# A TIME-LIKE SURFACE IN MINKOWSKI 3-SPACE WHICH CONTAINS LIGHT-LIKE LINES

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# A TIME-LIKE SURFACE IN MINKOWSKI 3-SPACE WHICH CONTAINS LIGHT-LIKE LINES

**Shyuichi Izumiya and Akihiro Takiyama**

Simple characterizations of a pseudosphere or a time-like plane in Minkowski 3-space by the existence of light-like lines are given.

## 1. INTRODUCTION

There are many simple characterizations of a sphere in Euclidean 3-space [1,3,4]. For example "all geodesics are plane curves" characterizes a sphere or a plane. In [4] Takeuchi gave a much simpler and practical characterization of a sphere. Her characterization is as follows:if there exist four geodesics through each point of a complete surface such that they are plane curves, then the surface is a sphere or a plane. She also gave other characterizations of a sphere or a plane in [4].

In this paper we consider the similar problem for a pseudosphere $S_1^2(r, a)$ (for definition, see Section 2) and a time-like plane in Minkowski 3-space. The normal vector field on a pseudosphere in $\mathbb{M}^3$ is light-like. We say that a surface $S$ in $\mathbb{M}^3$ is *time-like* if the normal vector field on $S$ is space-like. It is easy to show that if all geodesics on a time-like surface are plane curves then the surface is an open subset of a pseudosphere or a time-like plane in Minkowski 3-space $\mathbb{M}^3$ (cf., [5]). We give characterizations which are much simpler and peculiar to the case for a pseudosphere or a time-like plane in Minkowski 3-space $\mathbb{M}^3$. Our main results are as follows.

**THEOREM A.** Let $S$ be a connected time-like surface in Minkowski 3-space $\mathbb{M}^3$. For each point $p$ of $S$, suppose that there exist two light-like curves on $S$ through $p$ such that they are plane curves. Then $S$ is a open subset of a pseudosphere or a time-like plane.

We have corollaries of the theorem as follows:

**COROLLARY A.1.** Let $S$ be a time-like surface in Minkowski 3-space $\mathbb{M}^3$. For each point $p$ of $S$, suppose that the intersection of $S$ and $T_pS$, which is considered as an affine plane in $\mathbb{M}^3$, is the two lines such that each direction is light-like. Then $S$ is a pseudosphere.

The assumption of the corollary can be interpreted as that if two lines can be pressed entirely at a point $p$ of the surface, then these are the light cone on the tangent plane.

We have a simple characterization of a time-like plane in $\mathbb{M}^3$ as an corollary of Theorem A.

**COROLLARY A.2.** Let $S$ be a connected time-like surface in $\mathbb{M}^3$. For each point $p$ of $S$, suppose that there exist two light-like curves of $S$ which are plane curve and one non-light-like line on $S$ through $p$. Then $S$ is a open set of a time-like plane.

Another characterization by the existence of planar geodesics is given by the following.

**THEOREM B.** Let $S$ be a connected time-like surface in Minkowski 3-space $\mathbb{M}^3$. For each point $p$ in $S$, suppose that there exist one light-like curve and two non-light-like geodesics on $S$ through $p$ such that they are plane curves. Then $S$ is an open subset of a pseudosphere or a time-like plane.

We also have the following simple characterization of a time-like plane in $\mathbb{M}^3$ as a corollary of Theorem B.

**COROLLARY B.1.** Let $S$ be a connected time-like surface in Minkowski 3-space $\mathbb{M}^3$. For each point $p$ in $S$, suppose that there exist one light-like curve, one non-light-like geodesics on $S$ such that they are plane curves and one non-light-like line through $p$. Then $S$ is an open subset of a time-like plane.

We remark that these theorems are the best possible in some sense (cf., Section 4). For space-like surfaces, the induced metric on the surface is positively definite, so that we can give characterizations of a hyperbolic surface $H_1^2(r,a)$ by exactly the same arguments as those in [1,3,4]. Thus we do not consider this case in this paper.

All surfaces and maps considered here are of class $C^\infty$ unless stated otherwise.

## 2. BASIC NOTIONS

Let $\mathbb{R}^3 = \{(x_1,x_2,x_3)|x_1,x_2,x_3 \in \mathbb{R}\}$ be the usual oriented 3-dimensional vector space and differential manifold, which is oriented by $e_1 = (1,0,0)$, $e_2 = (0,1,0)$ and $e_3 = (0,0,1)$ and given the Euclidean differentiable structure. *Minkowski 3-space* is defined by $\mathbb{M}^3 = \{\mathbb{R}^3, I_{(2,1)}\}$, where $I_{(2,1)} = dx_1^2 + dx_2^2 - dx_3^2$. Thus the metric tensor is given by

$< \mathrm{X}, \mathrm{Y} >= x_1y_1 + x_2y_2 - x_3y_3$, where $\mathrm{X}= (x_1, x_2, x_3)$ and $\mathrm{Y}= (y_1, y_2, y_3)$. A vector X in $\mathbb{M}^3$ is called *light-like* if $< \mathrm{X}, \mathrm{X} >= 0$, *space-like* if $< \mathrm{X}, \mathrm{X} >> 0$ and *time-like* if $< \mathrm{X}, \mathrm{X} >< 0$. A curve $\gamma$ is called *light-like* if its tangent vector field $\gamma'$ is always light-like. We also say that a curve $\gamma$ is *non-light-like* if its tangent vector field $\gamma'$ is always space-like or time-like. *The pseudosphere* is defined to be

$$S_1^2(r, a) = \{(x_1, x_2, x_3)|(x_1 - a_1)^2 + (x_2 - a_2)^2 - (x_3 - a_3)^2 = r^2\},$$

where $a = (a_1, a_2, a_3)$ is the center and $r > 0$ is the radial of $S_1^2(r, a)$.

The Levi-Civita connection of $\mathbb{M}^3$ is denoted by $\nabla$. Let $S$ be a surface in $\mathbb{M}^3$. We say that $S$ is *time-like* if the normal vector field to $S$ is space-like. Let $\Pi(\mathrm{X}, \mathrm{Y})$ be a second fundamental form tensor of $S$ in $\mathbb{M}^3$. Since $S$ is a time-like surface, there exists (at least locally) a unit normal vector field $\xi$ on $S$. We have the following formula:

$$< \Pi(\mathrm{X}, \mathrm{Y}), \xi >=< -\nabla_{\mathrm{X}}\xi, \mathrm{Y} > .$$

If we denote that $\Pi(\mathrm{X}, \mathrm{Y}) = \sigma(\mathrm{X}, \mathrm{Y})\xi$, we have $\sigma(\mathrm{X}, \mathrm{Y}) =< -\nabla_{\mathrm{X}}\xi, \mathrm{Y} >$, so that $-\nabla_{\mathrm{X}}\xi$ is the shape operator in this case It is well-known that the shape operator is self-adjoint with respect to $<, >$ (i.e., $< -\nabla_{\mathrm{X}}\xi, \mathrm{Y} >=< \mathrm{X}, -\nabla_{\mathrm{Y}}\xi >$).
We now state some lemmas for preparing the proof of main results.

**LEMMA 2.1** Let $V$ be a time-like plane in $\mathbb{M}^3$. If $< \mathrm{X}, \mathrm{X} >= 0$ and $< \mathrm{Y}, \mathrm{Y} >\neq 0$, then $< \mathrm{X}, \mathrm{Y} >\neq 0$.

*Proof.* Since $V$ has two light-like direction, there exists $\mathrm{Z} \in V$ such that $< \mathrm{Z}, \mathrm{Z} >= 0$ and $\mathrm{Z} \notin \langle \mathrm{X} \rangle_{\mathbb{R}}$. It follows that there exist real numbers $\lambda, \mu$ with $\mu \neq 0$ such that $\mathrm{Z} = \lambda\mathrm{X} + \mu\mathrm{Y}$. Thus we have

$$0 =< \mathrm{Z}, \mathrm{Z} >= 2\lambda\mu < \mathrm{X}, \mathrm{Y} > +\mu^2 < \mathrm{Y}, \mathrm{Y} > .$$

If $< \mathrm{X}, \mathrm{Y} >= 0$, then $< \mathrm{Y}, \mathrm{Y} >= 0$. This is a contradiction

Since the light direction on a time-like plane in Minkowski 3-space $\mathbb{M}^3$ is constant, we have the following simple lemma.

**LEMMA 2.2** Let $\gamma$ be a light-like plane curve in $\mathbb{M}^3$. Then it is a line, especially a geodesic in $\mathbb{M}^3$.

We also have the following lemma for light-like line on a surface in $\mathbb{M}^3$.

**LEMMA 2.3** Let $\gamma$ be a light-like line on a time-like surface in $\mathbb{M}^3$ and $\xi$ be a normal unit vector field on $S$. Then there exists an real number $k$ such that $-\nabla_{\mathrm{X}}\xi = k\mathrm{X}$, where X is a tangent vector of $\gamma$.

*Proof.* Since $\gamma$ is light-like line, we have $\sigma(\mathrm{X},\mathrm{X}) = 0$ (cf., [2], page 103 Corollary 9). It follows that

$$< -\nabla_{\mathrm{X}}\xi, \mathrm{X} >= \sigma(\mathrm{X},\mathrm{X}) = 0.$$

Suppose that there are no real numbers $k$ such that $-\nabla_{\mathrm{X}}\xi = k\mathrm{X}$. Since $\dim S = 2$, for any $\mathrm{Y} \in T_pS$, there exist real numbers $\lambda, \mu$ such that $Y = \lambda(-\nabla_{\mathrm{X}}\xi) + \mu k\mathrm{X}$. Thus we have

$$< \mathrm{X}, \mathrm{Y} >= \lambda < -\nabla_{\mathrm{X}}\xi, \mathrm{X} > +\mu < \mathrm{X}, \mathrm{X} >= 0.$$

Let $\mathrm{Y}$ be a tangent vector at $p$ which has another light-like direction. Then we have

$$\begin{aligned} < \mathrm{X}+\mathrm{Y}, \mathrm{X}+\mathrm{Y} > &= < \mathrm{X}, \mathrm{X} > +2 < \mathrm{X}, \mathrm{Y} > + < \mathrm{Y}, \mathrm{Y} > \\ &= 2 < \mathrm{X}, \mathrm{Y} >= 0, \end{aligned}$$

This contradicts to the fact that $\mathrm{X}+\mathrm{Y}$ is not light-like.

Let $\gamma$ be a non-light-like curve. We may assume that $\gamma$ is parametrized by arc length $s$. Thus we have $< \gamma'(s), \gamma'(s) >= \varepsilon(s) = \pm 1$. By the exactly same arguments as in the case for a curve in Euclidean 3-space, we have the following Frenet-Serret formula (cf., [5]): There exist field of vectors $\mathbf{n}(s), \mathbf{b}(s)$ with $< \mathbf{n}(s), \mathbf{n}(s) >= \delta(s) = \pm 1$, $< \mathbf{b}(s), \mathbf{b}(s) >= -\varepsilon(s)\delta(s)$ and functions $k = k(s)$, $\tau = \tau(s)$ along $\gamma$ such that

$$\begin{aligned} &\nabla_{\gamma'(s)}\gamma'(s) = k(s)\mathbf{n}(s) \\ &\nabla_{\gamma'(s)}\mathbf{n}(s) = -\varepsilon(s)\delta(s)k(s)\gamma'(s) + \varepsilon(s)\tau(s)\mathbf{b}(s) \\ &\nabla_{\gamma'(s)}\mathbf{b}(s) = \tau(s)\mathbf{n}(s). \end{aligned}$$

If $\gamma$ is a plane curve, then $\tau(s) \equiv 0$, so that we have

$$\nabla_{\gamma'(s)}\gamma'(s) = k(s)\mathbf{n}(s) \quad \text{and} \quad \nabla_{\gamma'(s)}\mathbf{n}(s) = -\varepsilon(s)\delta(s)k(s)\gamma'(s).$$

Suppose that $\gamma$ is also a geodesic of $S$, then

$$\nabla_{\gamma'(s)}\gamma'(s) = \Pi(\gamma'(s), \gamma'(s)) = \sigma(\gamma'(s), \gamma'(s))\xi(\gamma(s))$$

(cf., [2], page 103 Corollary 9). It follows that $\mathbf{n}(s) = \pm\xi(\gamma(s))$, so we have the following lemma.

**LEMMA 2.4** Let $S$ be a time-like surface in Minkowski 3-space $\mathbb{M}^3$ and $\gamma$ be a non-light-like geodesic on $S$ which is parametrized by arc length $s$ such that it is a plane curve. Then $\sigma(\gamma'(s), \gamma'(s)) = 0$ when $k(s) = 0$ and $\nabla_{\gamma'(s)}\xi(\gamma(s)) = \mp\varepsilon(s)\delta(s)k(s)\gamma'(s)$ when $k(s) \neq 0$.

## 3. PROOF OF RESULTS

We now give a proof of Theorem A.

*Proof of Theorem A.* Let $\gamma_i$ $(i = 1,2)$ be two light-like curve on $S$ through $p$ such that they are plane curves. By Lemma 2.3, we have $-\nabla_{\mathbf{X}_i}\xi = k_i\xi$ for some $k_i$, where $\mathbf{X}_i$ is the tangent vector of $\gamma_i$ $(i = 1,2)$. If $k_1 \neq k_2$, we have

$$\begin{aligned} &k_1 < \mathbf{X}_1, \mathbf{X}_2 >=< k_1\mathbf{X}_1, \mathbf{X}_2 >=< -\nabla_{\mathbf{X}_1}\xi, \mathbf{X}_2 > \\ &= < \mathbf{X}_1, -\nabla_{\mathbf{X}_2}\xi >=< \mathbf{X}_1, k_2\mathbf{X}_2 >= k_2 < \mathbf{X}_1, \mathbf{X}_2 >, \end{aligned}$$

so that $< \mathbf{X}_1, \mathbf{X}_2 >= 0$.

Since $\mathbf{X}_1 + \mathbf{X}_2$ is not light-like vector, we have

$$0 \neq < \mathbf{X}_1 + \mathbf{X}_2, \mathbf{X}_1 + \mathbf{X}_2 >=< \mathbf{X}_1, \mathbf{X}_1 > +2 < \mathbf{X}_1, \mathbf{X}_2 > + < \mathbf{X}_2, \mathbf{X}_2 >= 2 < \mathbf{X}_1, \mathbf{X}_2 > .$$

This is a contradiction, so that we have $k_1 = k_2$. Since $\dim S = 2$, $p$ is an umbilic point.

*Proof of Corollary A.1.* By the assumption, for each point $p \in S$, there exit two light-like line on $S$ which through $p$. By Theorem A, $S$ is a pseudosphere or a time-like plane, however, a time-like plane does not satisfy the assumption.

*Proof of Corollary A.2.* It follows from Theorem A that there exists real number $k$ such that $-\nabla_{\mathbf{X}}\xi = k\mathbf{X}$ for any $\mathbf{X} \in T_pS$. By the assumption, there exists a non-light-like line $\gamma$ on $S$ through $p$, so that we have $-\nabla_{\gamma'}\xi = k\gamma'$. Since $\gamma$ is a line, $\sigma(\gamma', \gamma') = 0$. Thus we have

$$k < \gamma', \gamma' >=< k\gamma', \gamma' >=< -\nabla_{\gamma'}\xi, \gamma' >= \sigma(\gamma', \gamma') = 0,$$

so that $k = 0$ because $\gamma$ is non-light-like. Therefore $p$ is a geodesic point. This completes the proof.

*Proof of Theorem B.* Let $\gamma$ be a light-like line on $S$ through p. By Lemma 2.3, there exists a real number k such that $-\nabla_{\gamma'}\xi = k\gamma'$. Let $\gamma_i$ $(i = 1,2)$ be non-light-like planar geodesics. By Lemma 2.4, we have $-\nabla_{\gamma_i'}\xi = k_i\gamma_i'$ $(k_i \neq 0)$ or $\sigma(\gamma_i', \gamma_i') = 0$. Thus we distinguish the following three cases.

Case 1) $k_1 \neq 0$ and $k_2 \neq 0$.
Assume that $k = 0$, so that $-\nabla_{\gamma'}\xi = 0$. Since $\dim S = 2$, there exist real numbers $\lambda$, $\mu$ such that $\gamma' = \lambda\gamma_1' + \mu\gamma_2'$. Thus we have

$$0 = -\nabla_{\gamma'}\xi = \lambda(-\nabla_{\gamma'}\xi) + \mu(-\nabla_{\gamma'}\xi) = \lambda k_1\gamma_1' + \mu k_2\gamma_2'.$$

Since $\gamma_1', \gamma_2'$ are linearly independent, $\lambda k_1 = \mu k_2 = 0$, then $\lambda = \mu = 0$. This contradicts the fact that $\gamma' \neq 0$. It follows that there exist three principal directions $\gamma', \gamma_1'$ and $\gamma_2'$, then $p$ is an umbilic point.

Case 2) $\sigma(\gamma_1', \gamma_1') = \sigma(\gamma_2', \gamma_2') = 0$.

In this case $< -\nabla_{\gamma_i'}\xi, \gamma_i' >= \sigma(\gamma_i', \gamma_i') = 0$ $(i = 1, 2)$ and $-\nabla_{\gamma'}\xi = k\gamma'$. Since $\gamma', \gamma_1', \gamma_2'$ are linearly independent, there exist non-zero real numbers $\lambda, \mu$ such that $\gamma' = \lambda\gamma' + \mu\gamma_2'$. Thus we have

$$0 =< -\nabla_{\gamma_1'}\xi, \gamma_1' >= 2\lambda\mu < -\nabla_{\gamma'}\xi, \gamma_2' >= 2\lambda\mu < k\gamma', \gamma_2' >= 2\lambda\mu k < \gamma', \gamma_2' >,$$

so that $2k < \gamma', \gamma_2' >= 0$. By the assumption, we have $< \gamma', \gamma' >= 0$ and $< \gamma_2', \gamma_2' >\neq 0$. It follows from Lemma 2.1 that $< \gamma', \gamma_2' >\neq 0$. Thus we have $k = 0, 4$ so that $-\nabla_{\gamma'}\xi = 0$. For any $\mathbf{X} \in T_pS$, there exist real numbers $\nu, \rho$ such that $\mathbf{X} = \nu\gamma_1' + \rho\gamma'$. We have

$$\sigma(\mathbf{X}, \mathbf{X}) = 2\nu\rho\sigma(\gamma', \gamma_1') = 2\nu\rho < -\nabla_{\gamma'}\xi, \gamma_1' >= 2\nu\rho < 0, \gamma' >= 0.$$

This means that $p$ is a geodesic point.

Case 3) $k_1 \neq 0$ and $\sigma(\gamma_2', \gamma_2') = 0$.
If $k \neq k_1$, we have

$$\begin{aligned} k < \gamma', \gamma_1' >&=< k\gamma', \gamma_1' >=< -\nabla_{\gamma'}\xi, \gamma_1' > \\ &=< \gamma', -\nabla_{\gamma'}\xi >=< \gamma', k_1\gamma_1' >= k_1 < \gamma', \gamma_1' > . \end{aligned}$$

It follows that $< \gamma', \gamma_1' >= 0$. This contradicts to Lemma 2.1, so that $k = k_1$. Since $\gamma', \gamma_1'$ are linearly independent, $p$ is an umbilic point. This completes the proof of Theorem C.

*Proof of Corollary B.1.* Since a non-light-like line on $S$ is a planar geodesic, $S$ is totally umbilic by Theorem B. By exactly the same arguments as those in the proof of Corollary A.2, $S$ is totally geodesic. This completes the proof.

## 4. EXAMPLES

In this section we give some examples which indicate that Theorems A, B and those corollaries are the best possible.

**EXAMPLE 4.1.** We consider the following parametrized surface in $\mathbb{M}^3$

$$\mathrm{X}(u, v) = (\cos u, \frac{1}{\sqrt{2}}(\sin u + v), \frac{1}{\sqrt{2}}(-\sin u + v)),$$

where $|u|, |v|$ are small enough. The surface is a part of a cylinder whose rulings are light-like in $\mathbb{M}^3$, so that it has only one light-like line through each point. This example describes that Theorem A is the best possible.

**EXAMPLE 4.2.** Since a pseudosphere has just two light-like curve through each point, Corollary A.2 is the best possible.

**EXAMPLE 4.3.** We consider the following ruled surface in $\mathbb{M}^3$

$$X(u, v) = (a(\cos u - v \sin u), b(\sin u + v \cos u), v),$$

where $a, b > 0$ and $a \neq b$. An outside of some closed subsets on the surface has two non-light-like line through each point. Of course these lines are planar geodesics on the surface, so that Theorem B and Corollary B.1 are the best possible.

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