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A TIME-LIKE SURFACE IN MINKOWSKI 3-SPACE WHICH CONTAINS LIGHT-LIKE LINES

Shyuichi Izumiya and Akihiro Takiyama

Simple characterizations of a pseudosphere or a time-like plane in Minkowski 3-space by the existence of light-like lines are given.

1. INTRODUCTION

There are many simple characterizations of a sphere in Euclidean 3-space [1,3,4]. For example "all geodesics are plane curves" characterizes a sphere or a plane. In [4] Takeuchi gave a much simpler and practical characterization of a sphere. Her characterization is as follows: if there exist four geodesics through each point of a complete surface such that they are plane curves, then the surface is a sphere or a plane. She also gave other characterizations of a sphere or a plane in [4].

In this paper we consider the similar problem for a pseudosphere $S_1^2(r, a)$ (for definition, see Section 2) and a time-like plane in Minkowski 3-space. The normal vector field on a pseudosphere in \mathbb{M}^3 is light-like. We say that a surface S in \mathbb{M}^3 is *time-like* if the normal vector field on S is space-like. It is easy to show that if all geodesics on a time-like surface are plane curves then the surface is an open subset of a pseudosphere or a time-like plane in Minkowski 3-space \mathbb{M}^3 (cf., [5]). We give characterizations which are much simpler and peculiar to the case for a pseudosphere or a time-like plane in Minkowski 3-space \mathbb{M}^3 . Our main results are as follows.

THEOREM A. Let S be a connected time-like surface in Minkowski 3-space \mathbb{M}^3 . For each point p of S , suppose that there exist two light-like curves on S through p such that they are plane curves. Then S is an open subset of a pseudosphere or a time-like plane.

We have corollaries of the theorem as follows:

COROLLARY A.1. Let S be a time-like surface in Minkowski 3-space \mathbb{M}^3 . For each point p of S , suppose that the intersection of S and $T_p S$, which is considered as an affine plane in \mathbb{M}^3 , is the two lines such that each direction is light-like. Then S is a pseudosphere.

The assumption of the corollary can be interpreted as that if two lines can be pressed entirely at a point p of the surface, then these are the light cone on the tangent plane.

We have a simple characterization of a time-like plane in \mathbb{M}^3 as an corollary of Theorem A.

COROLLARY A.2. Let S be a connected time-like surface in \mathbb{M}^3 . For each point p of S , suppose that there exist two light-like curves of S which are plane curve and one non-light-like line on S through p . Then S is an open set of a time-like plane.

Another characterization by the existence of planar geodesics is given by the following.

THEOREM B. Let S be a connected time-like surface in Minkowski 3-space \mathbb{M}^3 . For each point p in S , suppose that there exist one light-like curve and two non-light-like geodesics on S through p such that they are plane curves. Then S is an open subset of a pseudosphere or a time-like plane.

We also have the following simple characterization of a time-like plane in \mathbb{M}^3 as a corollary of Theorem B.

COROLLARY B.1. Let S be a connected time-like surface in Minkowski 3-space \mathbb{M}^3 . For each point p in S , suppose that there exist one light-like curve, one non-light-like geodesics on S such that they are plane curves and one non-light-like line through p . Then S is an open subset of a time-like plane.

We remark that these theorems are the best possible in some sense (cf., Section 4). For space-like surfaces, the induced metric on the surface is positively definite, so that we can give characterizations of a hyperbolic surface $H_1^2(r, a)$ by exactly the same arguments as those in [1,3,4]. Thus we do not consider this case in this paper.

All surfaces and maps considered here are of class C^∞ unless stated otherwise.

2. BASIC NOTIONS

Let $\mathbb{R}^3 = \{(x_1, x_2, x_3) | x_1, x_2, x_3 \in \mathbb{R}\}$ be the usual oriented 3-dimensional vector space and differential manifold, which is oriented by $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$ and given the Euclidean differentiable structure. *Minkowski 3-space* is defined by $\mathbb{M}^3 = \{\mathbb{R}^3, I_{(2,1)}\}$, where $I_{(2,1)} = dx_1^2 + dx_2^2 - dx_3^2$. Thus the metric tensor is given by

$\langle \mathbf{X}, \mathbf{Y} \rangle = x_1y_1 + x_2y_2 - x_3y_3$, where $\mathbf{X} = (x_1, x_2, x_3)$ and $\mathbf{Y} = (y_1, y_2, y_3)$. A vector \mathbf{X} in \mathbb{M}^3 is called *light-like* if $\langle \mathbf{X}, \mathbf{X} \rangle = 0$, *space-like* if $\langle \mathbf{X}, \mathbf{X} \rangle > 0$ and *time-like* if $\langle \mathbf{X}, \mathbf{X} \rangle < 0$. A curve γ is called *light-like* if its tangent vector field γ' is always light-like. We also say that a curve γ is *non-light-like* if its tangent vector field γ' is always space-like or time-like. The *pseudosphere* is defined to be

$$S_1^2(r, a) = \{(x_1, x_2, x_3) | (x_1 - a_1)^2 + (x_2 - a_2)^2 - (x_3 - a_3)^2 = r^2\},$$

where $a = (a_1, a_2, a_3)$ is the center and $r > 0$ is the radial of $S_1^2(r, a)$.

The Levi-Civita connection of \mathbb{M}^3 is denoted by ∇ . Let S be a surface in \mathbb{M}^3 . We say that S is *time-like* if the normal vector field to S is space-like. Let $\Pi(\mathbf{X}, \mathbf{Y})$ be a second fundamental form tensor of S in \mathbb{M}^3 . Since S is a time-like surface, there exists (at least locally) a unit normal vector field ξ on S . We have the following formula:

$$\langle \Pi(\mathbf{X}, \mathbf{Y}), \xi \rangle = \langle -\nabla_{\mathbf{X}}\xi, \mathbf{Y} \rangle.$$

If we denote that $\Pi(\mathbf{X}, \mathbf{Y}) = \sigma(\mathbf{X}, \mathbf{Y})\xi$, we have $\sigma(\mathbf{X}, \mathbf{Y}) = \langle -\nabla_{\mathbf{X}}\xi, \mathbf{Y} \rangle$, so that $-\nabla_{\mathbf{X}}\xi$ is the shape operator in this case. It is well-known that the shape operator is self-adjoint with respect to \langle, \rangle (i.e., $\langle -\nabla_{\mathbf{X}}\xi, \mathbf{Y} \rangle = \langle \mathbf{X}, -\nabla_{\mathbf{Y}}\xi \rangle$).

We now state some lemmas for preparing the proof of main results.

LEMMA 2.1 Let V be a time-like plane in \mathbb{M}^3 . If $\langle \mathbf{X}, \mathbf{X} \rangle = 0$ and $\langle \mathbf{Y}, \mathbf{Y} \rangle \neq 0$, then $\langle \mathbf{X}, \mathbf{Y} \rangle \neq 0$.

Proof. Since V has two light-like direction, there exists $\mathbf{Z} \in V$ such that $\langle \mathbf{Z}, \mathbf{Z} \rangle = 0$ and $\mathbf{Z} \notin \langle \mathbf{X} \rangle_{\mathbb{R}}$. It follows that there exist real numbers λ, μ with $\mu \neq 0$ such that $\mathbf{Z} = \lambda\mathbf{X} + \mu\mathbf{Y}$. Thus we have

$$0 = \langle \mathbf{Z}, \mathbf{Z} \rangle = 2\lambda\mu \langle \mathbf{X}, \mathbf{Y} \rangle + \mu^2 \langle \mathbf{Y}, \mathbf{Y} \rangle.$$

If $\langle \mathbf{X}, \mathbf{Y} \rangle = 0$, then $\langle \mathbf{Y}, \mathbf{Y} \rangle = 0$. This is a contradiction.

Since the light direction on a time-like plane in Minkowski 3-space \mathbb{M}^3 is constant, we have the following simple lemma.

LEMMA 2.2 Let γ be a light-like plane curve in \mathbb{M}^3 . Then it is a line, especially a geodesic in \mathbb{M}^3 .

We also have the following lemma for light-like line on a surface in \mathbb{M}^3 .

LEMMA 2.3 Let γ be a light-like line on a time-like surface in \mathbb{M}^3 and ξ be a normal unit vector field on S . Then there exists an real number k such that $-\nabla_{\mathbf{X}}\xi = k\mathbf{X}$, where \mathbf{X} is a tangent vector of γ .

Proof. Since γ is light-like line, we have $\sigma(\mathbf{X}, \mathbf{X}) = 0$ (cf., [2], page 103 Corollary 9). It follows that

$$\langle -\nabla_{\mathbf{X}}\xi, \mathbf{X} \rangle = \sigma(\mathbf{X}, \mathbf{X}) = 0.$$

Suppose that there are no real numbers k such that $-\nabla_{\mathbf{X}}\xi = k\mathbf{X}$. Since $\dim S = 2$, for any $\mathbf{Y} \in T_p S$, there exist real numbers λ, μ such that $\mathbf{Y} = \lambda(-\nabla_{\mathbf{X}}\xi) + \mu k\mathbf{X}$. Thus we have

$$\langle \mathbf{X}, \mathbf{Y} \rangle = \lambda \langle -\nabla_{\mathbf{X}}\xi, \mathbf{X} \rangle + \mu \langle \mathbf{X}, \mathbf{X} \rangle = 0.$$

Let \mathbf{Y} be a tangent vector at p which has another light-like direction. Then we have

$$\begin{aligned} \langle \mathbf{X} + \mathbf{Y}, \mathbf{X} + \mathbf{Y} \rangle &= \langle \mathbf{X}, \mathbf{X} \rangle + 2 \langle \mathbf{X}, \mathbf{Y} \rangle + \langle \mathbf{Y}, \mathbf{Y} \rangle \\ &= 2 \langle \mathbf{X}, \mathbf{Y} \rangle = 0, \end{aligned}$$

This contradicts to the fact that $\mathbf{X} + \mathbf{Y}$ is not light-like.

Let γ be a non-light-like curve. We may assume that γ is parametrized by arc length s . Thus we have $\langle \gamma'(s), \gamma'(s) \rangle = \varepsilon(s) = \pm 1$. By the exactly same arguments as in the case for a curve in Euclidean 3-space, we have the following Frenet-Serret formula (cf., [5]): There exist field of vectors $\mathbf{n}(s), \mathbf{b}(s)$ with $\langle \mathbf{n}(s), \mathbf{n}(s) \rangle = \delta(s) = \pm 1$, $\langle \mathbf{b}(s), \mathbf{b}(s) \rangle = -\varepsilon(s)\delta(s)$ and functions $k = k(s), \tau = \tau(s)$ along γ such that

$$\begin{aligned} \nabla_{\gamma'(s)}\gamma'(s) &= k(s)\mathbf{n}(s) \\ \nabla_{\gamma'(s)}\mathbf{n}(s) &= -\varepsilon(s)\delta(s)k(s)\gamma'(s) + \varepsilon(s)\tau(s)\mathbf{b}(s) \\ \nabla_{\gamma'(s)}\mathbf{b}(s) &= \tau(s)\mathbf{n}(s). \end{aligned}$$

If γ is a plane curve, then $\tau(s) \equiv 0$, so that we have

$$\nabla_{\gamma'(s)}\gamma'(s) = k(s)\mathbf{n}(s) \quad \text{and} \quad \nabla_{\gamma'(s)}\mathbf{n}(s) = -\varepsilon(s)\delta(s)k(s)\gamma'(s).$$

Suppose that γ is also a geodesic of S , then

$$\nabla_{\gamma'(s)}\gamma'(s) = \Pi(\gamma'(s), \gamma'(s)) = \sigma(\gamma'(s), \gamma'(s))\xi(\gamma(s))$$

(cf., [2], page 103 Corollary 9). It follows that $\mathbf{n}(s) = \pm\xi(\gamma(s))$, so we have the following lemma.

LEMMA 2.4 Let S be a time-like surface in Minkowski 3-space \mathbb{M}^3 and γ be a non-light-like geodesic on S which is parametrized by arc length s such that it is a plane curve. Then $\sigma(\gamma'(s), \gamma'(s)) = 0$ when $k(s) = 0$ and $\nabla_{\gamma'(s)}\xi(\gamma(s)) = \mp\varepsilon(s)\delta(s)k(s)\gamma'(s)$ when $k(s) \neq 0$.

3. PROOF OF RESULTS

We now give a proof of Theorem A.

Proof of Theorem A. Let γ_i ($i = 1, 2$) be two light-like curve on S through p such that they are plane curves. By Lemma 2.3, we have $-\nabla_{\mathbf{X}_i}\xi = k_i\xi$ for some k_i , where \mathbf{X}_i is the tangent vector of γ_i ($i = 1, 2$). If $k_1 \neq k_2$, we have

$$\begin{aligned} k_1 \langle \mathbf{X}_1, \mathbf{X}_2 \rangle &= \langle k_1 \mathbf{X}_1, \mathbf{X}_2 \rangle = \langle -\nabla_{\mathbf{X}_1}\xi, \mathbf{X}_2 \rangle \\ &= \langle \mathbf{X}_1, -\nabla_{\mathbf{X}_2}\xi \rangle = \langle \mathbf{X}_1, k_2 \mathbf{X}_2 \rangle = k_2 \langle \mathbf{X}_1, \mathbf{X}_2 \rangle, \end{aligned}$$

so that $\langle \mathbf{X}_1, \mathbf{X}_2 \rangle = 0$.

Since $\mathbf{X}_1 + \mathbf{X}_2$ is not light-like vector, we have

$$0 \neq \langle \mathbf{X}_1 + \mathbf{X}_2, \mathbf{X}_1 + \mathbf{X}_2 \rangle = \langle \mathbf{X}_1, \mathbf{X}_1 \rangle + 2 \langle \mathbf{X}_1, \mathbf{X}_2 \rangle + \langle \mathbf{X}_2, \mathbf{X}_2 \rangle = 2 \langle \mathbf{X}_1, \mathbf{X}_2 \rangle.$$

This is a contradiction, so that we have $k_1 = k_2$. Since $\dim S = 2$, p is an umbilic point.

Proof of Corollary A.1. By the assumption, for each point $p \in S$, there exist two light-like line on S which through p . By Theorem A, S is a pseudosphere or a time-like plane, however, a time-like plane does not satisfy the assumption.

Proof of Corollary A.2. It follows from Theorem A that there exists real number k such that $-\nabla_{\mathbf{X}}\xi = k\mathbf{X}$ for any $\mathbf{X} \in T_p S$. By the assumption, there exists a non-light-like line γ on S through p , so that we have $-\nabla_{\gamma'}\xi = k\gamma'$. Since γ is a line, $\sigma(\gamma', \gamma') = 0$. Thus we have

$$k \langle \gamma', \gamma' \rangle = \langle k\gamma', \gamma' \rangle = \langle -\nabla_{\gamma'}\xi, \gamma' \rangle = \sigma(\gamma', \gamma') = 0,$$

so that $k = 0$ because γ is non-light-like. Therefore p is a geodesic point. This completes the proof.

Proof of Theorem B. Let γ be a light-like line on S through p . By Lemma 2.3, there exists a real number k such that $-\nabla_{\gamma'}\xi = k\gamma'$. Let γ_i ($i = 1, 2$) be non-light-like planar geodesics. By Lemma 2.4, we have $-\nabla_{\gamma'_i}\xi = k_i\gamma'_i$ ($k_i \neq 0$) or $\sigma(\gamma'_i, \gamma'_i) = 0$. Thus we distinguish the following three cases.

Case 1) $k_1 \neq 0$ and $k_2 \neq 0$.

Assume that $k = 0$, so that $-\nabla_{\gamma'}\xi = 0$. Since $\dim S = 2$, there exist real numbers λ, μ such that $\gamma' = \lambda\gamma'_1 + \mu\gamma'_2$. Thus we have

$$0 = -\nabla_{\gamma'}\xi = \lambda(-\nabla_{\gamma'_1}\xi) + \mu(-\nabla_{\gamma'_2}\xi) = \lambda k_1 \gamma'_1 + \mu k_2 \gamma'_2.$$

Since γ'_1, γ'_2 are linearly independent, $\lambda k_1 = \mu k_2 = 0$, then $\lambda = \mu = 0$. This contradicts the fact that $\gamma' \neq 0$. It follows that there exist three principal directions γ', γ'_1 and γ'_2 , then p is an umbilic point.

Case 2) $\sigma(\gamma'_1, \gamma'_1) = \sigma(\gamma'_2, \gamma'_2) = 0$.

In this case $\langle -\nabla_{\gamma'_i} \xi, \gamma'_i \rangle = \sigma(\gamma'_i, \gamma'_i) = 0$ ($i = 1, 2$) and $-\nabla_{\gamma'} \xi = k\gamma'$. Since $\gamma', \gamma'_1, \gamma'_2$ are linearly independent, there exist non-zero real numbers λ, μ such that $\gamma' = \lambda\gamma'_1 + \mu\gamma'_2$. Thus we have

$$0 = \langle -\nabla_{\gamma'} \xi, \gamma'_1 \rangle = 2\lambda\mu \langle -\nabla_{\gamma'} \xi, \gamma'_2 \rangle = 2\lambda\mu \langle k\gamma', \gamma'_2 \rangle = 2\lambda\mu k \langle \gamma', \gamma'_2 \rangle,$$

so that $2k \langle \gamma', \gamma'_2 \rangle = 0$. By the assumption, we have $\langle \gamma', \gamma'_1 \rangle = 0$ and $\langle \gamma'_2, \gamma'_2 \rangle \neq 0$. It follows from Lemma 2.1 that $\langle \gamma', \gamma'_2 \rangle \neq 0$. Thus we have $k = 0, 4$ so that $-\nabla_{\gamma'} \xi = 0$. For any $X \in T_p S$, there exist real numbers ν, ρ such that $X = \nu\gamma'_1 + \rho\gamma'$. We have

$$\sigma(X, X) = 2\nu\rho\sigma(\gamma', \gamma'_1) = 2\nu\rho \langle -\nabla_{\gamma'} \xi, \gamma'_1 \rangle = 2\nu\rho \langle 0, \gamma'_1 \rangle = 0.$$

This means that p is a geodesic point.

Case 3) $k_1 \neq 0$ and $\sigma(\gamma'_2, \gamma'_2) = 0$.

If $k \neq k_1$, we have

$$\begin{aligned} k \langle \gamma', \gamma'_1 \rangle &= \langle k\gamma', \gamma'_1 \rangle = \langle -\nabla_{\gamma'} \xi, \gamma'_1 \rangle \\ &= \langle \gamma', -\nabla_{\gamma'} \xi \rangle = \langle \gamma', k_1\gamma'_1 \rangle = k_1 \langle \gamma', \gamma'_1 \rangle. \end{aligned}$$

It follows that $\langle \gamma', \gamma'_1 \rangle = 0$. This contradicts to Lemma 2.1, so that $k = k_1$. Since γ', γ'_1 are linearly independent, p is an umbilic point. This completes the proof of Theorem C.

Proof of Corollary B.1. Since a non-light-like line on S is a planar geodesic, S is totally umbilic by Theorem B. By exactly the same arguments as those in the proof of Corollary A.2, S is totally geodesic. This completes the proof.

4. EXAMPLES

In this section we give some examples which indicate that Theorems A, B and those corollaries are the best possible.

EXAMPLE 4.1. We consider the following parametrized surface in \mathbb{M}^3

$$X(u, v) = \left(\cos u, \frac{1}{\sqrt{2}}(\sin u + v), \frac{1}{\sqrt{2}}(-\sin u + v) \right),$$

where $|u|, |v|$ are small enough. The surface is a part of a cylinder whose rulings are light-like in \mathbb{M}^3 , so that it has only one light-like line through each point. This example describes that Theorem A is the best possible.

EXAMPLE 4.2. Since a pseudosphere has just two light-like curve through each point, Corollary A.2 is the best possible.

EXAMPLE 4.3. We consider the following ruled surface in M^3

$$X(u, v) = (a(\cos u - v \sin u), b(\sin u + v \cos u), v),$$

where $a, b > 0$ and $a \neq b$. An outside of some closed subsets on the surface has two non-light-like line through each point. Of course these lines are planar geodesics on the surface, so that Theorem B and Corollary B.1 are the best possible.

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