ON THE RESONANCE EQUATIONS OF LONG AND SHORT WAVES

Tohru Ozawa

Series ♯266. October 1994

HOKKAIDO UNIVERSITY

PREPRINT SERIES IN MATHEMATICS

♯ 240 C. Dohmen, Existence of Fast Decaying Solutions to a Haraux-Weissler Equation With a Prescribed Number of Zeroes, 12 pages. 1994.

♯ 241 K. Sugano, Note on H-separable Frobenius extensions, 8 pages. 1994.

♯ 242 J. Zhai, Some Estimates For The Blowing up Solutions of Semilinear Heat Equations, 11 pages. 1994.

♯ 243 N. Hayashi, K. Kato and T. Ozawa, Dilation Method and Smoothing Effect of the Schrödinger Evolution Group, 10 pages. 1994.

♯ 244 D. Lehmann, T. Suwa, Residues of holomorphic vector fields relative to singular invariant subvarieties, 26 pages. 1994.

♯ 245 H. Kubo, Slowly decaying solutions for semilinear wave equations in odd space dimensions, 30 pages. 1994.

♯ 246 T. Nakazi, M. Yamada, (A_2)-Conditions and Carleson Inequalities, 27 pages. 1994.

♯ 247 N. Hayashi, K. Kato and T. Ozawa, Dilation Method and smoothing Effect of Solutions to the Benjamin-ono Equation, 17 pages. 1994.

♯ 248 H. Kikuchi, Sheaf cohomology theory for measurable spaces, 12 pages. 1994.

♯ 249 A. Inoue, Tauberian theorems for Fourier cosine transforms, 9 pages. 1994.

♯ 250 S. Izumiya, G. T. Kossioris, Singularities for viscosity solutions of Hamilton-Jacobi equations, 23 pages. 1994.

♯ 251 H. Kubo, K. Kubota, Asymptotic behaviors of radially symmetric solutions of $\Box u = |u|^p$ for super critical values p in odd space dimensions, 51 pages. 1994.

♯ 252 T. Mikami, Large Deviations and Central Limit Theorems for Eyraud-Farlie-Gumbel-Morgenstern Processes, 9 pages. 1994.

♯ 253 T. Nishimori, Some remarks in a qualitative theory of similarity pseudogroups, 19 pages. 1994.

♯ 254 T. Suwa, Residues of complex analytic foliations relative to singular invariant subvarieties, 15 pages. 1994.

♯ 255 T. Tsujishita, On Triple Mutual Information, 7 pages. 1994.

♯ 256 T. Tsujishita, Construction of Universal Modal World based on Hyperset Theory, 15 pages. 1994.

♯ 257 A. Arai, Trace Formulas, a Golden-Thompson Inequality and Classical Limit in Boson Fock Space, 35 pages. 1994.

♯ 258 Y-G. Chen, Y. Giga, T. Hitaka and M. Honma, A Stable Difference Scheme for Computing Motion of Level Surfaces by the Mean Curvature, 18 pages. 1994.

♯ 259 K. Iwata, J. Schäfer, Markov property and cokernels of local operators, 7 pages. 1994.

♯ 260 T. Mikami, Copula fields and its applications, 14 pages. 1994.

♯ 261 A. Inoue, An Abel-Tauber theorem for Fourier sine transforms, 6 pages. 1994.

♯ 262 N. Kawazumi, Homology of hyperelliptic mapping class groups for surfaces, 13 pages. 1994.

♯ 263 Y. Giga, M. E. Gurtin, A comparison theorem for crystalline evolution in the plane, 14 pages. 1994.

♯ 264 J. Wierzbicki, On Commutativity of Diagrams of Type II_1 Factors, 26 pages. 1994.

♯ 265 N. Hayashi, T. Ozawa, Schrödinger Equations with nonlinearity of integral type, 12 pages. 1994.

ON THE RESONANCE EQUATIONS OF LONG AND SHORT WAVES

T. OZAWA

Department of Mathematics, Hokkaido University
Sapporo 060, Japan

ABSTRACT. We study the Cauchy problem for the equations describing the resonant interactions between long and short waves. Global well-posedness of the problem is proved in the space $H^{1/2} \times L^2$, the first and second component of which correspond to the short and long waves, respectively.

1. Introduction.

Long and short waves in a dispersive media strongly interact and exchange their energies under the resonance conditions on the wave numbers and on the frequencies. The one-dimensional propagation of resonant long and short waves has been described by D.J. Benney [1], V.D. Djordjevic & L.G. Reddekopp [2], M.Funakoshi & M. Oikawa [3], R.H.J. Grinshaw [7] under the assumption of weak nonlinearity. In suitably rescaled coordinates, the resonance equations of long and short waves take the form

$$i\partial_t u + \frac{1}{2}\partial^2 u = vu + \lambda|u|^2 u, \tag{1a}$$

$$\partial_t v = \partial(|u|^2), \tag{1b}$$

where u and v are complex and real valued functions of $(t, x) \in \mathbb{R} \times \mathbb{R}$, respectively, $\partial_t = \partial/\partial t, \partial = \partial/\partial x, \lambda \in \mathbb{R}$. The functions u and v are related to the slowly varying envelope of the short wave and the amplitude of the long wave, respectively. The equations (1a),(1b) are known to be solvable by the inverse spectral method only in the case $\lambda = 0$ (see Ma [15]). Concerning the Cauchy problem for (1a), (1b) in the usual Sobolev spaces for (u, v), M. Tsutsumi & S. Hatano [18] proved the global well-posedness in the spaces $H^{k+1/2} \times H^k$ for all integers $k \geq 1$ and, under the restriction $\lambda = 0$, the unique global solvability for the Cauchy data in the space $H^{1/2} \times (L^2 \cap L^\infty)$. We note here that the last space seems rather restrictive for the persistence property.

The purpose in this paper is to prove that (1a), (1b) form a dynamical system on $H^{1/2} \times L^2$ by generating a continuous global flow, namely, that (1a), (1b) are globally well-posed in $H^{1/2} \times L^2$ without the restriction $\lambda = 0$. The proof uses a number of estimates associated with smoothing effects in the linear problem and therefore basically we follow the argument in [18].

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

In this paper we combine the estimates of Kato type due to Kenig, Ponce & Vega [12, 13], of Strichartz type due to Ginibre & Velo [6] and Yajima [20], and of maximal functions due to Kenig & Ruiz [14], the last two of which are not used in [18], however. The combination of these three ingredients eventually leads to the global well-posedness in $H^{1/2} \times L^2$.

In order to state our result precisely, we introduce some notation. The usual Lebesgue spaces of complex and real valued functions of the space variable x in $\mathbb{R}$ are denoted by $L^p(\mathbb{R})$, by L^p for simplicity, or by L^p_x for definiteness. The usual Sobolev spaces of order m are defined by $H^m = (1-\Delta)^{-m/2}L^2$. For a time interval $I \subset \mathbb{R}$ and a Banach space X we denote by $C(I;X)$ the space of strongly continuous functions from I to X and by $L^p(I;X)$ the space of measurable functions u from I to X such that $\|u(\cdot)\|_X \in L^p(I)$. The abbreviation such as $L^p_t L^q_x = L^p_t(I; L^q_x) = L^p(I; L^q(\mathbb{R}))$ and $L^p_x L^q_t = L^p_x(L^q_t(I)) = L^p(\mathbb{R}; L^q(I))$ will often be made when this causes no confusion.

Now we consider the Cauchy problem for (1a), (1b) in the following integral form:

$$u(t) = U(t)u_0 - i\int_0^t U(t-\tau)(vu + \lambda|u|^2u)(\tau)d\tau, \tag{2a}$$

$$v(t) = v_0 + \int_0^t \partial(|u|^2)d\tau, \tag{2b}$$

where (u_0, v_0) is the Cauchy data prescribed at $t=0$ and $U(t) = \exp(i(t/2)\Delta)$ is the free propagator which solves the free Schrödinger equation. The equations (2a), (2b) make a single equation of u with data (u_0, v_0), which will be solved by a contraction method in the function space $X(I)$ over the time interval I, defined by

$$X(I) = \{u \in C(I; H^{1/2}) \cap L^4_t(I; L^\infty_x) \cap L^4_x(L^\infty_t(I)); \partial u \in L^\infty_x(L^2_t(I))\}$$

with norm

$$|||u||| = \|u\|_{L^\infty_t(I;H^{1/2})} + \|u\|_{L^4_t(I;L^\infty_x)} + \|u\|_{L^4_x(L^\infty_t(I))} + \|\partial u\|_{L^\infty_x(L^2_t(I))}.$$

The basic existence and uniqueness result is the following:

Theorem 1. *For any $(u_0, v_0) \in H^{1/2} \times L^2$ (2a), (2b) have a unique pair of solutions $(u, v) \in C(\mathbb{R}; H^{1/2}) \times C(\mathbb{R}; L^2)$ such that $u \in X([-T,T])$ for any $T > 0$. Moreover, $\|u(t)\|_{L^2} = \|u_0\|_{L^2}$ for any $t \in \mathbb{R}$.*

Theorem 1 enables us to define the nonlinear propagator $W(t) : (u_0, v_0) \mapsto (u(t), v(t))$. The nonlinear propagator $W(t)$ forms a group on $H^{1/2} \times L^2$ under composition. The map $(u_0, v_0) \mapsto W(\cdot)(u_0, v_0)$ is well-defined from $H^{1/2} \times L^2$ to $C(\mathbb{R}; H^{1/2}) \times C(\mathbb{R}; L^2)$. Moreover, we have:

Theorem 2. *For any $T > 0$ and $(u_0, v_0) \in H^{1/2} \times L^2$ there exists $\varepsilon > 0$ such that the map $(u_0, v_0) \mapsto W(\cdot)(u_0, v_0)$ is Lipschitz from $B_\varepsilon(u_0, v_0) \equiv \{(\phi, \psi) \in H^{1/2} \times L^2; \|\phi - u_0\|_{H^{1/2}} < \varepsilon, \|\psi - v_0\|_{L^2} < \varepsilon\}$ to $X([-T,T]) \times L^\infty(-T,T;L^2)$.*

Remark 1. We have stated Theorem 1 with emphasis on the existence part. In $X([-T,T])$ in the statement of Theorem 1, we may replace $L^4_t(-T,T;L^\infty_x)$ by

$L_t^8(-T,T;L_x^4)$. In fact, we give the proof that works for that purpose with minor modifications. Here we note that $\|u\|_{L_t^8 L_x^4} \le \|u\|_{L_t^4 L_x^\infty}^{1/2} \|u\|_{L_t^\infty L_x^2}^{1/2}$.

Remark 2. Nothing is known about the large time behavior of solutions of (1a), (1b). Since (1a),(1b) are cubic with respect to u in one space dimension, one could expect the existence of modified wave operators on a dense set of small and sufficiently regular asymptotic states [4,5,8,9,16,19]. A preliminary argument proceeds as follows. If one tries to construct an appropriate modified dynamics for u in the form

$$w(t,x) = (it)^{-1/2} \exp(ix^2/(2t) + iS(t,x/t))\hat{u}_+(x/t)$$

with phase function S, where $\hat{u}_+$ is the Fourier transform of the asymptotic state u_+, then the large time behavior of the potential part $v + \lambda|u|^2$ are supposedly approximated by

$$(v_+ - \int_t^\infty \partial|w|^2 d\tau) + \lambda|w|^2 = v_+ - \frac{1}{x}(|\hat{u}_+|^2(\frac{x}{t}) - |\hat{u}_+|^2(0)) + \frac{\lambda}{t}|\hat{u}_+|^2(\frac{x}{t}),$$

where v_+ is the asymptotic state for v. The potential is thus of long-range type and the phase function becomes

$$S(t,\xi) = ((|\hat{u}_+|^2(\xi) - |\hat{u}_+|^2(0))/\xi + \lambda|\hat{u}_+|^2(\xi)) \log t,$$

provided that v_+ is short range. Then we need to solve (1a),(1b) around the modified free dynamics $(w, v_+ - (|\hat{u}_+|^2(x/t) - |\hat{u}_+|^2(0))/x)$ by a contraction method, though we have not been able to justify this.

Remark 3. The equations (1a),(1b) are regarded as a special case of the coupled Schrödinger-KdV equations

$$i\partial_t u + \frac{1}{2}\partial^2 u = vu + \lambda|u|^2 u, \tag{3a}$$

$$\partial_t v + \alpha\partial^3 v + \beta v \partial v = \partial(|u|^2), \tag{3b}$$

where $\alpha, \beta \in \mathbb{R}$. When $\alpha = \beta = 1$, M. Tsutsumi [17] proved the global well-posedness of the Cauchy problem for (3a),(3b) in the spaces $H^{k+1/2} \times H^k$ for all integers $k \ge 1$. In view of the results in this paper, it might be natural to expect the global well-posedness for (3a),(3b) in the space $H^{1/2} \times L^2$.

Remark 4. One-dimensional Zakharov equations of the form

$$i\partial_t E + \frac{1}{2}\partial^2 E = nE + \lambda|E|^2 E, \tag{4a}$$

$$\partial_t n \pm \partial n = \partial(|E|^2), \tag{4b}$$

are equivalent to (1a), (1b) through the relations

$$E(t,x) = e^{i(\mp x - t/2)} u(t, x \pm t),$$

$$n(t,x) = v(t, x \pm t).$$

This shows that the Cauchy problem for (4a), (4b) is globally well-posed in $H^{1/2} \times L^2$. We note here that the multiplication by $e^{\pm ix}$ leaves $H^{1/2}$ invariant and that $\|u\|_{L^\infty_x(L^2_t(-T,T))} \leq (2T)^{1/4}\|u\|_{L^4_t(-T,T;L^\infty_x)}$. When $\lambda = 0$, the equations (4a), (4b) are solvable by the inverse spectral method (see [21]).

This paper is organized as follows. In section 2 we prepare some basic estimates of the free propagator and of the nonlinear term. In section 3 we prove Theorems 1 and 2. For simplicity we restrict ourselves to positive times and the time variable t will often be omitted. Different positive constants might be denoted by the same letter C. For any r with $1 \leq r \leq \infty$ we denote by r' the exponent dual to r.

2. Preliminary Estimates.

In this section we collect some basic estimates related to the free propagator $U(t) = \exp(i(t/2)\Delta)$ and to the nonlinear term.

Lemma 1 ([6,10,11,20]). *U satisfies the following estimates.*
(1) *For any (q,r) with $0 \leq 2/q = 1/2 - 1/r \leq 1/2$*

$$\|U(\cdot)\phi\|_{L^q_t(\mathbb{R};L^r)} \leq C\|\phi\|_{L^2}.$$

(2) *For any (q_1,r_1) and (q_2,r_2) with $0 \leq 2/q_j = 1/2 - 1/r_j \leq 1/2, j = 1,2,$ and for any interval $I \subset \mathbb{R}$ with $0 \in \bar{I}$ the operator G defined by*

$$(Gf)(t) = \int_0^t U(t-\tau)f(\tau)d\tau$$

satisfies the estimate

$$\|Gf\|_{L^{q_1}_t(I;L^{r_1}_x)} \leq C\|f\|_{L^{q_2'}_t(I;L^{r_2'}_x)}$$

where C is independent of I. Moreover, $Gf \in C(\bar{I};L^2)$.

Lemma 2([12,13,14]). (1) *U satisfies the following estimates.*

$$\|(-\Delta)^{1/4}U(\cdot)\phi\|_{L^\infty_x(L^2_t(\mathbb{R}))} \leq C\|\phi\|_{L^2}.$$

$$\|U(\cdot)\phi\|_{L^4_x(L^\infty_t(\mathbb{R}))} \leq C\|(-\Delta)^{1/8}\phi\|_{L^2}.$$

(2) *For any interval $I \subset \mathbb{R}$ with $0 \in \bar{I}$ the operator G defined in Lemma 1 satisfies the estimate*

$$\|(-\Delta)^{1/4}Gf\|_{L^\infty_t(I;L^2_x)} \leq C\|f\|_{L^1_x(L^2_t(I))},$$

$$\|\partial Gf\|_{L^\infty_x(L^2_t(I))} \leq C\|f\|_{L^1_x(L^2_t(I))},$$

where C is independent of I.

Lemma 3 ([18]). *For any $f_1, f_2, f_3 \in X(I)$ with $I = [0,T]$*

$$\|\int_0^t f_1(s)f_2(s)ds f_3\|_{L^1_x(L^2_t(I))} \le T\|f_1\|_{L^\infty_x(L^2_t(I))}\prod_{j=2}^3 \|f_j\|_{L^\infty_t(I;L^2_x)},$$

$$\|\int_0^t f_1(s)f_2(s)ds\|_{L^\infty_t(I;L^2_x)} \le T^{1/2}\|f_1\|_{L^\infty_x(L^2_t(I))}\|f_2\|_{L^\infty_t(I;L^2_x)}.$$

Proof. By the Schwarz and Hölder inequalities, we have

$$\begin{aligned}
&\|\int_0^t f_1(s)f_2(s)ds\|_{L^\infty_t(I;L^2_x)}\\
\le&\|\prod_{j=1}^2 \|f_j\|_{L^2_t(I)}\|_{L^2_x} \le \|f_1\|_{L^\infty_x(L^2_t(I))}\|f_2\|_{L^2_t(I;L^2_x)},\\
&\|\int_0^t f_1(s)f_2(s)ds f_3\|_{L^1_x(L^2_t(I))}\\
\le&\|(\prod_{j=1}^2 \|f_j\|_{L^2_t(I)})f_3\|_{L^1_x(L^2_t(I))}\\
=&\|\prod_{j=1}^3 \|f_j\|_{L^2_t(I)}\|_{L^1_x} \le \|f_1\|_{L^\infty_x(L^2_t(I))}\prod_{j=2}^3 \|f_j\|_{L^2_t(I;L^2_x)}.
\end{aligned}$$

The lemma follows from these inequalities. QED

3. Proof of Theorems 1 and 2.

Throughout this section we put $I = [0,T]$ with $T > 0$. In view of the results and arguments in [18], it suffices to prove the local version of the theorems. We begin with additional notation. For $v_0 \in L^2$ and $w \in X(I)$ we define

$$F(w) = F_1(w) + F_2(w) + F_3(w)$$

where

$$\begin{aligned}
(F_1(w))(t,x) &= w(t,x)\int_0^t \partial|w|^2(s,x)ds,\\
(F_2(w))(t,x) &= v_0(x)w(t,x),\\
(F_3(w))(t,x) &= \lambda|w|^2 w(t,x).
\end{aligned}$$

In order to carry out a contraction argument, we single out the basic estimates.

Lemma 4. *For any $w_1, w_2 \in X(I)$,*

$$\begin{aligned}
\|F(w_1) - F(w_2)\|_{L^1_x(L^2_t(I))} \le{}& C(T+T^{1/2})(|||w_1|||^2 + |||w_2|||^2)|||w_1 - w_2|||\\
&+ CT^{1/2}\|v_0\|_{L^2}|||w_1 - w_2|||,\\
\|F(w_1) - F(w_2)\|_{L^{8/7}_t(I;L^{4/3}_x)} \le{}& C(T^{5/4}+T^{1/2})(|||w_1|||^2 + |||w_2|||^2)|||w_1 - w_2|||\\
&+ CT^{3/4}\|v_0\|_{L^2}|||w_1 - w_2|||.
\end{aligned}$$

Proof. For simplicity we prove the lemma in the case where $w_1 = w, w_2 = 0$, since the general case is proved similarly. By Lemma 3 and the Hölder inequality, we have

$$\begin{aligned}
\|F_1(w)\|_{L^1_x(L^2_t(I))} &\le 2T\|\partial w\|_{L^\infty_x(L^2_t(I))}\|w\|^2_{L^\infty_t(I;L^2_x)},\\
\|F_1(w)\|_{L^{8/7}_t(I;L^{4/3}_x)} &\le T^{3/4}\|\int_0^t \partial|w|^2(s)ds\|_{L^\infty_t(I;L^2_x)}\|w\|_{L^8_t(I;L^4_x)}\\
&\le 2T^{5/4}\|\partial w\|_{L^\infty_x(L^2_t(I))}\|w\|_{L^\infty_t(I;L^2_x)}\|w\|_{L^8_t(I;L^4_x)}\\
&\le 2T^{5/4}\|\partial w\|_{L^\infty_x(L^2_t(I))}\|w\|^{3/2}_{L^\infty_t(I;L^2_x)}\|w\|^{1/2}_{L^4_t(I;L^\infty_x)}.
\end{aligned}$$

By the Schwarz and Hölder inequalities, we have

$$\begin{aligned}
\|F_2(w)\|_{L^1_x(L^2_t(I))} &\le \|v_0\|_{L^2}\|w\|_{L^2_x(L^2_t(I))} \le T^{1/2}\|v_0\|_{L^2}\|w\|_{L^\infty_t(I;L^2_x)},\\
\|F_2(w)\|_{L^{8/7}_t(I;L^{4/3}_x)} &\le T^{3/4}\|v_0\|_{L^2}\|w\|_{L^8_t(I;L^4_x)}\\
&\le T^{3/4}\|v_0\|_{L^2}\|w\|^{1/2}_{L^\infty_t(I;L^2_x)}\|w\|^{1/2}_{L^4_t(I;L^\infty_x)}.
\end{aligned}$$

By the Hölder inequality, we have

$$\begin{aligned}
\|F_3(w)\|_{L^1_x(L^2_t(I))} &\le |\lambda|\|w\|^2_{L^4_x(L^\infty_t(I))}\|w\|_{L^2_x(L^2_t(I))}\\
&\le |\lambda|T^{1/2}\|w\|^2_{L^4_x(L^\infty_t(I))}\|w\|_{L^\infty_t(I;L^2_x)},\\
\|F_3(w)\|_{L^{8/7}_t(I;L^8_x)} &\le |\lambda|T^{1/2}\|w\|^3_{L^8_t(I;L^4_x)}\\
&\le |\lambda|T^{1/2}\|w\|^{3/2}_{L^\infty_t(I;L^2_x)}\|w\|^{3/2}_{L^4_t(I;L^\infty_x)}.
\end{aligned}$$

Collecting these estimates, we obtain the lemma. QED

We are now in a position to prove the theorems. For $(u_0, v_0) \in H^{1/2} \times L^2$ and $w \in X(I)$, we define

$$\Phi(w) = U(\cdot)u_0 - iGF(w).$$

Since $(\infty, 2), (4, \infty)$, and $(8,4)$ are admissible pairs in Lemma 1, it follows from Lemmas 1 and 4 that

$$\begin{aligned}
&\|\Phi(w)\|_{L^\infty_t(I;L^2_x)} + \|\Phi(w)\|_{L^4_t(I;L^\infty_x)}\\
&\le C\|u_0\|_{L^2} + C(T^{5/4}+T^{1/2})|||w|||^3 + CT^{3/4}\|v_0\||||w|||.
\end{aligned}$$

By Lemmas 1, 2, and 4, we obtain

$$\begin{aligned}
&\|\Phi(w)\|_{L^4_x(L^\infty_t(I))} = \|U(\cdot)(u_0 - iU^{-1}GF(w))\|_{L^4_x(L^\infty_t(I))}\\
&\le C\|(-\Delta)^{1/8}u_0\|_{L^2} + C\|(-\Delta)^{1/8}U^{-1}GF(w)\|_{L^\infty_t(I;L^2_x)}\\
&\le C\|(-\Delta)^{1/8}u_0\|_{L^2} + C\|(-\Delta)^{1/4}GF(w)\|^{1/2}_{L^\infty_t(I;L^2_x)}\|GF(w)\|^{1/2}_{L^\infty_t(I;L^2_x)}\\
&\le C\|(-\Delta)^{1/8}u_0\|_{L^2} + C\|F(w)\|^{1/2}_{L^1_x(L^2_t(I))}\|F(w)\|^{1/2}_{L^{8/7}_t(I;L^{4/3}_x)}\\
&\le C\|(-\Delta)^{1/8}u_0\|_{L^2} + C(T^{5/4}+T^{1/2})|||w|||^3 + C(T^{3/4}+T^{1/2})\|v_0\||||w|||,
\end{aligned}$$

$$
\begin{aligned}
&\|\partial\Phi(w)\|_{L^\infty_x(L^2_t(I))}\\
&\le \|(-\Delta)^{1/4}U(\cdot)\partial(-\Delta)^{-1/4}u_0\|_{L^\infty_x(L^2_t(I))} + C\|F(w)\|_{L^1_x(L^2_t(I))}\\
&\le C\|(-\Delta)^{1/4}u_0\|_{L^2} + C(T+T^{1/2})|||w|||^3 + CT^{1/2}\|v_0\|_{L^2}|||w|||,
\end{aligned}
$$

$$
\begin{aligned}
&\|(-\Delta)^{1/4}\Phi(w)\|_{L^\infty_t(I;L^2_x)}\\
&\le C\|u_0\|_{L^2} + C(T+T^{1/2})|||w|||^3 + CT^{1/2}\|v_0\|_{L^2}|||w|||.
\end{aligned}
$$

Collecting these estimates, we have

$$
|||\Phi(w)||| \le C\|u_0\|_{H^{1/2}} + C(T^{5/4}+T^{1/2})|||w|||^3 + C(T^{3/4}+T^{1/2})\|v_0\|_{L^2}|||w|||.
$$

Similarly, we have for $w_1, w_2 \in X(I)$

$$
\begin{aligned}
|||\Phi(w_1)-\Phi(w_2)||| \le\ & C(T^{5/4}+T^{1/2})(|||w_1|||^2 + |||w_2|||^2)|||w_1-w_2|||\\
&+ C(T^{3/4}+T^{1/2})\|v_0\|_{L^2}|||w_1-w_2|||.
\end{aligned}
$$

The last two estimates prove the map $\Phi : w \mapsto \Phi(w)$ is a contraction on a closed ball in $X(I)$ provided that $T > 0$ is sufficiently small. We denote by u the unique fixed point of Φ and then we define v by the equation (2b). We prove that $v \in C(I; L^2_x)$. Let $t, s \in I$. By the Hölder inequality, we have

$$
\begin{aligned}
\|\int_s^t \partial|u|^2(\tau)d\tau\|_{L^2_x} &\le 2\|(\int_s^t |u|^2 d\tau)^{1/2}\|\partial u\|_{L^2_t(I)}\|_{L^2_x}\\
&\le 2|t-s|\|u\|_{L^\infty_t(I;L^2_x)}\|\partial u\|_{L^\infty_x(L^2_t(I))},
\end{aligned}
$$

so that $v \in C(I; L^2_x)$, as was to be shown. This proves the essential part of the theorems and the rest of the proof proceeds in the same way as in [18]. QED

Remark. After this paper has been completed, the author noticed the reference [22], where related results on (1) and (4) are obtained by a rather different method in some respects.

References

[1] D. J. Benney, A general theory for interactions between short and long waves, Stud. Appl. Math. **56** (1977), 81–94.

[2] V. D. Djordjevic & L. G. Redekopp, On two-dimensional packets of capillary-gravity waves, J. Fluid Mech. **79** (1977), 703–714.

[3] M. Funakoshi & M. Oikawa, The resonant interactions between a long internal gravity wave and a surface gravity wave packet, J. Phys. Soc. Japan **52** (1983), 1982–1995.

[4] J. Ginibre & T. Ozawa, Long range scattering for nonlinear Schrödinger and Hartree equations in space dimension $n \ge 2$, Commun. Math. Phys. **151** (1993), 619–645.

[5] J. Ginibre, T. Ozawa & G. Velo, On the existence of the wave operators for a class of nonlinear Schrödinger equations, Ann. Inst. Henri Poincaré, Physique théorique **60** (1994), 211–239.

[6] J. Ginibre & G. Velo, Scattering theory in the energy space for a class of nonlinear Schrödinger equations, J. Math. Pures Appl. **64** (1985), 363–401.

[7] R. H. J. Grimshaw, The modulation of an internal gravity-wave packet, and the resonance with the mean motion, Stud. Appl. Math. **56** (1972), 241–266.

[8] N. Hayashi & T. Ozawa, Modified wave operators for the derivative nonlinear Schrödinger equation, Math. Annalen **298** (1994), 557–576.
[9] N. Hayashi & T. Ozawa, Schrödinger equations with nonlinearity of integral type, preprint, Hokkaido University (1994).
[10] T. Kato, On nonlinear Schrödinger equations, Ann. Inst. Henri Poincaré, Physique théorique **46** (1987), 113–129.
[11] T. Kato, Nonlinear Schrödinger equations, in *Schrödinger Operators*, Lecture Notes in Physics **345** (1989), Springer, 218–263.
[12] C. Kenig, G. Ponce & L. Vega, Oscillatory integrals and regularity of dispersive equations, Indiana Univ. Math. J. **40** (1991), 33–67.
[13] C. Kenig, G. Ponce & L. Vega, Small solutions to nonlinear Schrödinger equations, Ann. Inst. Henri Poincaré, Analyse nonlinéaire **10** (1993), 255–288.
[14] C. Kenig & A. Ruiz, A strong type (2,2) estimate for the maximal function associated to the Schrödinger equations, Trans. Amer. Math. Soc. **280** (1983), 239–246.
[15] Y. C. Ma, The complete solution of the long-wave-short-wave resonance equations, Stud. Appl. Math. **59** (1978), 201–221.
[16] T. Ozawa, Long range scattering for nonlinear Schrödinger equations in one space dimension, Commun. Math. Phys. **139** (1991), 479–493.
[17] M. Tsutsumi, Well-posedness of the Cauchy problem for a coupled Schrödinger-KdV equation, in *Nonlinear Mathematical Problems in Industry,* GAKUTO International Series, Mathematical Sciences and Applications **2** (1993), 513–528.
[18] M. Tsutsumi & S. Hatano, Well-posedness of the Cauchy problem for the long wave-short wave resonance equations, Nonlinear Analysis, TMA **22** (1994), 155–171.
[19] Y. Tsutsumi, Global existence and asymptotic behavior of solutions for the Maxwell-Schrödinger equations in three space dimensions, Commun. Math. Phys. **151** (1993), 543–576.
[20] K. Yajima, Existence of solutions for Schrödinger evolution equations, Commun. Math. Phys. **110** (1987), 415–426.
[21] N. Yajima & M. Oikawa, Formation and interaction of sonic Langmuir solitons, Prog. Theoretical Phys. **56** (1976), 1719–1739.
[22] M. Tsutsumi & S. Hatano, Well-posedness of the Cauchy problem for Benney's first equations of long wave short wave interactins, Funkcialaj Ekvacioj **37** (1994), 289–316.