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K. Goto, A. Yamaguchi and I. Tsuda

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Nine-bit states cellular automata are capable of simulating the pattern dynamics of coupled map lattice

Kentaro Goto, Akihiro Yamaguchi and Ichiro Tsuda

Department of Mathematics, Hokkaido University, Sapporo, 060, Japan

Abstract

Cellular automata induced by quantization of coupled map lattice are investigated. The following results were obtained by the numerical experiments. The 2^9 states on each cell is sufficient for equivalent dynamics, and at least 2^6 states are necessary for qualitative dynamics. We also develop a theory to assure the conditions.

1 Introduction

Cellular automata (CA) have been studied as a model of self-reproduction, chemical turbulence, and even parallel computations [1]. Coupled map lattices (CML) [2, 3] are defined similarly to CA except its elementary dynamics, namely in CML elementary dynamics is chaotic, thus CML is defined on the real numbers, whereas CA defined on the integers. We investigate CA induced from CML by quantization in order to show what extent CA can simulate CML. CML is capable of describing a wide class of physical, chemical, and biological phenomena [4], thus the study of CML as CA leads us to the notion of spatio-temporal computation by CA.

The CML in this paper is given as follows:

$$x_{n+1}^{i} = (1-\epsilon)f(x_n^i) + \frac{\epsilon}{2}\{f(x_n^{i-1}) + f(x_n^{i+1})\}, \tag{1}$$

where f is given by the logistic map $f(x) = 1 - ax^2$. Here x_n^i $(i = 1, \ldots, N)$ denote the state at i-th site at time n, and a and ϵ the degree parameters of nonlinearity and the strength of coupling, respectively.

We briefly sketch typical dynamics of the model (1) (see ref. [3]). In case of low or intermediate coupling strength ($\epsilon < 0.5$), domains separated by fixed point of single logistic map are formed by spatial period-doubling bifurcations, and only a small size of domain becomes to be selected, as increasing the nonlinearity. A further increase of nonlinearity destroys these domains, namely chaotic bursts occur intermittently in both spatial and temporal dimensions, and eventually a fully developed turbulence occurs.

We will show how to construct CA which simulates the dynamics of CML for almost values of nonlinearity a. We will also evaluate the number of states of such CA.

2 Converting CML to CA

Since all the methods of observing CML dynamics are numerical simulation on digital computer with finite precision, every CML can be considered as CA. In order to study whether or not higher precision is necessary for the categorized phases reviewed in the previous section, we present a method of converting a given CML to CA, specifying the precision explicitly.

CA are here defined by (Q, δ) where $Q = \{q_0, \ldots, q_M\}$ is an ordered discrete set and $\delta : Q \times Q \times Q \to Q$, expressing the nearest neighbor interactions in CML. Each element of Q represents the state of a single cell and δ is a local transition rule.

On the other hand, CML of model (1) are given by (I, f) where f is the logistic map defined on the interval $I = [I_{min}, I_{max}]$ ($I_{min} = (-1 - \sqrt{4a})/2a, I_{max} = (1 + \sqrt{4a})/2a$). Now let $q_0 = I_{min}$, $q_M = I_{max}$, and $q_i = q_{i-1} + (I_{max} - I_{min})/M$ for $i = 1, \ldots, M$.

Now, what we should determine is only the local transition rule δ. The translation f to δ is as follows: for any $u, v, w \in Q$, $\delta(u, v, w) := q \in Q$ such that $q - (I_{max} - I_{min})/2M \leq (1 - \epsilon)f(v) + \frac{\epsilon}{2}\{f(u) + f(w)\} < q + (I_{max} - I_{min})/2M$.

Here we take M as in the form of $2^\eta + 1$. The numeral η has the meaning of *precision* like "bit" in the computer terminology.

3 Numerical experiment

A coarse-gaining of the orbits often leads to the appropriate classification of qualitative dynamics. In particular, computer experiments in CML shows that the coarse-grained domain structure is a basis of such a classification, where the fixed point x_* of elementary dynamics is adopted.

It is convenient to introduce *binary coding* $\sigma(q)$ as

$$\begin{cases} \sigma(q) = 1 & (\text{if } q \geq x_*) \\ \sigma(q) = 0 & (\text{otherwise}), \end{cases} \tag{2}$$

where q denotes a state of site. In the graph of spatio-temporal evolution, cells q with $\sigma(q) = 0$ are black and cells q with $\sigma(q) = 1$ are white, respectively. By this coding scheme, a domain is a region consisted of the same symbols. Throughout this paper the analyses are based on this coding scheme.

We use periodic boundary conditions in the present numerical experiments.

3.1 Qualitative dynamics

If the precision η is sufficiently fine, then CA qualitatively approximate the dynamics of CML. It is interesting to ask how much precision is necessary to show similar dynamics. For this question, we found small η is enough i.e. $\eta = 9$.

In CML with the odd size N and $a = 1.8, \epsilon = 0.1$, every selected domain size is one (i.e. σ sequence is $\ldots 010101 \ldots$) but the pattern has a defect in each time at one point since N is odd, and this defect moves like a Brownian motion whose phase is called Brownian motion of defect (BD). Figure 1 shows collapse of BD when changing the precision $\eta = 9, 8, \ldots, 3$. In case of precision $\eta = 7$, BD phase typically becomes pattern selection (PS) — the formation of stationary domain — after transient motions which is quite similar to BD. Note that the variety of domain size in a fixed parameter is lost, thus almost the same domain size is selected.

<< *Figure 1* >>

<< *Figure 2* >>

Similarly, the pattern competition intermittency (PCI) collapses for η less than 8 (see Fig.2). Defect turbulence (DT) also collapses when decreasing η. All the three phases BD, DT and PCI become unstable in almost all cases according to a decrease of η, and consequently the frozen random pattern (FR) phase emerges via PS phase. It should be, however, noted that there are some exceptional cases where unclassified phases can appear. For example see Fig.1(e,f).

Other phases are more robust against quantization. FR, which appears in case of weak nonlinearity, is never destroyed by changing η, and fully developed turbulence (FDT) appeared in case of strong nonlinearity, are kept up to $\eta = 6$.

These results are summarized in table 1.

<< *Table 1* >>

3.2 Quantitative aspects

In this section we discuss some quantitative aspects of the dynamics. We study the properties of dynamics of our model in more details.

In order to assure that the spatio-temporal developments of the quantized CML are qualitatively the same as that of original CML, we calculated some quantifiers from the binary code. The quantifiers are based on the domain size, which is obtained by counting the spatial length of the σ-sequence consisted of the same symbol. For example, a binary sequence "000110" has three domains i.e. the length three of '0' , the length two of '1' and the length one of '0'.

Pattern distributions $Q(j)$ are defined[3] by the following way. Count the number of sites which belong to the domain of length j through the entire lattice and for sufficiently long time evolution, and normalize it by the number of whole spatio-temporal samplings. Then we can get the probability that a lattice point belongs to the domain of size j.

A *transition probability matrix* $T(j|i)$ are defined[3] for pattern dynamics. Take one lattice point k at a time step n and check what size of domain it belongs to (let us assume that the size of domain is i). After a given time step, we again calculate the size of domain which the lattice point belongs to (let us assume it is j). This gives an event of the transition i to j. By taking the spatio-temporal samplings, we obtain probabilities of the transition.

Using $Q(i)$ and $T(j|i)$, the quantifiers are defined as following. †

Static entropy:

$$S_p = -\sum_j Q(j)\log Q(j).$$

Dynamical entropy:

$$S_d = -\sum_{i,j} Q(i)T(j|i)\log T(j|i).$$

We calculated these quantifiers for our model by changing the value of parameters and the precision systematically. The results justified the qualitative observations.

<< *Figure 3* >>

<< *Figure 4* >>

Figure 3 shows $Q(k)$, and $S_p(k)$ and $S_d(k)$ are shown in Fig.4 for $\epsilon = 0.1$.†† For instance BD takes place in the range $1.78 < a < 1.85$ at $\epsilon = 0.1$, this phase has small S_p (i.e. $\ll 1$) because almost all lattice points are static for almost times except the one point of defect. However, it becomes locally periodic and defect disappears for η less than 8, thus increasing S_d. Although a selected domain size k is mainly 1 as seen in $Q(k)$ for CML, there are some other varieties than 1 in its size for η less than 7. Alternatively, FR, PS and FDT are almost conserved for $\eta = 8, 7, 6$. A drastically destroying of dynamics in almost parameter values at $\eta = 5$ are shown

†Physical meanings of them are discussed in [3]

††We show only in the case of $\epsilon = 0.1$, but essential part of results is held for other values of ϵ.

by Q, S_p and S_d (see Fig.3(c,d) and Fig.4 (c,d)).

Consequently, the number of states to describe CML is 2^9 with respect to binary observation scheme.

4 Irreducibility

In this section, we show, using the automata theory, that CA of our model cannot be reduced to be less than half the number of states.

Myhill-Nerode's theorem [7] provides a way to construct the minimal automaton equivalent to a given automaton.[†††] Extending it, we can also obtain the procedure to minimize CA that is a special version such that each transition is associated with output. Now we take the binary scheme σ as output, then we can obtain the minimal one in the equivalent CA with respect to σ. By the result of this section, it will be concluded that one cannot construct the CA that is equivalent to the one induced from quantization of precision η, under the restriction of the number of its states being less than $2^{\eta-1}$.

We use a notation X^* to represent all strings which are finite concatenations of members in X, and $\varepsilon \in X^*$ denotes the *null string* of which length is 0. Namely, by Γ^*, we denote $\Gamma^* = \bigcup_{i=0}^{\infty} \Gamma^i$, $\Gamma^i = \Gamma(\Gamma^{i-1})$, and $\Gamma^0 = \{\varepsilon\}$.

Note that the definition of CA in this section differs from the one in section 2, especially in terms of the transition function δ. The present δ depends on the states of cell and the sequences of signals from neighbors determined according to their states, while the previous δ depends on simply the states of both cell and its

[†††] Our treatments are as *Moore sequential machines* rather than as automata. The difference between sequential machines and automata are whether an output at each transient are defined or not. Since components of our model has output as signal it is called a sequential machine. Details are described, including verification of our minimization algorithm, in e.g. [7] and [6].

neighborhood. To emphasize this distinction, we use the notation $\tilde{\delta}$.

Definition 1 The *CA with observation* is defined by a 6-tuple $C = (Q, \Gamma, \Sigma, \tilde{\delta}, \gamma, \sigma)$. The meaning of each element is as follows. Q is a non-empty finite set and its members are called states; Γ is a non-empty finite set of finite symbols, each of which is called *signals*, what we use for interaction between the neighbor cells. $\Sigma = \{0, 1\}$ is a set of symbols which are used for observation. $\tilde{\delta} : Q \times (\Gamma \times \Gamma)^* \to Q$ is the local transition rule such that

1. $\tilde{\delta}(q, \varepsilon) = q \qquad (q \in Q)$
2. $\tilde{\delta}(q, (a, b)(x, y)) = \tilde{\delta}(\tilde{\delta}(q, (a, b)), (x, y)). \qquad (\ q \in Q,\ a, b \in \Gamma,\ (x, y) \in (\Gamma \times \Gamma)^*\ )$.

$\gamma : Q \to \Gamma$ is a map such that $\gamma(q)$ is the signal sent to the neighborhood when a state of cell is $q \in Q$. In this paper, we use $Q = \Gamma$, namely γ is identical. $\sigma : Q \to \Sigma$ is a map such that the $\sigma(q)$ is the observation concerning whether or not the state of cell is greater than x_*(see section 3).

The difference of $\tilde{\delta}$ from δ implies the notion of reducibility of the states, since different states may be identified in the sense of signals.

Definition 2 We say that *two signals a and b are equivalent*, which is denoted here by $a \equiv_\Gamma b (a, b \in \Gamma)$, if $\tilde{\delta}(q, (a, x)) = \tilde{\delta}(q, (b, x))$ and $\tilde{\delta}(q, (x, a)) = \tilde{\delta}(q, (x, b))$ for any $q \in Q$ and any $x \in \Gamma$.

Definition 3 We say that *two states p and q are equivalent*, which is denoted by $p \equiv_Q q$ if two symbol sequences from p and q are identical under observation. Namely $p \equiv_Q q$ is defined by $\gamma(\tilde{\delta}(p, x)) = \gamma(\tilde{\delta}(q, x))$ and $\sigma(\tilde{\delta}(p, x)) = \sigma(\tilde{\delta}(q, x))$ for any $x \in (\Gamma \times \Gamma)^*$.

Definition 4 We say that *two CA with observation*, $C = (Q, \Gamma, \Sigma, \tilde{\delta}, \gamma, \sigma)$ *and* $C' = (Q', \Gamma, \Sigma, \tilde{\delta}', \gamma', \sigma')$ *are equivalent* which is denoted $C \equiv C'$, if for any $q \in Q'$ there exists $p \in Q$ such that $p \equiv_Q q$, and if for any $p \in Q$ there exists $q \in Q'$ such that $p \equiv_Q q$.

Definition 5 We say that CA with observation is *irreducible* if $p \equiv_Q q$ implies $p = q$ for $p, q \in Q$.

Theorem 1 (1) For any CA with observation C there exists an irreducible C' such that $C \equiv C'$.

(2) The number of states of an irreducible CA with observation is minimal among the members of its equivalent class.

A proof is given in Appendix.

By theorem 1, we obtain that the minimal CA with observation can be immediately found by constructing irreducible one. Moreover, the next theorem enables us to construct minimal one for a given CA with observation.

Definition 6 $p \overset{k}{\equiv}_Q q$ represents that the states $p, q \in Q$ are equivalent for any signals of length less than k, and defined by $\gamma(\tilde{\delta}(p, x)) \equiv_\Gamma \gamma(\tilde{\delta}(q, x))$, where the length of x is less than k.

Let $Q/\overset{k}{\equiv}_Q$ be the whole of equivalent classes by the relation $\overset{k}{\equiv}_Q$.

Theorem 2 (1) $p \overset{k+1}{\equiv}_Q q$ iff $p \overset{k}{\equiv}_Q q$ and $\tilde{\delta}(p, a) \overset{k}{\equiv}_Q \tilde{\delta}(q, a)$ for any $a \in \Gamma$.

(2) If $(Q/\overset{k+1}{\equiv}_Q) = (Q/\overset{k}{\equiv}_Q)$, then $p \equiv q$ iff $p \overset{k}{\equiv}_Q q$

(3) If $(Q/\overset{k+1}{\equiv}_Q) = (Q/\overset{k}{\equiv}_Q)$, then $(Q/\overset{k}{\equiv}_Q) = (Q/\overset{j}{\equiv}_Q)$ for any $j \geq k$.

A proof is given in Appendix.

By this theorem, the algorithm minimizing CA is achieved in the following way. At first, one constructs the equivalent classes $Q/\overset{0}{\equiv}_Q$ the relations checking $\gamma(p)\equiv_\Gamma\gamma(q)$ and $\sigma(p) = \sigma(q)$ for any pair $p, q \in Q$. Second, make equivalent classes $Q/\overset{k+1}{\equiv}_Q$ inductively from equivalent classes $Q/\overset{k}{\equiv}_Q$ by using theorem 2(1) until the condition of theorem 2(2) is hold. Then we will eventually construct equivalent classes $Q/\equiv_Q$. Actually one can construct $C' = (Q', \Gamma, \Sigma, \tilde{\delta}', \gamma', \sigma)$ such that $Q' = Q/\equiv_Q$, $\tilde{\delta}'([p], a, b) = [\tilde{\delta}(p, a, b)]$, $\gamma'([p], a, b) = \gamma(p, a, b)$ and $\sigma'([p], a, b) = \sigma(p, a, b)$, where $[p]$ is an equivalence class of the relation $\equiv_Q$ including p. C' is the minimal CA with observation which is equivalent to given CA, i.e., C. The computational cost of this procedure is $O(|Q|^2)$.

By numerical computations the following hypotheses for minimal CA are obtained.

1. If two states $p, q \in Q$ are equivalent, $p = -q$ and $|p|, |q| \leq x_*$.

2. No distinct 3 states in Q' are equivalent except several states near the critical point of the single logistic map i.e. $p = 0$.

Let us explain the hypothesis 1. The relation $\equiv_Q$ needs the conditions (3) for signal γ and (4) for observation σ:

For null string ε

$$\gamma(\tilde{\delta}(p, \varepsilon))\equiv_\Gamma\gamma(\tilde{\delta}(q, \varepsilon)) \tag{3}$$

and

$$\sigma(\tilde{\delta}(p, \varepsilon)) = \sigma(\tilde{\delta}(q, \varepsilon)). \tag{4}$$

The condition (3), by definition, requires

$$\begin{cases} \tilde{\delta}(r,(\gamma(p),x)) = \tilde{\delta}(r,(\gamma(q),x)) \\ \text{and} \\ \tilde{\delta}(r,(x,\gamma(p))) = \tilde{\delta}(r,(x,\gamma(q))) \\ \text{for any } r \in Q \text{ and any } x \in \Gamma \end{cases} \tag{5}$$

Recall that $p, q, r \in Q$ and $x \in \Gamma$ are also viewed as a point in $[I_{min}, I_{max}]$ by our converting scheme, thus condition (5) can be rewritten in the form that

$$\begin{cases} (1-\epsilon)f(r) + \frac{\epsilon}{2}\{f(p)+f(x)\} = (1-\epsilon)f(r) + \frac{\epsilon}{2}\{f(q)+f(x)\} \\ \text{and} \\ (1-\epsilon)f(r) + \frac{\epsilon}{2}\{f(x)+f(p)\} = (1-\epsilon)f(r) + \frac{\epsilon}{2}\{f(x)+f(q)\} \\ \text{for any } r \in Q \text{ and any } x \in \Gamma. \end{cases} \tag{6}$$

After omitting every trivial identical term from both sides of (6), we simply consider the condition

$$f(p) + f(x) = f(q) + f(x). \tag{7}$$

The candidate for the pair p and q in (7) are

$$p = -q \tag{8}$$

because of the property of a logistic map, $f(p) = f(-p)$. This is a necessary condition for γ's relation (3). Furthermore from condition (4), p and q must satisfy

$$\begin{cases} p > x_* \text{ and } q > x_* \\ \text{or} \\ p < x_* \text{ and } q < x_* \\ \text{or} \\ p = q = x_*. \end{cases} \tag{9}$$

From (8) and (9), it is necessary for $p \equiv_Q q$ that

$$\begin{cases} p = -q \\ \text{and} \\ -x_* < p < x_*, -x_* < q < x_*. \end{cases} \tag{10}$$

Indeed, the computational results completely satisfy the condition (10) and also the converse are true excepted for a few cases.†††† Then the decreasing ratio of

††††The equivalent class has more than three members when p and q closed each other in $[I_{min}, I_{max}]$ cannot be distinguished in Q for coarse precision.

number of states by minimization depends only on the nonlinearity parameter a of the logistic map. If $|Q|$ and $|Q'|$ denotes the number of the states reduced before and after, respectively, then this ratio is written as

$$r = 1 - |Q'|/|Q|. \tag{11}$$

The quantity $|Q'|/|Q|$ is approximately estimated by calculating the ratio of the length of interval $I_{(10)}$ to the length of I, where $I_{(10)}$ is an interval such that for each member $p \in I_{(10)}$ there exists $q \neq p$ which holds condition (10). Thus it is obtained that

$$r \approx r' = \frac{1 + 2a - \sqrt{1+4a}}{4a}. \tag{12}$$

For $a \leq 2.0$ we obtain

$$r' \leq 0.25. \tag{13}$$

Therefore, one cannot reduce the precision by more than one bit in the present CA with observation. In the present study, we empirically obtained 9-bit CA to mimic CML. The above result (13) means that one cannot reduce the CA to that of 8-bit. Thus, η is a good measure on structural complexity of our model for any values of parameters. By this measure the comparisons of minimal η for each parameter value is possible, in which parameter the original behavior does not change.

5 Summary

In this paper, CA induced from CML by quantization were studied. Computational results obtained here are that CA with 9-bit states is enough to describe CML for almost every value of the nonlinearity parameter and the coupling parameter, and 6 bit states is necessary to describe qualitative dynamics. The present arguments are based on the binary observation scheme but observation with finer precision is remained open. Critical phases need much more states to be represelented than other phases, i.e., both static ones and fully developed turbulence. It was even shown that as far as their observed dynamics are conserved, induced CA cannot be reduced the number of its states by formalizing the observation of CA.

The fact of the irreducibility guarantees that the number of states is a good measure of the complexity. Generally, the complexity of two regimes can be compared by using the number of states necessary to describe their behavior. For instance, one can say that critical phase is more complex than the others since larger η is needed to constructively represent their dynamics. It must be, however, emphasized that this complexity is based on the constructing method of minimal CA, namely, it is obtained by throughout observation and classification with binary scheme σ and *stochastic* quantifiers.

The notion of a minimal CA mimicking CML discussed in this paper may provide a mechanism of minimal controllers (i.e., "machine demon") of emergent complex systems. The related results will be published in near future.

References

[1] e.g., H.A. Gutowitz eds., Cellular automata: theory and experiment, Proc. of a workshop sponsored by the center for nonlinear studies Los Alamos National Laboratory, *Pysica D 45* (1990).

[2] K. Kaneko. *Prog. Theor. Phys.* 72 (1984) 480.

[3] K. Kaneko. *Physica D*, 34 (1989) 1.

[4] K. Kaneko. Simulating Physics with Coupled Map Lattice, in *Formation, Dynamics, and Statistics of Patterns* (K. Kawasaki et. al., eds., World Scientific 1989).

[5] S. Wolfram. *Physica 10 D* (1984) 1-35.

[6] S. Arikawa, S. Miyano. *Automata and Computability*, Baihukan, Tokyo (1986)(in Japanese).

[7] J. E. Hopcroft, J. D. Ullman. *Introduction to Automata Theory, Languages, and Computation*, Addison-Wesley, Reading, Mass (1979).

Appendix. Proofs

The following proofs are led from the literature [6] straightforwardly.

Proof of theorem 1

(1). Let $C = (Q, \Gamma, \Sigma, \tilde{\delta}, \gamma, \sigma)$ and

$$\begin{cases} Q' = \{[p] | p \in Q\} \\ \tilde{\delta}'([p], a) = [\tilde{\delta}(p, a)] \\ \gamma'([p]) = \gamma(p) \\ \sigma'([p]) = \sigma(p), \end{cases} \tag{A1}$$

where $[p]$ denotes the equivalence class which contains $p \in Q$ with respect to relation $\equiv_Q$. Now let $C' = (Q', \Gamma', \Sigma, \tilde{\delta}', \gamma', \sigma')$, then for $\forall x \in (\Gamma \times \Gamma)^*$

$$\begin{aligned} [p] \equiv_Q [q] \quad &\Rightarrow \quad \gamma'(\tilde{\delta}'([p], x)) \equiv_\Gamma \gamma'(\tilde{\delta}'([q], x)), \\ &\qquad\; \sigma'(\tilde{\delta}'([p], x)) = \sigma'(\tilde{\delta}'([q], x)) \\ &\Rightarrow \quad \gamma(\tilde{\delta}(p, x)) \equiv_\Gamma \gamma(\tilde{\delta}(q, x)), \\ &\qquad\; \sigma(\tilde{\delta}(p, x)) = \sigma(\tilde{\delta}(q, x)) \\ &\Rightarrow \quad p \equiv_Q q \\ &\Rightarrow \quad [p] = [q]. \end{aligned}$$

Hence $C \equiv C'$.

(2). Let $C = (Q, \Gamma, \Sigma, \tilde{\delta}, \gamma, \sigma)$ be an irreducible CA with observation and $C' = (Q', \Gamma, \Sigma, \tilde{\delta}', \gamma', \sigma')$ be arbitrary one such that $C \equiv C'$. Suppose $|Q| > |Q'|$, then there exist $p, q \in Q$, $r \in Q'$ such that $p \equiv_Q r$, $q \equiv_Q r$ by $C \equiv C'$. Thus since $\equiv_Q$ is equivalence relationship, $p \equiv_Q q$ holds, but it contradicts to the assumption that C is irreducible. Therefore $|Q| \leq |Q'|$. ∎

Proof of theorem 2

(1). $(\Rightarrow)$ Let $p, q \in Q$ such that $p \overset{k+1}{\equiv}_Q q$. Then clearly $p \overset{k}{\equiv}_Q q$, and for $\forall a \in \Gamma \times \Gamma$

$$\begin{cases} \gamma(\tilde{\delta}(p, ax)) = \gamma(\tilde{\delta}(q, ax)) \\ \sigma(\tilde{\delta}(p, ax)) = \sigma(\tilde{\delta}(q, ax)). \end{cases} \tag{A2}$$

Also we can obtain

$$\begin{cases} \gamma(\tilde{\delta}(p, a)) = \gamma(\tilde{\delta}(q, a)) \\ \sigma(\tilde{\delta}(p, a)) = \sigma(\tilde{\delta}(q, a)) \end{cases} \tag{A3}$$

by

$$\left\{\begin{array}{l}\gamma(\tilde{\delta}(p, ax)) = \gamma(\tilde{\delta}(p, a))\gamma(\tilde{\delta}(\tilde{\delta}(p, a), x)) \\ \gamma(\tilde{\delta}(q, ax)) = \gamma(\tilde{\delta}(q, a))\gamma(\tilde{\delta}(\tilde{\delta}(q, a), x))\end{array}\right. \tag{A4}$$

$$\left\{\begin{array}{l}\sigma(\tilde{\delta}(p, ax)) = \sigma(\tilde{\delta}(p, a))\sigma(\tilde{\delta}(\tilde{\delta}(p, a), x)) \\ \sigma(\tilde{\delta}(q, ax)) = \sigma(\tilde{\delta}(q, a))\sigma(\tilde{\delta}(\tilde{\delta}(q, a), x)).\end{array}\right. \tag{A5}$$

Hence

$$\left\{\begin{array}{l}\gamma(\tilde{\delta}(\tilde{\delta}(p, a), x)) = \gamma(\tilde{\delta}(\tilde{\delta}(q, a), x)) \\ \sigma(\tilde{\delta}(\tilde{\delta}(p, a), x)) = \sigma(\tilde{\delta}(\tilde{\delta}(q, a), x)).\end{array}\right. \tag{A6}$$

Therefore $\tilde{\delta}(p, a)\overset{k}{\equiv}_Q\tilde{\delta}(q, a)$.

($\Leftarrow$) Let $p, q \in Q$ such that $p\overset{k}{\equiv}_Q q$ and for $\forall a \in \Gamma \times \Gamma$ $\tilde{\delta}(p, a)\overset{k}{\equiv}_Q\tilde{\delta}(q, a)$. Then for $\forall x \in (\Gamma \times \Gamma)^{[k]}$

$$\left\{\begin{array}{l}\gamma(\tilde{\delta}(p, a)) = \gamma(\tilde{\delta}(q, a)) \\ \sigma(\tilde{\delta}(p, a)) = \sigma(\tilde{\delta}(q, a))\end{array}\right. \tag{A7}$$

$$\left\{\begin{array}{l}\gamma(\tilde{\delta}(\tilde{\delta}(p, a), x)) = \gamma(\tilde{\delta}(\tilde{\delta}(q, a), x)) \\ \sigma(\tilde{\delta}(\tilde{\delta}(p, a), x)) = \sigma(\tilde{\delta}(\tilde{\delta}(q, a), x)),\end{array}\right. \tag{A8}$$

where $X^{[k]}$ is defined by $X^{[k]} = \bigcup_{i=0}^{k} X^i$ where X^i denotes i times product of X and so each member in X^i is a sequence which is consisted of i's members of X, namely, $X^{[k]}$ is a set of all sequences on X such that its length is at most k.

Hence

$$\left\{\begin{array}{l}\gamma(\tilde{\delta}(p, ax)) = \gamma(\tilde{\delta}(q, ax)) \\ \sigma(\tilde{\delta}(p, ax)) = \sigma(\tilde{\delta}(q, ax)).\end{array}\right. \tag{A9}$$

Therefore $p\overset{k+1}{\equiv}_Q q$.

(2). Suppose $(Q/\overset{k+i}{\equiv}_Q) = (Q/\overset{k}{\equiv}_Q)$ for $i \leq m$. Let $p, q \in Q$, then $\hat{\delta}(p,a) \overset{k+m-1}{\equiv}_Q \hat{\delta}(q,a)$ for $\forall a \in \Gamma \times \Gamma$ by (1). By assumption, $(Q/\overset{k+m-1}{\equiv}_Q) = (Q/\overset{k+m}{\equiv}_Q)$, so $p \overset{k+m}{\equiv}_Q q$ and $\tilde{\delta}(p,a) \overset{k+m}{\equiv}_Q \tilde{\delta}(q,a)$, thus $p \overset{k+m+1}{\equiv}_Q q$ by (1). Hence $(Q/\overset{k+m+1}{\equiv}_Q) = (Q/\overset{k+m}{\equiv}_Q) = (Q/\overset{k}{\equiv}_Q)$. By using induction on m, for $j \geq k$ $(Q/\overset{k+j}{\equiv}_Q) = (Q/\overset{k}{\equiv}_Q)$

(3). $(\Rightarrow)$ Let $p, q \in Q$ such that $p \overset{k}{\equiv}_Q q$. Since $(Q/\overset{k}{\equiv}_Q) = (Q/\overset{k+1}{\equiv}_Q)$, for $\forall j \geq k$ $p \overset{k+j}{\equiv}_Q q$ by (2). Hence $p \equiv_Q q$.

$(\Leftarrow)$ Trivial. ∎

Table caption

Table 1: The qualitative relation between the precision and the global dynamics. The stars represent the degree of similarity to the original phase (i.e. in case of floating point calculation). The double star denotes a high similarity. By "PS" or "FR" in the right most columns, we denotes that the original phase typically changes to PS or RF. The dash denotes that the classification was disable. See text for the abbreviations.

Figure captions

Figure 1: Collapse of Brownian motion of defect as decreasing the value of precision η. Every 64 time steps are plotted in the time series from 0 (top) to 200×64 (bottom). Horizontal direction denotes a site ordered in index i. $a = 1.80, \epsilon = 0.1$. (a) $\eta = 9$. (b) $\eta = 8$. (c) $\eta = 7$. (d) $\eta = 6$. (e) $\eta = 5$. (f) $\eta = 4$. (g) $\eta = 3$.

Figure 2: Collapse of pattern competition intermittency as decreasing the value of precision η. Every 64 time steps are plotted in the time series from 0 (top) to 200×64 (bottom). Horizontal direction denotes a site ordered in index i. $a = 1.80, \epsilon = 0.1$. (a) $\eta = 9$. (b) $\eta = 8$. (c) $\eta = 7$. (d) $\eta = 6$. (e) $\eta = 5$. (f) $\eta = 4$. (g) $\eta = 3$.

Figure 3: Pattern distribution $Q(k)$. All plotted as a function of nonlinearity a. Every $Q(k)$ is overlaid for $k = 1, \ldots, 5$ ($Q(1) : \diamond$, $Q(2) : +$, $Q(3) : \square$, $Q(4) : \times$, $Q(5) : \triangle$). $\epsilon = 0.10$. Framed graph (a) shows $Q(k)$ for CML. Other graphs for the present CA are shown in (b) $\eta = 9$, (c) $\eta = 6$ and (d) $\eta = 5$.

Figure 4: Static and dynamic entropies (S_p and S_d respectively). S_p denoted by $\diamond$ and S_d by $+$. Framed graph (a) shows S_p and S_d for CML. Other graphs for the present CA are shown in (b) $\eta = 9$, (c) $\eta = 6$ and (d) $\eta = 5$.

Table 1

original behavior		similarity in each precision η						
		9	8	7	6	5	4	3
static phase	FR	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$
	PS	$\star\star$	$\star\star$	$\star\star$	$\star$	$\star$	—	FR
critical region	BD	$\star\star$	$\star$	PS	PS	—	—	FR
	DT	$\star\star$	$\star$	PS	PS	—	—	FR
	PCI	$\star\star$	$\star\star$	PS	PS	—	—	FR
turbulent phase	FDT	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star$	—	FR

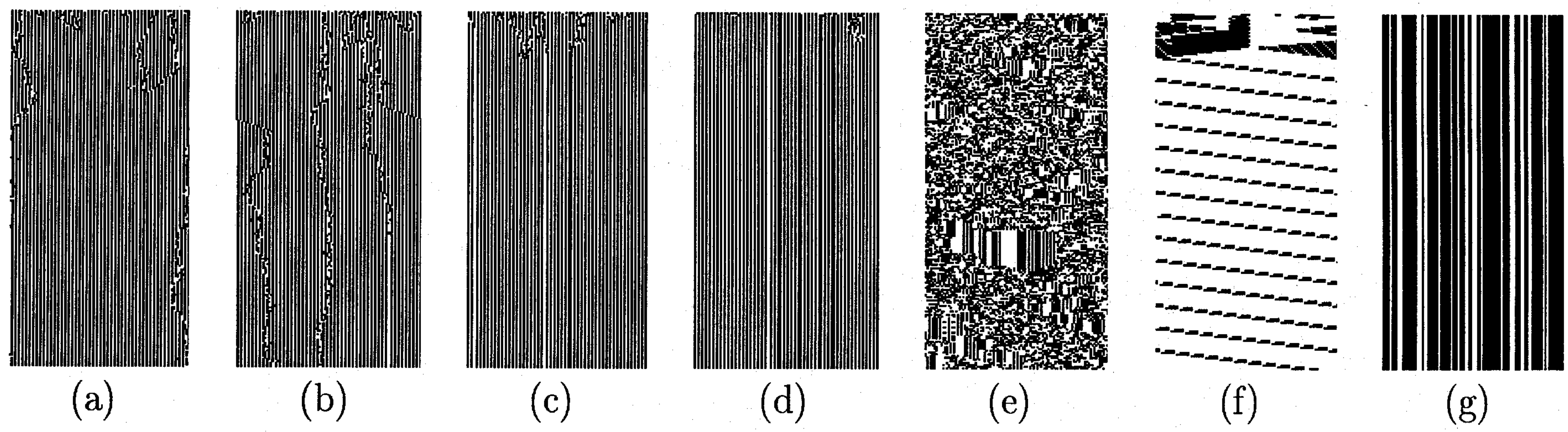


Figure 1

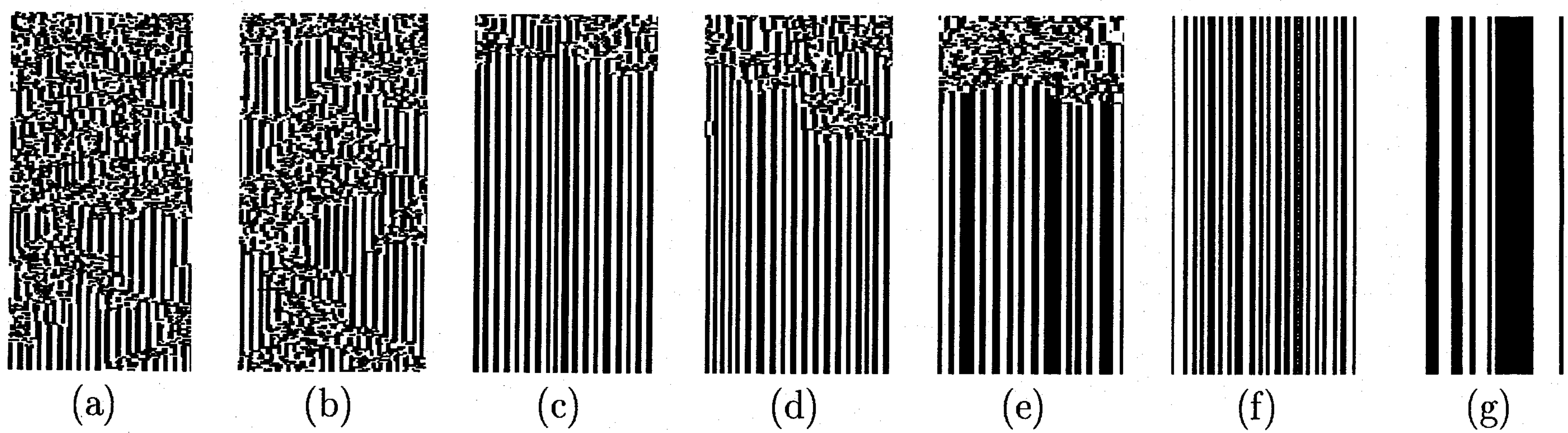
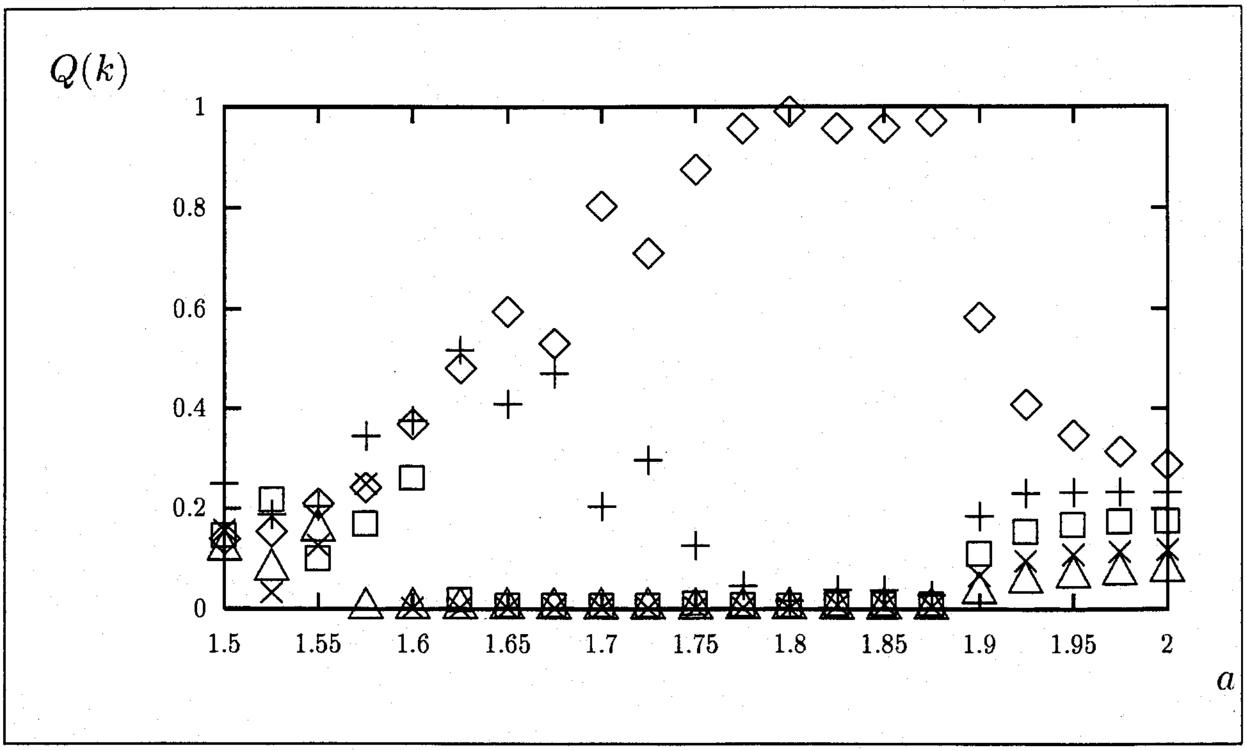
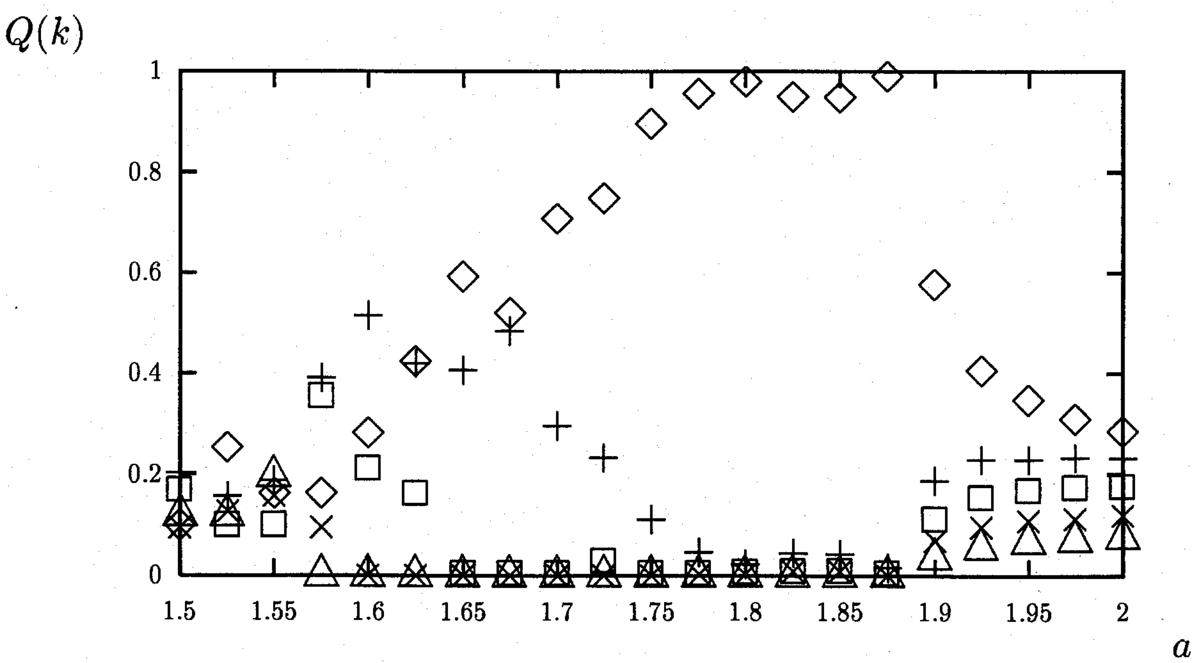


Figure 2

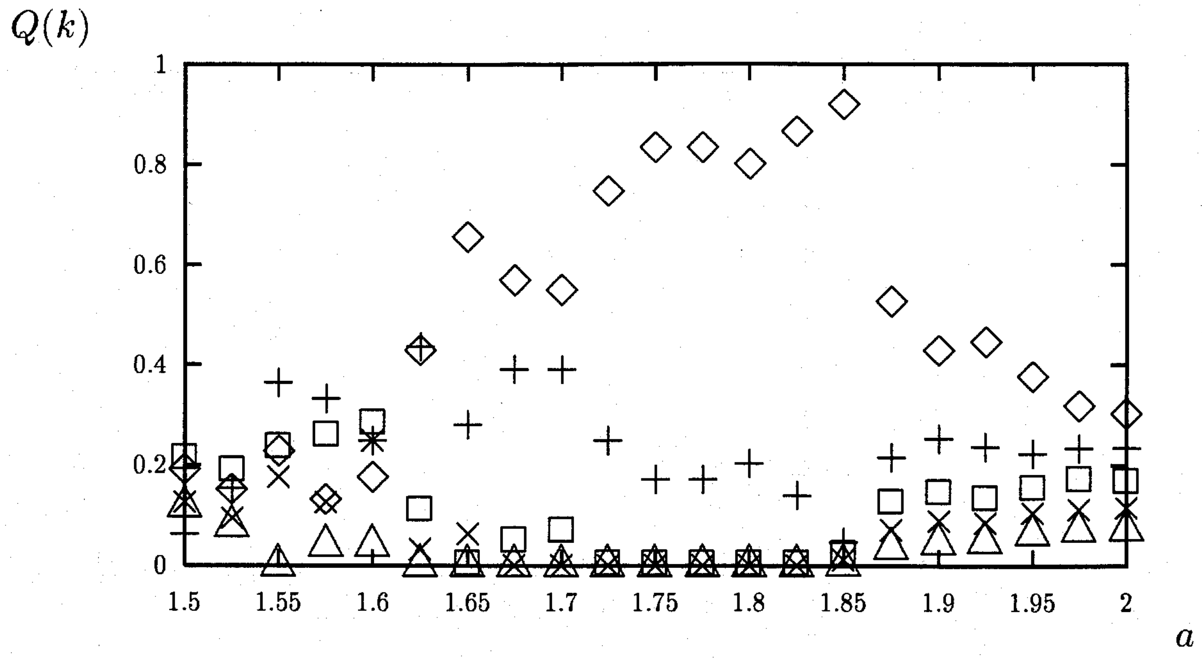


(a) $Q(k)$:**CML**

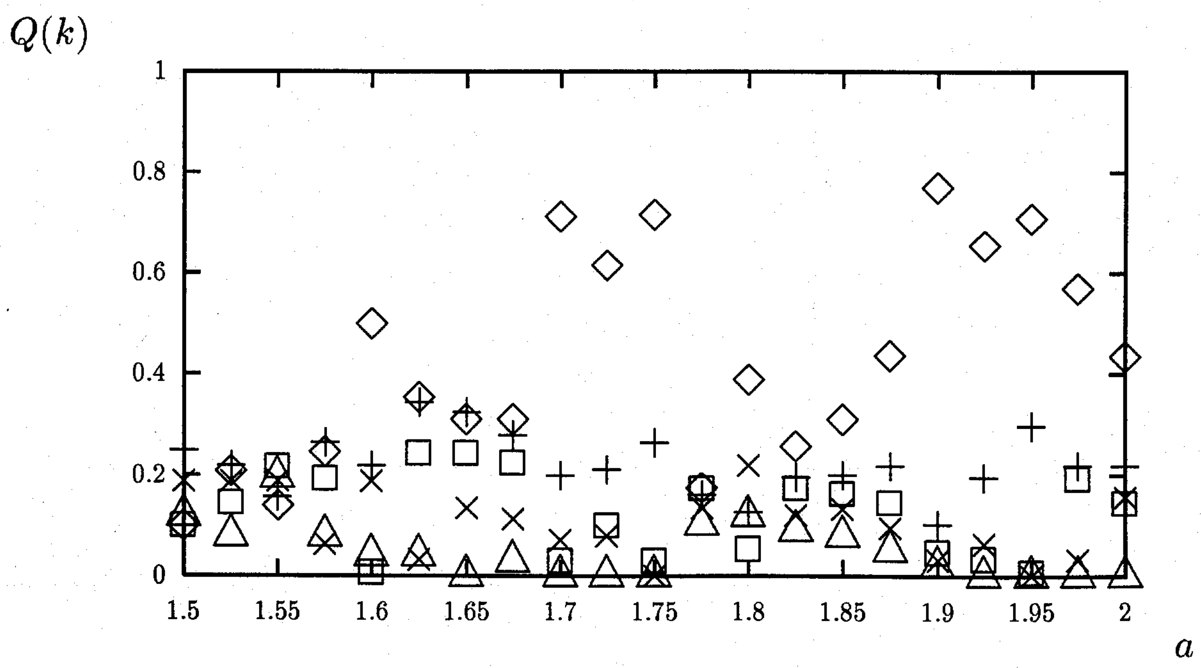


(b) $Q(k) : \eta = 9$

Figure 3

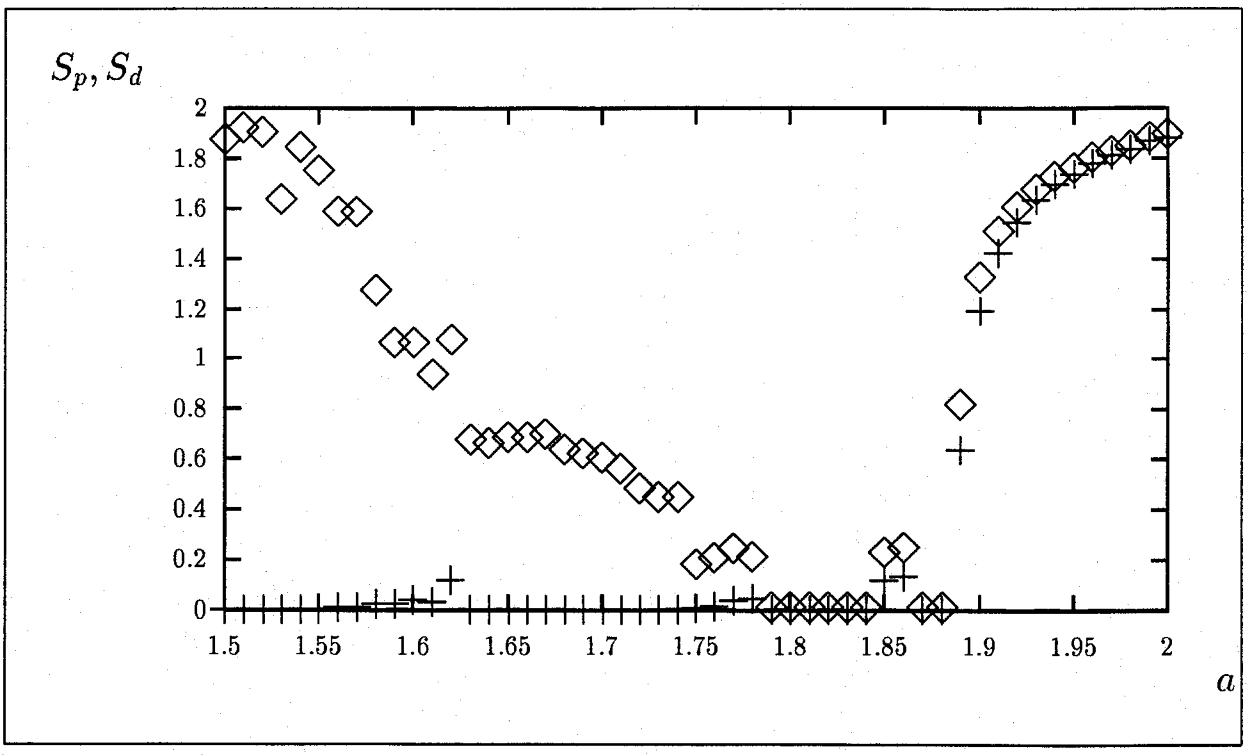


(c) $Q(k) : \eta = 6$

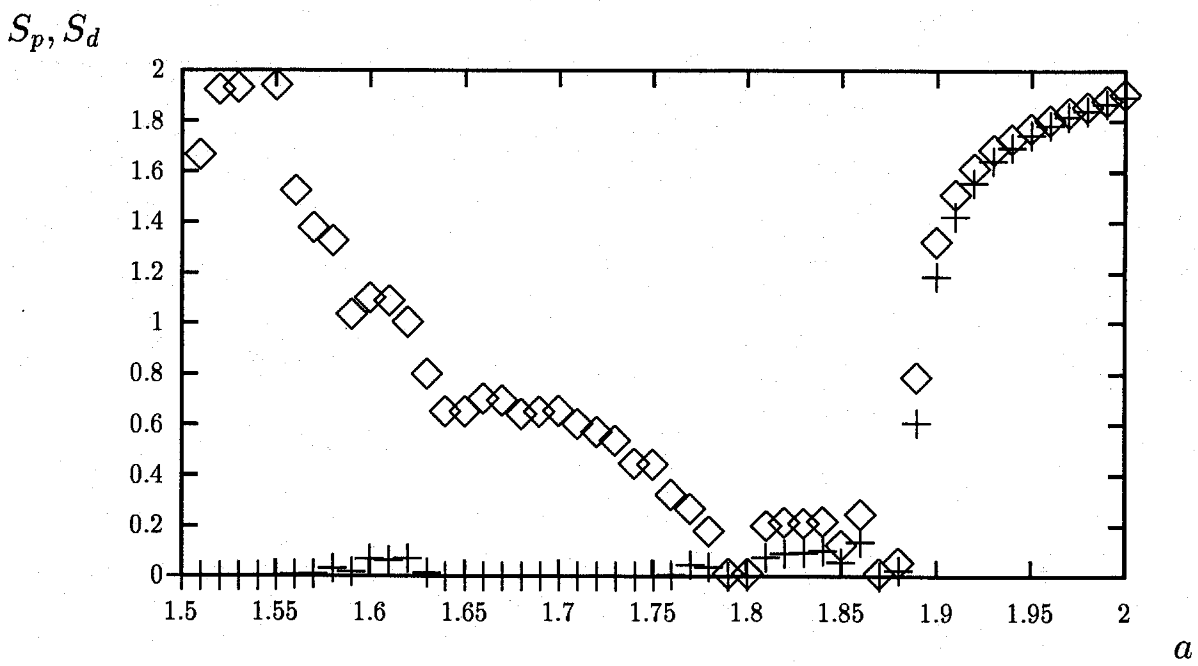


(d) $Q(k) : \eta = 5$

Figure 3 — continued

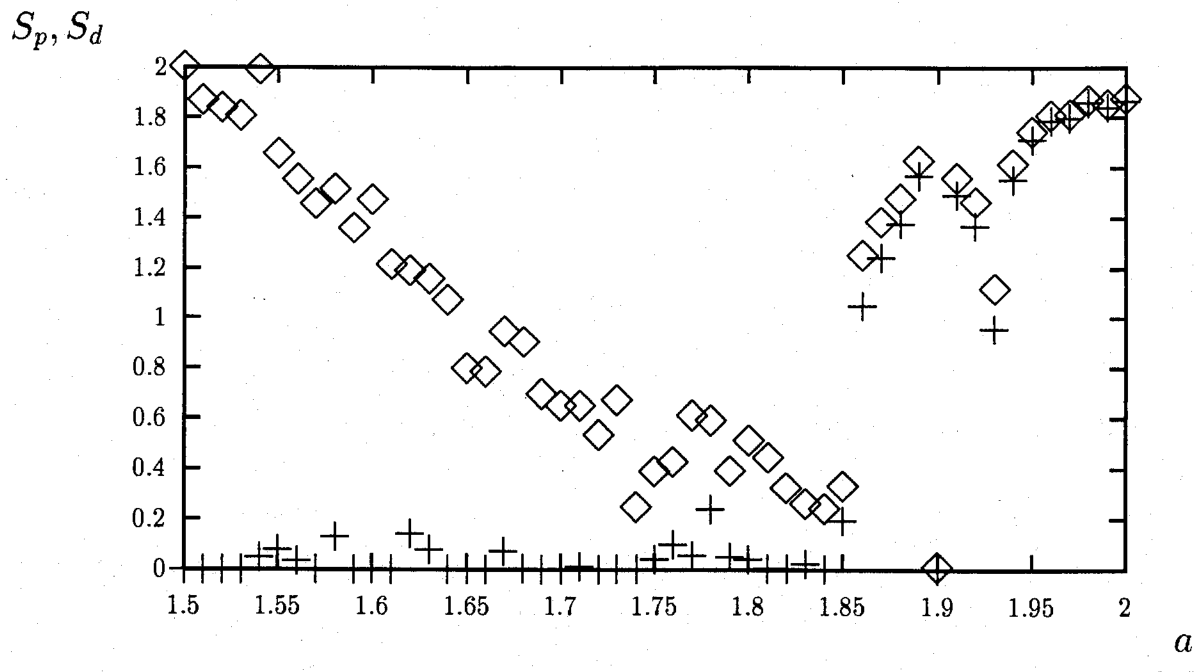


(a) S_p, S_d :**CML**

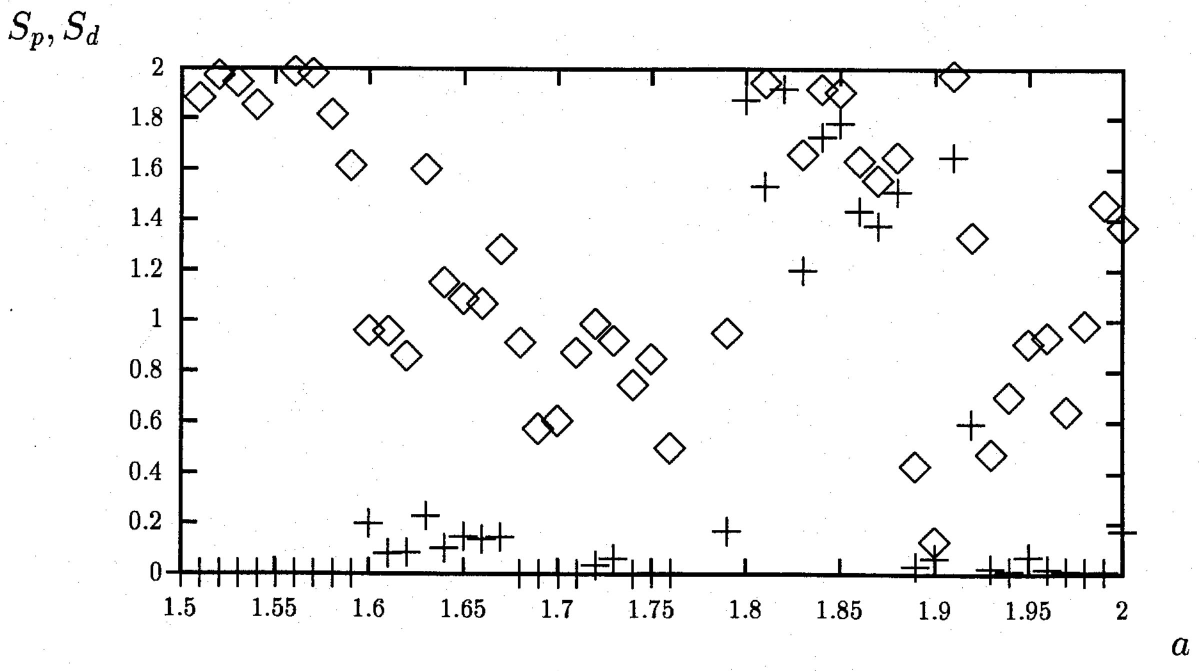


(b) $S_p, S_d : \eta = 9$

Figure 4



(c) $S_p, S_d : \eta = 6$



(d) $S_p, S_d : \eta = 5$

Figure 4 — continued