag{1.1}$$

Here, $\rho = \rho(x,t)$ and $v = (v^1(x,t), \cdots, v^n(x,t))$ denote the unknown density and the unknown velocity field, respectively, at time $t \geq 0$ and position $x \in \mathbb{R}^n$; $P = P(\rho)$ is the pressure that is assumed to be a smooth function of ρ satisfying $P'(\rho_*) > 0$ for a given constant $\rho_* > 0$; μ, μ' are the viscosity coefficients that are assumed to be constants and satisfy $\mu > 0$ and $\frac{2}{n}\mu + \mu' \geq 0$; and $g = g(x,t)$ is a given external force periodic in t with period $T > 0$.

The purpose of this talk is to investigate the existence and stability of a time-periodic solution of system (1.1).

Ma, Ukai, and Yang (2010) [1] showed that if $n \geq 5$, there exists a time-periodic solution $(\rho_{per}(t), v_{per}(t))$ around $(\rho_*, 0)$ of (1.1) for sufficiently small g. Furthermore, it was shown that the time-periodic solution is stable under sufficiently small initial perturbations and that the perturbation $(\rho(t) - \rho_{per}(t), v(t) - v_{per}(t))$ satisfies

$$\|(\rho(t) - \rho_{per}(t), v(t) - v_{per}(t))\|_{L^2} \leq C(1+t)^{-\frac{n}{4}}. \tag{1.3}$$

We will show the existence of a time-periodic solution $(\rho_{per}(t), v_{per}(t))$ for $n \geq 3$, if the external force g satisfies the condition $g(-x,t) = -g(x,t)$ $(x \in \mathbb{R}^n,\ t \in \mathbb{R})$ and g is small enough in some weighted Sobolev space. In addition, we will prove that the time-periodic solution $(\rho_{per}(t), v_{per}(t))$ is stable under sufficiently small initial perturbations and that the perturbation $(\rho(t) - \rho_{per}(t), v(t) - v_{per}(t))$ satisfies the decay estimate (1.3).

The results of this talk were obtained in a joint work with Kazuyuki Tsuda (Kyushu University).

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Maria. Schonbek, UCSC

Asymptotic stability of mild solutions to the Navier-Stokes equations

Abstract We consider the initial value problem for the Navier-Stokes equations modeling an incompressible fluid in three dimensions:

$$\begin{aligned} u_t + u \cdot \nabla u + \nabla p = \Delta u + F, \quad (x,t) \in \mathbb{R}^3 \times (0,\infty), \\ \operatorname{div} u = 0, \\ u(x,0) = u_0(x). \end{aligned}$$

It is well-known that this problem has a unique global-in-time mild solution for a sufficiently small initial condition u_0 and for a small external force F in suitable scaling invariant spaces. We show that these global-in-time mild solutions are asymptotically stable under every (arbitrary large) L^2-perturbation of their initial conditions.
The work is joint with Grsegorz Karch and Dominika Pilarczyk .

Stability of equilibria for incompressible two-phase flows with phase transitions

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Abstract

A basic model for incompressible two-phase flows with phase transitions consistent with thermodynamics in a bounded domain in the case of constant but non-equal densities of the phases is considered. We briefly discuss the well-posedness of the model in an L_p-setting which is based on *maximal regularity.* The main part of the talk is devoted to the stability of the equilibria. The negative total entropy of the problem serves as a Ljapunov functional and hence we know that the equilibria without boundary contact are zero velocity, constant temperature, constant pressure in each phase, and a subdomain, which forms one phase in the bounded domain, consists of a finite number of nonintersecting balls of equal size. We prove that an equilibrium is stable if and only if the phases are connected, otherwise it is unstable. This is a joint work with J. Prüss (Halle) and M. Wilke (Halle).

Classical solutions of mean-field games

Diogo A. Gomes*

April 30, 2013

A model problem for mean-field games [LL06a, LL06b, LL07, HMC06, HCM07] is the system

$$-V_t + H(D_xV, x) = \Delta V + g(\theta), \quad \theta_t - \mathrm{div}(\theta D_x V) = \Delta V. \tag{1}$$

together with initial-terminal conditions $V(x,T) = \psi(x)$, $\theta(x,0) = \theta_0$, and periodic boundary conditions in x. In [LL06b, LL07] the authors give conditions for existence of weak solutions to (1). In this talk we prove the following results (joint work with H. S. Morgado and G. Pires [GPSM13]):

Theorem 1. *Let $g(m) = m^\alpha$ and $H(p,x) = \frac{|p|^2}{2} + V(x)$, ψ and θ smooth, $\theta > 0$ If $d = 2$ and $\alpha > 0$, or if $d = 3$ and $\alpha < \frac{1}{2}$, then V is Lipschitz.*

Once this regularity is obtained then further regularity results can also be obtained by bootstrapping and using standard methods:

Theorem 2. *Under the conditions of Theorem 1 If $d = 2$ and $\alpha > 0$, or if $d = 3$ and $\alpha < \frac{1}{2}$, then $\ln m$ is Lipschitz.*

From the regularity obtained in the previous theorem then it is a routine matter to prove existence of smooth solutions. A number of further extensions are possible by considering more general Hamiltonians. A number of similar results for the time independent case can also be addressed similarly.

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Forward Discretely Self-Similar Solutions of the Navier-Stokes Equations

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Abstract

Denote $\mathbb{R}^4_+ = \mathbb{R}^3 \times (0, \infty)$. Consider the 3D incompressible Navier-Stokes equations for velocity $u : \mathbb{R}^4_+ \to \mathbb{R}^3$ and pressure $p : \mathbb{R}^4_+ \to \mathbb{R}$,

$$\partial_t u - \Delta u + (u \cdot \nabla)u + \nabla p = 0, \quad \operatorname{div} u = 0, \tag{1}$$

in $\mathbb{R}^4_+$, coupled with the initial condition $u|_{t=0} = u_0$ with $\operatorname{div} u_0 = 0$. The system (1) enjoys a scaling property: If $u(x, t)$ is a solution, then so is

$$u^{(\lambda)}(x, t) := \lambda u(\lambda x, \lambda^2 t) \tag{2}$$

for any $\lambda > 0$. We say $u(x, t)$ is **self-similar** (SS) if $u = u^{(\lambda)}$ for every $\lambda > 0$. In that case, the value of $u(x, t)$ is decided by its value at $t = 1$. On the other hand, if $u = u^{(\lambda)}$ only for one particular $\lambda > 1$, we say u is **discretely self-similar** (DSS) with **factor** λ. Its value in $\mathbb{R}^4_+$ is decided by its value in the strip $x \in \mathbb{R}^3$ and $1 \le t < \lambda^2$. They are called **forward** because $0 < t < \infty$. We can also consider (1) for $-\infty < t < 0$ or for time independent u. For both cases the scaling law (2) still holds, and we define **backward** and **stationary** SS and DSS solutions in the same manner.

When $u(x, t)$ is either SS or DSS, then so is $u_0(x)$. Thus it is natural to assume $|u_0(x)| \le \frac{C_*}{|x|}$ for some constant $C_* > 0$ and look for solutions satisfying

$$|u(x, t)| \le \frac{C(C_*)}{|x|}, \quad \text{or} \quad \|u(\cdot, t)\|_{L^{3,\infty}} \le C(C_*). \tag{3}$$

Here by $L^{q,r}$, $1 \le q, r \le \infty$, we denote the Lorentz spaces. In such classes, with sufficiently small C_*, the unique existence of mild solutions – solutions of the integral equation version of (1) via contraction mapping argument, was obtained by Giga-Miyakawa and refined by Cannone-Meyer-Planchon. As a consequence of the uniqueness, if $u_0(x)$ is SS or DSS with small C_*, the corresponding small mild solution is also SS or DSS. For large C_*, the existence theory for mild solutions is not available, and one may extend the concept of weak solutions and consider local-Leray solutions constructed by Lemarié-Rieusset. However, there is no uniqueness theorem for them to guarantee self-similarity.

In a surprising recent preprint [1], Jia and Šverák constructed SS solutions for every SS u_0 which is locally Hölder continuous. Their main tool is a local Hölder estimate of the solution near $t = 0$, assuming minimal control of u_0 in the large. This estimate enables them to prove a priori estimates of SS solutions, and then show the existence by applying the Leray-Schauder degree theorem. Note that this existence theorem does not assert uniqueness.

In this talk, I will present results asserting the existence of DSS solutions for DSS initial data u_0, assuming either the DSS factor λ is sufficiently close to 1 according to C_*, or if u_0 is axisymmetric with no swirl. This extends the result of [1] to DSS setting. I will also discuss their relevance to the uniqueness problem.

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Convergence for a 2D elliptic problem with large exponent in nonlinearity

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In this talk, we are concerned with the problem

$$(E_p) \quad \begin{cases} -\Delta u = K(x)u^p & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^2$, $K \in C^1(\overline{\Omega})$, $\inf_{\overline{\Omega}} K > 0$ is a given positive weight function, and $p > 1$ is a nonlinear exponent.

Let $\{u_p\}$ be a sequence of solutions to (E_p), not necessarily least energy ones. The main purpose of this talk is to investigate the asymptotic behavior of general solutions u_p when the nonlinear exponent p gets large.

Let $\{p_n\}$ be a sequence of exponents with $p_n > 1, p_n \to +\infty$, and $\{u_{p_n}\}$ be a solution sequence of (E_p) for $p = p_n$. In this talk, we prove that along a subsequence (again denoted by $\{p_n\}$), mass quantization occurs in the sense that

$$p_n \int_\Omega K(x) u_{p_n}^{p_n} dx \to 8\pi\sqrt{e}N, \quad N \in \mathbb{N} \cup \{+\infty\}.$$

Furthermore, we have the entire blow-up if $N = +\infty$, or, N-points concentration holds if $N \in \mathbb{N}$, in the sense that there exists a set of N points $\mathcal{S} = \{x_1, \cdots, x_N\} \subset \Omega$ such that, as $p_n \to \infty$,

$$\lim_{n\to\infty} \sup_{B_r(x_i)} u_{p_n} = \lim_{n\to\infty} \|u_{p_n}\|_{L^\infty(\Omega)} = \sqrt{e} \quad (r > 0 \text{ small}),$$

$$p_n K(x) u_{p_n}^{p_n} \overset{*}{\rightharpoonup} 8\pi\sqrt{e} \sum_{i=1}^N \delta_{x_i} \text{ in the sense of measures on } \overline{\Omega},$$

$$p_n u_{p_n} \to 8\pi\sqrt{e} \sum_{i=1}^N G(\cdot, x_i) \text{ in } C^1_{loc}(\overline{\Omega} \setminus \mathcal{S}).$$

Also we obtain a characterization of each concentration point as a critical point of some function defined by the Green function and the coefficient function K.

These results are obtained by using ideas and techniques from the recent paper by S. Santra and J.C. Wei (J. d'Analyse Math. 2011) with suitable modifications.

Homogenization of Nonlinear Equations in nondivergence type with Neumann data in Perforated Domains

Ki-ahm Lee, Seoul National University

Abstract:
In this talk we are going to discuss the homogenization of Nonlinear Equations with Neumann data in Perforated Domains, which is a generalization of the homogenization of soft inclusions. Some application comes from the stochastic control or game theory with soft inclusion. Main difficulties come from the scale difference between Diffusion equation and Neumann data since diffusion equation is invariant under quadratic scaling while the Neumann data is invariant under the linear scaling. Another difficulty is the concept of compatibility condition for Nonlinear equations of nondivergence type with nonlinear Neumann data, since the natural compatible Neumann data comes from the integration by part for the operators of divergence type.

In this talk, we will show these two questions are actually related and can be solved by the existence of first corrector which will correct the limit profile in $¥epsilon$-order and gives a concept of compatibility condition. And the second corrector will give us the homogenized (or effective) equation for the limit profile.

Fast diffusion and geometry of domain

Shigeru Sakaguchi (RCPAM, Tohoku University)

We consider two fast diffusion equations $\partial_t u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ and $\partial_t u = \Delta u^m$, where $1 < p < 2$ and $0 < m < 1$. Let Ω be a domain in $\mathbb{R}^N$ with $N \geq 2$, and let $u = u(x,t)$ be the solution of either the initial-boundary value problem over Ω, where the initial value equals zero and the boundary value equals 1, or the Cauchy problem where the initial datum is the characteristic function of the set $\mathbb{R}^N \setminus \Omega$. Choose an open ball B in Ω whose closure intersects $\partial\Omega$ only at one point, and let $\alpha > \frac{(N+1)(2-p)}{2p}$ or $\alpha > \frac{(N+1)(1-m)}{4}$. Then, we derive asymptotic estimates for the integral of u^α over B for short times in terms of principal curvatures of $\partial\Omega$ at the point, which tells us about the interaction between fast diffusion and geometry of domain. These kinds of research, which were motivated by C. Cortázar, M. Del Pino and M. Elgueta [CDE], have been done in [MS1, MS2] for the case where $p \geq 2$ or $m \geq 1$. Here in [S] we take into account that the short time behavior of the solutions u is described by the boundary blow-up solutions studied in [BM, M].

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Applications of a new calculus for ideal hydrodynamics

Darren G. Crowdy, Imperial College London

Abstract:
In classical fluid dynamics, an important basic problem is to understand how solid bodies (e.g. aerofoils, obstacles or stirrers) immersed in an ideal fluid interact by “communicating” with each other through the ambient fluid. In recent work the speaker has shown that there is a way to formulate the theory of ideal hydrodynamics so that the relevant fluid dynamical formulae are exactly the same irrespective of the number of interacting objects. This talk will outline the idea of the approach and survey some interesting recent uses of it in applications.

Encoding of streamline topologies for incompressible vortex flows in 2D multiply connected domains

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Let us consider the flows generated by many vortex structures in the presence of the uniform flow in two-dimensional multiply connected exterior domains. They are regarded as mathematical models for biofluids such as insect flights and vertical descend of rotating plant seeds and for environmental flows in rivers and coastal flows. For the sake of theoretical simplicity, we suppose that the fluid is incompressible and inviscid (or slightly viscid). Then the instantaneous flow field $\boldsymbol{u}(t, x, y) = (u(t, x, y), v(t, x, y))$ in the two-dimensional (x, y)-plane is constructed from the stream function $\psi(t, x, y)$ by $u = \partial_y \psi$ and $v = -\partial_x \psi$, whose corresponding streamlines correspond to the contour lines of the stream function.

Here, we are concerned with the global topological structure of the streamline patterns generated by the vortex structures in the uniform flow. Classification of streamline patterns for the unbounded plane has been investigated by Aref and Brøns[1] and that for the sphere was done by Kidambi and Newton[2], in which no boundary is contained in the flow domains. The present study is not only an extension to the case of planar flow domains with many boundaries, but it also adds the following new aspects that have not been considered so far. First, the uniform flow is taken into considerations. This is an essential element in the study of biofluids and environmental flows, and it adds a new streamline structure to the flow profile topologically. Second we focus on *the structurally stable vector fields*, which are of significance since the structurally stable flows are more probable to be observed in many real flow phenomena. Third, the existence of separation points at the boundaries appends another structurally stable streamline structure. Thus the topological classification of the streamline patterns in multiply connected domains provide us with non-trivial new results mathematically that are applicable to many fluid phenomena.

In the present talk, let us first introduce an inductive procedure to construct structurally stable streamline patterns generated by finitely many vortex structures in the presence of the uniform flow[3]. That is to say, starting from some basic structurally stable streamline patterns in the disk of low genus, we repeat some fundamental operations that append a streamline pattern with increasing one genus to them. Owing to the procedure, one can regard a sequence of the operations as *a representing word* of each structurally stable streamline pattern. We then give an encoding algorithm to assign an unique canonical word expression for a given structurally stable vector field, which allows us to determine all possible structurally stable streamline patterns in a combinatorial manner. Finally the applications of the present encoding theory to some fluid problems are provided.

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Incompressible fluids on foliated manifolds

Yoshihiko MITSUMATSU (Chuo University, Tokyo)

In order to understand the topology and geometry of manifolds or geometric structures on them, it might be a natural idea to look at how fluids flow on them. Unfortunately, the analytical foundations for the fluid mechanics is very hard to establish and not yet enough to employ this idea. However, it is still tempting the author, even to investigate foliations on manifolds.

We have already a difficulty in writing down the proper Euler equation, namely, the equation of motion for incompressible fluids without viscosity whose flow lines are contained in leaves. If the time allows, we will discuss on this difficulty in the talk.

The main topic of this talk is concerning a more primitive stage than the genuine fluid mechanics. We try to understand in the case of codimension 1 foliations on closed oriented 3-manifolds the space of velocity fields of such fluids by using an interpretation of the notion of *helicity* as a symmetric bi-linear form on the space (not of velocity fields but) of volticity fields, which is so called the *asymptotic linking*.

Fix a volume form $dvol$ on a closed oriented manifold M. Let $\mathcal{X}$ denote the set of smooth vector fields on M and $\mathcal{X}_d$ the set of divergence free vector fields. By taking so called the *asymptotic cycle* $\mathcal{X}_d$ surjects to $H_1(M;\mathbb{R})$ and its kernel $\mathcal{X}_h$ is called homology free vector fields. $\mathcal{X}_h$ coincides with the span of locally supported ones in $\mathcal{X}_d$. The asymptotic linking lk is a symmetric bi-linear form on $\mathcal{X}_h$.

Now take a codimension 1 foliation $\mathcal{F}$ on a closed orinted 3-manifold M and set $\mathcal{X}(M,\mathcal{F}) = \{X \in \mathcal{X}\,;\, X /\!\!/ \mathcal{F}\}$. The velocity fields of our foliated incompressible fluids are those who belong to $\mathcal{X}_d(M;\mathcal{F}) = \mathcal{X}(M,\mathcal{F}) \cap \mathcal{X}_d$. Also consider holomogy free ones $\mathcal{X}_h(M;\mathcal{F}) = \mathcal{X}(M,\mathcal{F}) \cap \mathcal{X}_h$ and locally supported ones $\mathcal{X}_{loc}(M;\mathcal{F})$. As in a reasonable sense $\mathcal{X}_{loc}(M;\mathcal{F})$ is understandable, the understanding of $\mathcal{X}_d(M;\mathcal{F})$ may reduces to seeing the structures of $\mathcal{X}_d(M;\mathcal{F})/\mathcal{X}_h(M;\mathcal{F})$ and of $\mathcal{X}_h(M;\mathcal{F})/\mathcal{X}_{loc}(M;\mathcal{F})$.

Main Result 1 0) $\mathcal{X}_d(M;\mathcal{F})/\mathcal{X}_h(M;\mathcal{F})$ naturally injects to $H_1(M;\mathbb{R})$ and its image depends on each case. (This is almost trivial.)
1) $\mathcal{X}_{loc}(M;\mathcal{F})$ is a null subspace of $(\mathcal{X}_h, lk)$.
2) The orthogonal complement of $\mathcal{X}_{loc}(M;\mathcal{F})$ with respect to lk is exactly $\mathcal{X}_h(M;\mathcal{F})$.
3) On the quotient $\mathcal{X}_h(M;\mathcal{F})/\mathcal{X}_{loc}(M;\mathcal{F})$ a symmetic bi-liniar form (which is also denoted by lk) is naturally induced from lk.

Here is one more ingredient to analyse $(\mathcal{X}_h(M;\mathcal{F})/\mathcal{X}_{loc}(M;\mathcal{F}), lk)$, that is, the foliated (de Rham) cohomolgy $H^*(M,\mathcal{F})$, which is not necessarily of finite dimension and is quite often very hard to compute, and the characteristic pairing $CJ : H^1(M;\mathcal{F}) \otimes H^1(M;\mathcal{F}) \to H^3(M)$ ($\cong \mathbb{R}$ in our case).

Main Result 2 1) There exists a natural surjection $\Phi : H^1(M;\mathcal{F}) \to \mathcal{X}_h(M;\mathcal{F})/\mathcal{X}_{loc}(M;\mathcal{F})$ which intertwines the pairings CJ and lk.
2) On some particular cases we can compute $H^1(M;\mathcal{F})$ and Φ explicitely as well as the quotient $\mathcal{X}_d(M;\mathcal{F})/\mathcal{X}_h(M;\mathcal{F})$.

All these are computed in terms of differentital forms, volume dual to vector fields. The conservation laws should be studied.

KNOTTED VORTEX TUBES IN STEADY SOLUTIONS TO THE EULER EQUATION

ALBERTO ENCISO

The motion of particles in an ideal fluid in $\mathbb{R}^3$ is described by its velocity field $u(x,t)$, which satisfies the Euler equation

$$\partial_t u + (u \cdot \nabla) u = -\nabla P\,, \qquad \operatorname{div} u = 0$$

for some pressure function $P(x,t)$. The trajectories of the vorticity $\omega(x,t)$ for fixed t are usually called *vortex lines*. A solution u to the Euler equation is called *steady* when it does not depend on time.

A domain in $\mathbb{R}^3$ that is the union of vortex lines and whose boundary is an embedded torus is a (closed) *vortex tube*. The analysis of *thin* vortex tubes for solutions to the Euler equation has attracted considerable attention. A long-standing problem in this direction is Lord Kelvin's conjecture that knotted and linked thin vortex tubes can arise in steady solutions to the Euler equation. This is basically a question on the existence of knotted invariant tori in steady solutions of the Euler equation.

In this talk we will show the existence of steady solutions to the Euler equation in $\mathbb{R}^3$ having thin vortex tubes of any link and knot type. The steady solutions we construct are *Beltrami fields* (that is, they satisfy the equation $\operatorname{curl} u = \lambda u$ in $\mathbb{R}^3$ for some nonzero real constant λ) and fall off at infinity as $|u(x)| < C/|x|$.

The structure of the vortex lines inside each vortex tube is extremely rich. In the first place, it an be shown that the cores of the thin tubes are knotted vortex lines, so previous results on the existence of knotted trajectories are recovered [1]. In the second place, the vortex tubes we construct are *stable*, in a sense that we will make precise. This is related to the existence of infinitely many invariant tori in the interior of the vortex tube. The proof of these results combines fine estimates for the Beltrami equation in a thin tube with ideas from KAM theory.

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Energy, pseudomomentum and Stokes drift of inertial waves and their application to stability of a columnar vortex

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Abstract

A steady Euler flow of an inviscid incompressible fluid is characterized as an extremum of the total kinetic energy (=the Hamiltonian) with respect to perturbations constrained to an isovortical sheet (=coadjoint orbits). We exploit the criticality in the Hamiltonian to calculate the energy of the inertial waves or Kelvin waves, three-dimensional waves on a steady vortical flow [1, 2, 3], and, as a by-product, to calculate the mean flow of second order in amplitude, induced by nonlinear interaction of waves with themselves [4]. We pursue the relation of this mean flow with the pseudomomentum and the Stokes drift [5].

We then apply these formulas to the linear and weakly nonlinear stability of a rotating flow confined in a cylinder of elliptic cross-section. The linear instability is known as the Moore-Saffman-Tsai-Widnall (MSTW) instability, and its characteristics as parametric resonance between a pair of Kelvin waves is well captured from the viewpoint of Krein's theory of Hamiltonian spectra [1]. The wave-induced mean flow is indispensable for proceeding to the weakly nonlinear stage. A hybrid method of combining the Eulerian and the Lagrangian approaches is developed to deduce the amplitude equations to third order [6, 7, 5]. By an appropriate normalization of dependent variables, the resulting amplitude equations are made into Hamiltonian form [5]. The linear instability is connected to the Hamiltonian pitchfork and Hopf bifurcations.

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Using persistence to quantify vortex significance

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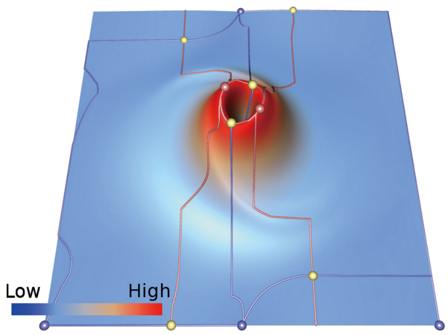
The extraction of vortices plays a fundamental role in the analysis of fluid flows. They are one of the main structures that are relevant for different engineering tasks such as the construction of airfoils or streamlined cars. Unfortunately, the number of vortices rises with the complexity of the flow. Anyhow, not all vortices are equally important. Their influence on the flow field is related to their strength, i.e., the energy transported by the vortex. It is therefore helpful to discriminate between individual vortices by assigning an importance measure to theses structures.

The first step to extract vortex cores is to determine a set of feature points. Typically, these points are defined as extremal structures of a certain scalar feature identifier. Different feature identifiers have been presented in the past. Well-known examples are the Okubo-Weiss criterion (Okubo 1970, Weiss 1991), vorticity, the λ_2 criterion (Jeong and Hussain 1995). In this work, we consider the extraction of vortices in two-dimensional time-dependent flow fields. Here, we want to use the scalar quantity not only as a feature identifier but also as a measure that resembles the importance or strength of a vortex. We use the acceleration magnitude of the flow field. The acceleration is a fundamental part of the Navier-Stokes equations. In the right hand figure, the acceleration magnitude of a Lundgren vortex is shown. It can be seen that the vortex core is surrounded by a particularly pronounced ridge. The height of this ridge varies with the strength of the vortex. It resembles the ability of the vortex to attract particles towards the vortex core.



Using the fact that the strength of the vortex is related to the height of the surrounding ridge in the acceleration magnitude, we now have to find a robust approach to quantify this height difference. To do so, persistent homology as proposed by Edelsbrunner [1] can be used. Loosely speaking, persistence measures the robustness of a critical point against perturbations of the scalar values within its vicinity. In the two-dimensional case, the persistence of a minimum is determined by the height difference to its connected saddle. In the case of the acceleration magnitude, the vortex core is given by a minimum in the scalar field topology. The connected saddle point lies on the ridge surrounding the minimum. Thus, persistence indeed measures the strength of the vortex.

In addition to analyzing slices of a two-dimensional time-dependent data set, the importance measure given by persistence can be extended to the temporal domain by incorporating the lifetime of a vortex. Integrating the persistence value along a tracked vortex core line results in a measure that marks spatially important and long-living vortices.

The extraction of persistence can be done in a combinatorial setting that also includes the extraction of the minima of the acceleration magnitude. We therefore have a unified framework to extract the vortex cores and their strength. The tracking can also be accomplished in the same combinatorial context.

More information about the extraction of vortices using the acceleration magnitude and tracking vortex core in time-dependent flow fields can be found in [2].

Acknowledgements: I thank Oliver Kamps (University of Münster, Germany) for providing the Lundgren data set. Furthermore, I thank Hans-Christian Hege, Ingrid Hotz (both Zuse Institute Berlin), Bernd R. Noack (Institute PPrime, Poitiers, France) and Jan Reininghaus (IST Austria) for helping me to develop the ideas described above.
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Strong turbulence in nonlinear Schrödinger equation

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The order parameter $\psi(\boldsymbol{x},t)$ for the condensed phase of a Bose gas satisfies a nonlinear-Schrödinger (NLS) equation

$$\mathrm{i}\frac{\partial}{\partial t}\psi = -\frac{1}{2m}\nabla^2\psi - \mu\psi + g|\psi|^2\psi, \tag{1}$$

which is also called the Gross-Pitaevskii equation, under a certain approximation. Here m is the mass of the particles, μ the chemical potential, g the coupling constant and the unit of $\hbar = 1$, $\hbar$ is the Planck constant divided by 2π, is used.

When the nonlinear term, the last term in the r.h.s. of (1), is small enough compared to the first term in the r.h.s. of (1), the statistical properties of the turbulent solutions of NLS equation is well described by the weak wave turbulence (WWT) theory[1,2]. Within the WWT theory, the spectrum $F(k)$ [see (4) for the definition] obeys k^{-1} law for the energy-transfer range and $k^{-1/3}$ law for the particle-number-transfer range. The former is also observed in a numerical simulation of the NLS equation accompanied with external forcing and dissipation[3].

In the present study, we attempt to derive theoretically the spectrum $F(k)$ of the turbulence obeying the NLS equation not only in the WWT range but also in the strong turbulence (ST) range where the nonlinear term becomes dominant in the r.h.s. of (1), by means of a spectral closure approximation, or in other words, a two-point closure approximation.

Let $\psi_{\boldsymbol{k}}(t)$ be the Fourier transform of $\psi(\boldsymbol{x},t)$ with respect to the coordinate variable $\boldsymbol{x}$. It is convenient to introduce a doublet

$$\begin{pmatrix}\psi^{+}_{\boldsymbol{k}}(t)\\ \psi^{-}_{\boldsymbol{k}}(t)\end{pmatrix} := \begin{pmatrix}\mathrm{e}^{\mathrm{i}(k^2/2m-\mu)t}\psi_{\boldsymbol{k}}(t)\\ \mathrm{e}^{-\mathrm{i}(k^2/2m-\mu)t}\psi^{*}_{-\boldsymbol{k}}(t)\end{pmatrix}. \tag{2}$$

By assuming statistical homogeneity in space, the two-point correlation function Q and the two-point response function G can be defined by

$$\langle \psi^{\alpha}_{\boldsymbol{k}}(t)\psi^{\beta}_{-\boldsymbol{k}'}(t')\rangle = Q^{\alpha\beta}_{\boldsymbol{k}}(t,t')(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}'), \qquad \left\langle \frac{\delta\psi^{\alpha}_{\boldsymbol{k}}(t)}{\delta\psi^{\beta}_{\boldsymbol{k}'}(t')}\right\rangle = G^{\alpha\beta}_{\boldsymbol{k}}(t,t')(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}'), \tag{3}$$

where $\langle\cdot\rangle$ denotes an ensemble average and upper Greece indices denote $\{+,-\}$. The spectrum $F(k)$ is defined by

$$F(k,t) = \frac{1}{2}\int\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3}\delta(|\boldsymbol{k}'|-k)\left[Q^{+-}_{\boldsymbol{k}'}(t,t) + Q^{-+}_{\boldsymbol{k}'}(t,t)\right]. \tag{4}$$

Closed equations for Q and G can be obtained by the method of renormalized expansion and truncation. The closure approximation is essentially NLS equation equivalent of the direct interaction approximation (DIA)[4] of the Navier-Stokes equation.

It is found that, for the energy-transfer range, the time scale $T_{\mathrm{NL}}(k)$ associated with $Q_k(t,t')$ and $G_k(t,t')$ is given by $T_{\mathrm{NL}}(k) = g^{-1}n^{-1}$. The time scale associated to the linear wave is given by $T_{\mathrm{L}}(k) = 2mk^{-2}$. In the WWT range where $T_{\mathrm{NL}}(k) \gg T_{\mathrm{L}}(k)$, the spectral closure reduce to the WWT theory. In the ST range where $T_{\mathrm{NL}}(k) \ll T_{\mathrm{L}}(k)$, we obtained a new scaling law $F(k) \propto k^{-2}$. Similar analysis is also done for the particle-number-transfer range.

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CONSTRAINED MECHANICS AND IDEALIZED MODELS FOR AQUATIC LOCOMOTION VIA VORTEX SHEDDING

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A mature formalism exists for the realization of reduced-order models for the dynamics of mechanical systems that exhibit symmetries. In the context of finite-dimensional systems, this formalism encompasses the treatment of systems subject to integrable and nonintegrable velocity constraints [1]. In the context of fluid-body interactions, localized velocity constraints can be used to represent the physics underlying vortex shedding in an simplified way. The application of such constraints discontinuously in time leads to models in which discrete distributions of vorticity interact with free bodies according to noncanonical Hamiltonian equations like those documented in [2, 3, 4]. This talk will describe idealized models for several problems concerning the self-propulsion of deformable bodies in fluids, highlighting the role played by symmetry-breaking constraints.

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New computational approaches to curvature driven interface evolution with applications to crystal growth and image analysis

Harald Garcke(University of Regensburg, Germany)

Abstract:

In crystal growth and image segmentation, one of the most important tasks in image analysis, interfaces move by a speed involving the curvature of the interface. We present new computational approaches which allow for an efficient computation of curvature driven interface motion. The approach is based on a parametric finite element method (PFEM) and leads to evolving meshes which do not deteriorate - a common feature of traditional interface tracking methods. We also demonstrate how junctions and topology changes can be incorporated into the methodology.

As one application of the approach we explain how the parametric finite element method can be used to compute anisotropic nearly crystalline motion in solidification. In particular we present computations of snow crystal growth. As a second application we discuss geometric active contour models of Chan-Vese type. Instead of level set methods, which are typically used in this context, we also use the parametric finite element approach for the evolution of the active contours. Since we allow for junctions, we can also deal with multi-phase situations and as topology changes are included we can handle almost all situations of practical relevance.

A mesoscopic model for snow crystal growth

Janko Gravner (University of California, Davis, USA)

Abstract:
The talk will review a mesoscopic approach to crystal growth modeling, developed jointly with David Griffeath. A three-dimensional, computationally feasible, mesoscopic model is based on diffusion of vapor, anisotropic attachment, and a boundary layer.

Several case studies will be presented that faithfully replicate most observed snow crystal morphology. In particular, many of the most striking physical specimens feature both facets and branches and our model provides an explanation for this phenomenon.

We also duplicate many other observed traits, including ridges, ribs, sandwich plates, and hollow columns, as well as various dynamic instabilities. The concordance of observed phenomena suggests that diffusion with anisotropic attachment is the most important ingredient in the development of physical snow crystals.

Anisotropic models: analysis of flat regions of solutions

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Anisotropic systems in class of singular parabolic equations generates unusual structures of solutions. One of the most spectacular phenomena are flat regions of solutions, called by the theory: *facets*. Such effects are consequences of the very high singularity of the nonlinear elliptic operator. If the system is anisotropic then the shape of the facets is determined by the anisotropy. From that reason such models find naturally a place in the theory of crystal growth and image processing.

The goal of my talk is to present some current results concerning model problems. I plan to consider the mono-dimensional system

$$u_t - \partial_x(L(u_x)) = 0$$

on an interval with Dirichlet boundary conditions. Fundamental examples are

$$L_0(p) = \operatorname{sgn} p \quad \text{and} \quad L_1(p) = p + \operatorname{sgn} p.$$

For such systems we are able to construct a complete theory explaining the qualitative features of solutions.

The second part is dedicated to two dimensional systems. There are distinguished two equations:

$$u_t - \partial_{x_1}(\operatorname{sgn} u_{x_1}) - \partial_{x_2}(\operatorname{sgn} u_{x_2}) = 0,$$
$$u_t - \partial_{x_1}(\operatorname{sgn} u_{x_1}) - \partial_{x_2}(\operatorname{sgn} u_{x_2}) - \Delta u = 0.$$

Partial analytical results show that the extra linear diffusion in the second equation does not change the main qualitative properties which are characteristic for the first equation. Here we are able to observe not only facets, but also ruled surfaces.

My talk is based on joint results with Piotr Rybka (Warszawa), Monika Muszkieta (Wroclaw) and Karolina Kielak (Warszawa).

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Asymptotically self-similar solutions to curvature flow equations with prescribed contact angle

Nao Hamamuki, University of Tokyo

Abstract:
We study the asymptotic behavior of solutions to fully nonlinear second order parabolic equations including a generalized curvature flow equation which was introduced by W. W. Mullins in 1957 as a model of evaporation-condensation. We prove that, in the multi-dimensional half space, solutions of the problem with prescribed contact angle asymptotically converge to a self-similar solution of the associated problem. We also give estimates for the depth of the thermal groove, which is represented by the value of the self-similar solution at the boundary.

Topological approach to pattern formation problems arising in materials science

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Topological approach is a new non-invasive mathematical measurement potentially applicable to various fields including materials science. In this talk I will present two case studies in which computational homology and its variants play a key role to extract essential information on morphological dynamics arising in materials science.

One is the diblock copolymer problem in a three-dimensional space. It is known that the double gyroid and orthorhombic morphologies are obtained as energy minimizers. By investigating the geometric properties of these bicontinuous morphologies, we demonstrate the underlying mechanism affecting the triply periodic energy minimizers in terms of a balanced scaling law. Then we apply computational homology to characterize the topological changes during the morphology transition in three-dimensional space. It should be noted that even the “transient” morphologies can be detected during the time course of the transition, since integer-valued index (Betti numbers) can be observed for certain time before reaching the final state, for instance, transient perforated layers are found from layers to cylinders. The scaling law for Betti number in the phase ordering process is also presented.

The other study is about the bulk metallic glasses (BMGs). BMGs have been extensively studied because certain mechanical properties, such as strength, can be significantly improved over their crystalline counterparts. Although BMGs are metallic alloys, they do not have crystalline structure nor random one. Numerous rules, criteria, and mechanisms have been proposed to guide the development of metallic alloys with high glass-forming ability (GFA), however there is no general principle characterizing the basic features of BMGs. For instance, it is known experimentally that the best glass former in an alloy system is located at a pinpoint composition for two component alloy like Cu-Zr. The existing mechanisms and criteria for glass formation fail to explain why the best glass former only occurs at a pinpoint composition. The atomic-level microstructure of metallic glasses is a long-standing subject that has been attracting large interest. Although the atomic structural picture is far from being established, it has been realized that clusters should be the building block in glassy alloys. I will present a progress report on the issue of pinpoint composition by using the persistent homology, especially focusing on the topological characterization of such a pinpoint.

The first part is a joint work with T. Teramoto (Asahikawa Medical University) and the second one is with A. Hirata (WPI-AIMR, Tohoku Univ.) and his collaborators.

Title: Non-stationary blind image de-deconvolution

Speaker: Dr. Hui Ji

Affiliation: Department of Mathematics, National University of Singapore

Abstract: Blind image de-convolution problem is about how to remove the blurring from images without knowing the blurring process. Blind image de-blurring is a challenging ill-posed inverse problem, which has drawn a lot of attentions in recent years. Most existing approaches consider a spatially invariant convolution model. However, Often the practical motion blurring is a spatially varying blurring process over the image. In this talk, I will start with the introduction of several important results on various aspects of blind image de-convolution. Then, I will introduce a two-stage approach for spatially-varying blind image de-blurring.

Minimal Convex Combinations of Three Sequential Laplace-Dirichlet Eigenvalues

Chiu-Yen Kao (Claremont McKenna College)
Joint work with Braxton Osting (UCLA)

Abstract:
In this talk, the shape optimization problem where the objective function is a convex combination of three sequential Laplace-Dirichlet eigenvalues is presented. The domains which minimize the first few single Laplace-Dirichlet eigenvalues are known analytically and/or have been studied computationally and it is known that the optimal solution for the second eigenvalue have multiply connected components. Our computations based on the level set approach and the gradient descent method reproduce these previous results and extend these results to sequential problem, effectively capturing intermediate topology changes. Several properties of minimizers are studied computationally, including uniqueness, connectivity, symmetry, and eigenvalue multiplicity.

Multiscale contour shape analysis using the crystalline flow

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A method for a scale-space analysis of a contour figure based on a crystalline flow is proposed. A crystalline flow is a special family of evolving polygons, and is a discrete version of a curvature flow. Based on the crystalline flow of a given contour, the proposed method makes a scale-space representation of the given contour, and extracts several sets of dominant facets. By changing the shape of the Wulff shape that plays a role of a unit circle for computing the nonlocal curvature of each facet, the method analyzes the contour shape anisotropically.