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SOME ESTIMATES FOR THE BLOWING UP SOLUTIONS OF SEMILINEAR HEAT EQUATIONS

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ABSTRACT. We prove an above and a below differential inequality for the solutions of semilinear heat equation.From them,we obtain the estimates of critical initial date,which guarantee that the solution will blow up or be bounded globally.We also get the estimates of the growth rate of blowing up solution and the blowing up time.

0. Introduction.

This paper is concerned with some estimates for the behavior of the solutions of semilinear heat equation

$$u_t = \Delta u + |u|^{p-1}u \quad \text{in} \quad \Omega$$

$$u = 0 \quad \text{on} \quad \partial\Omega \tag{0.1}$$

$$u(x,0) = u_0(x)$$

whear $\Omega \subset \mathbb{R}^n$,u is scalar-valued,and $p > 1$.It is well known that for a large class of inital date u_0,the solutions $u(x,t)$ blow up in finite time.It is very interested to research the behaviour of blowing up solutions and obtain the estimates of the blowing up time and the rate of blowing up solutions.When Ω is convex,many results have been known while we still know fewer in general domains.For example,assuming that Ω is convex,

$$\Delta u_0 + u_0^p \geq 0 \quad \text{and} \quad u_0 \geq 0$$

[7] get following estimate

$$u \leq C(T-t)^{-\frac{1}{p-1}} \tag{0.2}$$

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

In the case of that Ω is star shape about blowing point and $p > 1$ in $n = 1$ or $2, 1 < p < \frac{n+2}{n-2}$ in $n \geq 3$,[5,6] get more precise conclutions:assuming that $a \in \Omega$ is a blowing up point,$T > 0$ is blowing up time,then

$$\lim_{t \to T}(T-t)^{\frac{1}{p-1}}u(a+y(T-t)^{\frac{1}{2}},t) = (p-1)^{-\frac{1}{p-1}} \tag{0.3}$$

uniformly on compact set

$$|y| \leq C$$

In this paper we consider the problom in general domain,i.e.,we only assume that:

(D) The volume of Ω is bounded and on which Divergence theorem can be applied.

As being pointed in [3],there is a critical value such that if some norm of the initial date u_0 is less than it then the solution exists globaly.In this paper,we give the estimates of the critical value from above and below.We also obtain the estimates of the growth rate of blowing up solution and blowing up time.

There are extensive articles for the blowing up behavious of the solutions of semilinear heat equation recently.We refer to [5,6,8] and references cited there.

Notation We will use $vol(\Omega)$ to denote the volume of domain Ω, and for $v(x) \in L^q(\Omega)$ $(q \geq 1)$,use following notations

$$\| v \|_q = (\int_\Omega (v(x))^q dx)^{\frac{1}{q}}$$

and

$$\| v \|_q (t) = (\int_\Omega (v(x,t))^q dx)^{1/q}$$

when v also depends on time t.

The main conclution of this paper is following theorems

Theorem 0.1. *Assume that $1 < p < 1 + 4/n$,$u \in H_0^{1,2}(\Omega)$ and Ω satisfies (D).*

(1) If $u_0(x)$ satisfies $E(0) \gneqq 0$ and

$$\| u_0 \|_2 \gneqq (vol(\Omega))^{\frac{p-1}{2(p+1)}} \left(\frac{2(p+1)E(0)}{p-1} \right)^{\frac{1}{p+1}} \tag{0.4}$$

or $E(0) \leq 0$ *and*

$$\| u_0 \|_2 \neq 0$$

when $E(0) = 0$ *then* $\| u \|_2 (t)$ *can not be bounded in* $(0, \infty)$.

(2) If $u_0(x)$ *satisfies* $E(0) \gneqq 0$ *and*

$$\| u_0 \|_2 \lneqq R$$

then

$$\| u \|_2 (t) \lneqq R \tag{0.5}$$

in the existence time of solution $u(x,t)$. *Where*

$$E(0) = \frac{1}{2} \int_\Omega |\nabla u_0(x)|^2 dx - \frac{1}{p+1} \int_\Omega |u_0(x)|^{p+1} dx$$

R *is defined as*

$$R = \max_i r_i$$

where r_i *are the solutions of equation (0.6):*

$$2(C_{31} r^{\gamma 1} + C_{32} r^{\gamma_2})^{\gamma_3} - C_2 r^2 = 0 \tag{0.6}$$

where

$$C_2 = (\frac{2}{C_0^{2/\theta}})(\frac{1}{vol(\Omega)})^{\frac{p-1}{\theta(p+1)}}$$

$$C_{31} = \frac{2}{p+1} C_0^{2/\theta}$$

$$C_{32} = (2C_0^{2/\theta})^{\frac{2-\theta(p+1)}{2}} (E(0))^{\frac{2-\theta(p+1)}{2}}$$

$$\gamma_1 = (2/\theta) - 2, \gamma_2 = (\frac{1}{\theta} - 1)(2 - \theta(p+1)), \gamma_3 = \frac{\theta(p+1)}{2 - \theta(p+1)}.$$

$$\theta = \frac{n(p-1)}{2(p+1)}$$

$$C_0 = (\frac{5}{4})^\theta \quad for \quad n = 1,$$

$$C_0 = (\frac{p+1}{2})^\theta \quad for \quad n = 2,$$

$$C_0 = \left(\frac{2(n-1)}{n-2}\right)^\theta \quad for \quad n \geq 3.$$

Theorem 0.2. *Let $1 < p < 1 + 4/n$ and Ω satisfy (D).Assume that the solution $u(x,t)$ of (1.1) blows up at some finit time T and $u \in H_0^{1,2}(\Omega)$.Then*

(1)

$$\overline{\lim_{t\to T}}(T-t)^{\frac{2}{p-1}} \parallel u \parallel_2^2 (t) \le K_1 \tag{0.7}$$

$$\underline{\lim_{t\to T}}(T-t)^{\frac{1}{\gamma-1}} \parallel u \parallel_2^2 (t) \ge K_2 \tag{0.8}$$

with $K_1 < \infty$, $K_2 > 0$.In fact,we can take

$$K_1 = 2^{\frac{1}{\gamma-1}} \left(\frac{2(p+1)}{(p-1)^2}\right)^{\frac{2}{p-1}} vol(\Omega) = K_1(n,p)vol(\Omega)$$
$$K_2 = K_2(n,p)vol(\Omega)$$

(2)In case of $E(0) \le 0$,we have

$$\int_{\|u_0\|_2^2}^{\infty} \frac{1}{-C_2 y + C_3 y^{2\gamma}} dy \le T \le \int_{\|u_0\|_2^2}^{\infty} \frac{1}{-4E(0) + C_1 y^{(p+1)/2}} dy \tag{0.9}$$

where

$$\gamma = \frac{2(p+1) - n(p-1)}{4 - n(p-1)}$$
$$C_1 = 2(\frac{p-1}{p+1})(\frac{1}{vol(\Omega)})^{\frac{p-1}{2}}$$

C_2 is same as Theorem0.1,

$$C_3 = 2(\frac{2C_0^{2/\theta}}{p+1})^{\frac{\theta(p+1)}{2-\theta(p+1)}}$$

When $E(0) > 0$,the below estimate is replaced by

$$\int_{\|u_0\|_2^2}^{\infty} \frac{1}{-C_2 y + 2(C_{31} y^{\gamma_1} + C_{32} y^{\gamma_2})^{\gamma_3}} dy \le T \tag{0.10}$$

RemarkThe estimate (0.8) has been proved in [1,3,10] under the assumption that the boundary of Ω is smooth.This assumption is essential in there because the strong continuous semigroup method have been used.Here,we get (0.8) only under the assumption (D),which comes from Proposition1.1(cf.[9] pp.60-63).

1.Differential inequalities

We will establish above and below differential inequalities about $\parallel u \parallel_2^2 (t)$,which are the fundamental of the proof of Theorem0.1-0.2.In fact,the below differential inequality has been used in several papers,for example,[2,5,6].The above inequality seems first to be used for the blowing up problom of (0.1).We begin by stating Gagliardo- Nireberg inequality(cf.[9] pp.62).

Proposition1.1. *For function $v \in H_0^{1,2}(\Omega)$,the inequality*

$$\| v \|_q \leq C_0 \| \nabla v \|_2^{\theta} \| v \|_2^{1-\theta} \tag{1.1}$$

is valid,in which

$$2 \leq q \lneqq \infty$$

for n=1 or 2,

$$2 \leq q \leq 2n/(n-2)$$

for $n \geq 3$,

$$C_0 = C_0(n, q),$$

$$\theta = n(q-2)/2q$$

.

Let u(x,t) be the solution of (0.1),we define the energy function E(t) about (0.1) as

$$E(t) = \frac{1}{2} \int_\Omega |\nabla u(x,t)|^2 dx - \frac{1}{p+1} \int_\Omega |u(x,t)|^{p+1} dx$$

Proposition 1.2.

$$\frac{dE}{dt} \leq 0 \tag{1.2}$$

Proof.Multiplying the furst equation of (0.1) by u_t and integrating it,we get

$$\begin{aligned}
\int_\Omega u_t^2 dx &= \int_\Omega u_t \Delta u dx + \int_\Omega u_t |u|^{p-1} u dx \\
&= \frac{d}{dt} \left(\frac{1}{2} \int_\Omega |\nabla u|^2 dx - \frac{1}{p+1} \int_\Omega |u|^{p+1} dx \right) \\
&= -\frac{dE}{dt}
\end{aligned}$$

□

Now we state below differential inequality.For completion,we give its proof.

Lemmer 1.3. *Let u(x,t) be the solution of (0.1).Then we have*

$$\frac{d}{dt} \| u \|_2^2 (t) \geq -4E(0) + C_1 \| u \|_2^{p+1} (t) \tag{1.3}$$

where C_1 is same as Theorem0.2.

Proof:Multiplying the first equation of (0.1) by u and integrating it,we get

$$\frac{1}{2}\frac{d}{dt} \| u \|_2^2 (t) = - \| \nabla u \|_2^2 (t) + \| u \|_{p+1}^{p+1} (t) \tag{1.4}$$

Using (1.2),we have

$$\| \nabla u \|_2^2 (t) \leq 2E(0) + \frac{2}{p+1} \| u \|_{p+1}^{p+1} (t)$$

So

$$\begin{aligned}\frac{1}{2}\frac{d}{dt} \| u \|_2^2 (t) &\geq -2E(0) + (\frac{p-1}{p+1}) \| u \|_{p+1}^{p+1} (t) \\ &\geq -2E(0) + (\frac{p-1}{p+1})(\frac{1}{vol(\Omega)})^{\frac{p-1}{2}} \| u \|_2^{p+1} (t)\end{aligned}$$

where we have used Jensen's inequality in last step. □

Next we prove above estimate which is a key step in this paper.

Lemmer 1.4. *Let*

$$1 < p < 1 + 4/n$$

.Assume that u(x,t) satisfies (0.1).Then

(1)For $E(0) \leq 0$,we have

$$\frac{d}{dt} \| u \|_2^2 \leq -C_2 \| u \|_2^2 + C_3 \| u \|_2^{2\gamma} \tag{1.5}$$

(2)For $E(0) > 0$,we have

$$\frac{d}{dt} \| u \|_2^2 \leq -C_2 \| u \|_2^2 + 2(C_{31} \| u \|_2^{\gamma_1} + C_{32} \| u \|_2^{\gamma_2})^{\gamma_3} \tag{1.5'}$$

where γ_i,C_i are same as Theorem 0.2.

Proof.Fome Proposition 1.2,we have

$$\frac{1}{2} \| \nabla u \|_2^2 (t) - \frac{1}{p+1} \| u \|_{p+1}^{p+1} (t) \leq E(0) \tag{1.6}$$

for $t > 0$.Using Proposition 1.2 to u as the function of x with q=p+1,we get

$$\| \nabla u \|_2^2 \geq \frac{\| u \|_{p+1}^{2/\theta}}{C_0^{2/\theta} \| u \|_2^{2(1-\theta)/\theta}} \tag{1.7}$$

So

$$\frac{\| u \|_{p+1}^{2/\theta}}{2C_0^{2/\theta} \| u \|_2^{2(1-\theta)/\theta}} - (\frac{1}{p+1}) \| u \|_{p+1}^{p+1} \leq E(0) \tag{1.8}$$

Remark that if p satisfies:

$$1 < p < 1 + (4/n)$$

then,from

$$\theta = n(\frac{1}{2} - \frac{1}{p+1})$$

we have

$$\frac{2}{\theta} > p + 1 \tag{1.9}$$

(1)In case of $E(0) \leq 0$*.From (1.8)-(1.9),we get*

$$\begin{aligned} \| u \|_{p+1}^{p+1} &\leq \left(\frac{2C_0^{2/\theta}}{p+1} \right)^{\frac{\theta(p+1)}{2-\theta(p+1)}} \| u \|_2^{\frac{2(1-\theta)(p+1)}{2-\theta(p+1)}} \\ &= (\frac{C_3}{2}) \| u \|_2^{2\gamma} \end{aligned} \tag{1.10}$$

Using (1.4),(1.6)-(1.10),we obtain

$$\begin{aligned} \frac{1}{2}\frac{d}{dt} \| u \|_2^2 &\leq - \frac{\| u \|_{p+1}^{2/\theta}}{C_0^{2/\theta} \| u \|_2^{2(1/\theta - 1)}} \\ &+ (\frac{C_3}{2}) \| u \|_2^{2\gamma} \\ &\leq - \frac{(\frac{1}{vol(\Omega)})^{\frac{p-1}{\theta(p+1)}} \| u \|_2^{2/\theta}}{C_0^{2/\theta} \| u \|_2^{2(1/\theta-1)}} \\ &+ (\frac{C_3}{2}) \| u \|_2^{2\gamma} \\ &= -(\frac{C_2}{2}) \| u \|_2^2 + (\frac{C_3}{2}) \| u \|_2^{2\gamma} \end{aligned}$$

where we have used Jensen's inequality to obtain the last inequality.

(2)In case $E(0) > 0$*.From (1.8),either (1.10) holds or*

$$\begin{aligned} \| u \|_{p+1}^{p+1} &\leq (2C_0^{2/\theta} \| u \|_2^{2(\frac{1}{\theta}-1)})^{\frac{p+1}{2/\theta}} \\ &\left(\frac{1}{p+1} (2C_0^{2/\theta} \| u \|_2^{2(\frac{1}{\theta}-1)})^{\frac{p+1}{2/\theta}} + E(0)^{\frac{2-\theta(p+1)}{2}} \right)^{\frac{\theta(p+1)}{2-\theta(p+1)}} \\ &= (C_{31} \| u \|_2^{\gamma_1} + C_{32} \| u \|_2^{\gamma_2})^{\gamma_3} \end{aligned} \tag{1.11}$$

Because (1.11) is stronger then (1.10), we can use (1.11) to replace (1.10) in the case of $E(0) > 0$. From (1.4),(1.7)and(1.11), we get (1.15'). □

2.The Proofs of Theorem0.1 and Theorem0.2

For proving Theorem 1 and Theorem 2,we compare

$$\| u \|_2^2 (t)$$

with the solutions of following ordinary differentional equations

$$\frac{d}{dt} J = -C_2 J + C_3 J^\gamma \tag{2.1}$$

and

$$\frac{d}{dt} J_1 = 4E(0) + C_1 J_1^{(p+1)/2} \tag{2.2}$$

Claim 2.1. *(1)Equation(2.1) has tow equilibrium points*

$$0, (\frac{C_2}{C_3})^{\frac{1}{\gamma-1}}.$$

0 is stable while the second is unstable. (2)Equation(2.2) has one equilibrium point

$$(\frac{-4E(0)}{C_1})^{\frac{2}{p+1}}$$

which is unstable.

In fact,in case of $E(0) > 0$,we must replace (2.1) by

$$\frac{dJ}{dt} = -C_2 J + 2(C_{31} J^{\gamma_1/2} + C_{32} J^{\gamma_2/2})^{\gamma_3} \tag{2.1'}$$

The similar conclusions can be obtained.We omit it.

Claim 2.2. *Let*

$$J(0) = J_1(0) = \| u_0 \|_2^2 .$$

Then for $t \in [0, T)$, we have

$$J_1(t) \leq \| u \|_2^2 (t) \leq J(t) \tag{2.3}$$

where T is the maximum existing time of solution $u(x,t)$.

From above claims and Lemmer1.3-1.4,we obtain the conclusions of Theorem0.1.

To prove Theorem0.2,we first prove following lemmer.

Lemmer2.3. *Assume that $Y(t) \geq 0$ satisfies following tow differential inequalities*

$$\frac{d}{dt}Y \leq -b_1 Y + b_2 Y^{\gamma} \tag{2.4}$$

$$\frac{d}{dt}Y \geq b_3 + b_4 Y^{\frac{p+1}{2}} \tag{2.5}$$

and Y(t) blows up at $T > 0$.Where $b_i > 0$,$b_2 > b_1$ and $\gamma > 1$.Then

$$\overline{\lim_{t \to T}}(T-t)^{\frac{2}{p-1}} Y(t) \leq \left(\frac{b_2}{b_2 - b_1}\right)^{\frac{1}{\gamma-1}} \left(\frac{2}{b_4(p-1)}\right)^{\frac{2}{p-1}} \tag{2.6}$$

$$\underline{\lim_{t \to T}}(T-t)^{\frac{1}{\gamma-1}} Y(t) \geq \left(\frac{1}{\gamma-1}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{2b_4}\right)^{\frac{2}{p-1}} b_2^{\frac{2}{p-1}-\frac{1}{\gamma-1}} \tag{2.7}$$

Proof.We first consider following differential equation

$$\frac{d}{dt}J = -b1J + b_2 J^{\gamma} \tag{2.8}$$

$$J|_{t=t_0 \geq 0} = Y(t_0)$$

From the assumption that Y(t) blows up and (2.4),J(t) must blow up at some time

$$T(t_0) \in (t_0, T)$$

and

$$Y(t) \leq J(t) \quad in \quad [t_0, T(t_0)).$$

Without loose anything,we assume

$$Y(t_0) \geq 1.$$

Notice that $\gamma > 1$ and

$$J(t) \geq J(t_0) = Y(t_0) \geq 1$$

we have

$$\begin{aligned} T(t_0) - t &= \int_{J(t)}^{\infty} \frac{dJ}{-b_1 J + b_2 J^{\gamma}} \\ &\leq \frac{1}{(\gamma-1)(b_2-b_1)J^{\gamma-1}(t)} \end{aligned} \tag{2.9}$$

So,for

$$t \in [t_0, T(t_0)),$$

we obtain

$$\overline{\lim_{t_0 \to T}}(T(t_0) - t_0)Y^{\gamma-1}(t_0) \leq \frac{1}{(\gamma-1)(b_2 - b_1)} \tag{2.10}$$

Second,we consider differential equation

$$\frac{dJ_1}{dt} = b_3 + b_4 J_1^{\frac{p+1}{2}} \tag{2.11}$$

$$J_1|_{t=t_0 \geq 0} = Y(t_0)$$

The solution $J_1(t)$ must blow up at some time

$$T_2(t_0)$$

which is larger then T.That is

$$\begin{aligned} T - t_0 &\leq T_2(t_0) - t_0 \\ &= \int_{J_1(t_0)}^{\infty} \frac{dJ_1}{b_3 + b_4 J_1^{\frac{p+1}{2}}} \\ &\leq \int_{J_1(t_0)}^{\infty} \frac{dJ_1}{b_4 J_1^{\frac{p+1}{2}}} \\ &= \frac{2}{b_4(p-1)} \left(\frac{1}{J_1(t_0)}\right)^{\frac{p-2}{2}} \end{aligned} \tag{2.12}$$

From (2.9),(2.12),we have

$$\frac{(T-t_0)^{\gamma-1}}{(T(t_0)-t_0)^{\frac{p-1}{2}}} \leq (\frac{2}{b_4(p-1)})^{\gamma-1}(b_2(\gamma-1))^{\frac{p-1}{2}} \tag{2.13}$$

From (2.10) and (2.13),we get (2.6).Similarly,we can get (2.7). □

Using Lemmer 1.3-1.4,2.3,we can prove Theorem0.2.

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