Title	An estimation of the depth from an intermediate subfactor
Author(s)	Wierzbicki, Jerzy
Citation	Hokkaido University Preprint Series in Mathematics, 204, 1-7
Issue Date	1993-06
DOI	10.14943/83348
Doc URL	http://hdl.handle.net/2115/68950
Type	bulletin (article)
File Information	pre204.pdf

An estimation of the depth from
an intermediate subfactor

Jerzy Wierzbicki

Series ♯204. June 1993

HOKKAIDO UNIVERSITY
PREPRINT SERIES IN MATHEMATICS

♯ 177: K. Sugano, On bicommutators of modules over H-separable extension rings III, 9 pages. 1993.

♯ 178: T. Nakazi, Toeplitz operators and weighted norm inequalities, 17 pages. 1993.

♯ 179: O. Ogurisu, Existence and structure of infinitely degenerate zero-energy ground states of a Wess-Zumino type model in supersymmetric quantum mechanics, 26 pages. 1993.

♯ 180: O. Ogurisu, Ground state of a spin 1/2 charged particle in an even dimensional magnetic field, 9 pages. 1993.

♯ 181: K. Sugano, Note on H-separable Galois extension, 6 pages. 1993.

♯ 182: M. Yamada, Distance formulas of asymptotic Toeplitz and Hankel operators, 13 pages. 1993

♯ 183: G. Ishikawa, T. Ozawa, The genus of a connected compact real algebraic surface in the affine three space, 11 pages. 1993.

♯ 184: T. Hibi, Canonical modules and Cohen-Macaulay types of partially ordered sets, 6 pages. 1993.

♯ 185: Y. Giga, K. Yama-uchi, On a lower bound for the extinction time of surfaces moved by mean curvature, 16 pages. 1993.

♯ 186: Y. Kurokawa, On functional moduli for first order ordinary differential equations, 9 pages. 1993.

♯ 187: O. Ogurisu, Unitary equivalence between a spin 1/2 charged particle in a two-dimensional magnetic field and a spin 1/2 neutral particle with an anomalous magnetic moment in a two-dimensional electric field, 4 pages. 1993.

♯ 188: A. Jensen, T. Ozawa, Existence and non-existence results for wave operators for perturbations of the laplacian, 30 pages. 1993.

♯ 189: T. Nakazi, Multipliers of invariant subspaces in the bidisc, 12 pages. 1993.

♯ 190: S. Izumiya, G.T. Kossioris, Semi-local classification of geometric singularities for Hamilton-Jacobi equations, 24 pages. 1993.

♯ 191: N. Hayashi, T. Ozawa, Finite energy solutions of nonlinear Schrödinger equations of derivative type, 21 pages. 1993.

♯ 192: J. Seade, T. Suwa, A residue formula for the index of a holomorphic flow, 22 pages. 1993.

♯ 193: H. Kubo, Blow-up of solutions to semilinear wave equations with initial data of slow decay in low space dimensions, 8 pages. 1993.

♯ 194: F. Hiroshima, Scaling limit of a model of quantum electrodynamics, 52 pages. 1993.

♯ 195: T. Ozawa, Y. Tsutsumi, Global existence and asymptotic behavior of solutions for the Zakharov equations in three space dimensions, 34 pages. 1993.

♯ 196: H. Kubo, Asymptotic behaviors of solutions to semilinear wave equations with initial data of slow decay, 25 pages. 1993.

♯ 197: Y. Giga, Motion of a graph by convexified energy, 32 pages. 1993.

♯ 198: T. Ozawa, Local decay estimates for Schrödinger operators with long range potentials, 17 pages. 1993.

♯ 199: A. Arai, N. Tominaga, Quantization of angle-variables, 31 pages. 1993.

♯ 200: S. Izumiya, Y. Kurokawa, Holonomic systems of Clairaut type, 17 pages. 1993.

♯ 201: K.-S. Saito, Y. Watatani, Subdiagonal algebras for subfactors, 7 pages. 1993.

♯ 202: K. Iwata, On Markov properties of Gaussian generalized random fields, 7 pages. 1993.

♯ 203: A. Arai, Characterization of anticommutativity of self-adjoint operators in connection with Clifford algebra and applications, 13 pages. 1993.

An estimation of the depth from an intermediate subfactor.

Jerzy Wierzbicki

Abstract. We show that for a triple $K \subset N \subset M$ of type II_1 factors the depth of the inclusion "$K \subset M$" is not greater than the maximum of depths of the inclusions "$K \subset N$" and "$N \subset M$", provided there is such a factor P, that the diagram $\begin{array}{ccc} P & \subset & M \\ \cup & & \cup \\ K & \subset & N \end{array}$ is commuting and co-commuting square (or a non-degenerate commuting square) of type II_1 factors. We give also a characterization of the above condition.

In [B] Bisch proved, that if depth of inclusion $K \subset M$ of two type II_1 factors is finite then for any intermediate subfactor N, the depths of "$K \subset N$" and of "$N \subset M$" are finite too. In this note we give a converse to the assertion. Similar result as that of ours was obtained recently in the case of the depth two irreducible inclusions in [S] by T. Sano, who used different method.

Preliminaries. We recall here shortly the basic notions we need. We will follow here [GHJ], [P1], [SW] and [PP2].

Let $N \subset M$ be an inclusion of the type II_1 factors with $[M : N] < \infty$. We have the corresponding Jones' tower

$$N \subset M \subset^{e_0} M_1 \subset^{e_1} M_2 \dots$$

with Jones' projections $e_i \in M_{i+1}$. Consider also the tower of relative commutants $\{Y_i\}_{i \geq -1}$, $Y_{-1} = N' \cap N$, $Y_0 = N' \cap M$, $Y_i = N' \cap M_i$. If "$N \subset M$" is of finite depth then for some $i > 1$, Y_i gets the basic construction for "$Y_{i-2} \subset Y_{i-1}$". From [GHJ] or [P3] we know that it happens, if and only if the central support $C(e_{i-1}, Y_i)$ of e_{i-1} in Y_i becomes identity.

Definition 1. The integer $\min\{ \, i \mid C(e_{i-1}, Y_i) = 1\}$ is called the depth of the inclusion "$N \subset M$". Let us denote it by $\mathrm{d}(N \subset M)$.

In [WW] we defined notion of co-commuting square of type II_1 factors.

Definition 2. A diagram $\begin{array}{ccc} M & \subset & L \\ \cup & & \cup \\ K & \subset & N \end{array}$ of finite factors is a co-commuting square, if their commutants $\begin{array}{ccc} M' & \subset & K' \\ \cup & & \cup \\ L' & \subset & N' \end{array}$ form a commuting square.

It was first analyzed in [SW] in terms of opposite angles between two subfactors. The opposite angles $\text{Op-ang}_L(M,N) = \frac{\pi}{2}$ iff the diagram $\begin{array}{ccc} M & \subset & L \\ \cup & & \cup \\ K & \subset & N \end{array}$ is a co-commuting square.

From [SW] we see, that the co-commuting square, which is also a commuting square of type II_1 factors coincides with the notion of non-degenerate commuting square of II_1 factors, which was introduced by S.Popa in [P1]. Let us recall a few basic properties, which can be found in [SW] or in [P1].

Proposition 3. Suppose that the diagram $\mathcal{S} = \begin{array}{ccc} N & \subset & L \\ \cup & & \cup \\ K & \subset & M \end{array}$ is a commuting square of type II_1 factors. Let also $[L:K] < \infty$. Then the following conditions are equivalent:

(i) $\mathcal{S}$ is nondegenerate commuting square.

(ii) $L = \text{Span } N \cdot M$

(iii) $[L:N] = [M:K]$

We will also use the notion of algebraic basic construction as it was introduced in [PP2].

Definition 4. Suppose that N is a subfactor of a type II_1 factor M with $[M:N] < \infty$. Let M_1 be a von Neumann with a finite, faithful and normal trace τ_1. Assume that e is a projection in M_1. If there is a trace preserving *-isomorphism $\phi : \langle M, e_N^M \rangle \longrightarrow M_1$ of the basic construction of "$N \subset M$" onto M_1 such that $\forall x \in M$ $\phi(x) = x$ and $\phi(e_N^M) = e$ then we call M_1 algebraic basic construction for the inclusion "$N \subset M$". For convenience we will write $M_1 = \langle M, N, e \rangle$.

The following proposition is a slight modification of [PP2]Proposition 1.2 and it gives more on the algebraic basic construction.

Proposition 5. Let N be a subfactor of a type II_1 factor M with $[M:N] < \infty$. Let M_1 be a von Neumann with a finite, faithful and normal trace τ_1. Assume that e is a projection in M_1. E_M - τ_1 preserving conditional expectation of M_1 onto M. The following conditions are equivalent:

(1) $M_1 = \langle M, N, e \rangle$

(2) M_1 is a factor, $[e, N] = 0$ and $E_M(e) = [M_1 : M]^{-1} = [M : N]^{-1}$.

(3) $M_1 = M \vee \{e\}$, $\tau(e) \geq [M : N]^{-1}$ and $exe = E_N(x)e$ for $x \in M$.

The main result.

We construct a system of type II_1 factors from a given commuting and co-commuting square of II_1 factors: $\begin{array}{ccc} Q_{1,0} & \subset & Q_{1,1} \\ \cup & & \cup \\ Q & \subset & Q_{0,1} \end{array}$, where the Jones' index $[Q_{1,1} : Q]$ is finite. At first step we define $Q_{2,1} = \langle Q_{1,1}, e^{Q_{1,1}}_{Q_{0,1}} \rangle$ as the basic construction for the pair $Q_{0,1} \subset Q_{1,1}$ and also $Q_{1,2} = \langle Q_{1,1}, e^{Q_{1,1}}_{Q_{1,0}} \rangle$ and $Q_{2,2} = \langle Q_{1,1}, e^{Q_{1,1}}_{Q} \rangle$. This way we obtained the following "bigger" diagram $\begin{array}{ccc} Q_{2,1} & \subset & Q_{2,2} \\ \cup & & \cup \\ Q_{1,1} & \subset & Q_{1,2} \end{array}$, which by [SW] is a commuting and co-commuting square too. Suppose that the diagram $\begin{array}{ccc} Q_{n,n-1} & \subset & Q_{n,n} \\ \cup & & \cup \\ Q_{n-1,n-1} & \subset & Q_{n-1,n} \end{array}$ was already defined. Similarly as at the first step, we put $Q_{n+1,n} = \langle Q_{n,n}, e^{Q_{n,n}}_{Q_{n-1,n}} \rangle$, $Q_{n,n+1} = \langle Q_{n,n}, e^{Q_{n,n}}_{Q_{n,n-1}} \rangle$ and $Q_{n+1,n+1} = \langle Q_{n,n}, e^{Q_{n,n}}_{Q_{n-1,n-1}} \rangle$. Let us use the following notation: $e_n = e^{Q_{n,n}}_{Q_{n-1,n}}$, $f_n = e^{Q_{n,n}}_{Q_{n,n-1}}$, $g_n = e^{Q_{n,n}}_{Q_{n-1,n-1}}$, $\alpha = [Q_{0,1} : Q]$ and $\beta = [Q_{1,0} : Q]$.

The above construction of increasing commuting and co-commuting squares can be found in [SW]. The following properties are noted there too:

$$\text{for } i, j \geq 1 \ e_i f_j = f_j e_i \text{ and } g_i = e_i f_i.$$

We complete the above system by defining von Neumann subalgebras $Q_{i,j}$ with the following iterative formulas:

$$Q_{i+1,j} = Q_{i,j} \vee \{f_i\} \text{ and } Q_{i,j+1} = Q_{i,j} \vee \{e_j\},$$

where "$\vee$" denotes generation of a von Neumann algebra. It is obvious, that the definitions agree with the previous construction.

Lemma 6. With the above notations we have:

(i) $\forall i, j \geq 1$, $Q_{i,j}$ are type II_1 factors.

(ii) $\forall i, j \geq 0$ $Q_{i+2,j} = \langle Q_{i+1,j}, Q_{i,j}, f_{i+1} \rangle$ and $Q_{i,j+2} = \langle Q_{i,j+1}, Q_{i,j}, e_{j+1} \rangle$.

Proof.

First we show that for any $i \geq 0$ $Q_{i,i+2} = \langle Q_{i,i+1}, Q_{i,i}, e_{i+1} \rangle$. We just check the condition (3) of the Proposition 5. Let us write $E_{i,j}$ for the trace preserving conditional expectation onto subalgebra $Q_{i,j}$. For $x \in Q_{i,i+1} \subset Q_{i+1,i+1}$,

$$e_{i+1} x e_{i+1} = E_{i+1,i}(x) e_{i+1} = E_{i+1,i} E_{i,i+1}(x) e_{i+1} = E_{i,i}(x) e_{i+1}.$$

The trace $\tau(e_{i+1}) = \alpha^{-1}$ and $Q_{i,i+2}$ is generated by e_{i+1} and $Q_{i,i+1}$ right from the definition. In particular, we see that $Q_{i,i+2}$ must be a type II_1 factor.

From the Proposition 5.(2) we see also that $Q_{i+2,i+2} = \langle Q_{i+1,i+2}, Q_{i,i+2}, f_{i+1}\rangle$. Indeed, since $[f_{i+1}, Q_{i,i+1}] = 0$ and $[f_{i+1}, e_{i+1}] = 0$, then $[f_{i+1}, Q_{i,i+2}] = 0$. Also,

$$E_{i+1,i+2}(f_{i+1}) = E_{i+1,i+2}E_{i+2,i+1}(f_{i+1}) = E_{i+1,i+1}(f_{i+1}) = \beta^{-1}.$$

We show now that all the diagrams $\begin{array}{ccc} Q_{i+1,i+1} & \subset & Q_{i+1,i+2} \\ \cup & & \cup \\ Q_{i,i+1} & \subset & Q_{i,i+2} \end{array}$ are nondegenerate commuting squares. By the Proposition 3.(iii) and [GHJ]Proposition 4.2.1 it is sufficient to check if $E_{i+1,i+1}(Q_{i,i+2}) \subset Q_{i,i+1}$. Take $x \in Q_{i,i+2}$. Then ([PP1]) $x = \sum_k x_k e_{i+1} y_k$, with $x_k, y_k \in Q_{i,i+1}$.

$$E_{i+1,i+1}(x) = \sum_k x_k E_{i+1,i+1}(e_{i+1}) y_k = \alpha^{-1} \sum_k x_k y_k \in Q_{i,i+1}.$$

Similarly, using the Proposition 5.(2) and [GHJ]4.2.1 we obtain $Q_{i+2,i+3} = \langle Q_{i+1,i+2}, Q_{i,i+1}, f_{i+1}e_{i+2}\rangle$.

So we see that the above argument can be repeated sequentially to get

$$Q_{i,i+k} = \langle Q_{i,i+k-1}, Q_{i,i+k-2}, e_{i+k-1}\rangle \text{ and } Q_{i+2,i+k} = \langle Q_{i+1,i+k}, Q_{i,i+k}, f_{i+1}\rangle,$$

for $k = 2, 3, 4 \dots$. By the symmetry we obtain analogous statements also for $Q_{i+k,i}$ with $k = 2, 3, 4 \dots$.

The next lemma can in fact be read off from the proof of [P3]Proposition 2.1. Let state it explicitly. If a von Neumann algebra A contains a projection e then we will write $C(e, A)$ for the central support of e in A. For a subalgebra B we define the following projection:

$$V(e, B) = \bigvee \{ueu^* \mid u \text{ is a unitary in } B\}.$$

Lemma 7. Let $N \subset M$ be type II_1 factors and $[M : N] < \infty$. B and A are such von Neumann subalgebras that the diagram $\begin{array}{ccc} N & \subset & M \\ \cup & & \cup \\ B & \subset & A \end{array}$ is commuting square. Suppose that a projection $e \in A$ satisfies $E_N^M(e) = [M : N]^{-1}$. Then $C(e, A) = V(e, B)$.

With the above preparation the proof of our main result becomes easy.

Theorem 8. Let $K \subset M \subset L$ be type II_1 factors with $[L : K] < \infty$. Suppose that $\mathrm{d}(K \subset M) < \infty$ and $\mathrm{d}(M \subset L) < \infty$. If there is such a type II_1 factor N, that $K \subset N \subset L$ and the diagram $\begin{array}{ccc} N & \subset & L \\ \cup & & \cup \\ K & \subset & M \end{array}$ is a nondegenerate commuting square, then

$$\mathrm{d}(K \subset L) \le \max(\mathrm{d}(K \subset M), \mathrm{d}(M \subset L)).$$

Proof. Let make a system of type II_1 factors $\{Q_{i,j}\}$ like in the Lemma 3. with $L = Q_{1,1}$, $N = Q_{1,0}$, $M = Q_{0,1}$ and $K = Q_{0,0} = Q$. Denote $g_i = e_i f_i$ and $q = \max(\mathrm{d}(K \subset M), \mathrm{d}(M \subset L))$. From the Lemmas 6, 7 and [P3]3.1 we obtain:

$$V(g_q, Q' \cap Q_{q,q}) \geq V(g_q, Q' \cap Q_{q,q} \cap \{f_q\}') =$$

$$(*) \quad \bigvee\{u e_q u^* f_q | u \in U(Q' \cap Q_{q-1,q})\} \geq V(e_q, Q' \cap Q_{0,q}) f_q =$$

$$C(e_q, Q' \cap Q_{0,q+1}) f_q = f_q,$$

so that

$$C(g_q, Q' \cap Q_{q+1,q+1}) = V(g_q, Q' \cap Q_{q,q}) = V(V(g_q, Q' \cap Q_{q,q}), Q' \cap Q_{q,q}) \geq$$

$$(**) \quad V(f_q, Q' \cap Q_{q,q}) \geq V(f_q, (Q_{0,1} \vee \{e_2, e_3 \ldots e_q\})' \cap Q_{q,q})$$
$$= V(f_q, Q'_{0,1} \cap Q_{q,1}) = C(f_q, Q'_{0,1} \cap Q_{q+1,1}) = 1.$$

Remark. The example $\begin{array}{ccc} L \otimes K & \subset & L \otimes M \\ \cup & & \cup \\ N \otimes K & \subset & N \otimes M \end{array}$, where $K \subset M$ and $N \subset L$ are finite index inclusions of type II_1 factors, shows that the above estimation is sharp. However in some cases there is no equality. Let α be an outer action of the symmetric group S_3 on a type II_1 factor K. Consider the following commuting and co-commuting square:

$$\begin{array}{ccc} K \rtimes_\alpha \langle(1,2,3)\rangle & \subset & K \rtimes_\alpha S_3 \\ \cup & & \cup \\ K & \subset & K \rtimes_\alpha S_2 \end{array}.$$

We see that

$$\mathrm{d}(K \subset K \rtimes_\alpha S_3) = 2 < \max(\mathrm{d}(K \subset K \rtimes_\alpha S_2), \mathrm{d}(K \rtimes_\alpha S_2 \subset K \rtimes_\alpha S_3)) = \max(2, 4).$$

Using the above method we can show a little more.

Corollary 9. Let the diagram $\begin{array}{ccc} N & \subset & L \\ \cup & & \cup \\ K & \subset & M \end{array}$ be as in the Theorem 8. We denote $a = \mathrm{d}(N \subset L)$, $b = \mathrm{d}(K \subset M)$, $c = \mathrm{d}(K \subset N)$ and $d = \mathrm{d}(M \subset L)$. Then

$$\mathrm{d}(K \subset L) \leq \max(\min(a, b), \min(c, d)) =$$

$$\min(\max(a, c), \max(a, d), \max(b, c), \max(b, d)).$$

Proof. If q is the right-hand side of the above inequality then the inequality $(*)$ in the proof of 8. may be replaced by the following one:

$$(*)' \quad \bigvee\{u e_q u^* f_q | u \in U(Q' \cap Q_{q-1,q})\} \geq V(e_q, Q'_{1,0} \cap Q_{1,q}) f_q,$$

except the case $q = 1$ which we consider later. Also the inequality $(**)$ above may be replaced by:

$$(**)' \quad V(f_q, Q' \cap Q_{q,q}) \geq V(f_q, Q' \cap Q_{q,0}).$$

By the symmetry this ends the proof in all cases except when $a = d = 1$ and $c, d > 1$. It can be solved by Theorem 8. and the next remark.

Remark. Let $N \subset M$ be a pair of type II_1 factors with $[M : N] < \infty$. Then $\mathrm{d}(N \subset M) = 1$, iff "$N \subset M$" $\cong$ "$N \subset N \otimes M_n(\mathbb{C})$".

Indeed, if $\mathrm{d}(N \subset M) = 1$ then the second relative commutant $N' \cap \langle M, e_N^M \rangle$ is a factor. So, there is a Pimsner-Popa basis of M over N composed of the elements of $N' \cap M$. Therefore, $M = N \vee (N' \cap M) \cong N \otimes (N' \cap M) = N \otimes M_n(\mathbb{C})$, where the last equality comes from the factorness of M.

We give a characterization of the condition used in Theorem 8. For a given pair $Q \subset N$ of II_1 factors with finite index A.Ocneanu ([O]) and also Bisch ([B]) considered a set of projections corresponding to intermediate subfactors between Q and N denoted in [B] by $IS(Q, N)$. We recall one of the equivalent definitions. If $N_1 = \langle N, e_Q^N \rangle$ is the basic construction for $Q \subset N$ then

$$IS(Q, N) = \{q \in P(Q' \cap N_1) \mid q e_Q^N = e_Q^N,\ E_N^{N_1}(q) \in \mathbb{C} \text{ and } qNq \subset Nq\}.$$

Proposition 10. Let $Q \subset K \subset N$ be a triple of type II_1 factors with $[N : Q] < \infty$ and $Q' \cap N = \mathbb{C}$. Then there exists such type II_1 factor B that $\begin{array}{ccc} B & \subset & N \\ \cup & & \cup \\ Q & \subset & K \end{array}$ is a nondegenerate commuting square, if and only if there is in $IS(Q, N)$ a Jones' projection e corresponding to the inclusion $Q \subset K$ i.e. such that $\forall x \in K, exe = E_Q(x)e$ and $E_K(e) = [K : Q]^{-1}$.

Proof. If such a factor B exists then we can put $e = e_B^N \in IS(Q, N)$. Let now $e \in IS(Q, N)$. If $B = \{e\}' \cap N$ then as in [B]4.2 we see that for $x \in N$, $exe = E_B(x)e$, and that $[N : B]^{-1} = \tau(e)$. Since $E_{K'}^{Q'}(e) = [K : Q]^{-1}$ and (by 5.(2)) $B' = \langle N', \langle N, B, e \rangle', e \rangle$, the diagram of commutants $\begin{array}{ccc} K' & \subset & Q' \\ \cup & & \cup \\ N' & \subset & B' \end{array}$ is a commuting square. By 3.(iii) it is also a co-commuting square, which ends the argument.

Remark. It is obvious that instead of $Q' \cap N = \mathbb{C}$ in the above proposition we could have demanded for $\{e_Q^K\}' \cap N$ to be a factor.

Bibliography

[B] D.Bisch, *A note on intermediate subfactors*, to appear in J.Func.Anal.

[GHJ] F.Goodman, P.de la Harpe and V.F.R.Jones, "Coxeter Graphs and Towers of Algebras", MSRI Publ. 14, Springer-Verlag, New York, 1989.

[J1] V.F.R.Jones, *Index for subfactors*, Invent.Math. v.72 (1983).

[O] A.Ocneanu, *Quantum symmetry, differential geometry of finite graphs and classification of subfactors,* Tokyo University Seminary Notes 45, (1991).

[P1] S.Popa, *Classification of amenable subfactors of type II*, Preprint 1992.

[P2] S.Popa, *Maximal injective subalgebras in factors associated with free groups*, Adv.Math. v.50 (1983), 27-48.

[P3] S.Popa, *Classification of subfactors: the reduction to commuting square,* Invent.Math. v.101 (1990), 19-43.

[PP1] M.Pimsner and S.Popa, *Entropy and index for subfactors*, Ann.Sci. Ec.Norm.Sup. 19 (1986), 57-106.

[PP2] ————, *Iterating the basic construction*, Trans. Am. Math. Soc. 310 (1988), 127-133.

[S] T.Sano, *Commuting co-commuting squares and finite dimensional Kac algebras*, Preprint (1993).

[SW] T.Sano and Y.Watatani, *Angles between two subfactors*, to appear in J. Operator Th.

[SZ] S.Stratila and L.Zsido, "Lectures on von Neumann Algebras", Abacus Press/ Ed. Academiei, Tunbridge Wells/ Bucuresti, 1979.

[T] M.Takesaki, "Theory of Operator Algebra I", Springer, Berlin, 1979.

[WW] J.Wierzbicki and Y.Watatani, *Commuting squares and relative entropy for two subfactors*, Preprint (1992).

Current address:
Jerzy Wierzbicki
Department of Mathematics
Hokkaido University
Sapporo, 060, Japan.