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# SOME ESTIMATES FOR THE BLOWING UP SOLUTIONS OF SEMILINEAR HEAT EQUATIONS

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**ABSTRACT.** We prove an above and a below differential inequality for the solutions of semilinear heat equation. From them, we obtain the estimates of critical initial data, which guarantee that the solution will blow up or be bounded globally. We also get the estimates of the growth rate of blowing up solution and the blowing up time.

## 0. Introduction.

This paper is concerned with some estimates for the behavior of the solutions of semilinear heat equation

$$u_t = \Delta u + |u|^{p-1}u \quad \text{in } \Omega$$

$$(0.1) \quad u = 0 \quad \text{on } \partial\Omega$$

$$u(x, 0) = u_0(x)$$

where  $\Omega \subset \mathbb{R}^n$ ,  $u$  is scalar-valued, and  $p > 1$ . It is well known that for a large class of initial data  $u_0$ , the solutions  $u(x, t)$  blow up in finite time. It is very interesting to research the behaviour of blowing up solutions and obtain the estimates of the blowing up time and the rate of blowing up solutions. When  $\Omega$  is convex, many results have been known while we still know fewer in general domains. For example, assuming that  $\Omega$  is convex,

$$\Delta u_0 + u_0^p \geq 0 \quad \text{and} \quad u_0 \geq 0$$

[7] get following estimate

$$(0.2) \quad u \leq C(T - t)^{-\frac{1}{p-1}}$$

In the case of that  $\Omega$  is star shape about blowing point and  $p > 1$  in  $n = 1$  or  $2, 1 < p < \frac{n+2}{n-2}$  in  $n \geq 3$ , [5,6] get more precise conclusions: assuming that  $a \in \Omega$  is a blowing up point,  $T > 0$  is blowing up time, then

$$(0.3) \quad \lim_{t \rightarrow T} (T-t)^{\frac{1}{p-1}} u(a + y(T-t)^{\frac{1}{2}}, t) = (p-1)^{-\frac{1}{p-1}}$$

uniformly on compact set

$$|y| \leq C$$

In this paper we consider the problem in general domain, i.e., we only assume that:

(D) The volume of  $\Omega$  is bounded and on which Divergence theorem can be applied.

As being pointed in [3], there is a critical value such that if some norm of the initial data  $u_0$  is less than it then the solution exists globally. In this paper, we give the estimates of the critical value from above and below. We also obtain the estimates of the growth rate of blowing up solution and blowing up time.

There are extensive articles for the blowing up behaviour of the solutions of semilinear heat equation recently. We refer to [5,6,8] and references cited there.

Notation We will use  $vol(\Omega)$  to denote the volume of domain  $\Omega$ , and for  $v(x) \in L^q(\Omega)$  ( $q \geq 1$ ), use following notations

$$\|v\|_q = \left( \int_{\Omega} (v(x))^q dx \right)^{\frac{1}{q}}$$

and

$$\|v\|_q(t) = \left( \int_{\Omega} (v(x,t))^q dx \right)^{1/q}$$

when  $v$  also depends on time  $t$ .

The main conclusion of this paper is following theorems

**Theorem 0.1.** Assume that  $1 < p < 1 + 4/n$ ,  $u \in H_0^{1,2}(\Omega)$  and  $\Omega$  satisfies (D).

(1) If  $u_0(x)$  satisfies  $E(0) \geq 0$  and

$$(0.4) \quad \|u_0\|_2 \geq (vol(\Omega))^{\frac{p-1}{2(p+1)}} \left( \frac{2(p+1)E(0)}{p-1} \right)^{\frac{1}{p+1}}$$

or  $E(0) \leq 0$  and

$$\|u_0\|_2 \neq 0$$

when  $E(0) = 0$  then  $\|u\|_2(t)$  can not be bounded in  $(0, \infty)$ .

(2) If  $u_0(x)$  satisfies  $E(0) \geq 0$  and

$$\|u_0\|_2 \leq R$$

then

$$(0.5) \quad \|u\|_2(t) \leq R$$

in the existence time of solution  $u(x, t)$ . Where

$$E(0) = \frac{1}{2} \int_{\Omega} |\nabla u_0(x)|^2 dx - \frac{1}{p+1} \int_{\Omega} |u_0(x)|^{p+1} dx$$

$R$  is defined as

$$R = \max_i r_i$$

where  $r_i$  are the solutions of equation (0.6):

$$(0.6) \quad 2(C_{31}r^{\gamma_1} + C_{32}r^{\gamma_2})^{\gamma_3} - C_2r^2 = 0$$

where

$$C_2 = \left(\frac{2}{C_0^{2/\theta}}\right) \left(\frac{1}{\text{vol}(\Omega)}\right)^{\frac{p-1}{\theta(p+1)}}$$

$$C_{31} = \frac{2}{p+1} C_0^{2/\theta}$$

$$C_{32} = (2C_0^{2/\theta})^{\frac{2-\theta(p+1)}{2}} (E(0))^{\frac{2-\theta(p+1)}{2}}$$

$$\gamma_1 = (2/\theta) - 2, \gamma_2 = \left(\frac{1}{\theta} - 1\right)(2 - \theta(p+1)), \gamma_3 = \frac{\theta(p+1)}{2 - \theta(p+1)}$$

$$\theta = \frac{n(p-1)}{2(p+1)}$$

$$C_0 = \left(\frac{5}{4}\right)^\theta \quad \text{for } n = 1,$$

$$C_0 = \left(\frac{p+1}{2}\right)^\theta \quad \text{for } n = 2,$$

$$C_0 = \left(\frac{2(n-1)}{n-2}\right)^\theta \quad \text{for } n \geq 3.$$

**Theorem 0.2.** Let  $1 < p < 1 + 4/n$  and  $\Omega$  satisfy (D). Assume that the solution  $u(x, t)$  of (1.1) blows up at some finite time  $T$  and  $u \in H_0^{1,2}(\Omega)$ . Then

(1)

$$(0.7) \quad \overline{\lim}_{t \rightarrow T} (T - t)^{\frac{2}{p-1}} \|u\|_2^2(t) \leq K_1$$

$$(0.8) \quad \underline{\lim}_{t \rightarrow T} (T - t)^{\frac{1}{p-1}} \|u\|_2^2(t) \geq K_2$$

with  $K_1 < \infty$ ,  $K_2 > 0$ . In fact, we can take

$$K_1 = 2^{\frac{1}{p-1}} \left( \frac{2(p+1)}{(p-1)^2} \right)^{\frac{2}{p-1}} \text{vol}(\Omega) = K_1(n, p) \text{vol}(\Omega)$$

$$K_2 = K_2(n, p) \text{vol}(\Omega)$$

(2) In case of  $E(0) \leq 0$ , we have

$$(0.9) \quad \int_{\|u_0\|_2^2}^{\infty} \frac{1}{-C_2 y + C_3 y^{2\gamma}} dy \leq T \leq \int_{\|u_0\|_2^2}^{\infty} \frac{1}{-4E(0) + C_1 y^{(p+1)/2}} dy$$

where

$$\gamma = \frac{2(p+1) - n(p-1)}{4 - n(p-1)}$$

$$C_1 = 2 \left( \frac{p-1}{p+1} \right) \left( \frac{1}{\text{vol}(\Omega)} \right)^{\frac{p-1}{2}}$$

$C_2$  is same as Theorem 0.1,

$$C_3 = 2 \left( \frac{2C_0^{2/\theta}}{p+1} \right)^{\frac{\theta(p+1)}{2-\theta(p+1)}}$$

When  $E(0) > 0$ , the below estimate is replaced by

$$(0.10) \quad \int_{\|u_0\|_2^2}^{\infty} \frac{1}{-C_2 y + 2(C_{31} y^{\gamma_1} + C_{32} y^{\gamma_2})^{\gamma_3}} dy \leq T$$

**Remark** The estimate (0.8) has been proved in [1,3,10] under the assumption that the boundary of  $\Omega$  is smooth. This assumption is essential in there because the strong continuous semigroup method have been used. Here, we get (0.8) only under the assumption (D), which comes from Proposition 1.1 (cf. [9] pp.60-63).

### 1. Differential inequalities

We will establish above and below differential inequalities about  $\|u\|_2^2(t)$ , which are the fundamental of the proof of Theorem 0.1-0.2. In fact, the below differential inequality has been used in several papers, for example, [2,5,6]. The above inequality seems first to be used for the blowing up problem of (0.1). We begin by stating Gagliardo-Nirenberg inequality (cf. [9] pp.62).

**Proposition 1.1.** For function  $v \in H_0^{1,2}(\Omega)$ , the inequality

$$(1.1) \quad \|v\|_q \leq C_0 \|\nabla v\|_2^\theta \|v\|_2^{1-\theta}$$

is valid, in which

$$2 \leq q \leq \infty$$

for  $n=1$  or  $2$ ,

$$2 \leq q \leq 2n/(n-2)$$

for  $n \geq 3$ ,

$$C_0 = C_0(n, q),$$

$$\theta = n(q-2)/2q$$

Let  $u(x, t)$  be the solution of (0.1), we define the energy function  $E(t)$  about (0.1) as

$$E(t) = \frac{1}{2} \int_{\Omega} |\nabla u(x, t)|^2 dx - \frac{1}{p+1} \int_{\Omega} |u(x, t)|^{p+1} dx$$

**Proposition 1.2.**

$$(1.2) \quad \frac{dE}{dt} \leq 0$$

*Proof.* Multiplying the first equation of (0.1) by  $u_t$  and integrating it, we get

$$\begin{aligned} \int_{\Omega} u_t^2 dx &= \int_{\Omega} u_t \Delta u dx + \int_{\Omega} u_t |u|^{p-1} u dx \\ &= \frac{d}{dt} \left( \frac{1}{2} \int_{\Omega} |\nabla u|^2 dx - \frac{1}{p+1} \int_{\Omega} |u|^{p+1} dx \right) \\ &= -\frac{dE}{dt} \end{aligned}$$

□

Now we state below differential inequality. For completion, we give its proof.

**Lemmer 1.3.** Let  $u(x, t)$  be the solution of (0.1). Then we have

$$(1.3) \quad \frac{d}{dt} \|u\|_2^2(t) \geq -4E(0) + C_1 \|u\|_2^{p+1}(t)$$



where  $C_1$  is same as Theorem 0.2.

*Proof:* Multiplying the first equation of (0.1) by  $u$  and integrating it, we get

$$(1.4) \quad \frac{1}{2} \frac{d}{dt} \|u\|_2^2(t) = -\|\nabla u\|_2^2(t) + \|u\|_{p+1}^{p+1}(t)$$

Using (1.2), we have

$$\|\nabla u\|_2^2(t) \leq 2E(0) + \frac{2}{p+1} \|u\|_{p+1}^{p+1}(t)$$

So

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} \|u\|_2^2(t) &\geq -2E(0) + \left(\frac{p-1}{p+1}\right) \|u\|_{p+1}^{p+1}(t) \\ &\geq -2E(0) + \left(\frac{p-1}{p+1}\right) \left(\frac{1}{\text{vol}(\Omega)}\right)^{\frac{p-1}{2}} \|u\|_2^{p+1}(t) \end{aligned}$$

where we have used Jensen's inequality in last step.  $\square$

Next we prove above estimate which is a key step in this paper.

**Lemmer 1.4.** Let

$$1 < p < 1 + 4/n$$

.Assume that  $u(x,t)$  satisfies (0.1). Then

(1) For  $E(0) \leq 0$ , we have

$$(1.5) \quad \frac{d}{dt} \|u\|_2^2 \leq -C_2 \|u\|_2^2 + C_3 \|u\|_2^{2\gamma}$$

(2) For  $E(0) > 0$ , we have

$$(1.5') \quad \frac{d}{dt} \|u\|_2^2 \leq -C_2 \|u\|_2^2 + 2(C_{31} \|u\|_2^{\gamma_1} + C_{32} \|u\|_2^{\gamma_2})^{\gamma_3}$$

where  $\gamma_i, C_i$  are same as Theorem 0.2.

*Proof.* From Proposition 1.2, we have

$$(1.6) \quad \frac{1}{2} \|\nabla u\|_2^2(t) - \frac{1}{p+1} \|u\|_{p+1}^{p+1}(t) \leq E(0)$$

for  $t > 0$ . Using Proposition 1.2 to  $u$  as the function of  $x$  with  $q=p+1$ , we get

$$(1.7) \quad \|\nabla u\|_2^2 \geq \frac{\|u\|_{p+1}^{2/\theta}}{C_0^{2/\theta} \|u\|_2^{2(1-\theta)/\theta}}$$

So

$$(1.8) \quad \frac{\|u\|_{p+1}^{2/\theta}}{2C_0^{2/\theta} \|u\|_2^{2(1-\theta)/\theta}} - \left(\frac{1}{p+1}\right) \|u\|_{p+1}^{p+1} \leq E(0)$$

Remark that if  $p$  satisfies:

$$1 < p < 1 + (4/n)$$

then, from

$$\theta = n\left(\frac{1}{2} - \frac{1}{p+1}\right)$$

we have

$$(1.9) \quad \frac{2}{\theta} > p+1$$

(1) In case of  $E(0) \leq 0$ . From (1.8)-(1.9), we get

$$(1.10) \quad \begin{aligned} \|u\|_{p+1}^{p+1} &\leq \left(\frac{2C_0^{2/\theta}}{p+1}\right)^{\frac{\theta(p+1)}{2-\theta(p+1)}} \|u\|_2^{\frac{2(1-\theta)(p+1)}{2-\theta(p+1)}} \\ &= \left(\frac{C_3}{2}\right) \|u\|_2^{2\gamma} \end{aligned}$$

Using (1.4),(1.6)-(1.10), we obtain

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} \|u\|_2^2 &\leq -\frac{\|u\|_{p+1}^{2/\theta}}{C_0^{2/\theta} \|u\|_2^{2(1/\theta-1)}} \\ &\quad + \left(\frac{C_3}{2}\right) \|u\|_2^{2\gamma} \\ &\leq -\frac{\left(\frac{1}{\text{vol}(\Omega)}\right)^{\frac{p-1}{\theta(p+1)}} \|u\|_2^{2/\theta}}{C_0^{2/\theta} \|u\|_2^{2(1/\theta-1)}} \\ &\quad + \left(\frac{C_3}{2}\right) \|u\|_2^{2\gamma} \\ &= -\left(\frac{C_2}{2}\right) \|u\|_2^2 + \left(\frac{C_3}{2}\right) \|u\|_2^{2\gamma} \end{aligned}$$

where we have used Jensen's inequality to obtain the last inequality.

(2) In case  $E(0) > 0$ . From (1.8), either (1.10) holds or

$$(1.11) \quad \begin{aligned} \|u\|_{p+1}^{p+1} &\leq (2C_0^{2/\theta} \|u\|_2^{2(\frac{1}{\theta}-1)})^{\frac{p+1}{2/\theta}} \\ &\quad \left(\frac{1}{p+1} (2C_0^{2/\theta} \|u\|_2^{2(\frac{1}{\theta}-1)})^{\frac{p+1}{2/\theta}} + E(0)^{\frac{2-\theta(p+1)}{2}}\right)^{\frac{\theta(p+1)}{2-\theta(p+1)}} \\ &= (C_{31} \|u\|_2^{\gamma_1} + C_{32} \|u\|_2^{\gamma_2})^{\gamma_3} \end{aligned}$$

Because (1.11) is stronger than (1.10), we can use (1.11) to replace (1.10) in the case of  $E(0) > 0$ . From (1.4), (1.7) and (1.11), we get (1.15').  $\square$

## 2. The Proofs of Theorem 0.1 and Theorem 0.2

For proving Theorem 1 and Theorem 2, we compare

$$\|u\|_2^2(t)$$

with the solutions of following ordinary differential equations

$$(2.1) \quad \frac{d}{dt}J = -C_2J + C_3J^\gamma$$

and

$$(2.2) \quad \frac{d}{dt}J_1 = 4E(0) + C_1J_1^{(p+1)/2}$$

**Claim 2.1.** (1) Equation (2.1) has two equilibrium points

$$0, \left(\frac{C_2}{C_3}\right)^{\frac{1}{\gamma-1}}.$$

0 is stable while the second is unstable. (2) Equation (2.2) has one equilibrium point

$$\left(\frac{-4E(0)}{C_1}\right)^{\frac{2}{p+1}}$$

which is unstable.

In fact, in case of  $E(0) > 0$ , we must replace (2.1) by

$$(2.1') \quad \frac{dJ}{dt} = -C_2J + 2(C_{31}J^{\gamma_1/2} + C_{32}J^{\gamma_2/2})^{\gamma_3}$$

The similar conclusions can be obtained. We omit it.

**Claim 2.2.** Let

$$J(0) = J_1(0) = \|u_0\|_2^2.$$

Then for  $t \in [0, T)$ , we have

$$(2.3) \quad J_1(t) \leq \|u\|_2^2(t) \leq J(t)$$

where  $T$  is the maximum existing time of solution  $u(x, t)$ .

From above claims and Lemmas 1.3-1.4, we obtain the conclusions of Theorem 0.1.

To prove Theorem 0.2, we first prove following lemma.

**Lemmer2.3.** Assume that  $Y(t) \geq 0$  satisfies following tow differential inequalities

$$(2.4) \quad \frac{d}{dt}Y \leq -b_1Y + b_2Y^\gamma$$

$$(2.5) \quad \frac{d}{dt}Y \geq b_3 + b_4Y^{\frac{p+1}{2}}$$

and  $Y(t)$  blows up at  $T > 0$ . Where  $b_i > 0, b_2 > b_1$  and  $\gamma > 1$ . Then

$$(2.6) \quad \overline{\lim}_{t \rightarrow T} (T-t)^{\frac{2}{p-1}} Y(t) \leq \left( \frac{b_2}{b_2 - b_1} \right)^{\frac{1}{\gamma-1}} \left( \frac{2}{b_4(p-1)} \right)^{\frac{2}{p-1}}$$

$$(2.7) \quad \underline{\lim}_{t \rightarrow T} (T-t)^{\frac{1}{\gamma-1}} Y(t) \geq \left( \frac{1}{\gamma-1} \right)^{\frac{1}{\gamma-1}} \left( \frac{1}{2b_4} \right)^{\frac{2}{p-1}} b_2^{\frac{2}{p-1} - \frac{1}{\gamma-1}}$$

*Proof.* We first consider following differential equation

$$(2.8) \quad \frac{d}{dt}J = -b_1J + b_2J^\gamma$$

$$J|_{t=t_0} = Y(t_0)$$

From the assumption that  $Y(t)$  blows up and (2.4),  $J(t)$  must blow up at some time

$$T(t_0) \in (t_0, T)$$

and

$$Y(t) \leq J(t) \quad \text{in } [t_0, T(t_0)).$$

Without loose anything, we assume

$$Y(t_0) \geq 1.$$

Notice that  $\gamma > 1$  and

$$J(t) \geq J(t_0) = Y(t_0) \geq 1$$

we have

$$(2.9) \quad \begin{aligned} T(t_0) - t &= \int_{J(t)}^{\infty} \frac{dJ}{-b_1J + b_2J^\gamma} \\ &\leq \frac{1}{(\gamma-1)(b_2 - b_1)J^{\gamma-1}(t)} \end{aligned}$$

So,for

$$t \in [t_0, T(t_0)),$$

we obtain

$$(2.10) \quad \overline{\lim}_{t_0 \rightarrow T} (T(t_0) - t_0) Y^{\gamma-1}(t_0) \leq \frac{1}{(\gamma - 1)(b_2 - b_1)}$$

Second,we consider differential equation

$$(2.11) \quad \frac{dJ_1}{dt} = b_3 + b_4 J_1^{\frac{p+1}{2}}$$

$$J_1|_{t=t_0} = Y(t_0)$$

The solution  $J_1(t)$  must blow up at some time

$$T_2(t_0)$$

which is larger then  $T$ .That is

$$(2.12) \quad \begin{aligned} T - t_0 &\leq T_2(t_0) - t_0 \\ &= \int_{J_1(t_0)}^{\infty} \frac{dJ_1}{b_3 + b_4 J_1^{\frac{p+1}{2}}} \\ &\leq \int_{J_1(t_0)}^{\infty} \frac{dJ_1}{b_4 J_1^{\frac{p+1}{2}}} \\ &= \frac{2}{b_4(p-1)} \left( \frac{1}{J_1(t_0)} \right)^{\frac{p-2}{2}} \end{aligned}$$

From (2.9),(2.12),we have

$$(2.13) \quad \frac{(T - t_0)^{\gamma-1}}{(T(t_0) - t_0)^{\frac{p-1}{2}}} \leq \left( \frac{2}{b_4(p-1)} \right)^{\gamma-1} (b_2(\gamma - 1))^{\frac{p-1}{2}}$$

From (2.10) and (2.13),we get (2.6).Similarly,we can get (2.7).  $\square$

Using Lemmer 1.3-1.4,2.3,we can prove Theorem0.2.

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