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Théorie de Hodge I, II, III pour cohomologies p -adiques.

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Resumé: Dans ce rapport, nous présentons quelques résultats démontrés dans [NS] et [Nak2]. Nous avons construit deux objets filtrés fondamentaux: *un complexe cristallin filtré par le préponds* et *un complexe zariskian par le préponds*. Nous avons construit aussi la filtration par le poids sur la cohomologie rigide d'un schéma séparé de type fini sur un corps parfait de caractéristique $p > 0$.

Mots-clefs: cohomologies rigides, cohomologies log-cristallines, suites p -adiques spectrales des poids.

1 Introduction.

Motivated by a conjectural theory of motives due to A. Grothendieck, P. Deligne has given a translation of concepts on mixed Hodge structures into those on l -adic cohomologies and vice versa in [D1], and he has proved his conjectures on mixed Hodge structures in [D2] and [D3].

Let U be a separated scheme of finite type over a field κ . Then there is the following translation [D1], [D2], [D3]:

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(1.0.1)

l -adic objects/ \mathbb{F}_q (for simplicity), $(l, q) = 1$	objects/ \mathbb{C}
$H_{\text{et}}^h(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$ ($h \in \mathbb{Z}$)	$H^h(U^{\text{an}}, \mathbb{Q})$ ($h \in \mathbb{Z}$)
F : geometric Frobenius $\curvearrowright H_{\text{et}}^h(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$; there exists a unique finite increasing filtration $\{P_k\}_{k \in \mathbb{Z}}$ characterized by the following: $P_k H_{\text{et}}^h(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$ is the principal subspace of $H_{\text{et}}^h(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$ where the eigenvalues α of F satisfy the following: $ \sigma(\alpha) \leq q^{k/2}$ ($\forall \sigma: \overline{\mathbb{Q}} \xrightarrow{\subset} \mathbb{C}$) (cf. [dJ]).	(weight filtrations)+ (Hodge ones) =: (mixed Hodge structures).
A morphism commuting F 's is strictly compatible with P_k 's.	A morphism in the category (MHS/ \mathbb{Q}) is strictly compatible with the weight filtration and the Hodge filtration.

Let us consider a more special case as in [D2]. Let (X, D) be a smooth scheme over κ with a SNCD (=simple normal crossing divisor) over κ . Put $U := X \setminus D$ and let $j: U \xrightarrow{\subset} X$ be the natural open immersion. Put $M_D := \{g \in \mathcal{O}_X \mid g \text{ is invertible outside } D\}$. Let $\epsilon: \tilde{X}_{\text{et}}^{\log} \rightarrow \tilde{X}_{\text{et}}$ be the forgetting log morphism induced by a natural morphism $(X, M_D) \rightarrow (X, \mathcal{O}_X^*)$ of log schemes in the sense of Fontaine-Illusie-Kato [K]. Let Y be an analytic variety over \mathbb{C} , and let us also denote by ϵ the real blow up $Y^{\log} \rightarrow Y$ [KN] which is denoted by τ in [loc. cit.]. Then we have the following translation:

(1.0.2)

objects/ \mathbb{C}	l -adic objects
$U^{\text{an}}, (X^{\text{an}})^{\log}$	$\tilde{U}_{\text{et}}, \tilde{X}_{\text{et}}^{\log}$
X^{an}	\tilde{X}_{et}
$j^{\text{an}}: U^{\text{an}} \xrightarrow{\subset} X^{\text{an}}, \quad \epsilon: (X^{\text{an}})^{\log} \rightarrow X^{\text{an}}$	$j_{\text{et}}: \tilde{U}_{\text{et}} \rightarrow \tilde{X}_{\text{et}}, \quad \epsilon: \tilde{X}_{\text{et}}^{\log} \rightarrow \tilde{X}_{\text{et}}$
$Rj_*^{\text{an}} \mathbb{Z} = R\epsilon_* \mathbb{Z}$ [KN]	$Rj_{\text{et}*}(\mathbb{Z}/l^n) = R\epsilon_* (\mathbb{Z}/l^n)$ [Fu], [FK]
$X^{\text{an}} \rightarrow X$	$\text{id}: \tilde{X}_{\text{et}} \rightarrow \tilde{X}_{\text{et}}$
$(\Omega_{X/\mathbb{C}}^{\bullet}(\log D), P_k)$?
$(\Omega_{X^{\text{an}}/\mathbb{C}}^{\bullet}(\log D^{\text{an}}), \tau_k) = (\Omega_{X^{\text{an}}/\mathbb{C}}^{\bullet}(\log D^{\text{an}}), P_k)$?

The purpose of this report is to inform the reader that we have succeeded in making two translations (1.0.1) and (1.0.2) for p -adic cohomologies.

This report came out of two preprints [NS], [Nak2] and my lecture entitled with *Théorie de Hodge III pour cohomologies p -adiques* at a symposium *Hodge theory and algebraic geometry* at Hokkaido-University in October in 2002. We give no proof in this report; we have given proofs in [NS] and [Nak2].

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Conventions. Let \mathcal{A} be an abelian category.

(1) For a complex (E^\bullet, d^\bullet) of objects in \mathcal{A} and for an integer n , $(E^{\bullet+n}, d^{\bullet+n})$ or $(E^\bullet\{n\}, d^\bullet\{n\})$ denotes the following complex: $\cdots \longrightarrow E^{q+n} \xrightarrow[q]{d^{q+n}} E^{q+1+n} \xrightarrow[q+1]{d^{q+1+n}} \cdots$. Here the numbers under the objects above in \mathcal{A} mean the degrees.

(2) For a complex (E^\bullet, d^\bullet) of objects in \mathcal{A} and for an integer n , $(E^\bullet[n], d^\bullet[n])$ denotes the following complex as usual: $(E^\bullet[n])^q := E^{q+n}$ with boundary morphisms $d^\bullet[n] = (-1)^n d^{\bullet+n}$.

(3) For a complex (E^\bullet, d^\bullet) of objects in \mathcal{A} , the identity $\text{id}: E^q \longrightarrow E^q$ ($\forall q \in \mathbb{Z}$) induces an isomorphism $\mathcal{H}^q((E^\bullet, -d^\bullet)) \xrightarrow{\sim} \mathcal{H}^q((E^\bullet, d^\bullet))$ ($\forall q \in \mathbb{Z}$) of cohomologies.

2 Constant simplicial open schemes.

In this section we state some results which Shiho and I have obtained in [NS].

Let (S, \mathcal{I}, γ) be a PD-scheme with quasi-coherent ideal $\mathcal{I} \subset \mathcal{O}_S$. Put $S_0 := \text{Spec}_S(\mathcal{O}_S/\mathcal{I})$. Let $f: (X, D) \longrightarrow S_0$ be a smooth scheme with a relative SNCD (=simple normal crossing divisor) over S_0 . By abuse of notation, we denote the composite morphism $(X, D) \longrightarrow S_0 \xrightarrow{\subset} S$ also by f . Then (X, D) gives an fs log structure M_D on X [NS] (cf. [K, p. 222–223], [Fa, §2 (c)]). Let $j: X \setminus D \xrightarrow{\subset} X$ be the natural open immersion. Note that $M_D \not\subseteq \mathcal{O}_X \cap j_*(\mathcal{O}_U^*)$ in general; indeed, the sections of $(\mathcal{O}_X \cap j_*(\mathcal{O}_U^*)) / \mathcal{O}_X^*$ over an affine open subscheme of X is not even finitely generated in general [NS].

Let $\Delta := \{D_\lambda\}_{\lambda \in \Lambda}$ be a decomposition of D by smooth components of D , where Λ is a set: $D = \bigcup_{\lambda \in \Lambda} D_\lambda$ and each D_λ is smooth over S_0 . Let k be a positive integer. Put $D^{(k)} := \coprod_{\{\lambda_1, \dots, \lambda_k \mid \lambda_i \neq \lambda_j \text{ (} i \neq j \text{)}\}} D_{\lambda_1} \cap \cdots \cap D_{\lambda_k}$. Then $D^{(k)}$ has been shown to be independent of the choice of the decomposition of D by smooth components of D [NS]. Put $D^{(0)} := X$.

For simplicity of notation, we denote the log crystalline topos $((X, \widetilde{M_D})/S)_{\text{crys}}^{\text{log}}$ simply by $(X/S)_{\text{crys}}^{\text{log}}$. Let $(X/S)_{\text{crys}}$ be the usual crystalline topos. Let $\mathcal{O}_{X/S}$ (resp. $\overset{\circ}{\mathcal{O}}_{X/S}$) be the structure sheaf in $(X/S)_{\text{crys}}^{\text{log}}$ (resp. $(X/S)_{\text{crys}}$).

Definition 2.1. ([NS]) Let $\iota: (X, D) \xrightarrow{\subset} (\mathcal{X}, \mathcal{D})$ be an exact closed immersion into a smooth scheme with a relative SNCD over S . Then we call ι an *admissible immersion with respect to Δ* if \mathcal{D} has a decomposition $\widetilde{\Delta} = \{\mathcal{D}_\lambda\}_{\lambda \in \Lambda}$ by smooth components of \mathcal{D} and if ι induces an isomorphism $D_\lambda \xrightarrow{\sim} X \times_{\mathcal{X}} \mathcal{D}_\lambda$ of schemes for all $\lambda \in \Lambda$.

Next let us recall and define orientation sheaves for $D^{(k)}/S_0$ and $D^{(k)}/S$ ($k \in \mathbb{N}$) (cf. [D2, (3.1.4)]).

Let E be a non-empty finite set with cardinality k . Put $\varpi_E := \wedge^k \mathbb{Z}^E$. Let k be a positive integer. Let P be a point of $D^{(k)}$. Let $\{D_\lambda\}_{\lambda \in \Lambda}$ be a decomposition of D by smooth components of D . Let $D_{\lambda_0}, \dots, D_{\lambda_k}$ be distinct smooth components of D such that $D_{\lambda_0} \cap \dots \cap D_{\lambda_k}$ contains P . Then, the set $\{D_{\lambda_0}, \dots, D_{\lambda_k}\}$ gives an orientation sheaf $\varpi_{\lambda_0 \dots \lambda_k, \text{zar}}(D/S_0)$ on a local neighborhood of P in $D^{(k)}$. The sheaf $\varpi_{\lambda_0 \dots \lambda_k, \text{zar}}(D/S_0)$ is globalized, and we denote this globalized sheaf by the same symbol $\varpi_{\lambda_0 \dots \lambda_k, \text{zar}}(D/S_0)$. Put $\varpi_{\text{zar}}^{(k)}(D/S_0) := \bigoplus_{\{\lambda_0, \dots, \lambda_k\}} \varpi_{\lambda_0 \dots \lambda_k, \text{zar}}(D/S_0)$. Put $\varpi_{\text{zar}}^{(k)}(D/S_0) := \mathbb{Z}_X$ for $k = 0$. The sheaf $\varpi_{\text{zar}}^{(k)}(D/S_0)$ is extended to a sheaf $\varpi_{\text{crys}}^{(k)}(D/S)$ in the usual crystalline topos $(D^{(k)}/S)_{\text{crys}}$.

Definition 2.2. We call $\varpi_{\text{zar}}^{(k)}(D/S_0)$ (resp. $\varpi_{\text{crys}}^{(k)}(D/S)$) the *zariskian orientation sheaf* (resp. *crystalline orientation sheaf*) of $D^{(k)}/S_0$ (resp. $D^{(k)}/(S, \mathcal{I}, \gamma)$).

The sheaves $\varpi_{\text{zar}}^{(k)}(D/S_0)$, $\varpi_{\text{crys}}^{(k)}(D/S)$ are defined by the local nature of D ; they are independent of the choice of the decomposition of D by smooth components of D .

If S_0 is of characteristic $p > 0$, we define the Frobenius action on the orientations sheaves above. Let $F: (X, D) \rightarrow (X', D')$ be the relative Frobenius morphism over S_0 . The morphism F induces the relative Frobenius morphism $F^{(k)}: D^{(k)} \rightarrow D'^{(k)} = D^{(k)'}$. Let $a^{(k)}: D^{(k)} \rightarrow X$ and $a^{(k)'}: D^{(k)'} \rightarrow X'$ be the natural morphisms. Then we define the following two Frobenius morphisms

$$(2.2.1) \quad \Phi^{(k)}: a_{\text{crys}*}^{(k)'} \varpi_{\text{crys}}^{(k)}(D'/S) \rightarrow F_{\text{crys}*} a_{\text{crys}*}^{(k)} \varpi_{\text{crys}}^{(k)}(D/S)$$

and

$$(2.2.2) \quad \Phi^{(k)}: a_*^{(k)'} \varpi_{\text{zar}}^{(k)}(D'/S_0) \rightarrow F_* a_*^{(k)} \varpi_{\text{zar}}^{(k)}(D/S_0)$$

by the multiplication by p^k under the natural identifications

$$\varpi_{\text{crys}}^{(k)}(D'/S) \xrightarrow{\sim} F_{\text{crys}*}^{(k)} \varpi_{\text{crys}}^{(k)}(D/S), \quad \varpi_{\text{zar}}^{(k)}(D'/S) \xrightarrow{\sim} F_*^{(k)} \varpi_{\text{zar}}^{(k)}(D/S).$$

In [NS] we have proved the following:

Theorem-Definition 2.3. ([NS]) Let (S, \mathcal{I}, γ) and $(X, D)/S_0$ be as in the beginning of this section. Then there exist two filtered objects

$$(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k) = (\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k \mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}))_{k \in \mathbb{Z}}$$

and

$$(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k) = (\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k \mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}))_{k \in \mathbb{Z}}$$

in the filtered derived categories $D^+F(\mathring{\mathcal{O}}_{X/S})$ of bounded below complexes of $\mathring{\mathcal{O}}_{X/S}$ -modules and $D^+F(f^{-1}(\mathcal{O}_S))$ of bounded below complexes of $f^{-1}(\mathcal{O}_S)$ -modules, respectively. We have called $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k)$ and $(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k)$ the *preweight-filtered crystalline complex* of $(X, D)/(S, \mathcal{I}, \gamma)$ and the *preweight-filtered zariskian complex* of $(X, D)/(S, \mathcal{I}, \gamma)$, respectively. They enjoy the following properties:

(1; c): $\{P_k \mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S})\}_{k \in \mathbb{Z}}$ is an increasing “filtration” on $\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S})$ which is finite locally on $(X/S)_{\text{crys}}$ such that $P_{-1} \mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}) = 0$, $P_0 \mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}) \xleftarrow{\sim} \mathring{\mathcal{O}}_{X/S}$, and $\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}) \xleftarrow{\sim} R\epsilon_{X/S*}(\mathcal{O}_{X/S})$.

(2; c): If (X, D) has an admissible immersion $(X, D) \xrightarrow{\subset} (\mathcal{X}, \mathcal{D})$ over S with respect to Δ , then

$$(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k) \simeq (\mathring{L}(\Omega_{\mathcal{X}/S}^\bullet(\log \mathcal{D})), \mathring{L}(P_k \Omega_{\mathcal{X}/S}^\bullet(\log \mathcal{D}))),$$

where \mathring{L} is the classical linearization functor for $\mathcal{O}_{\mathcal{X}}$ -modules [Be, Chap. IV, 3].

$$(3): (\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k) = Ru_{X/S*}(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k).$$

In particular,

(1; z): $\{P_k \mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S})\}_{k \in \mathbb{Z}}$ is an increasing “filtration” on $\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S})$ which is finite locally on X such that $P_{-1} \mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}) = 0$, $P_0 \mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}) \xleftarrow{\sim} Ru_{X/S*} \mathring{\mathcal{O}}_{X/S}$, and $\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}) \xleftarrow{\sim} Ru_{X/S*}(\mathcal{O}_{X/S})$,

and

(2; z): If (X, D) has an admissible immersion $(X, D) \xrightarrow{\subset} (\mathcal{X}, \mathcal{D})$ over S with respect to Δ ,

$$(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k) \simeq (\mathcal{O}_{\mathcal{Y}} \otimes_{\mathcal{O}_{\mathcal{X}}} \Omega_{\mathcal{X}/S}^\bullet(\log \mathcal{D}), \mathcal{O}_{\mathcal{Y}} \otimes_{\mathcal{O}_{\mathcal{X}}} P_k \Omega_{\mathcal{X}/S}^\bullet(\log \mathcal{D})),$$

where \mathcal{Y} is the PD-envelope of the closed immersion $X \xrightarrow{\subset} \mathcal{X}$.

(4; c): Let $\text{gr}_k^P: D^+F(\mathring{\mathcal{O}}_{X/S}) \rightarrow D^+(\mathring{\mathcal{O}}_{X/S})$ ($k \in \mathbb{Z}$) be the gr-functor [NS]. Then $\text{gr}_k^P((\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_l)) = a_{\text{crys}*}^{(k)}(\mathcal{O}_{D^{(k)}/S} \otimes_{\mathbb{Z}} \varpi_{\text{crys}}^{(k)}(D/S))\{-k\}$.

(4; z): Let $\text{gr}_k^P: D^+F(f^{-1}(\mathcal{O}_S)) \rightarrow D^+(f^{-1}(\mathcal{O}_S))$ ($k \in \mathbb{Z}$) be the gr-functor. Then $\text{gr}_k^P((\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_l)) = a_*^{(k)}(Ru_{D^{(k)}/S*}(\mathcal{O}_{D^{(k)}/S}) \otimes_{\mathbb{Z}} \varpi_{\text{zar}}^{(k)}(D/S_0))\{-k\}$.

(5): $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), \tau_k) \xrightarrow{\sim} (\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k)$, where τ_k is the canonical filtration.

(6): If S_0 is the spectrum of a perfect field κ of characteristic p and if S is the spectrum of the Witt ring W_n of κ of finite length $n > 0$, then $(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k)$ is canonically isomorphic to the filtered object $(W_n \Omega_X^\bullet(\log D), P_k W_n \Omega_X^\bullet(\log D))$ in [M1] and [M2].

We give important remarks on (2.3).

(5) is equivalent to the following *p-adic purity*:

$$(2.3.1) \quad R^k \epsilon_{X/S*} \mathcal{O}_{X/S} = a_{\text{crys}*}^{(k)} (\mathcal{O}_{D^{(k)}/S} \otimes_{\mathbb{Z}} \varpi_{\text{crys}}^{(k)}(D/S)).$$

Logically speaking, in [NS], we have first proved (2.3.1) by using $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k)$ and then we have proved (5).

By (5), $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k)$ and $(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k)$ are functorial; that is, for another smooth scheme Y with a relative SNCD E over S_0 and for a morphism $g: (X, D) \rightarrow (Y, E)$ of log schemes in the sense of Fontaine-Illusie-Kato, we have natural morphisms

$$g_{\text{crys}}^*: (\mathcal{C}_{\text{crys}}(\mathcal{O}_{Y/S}), P_k) \rightarrow Rg_{\text{crys}*}(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k)$$

and

$$g^*: (\mathcal{C}_{\text{zar}}(\mathcal{O}_{Y/S}), P_k) \rightarrow Rg_*(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k).$$

By (1; c) and (4; c), we have the following spectral sequence:

Corollary-Definition 2.4. ([NS]) There exists the following functorial spectral sequence

$$(2.4.1) \quad E_1^{-k, h+k} = R^{h-k} f_{D^{(k)}/S*} (\mathcal{O}_{D^{(k)}/S} \otimes_{\mathbb{Z}} \varpi_{\text{crys}}^{(k)}(D/S)) \implies R^h f_{X/S*} (\mathcal{O}_{X/S}).$$

We call the spectral sequence (2.4.1) the *preweight spectral sequence* of $(X, D)/(S, \mathcal{I}, \gamma)$.

Remark 2.5. Though we have obtained deep other theorems in [NS], we can only mention them because of lack of spaces: (pre)weight-filtered base change formula, (pre)weight-filtered Künneth formula, weight-filtered Poincaré duality, a weight-filtered Berthelot-Ogus isomorphism, theory for compact support cohomology, the convergence of weight filtrations. See [NS] for details.

If S is a *p*-adic formal V -scheme in the sense of [O, §1], we obtain the following translation as a conclusion:

(2.5.1)

$/\mathbb{C}$	crystal
$U^{\text{an}}, (X^{\text{an}})^{\log}$	$\widetilde{(X/S)}_{\text{crys}}^{\log}$
X^{an}	$\widetilde{(X/S)}_{\text{crys}}$
$j^{\text{an}}: U^{\text{an}} \xrightarrow{\subset} X^{\text{an}}, \quad \epsilon: (X^{\text{an}})^{\log} \rightarrow X^{\text{an}}$	$\epsilon_{X/S}: \widetilde{(X/S)}_{\text{crys}}^{\log} \rightarrow \widetilde{(X/S)}_{\text{crys}}$
$Rj_*^{\text{an}} \mathbb{Z} = R\epsilon_* \mathbb{Z}$	$R\epsilon_{X/S*}(\mathcal{O}_{X/S})$
$X^{\text{an}} \rightarrow X$	$\mathring{u}_{X/S}: \widetilde{(X/S)}_{\text{crys}} \rightarrow \widetilde{X}_{\text{zar}}$
$(\Omega_{X/\mathbb{C}}^{\bullet}(\log D), P_k)$	$(\mathcal{C}_{\text{zar}}(\mathcal{O}_{X/S}), P_k)$
$(\Omega_{X^{\text{an}}/\mathbb{C}}^{\bullet}(\log D^{\text{an}}), \tau_k) = (\Omega_{X^{\text{an}}/\mathbb{C}}^{\bullet}(\log D^{\text{an}}), P_k)$	$(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), \tau_k) = (\mathcal{C}_{\text{crys}}(\mathcal{O}_{X/S}), P_k)$

Remark 2.6. Let \mathcal{V} be a complete discrete valuation ring of mixed characteristics $(0, p)$. Let κ (resp. K) be the (not necessarily perfect) residue (resp. fraction) field of \mathcal{V} . Let $\sigma \in \text{Aut } \mathcal{V}$ be a fixed lift of the p -th power endomorphism of κ . Let (X, D) be a smooth scheme with a SNCD over κ . Fix a decomposition $\{D_\lambda\}_{\lambda \in \Lambda}$ of D by smooth components of D and fix a total order on Λ . Assume that there exists a closed immersion $X \xrightarrow{\subset} \mathcal{P}$ into a formal V -scheme such that $\mathcal{P}/\text{Spf } V$ is formally smooth around X . In [CL, (3.8)], Chiarellotto and Le Stum have constructed the following p -adic weight spectral sequence by the method of local rigid cohomologies:

$$(2.6.1) \quad E_{1, \text{rig}}^{-k, h+k} = H_{\text{rig}}^{h-k}(D^{(k)}/K)(-k) \implies H_{\text{rig}}^h(U/K).$$

More generally, in (4.3.4) below, we shall construct a p -adic weight spectral sequence for the rigid cohomology of a separated scheme of finite type over κ ; our method is a complicated p -adic version of that in [D3], and it is different from theirs.

3 Simplicial open schemes.

Let (S, \mathcal{I}, γ) and S_0 be as in §2. Let $f: (X_\bullet, D_\bullet)_{\bullet \in \mathbb{N}} \rightarrow S_0$ be a simplicial smooth scheme with a simplicial relative SNCD over S_0 . The morphism f induces a morphism $f_\bullet: (X_\bullet, D_\bullet)_{\bullet \in \mathbb{N}} \rightarrow S_{0\bullet}$, where $S_{0\bullet}$ is the constant simplicial scheme defined by S_0 . By abuse of notation, we denote by f (resp. f_\bullet) the composite morphism $(X_\bullet, D_\bullet) \rightarrow S_0 \xrightarrow{\subset} S$ (resp. $(X_\bullet, D_\bullet) \rightarrow S_{0\bullet} \xrightarrow{\subset} S_\bullet$). Furthermore, for simplicity of notation, we sometimes denote simply by f the morphism $f_t: (X_t, D_t) \rightarrow S_0$ ($t \in \mathbb{N}$) and also by f the composite morphism $(X_t, D_t) \xrightarrow{f_t} S_0 \xrightarrow{\subset} S$. We have a log crystalline topos $(\widetilde{X_\bullet/S})_{\text{crys}}^{\text{log}} := ((X_\bullet, \widetilde{M_{D_\bullet}})/S)_{\text{crys}}^{\text{log}}$ and a usual crystalline topos $(\widetilde{X_\bullet/S})_{\text{crys}}$. Let $\mathcal{O}_{X_\bullet/S}$ (resp. $\mathring{\mathcal{O}}_{X_\bullet/S}$) be the structure sheaf in $(\widetilde{X_\bullet/S})_{\text{crys}}^{\text{log}}$ (resp. $(\widetilde{X_\bullet/S})_{\text{crys}}$). The morphisms f and f_\bullet induce morphisms of topoi

$$\begin{aligned} f_{X_\bullet/S}: (\widetilde{X_\bullet/S})_{\text{crys}}^{\text{log}} &\longrightarrow \widetilde{S}_{\text{zar}} & \text{and} & & f_{X_\bullet/S_\bullet}: (\widetilde{X_\bullet/S})_{\text{crys}}^{\text{log}} &\longrightarrow \widetilde{S}_{\bullet\text{zar}}, \\ \mathring{f}_{X_\bullet/S}: (\widetilde{X_\bullet/S})_{\text{crys}} &\longrightarrow \widetilde{S}_{\text{zar}} & \text{and} & & \mathring{f}_{X_\bullet/S_\bullet}: (\widetilde{X_\bullet/S})_{\text{crys}} &\longrightarrow \widetilde{S}_{\bullet\text{zar}}. \end{aligned}$$

Then we have a cosimplicial filtered object $Rf_{X_\bullet/S_\bullet}(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X_\bullet/S}), P_k) \in \text{D}^+\text{F}(f_\bullet^{-1}(\mathcal{O}_S))$ by the functoriality of $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X_\bullet/S}), P_k)$. Let (\mathcal{K}, P_k) be a representative of the filtered object $R\mathring{f}_{X_\bullet/S_\bullet}(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X_\bullet/S}), P_k)$. The filtered complex (\mathcal{K}, P_k) defines a filtered double complex $(\mathcal{K}^{\bullet\bullet}, P_k)$ (cf. [D3, (5.1.9) (IV)]). Here the first degree is the cosimplicial degree. We make the following convention on the signs of boundary morphisms of the single complex $\mathfrak{s}\mathcal{K}$ of $\mathcal{K}^{\bullet\bullet}$:

$$(3.0.1) \quad (\mathfrak{s}\mathcal{K})^n = \bigoplus_{i+j=n} \mathcal{K}^{ij}; \quad d(x^{ij}) = \sum_{s=0}^{i+1} (-1)^s \delta_s^*(x^{ij}) + (-1)^i d_{\mathcal{K}}(x^{ij}),$$

where $\delta_s : (X_{i+1}, D_{i+1}) \longrightarrow (X_i, D_i)$ ($0 \leq s \leq i+1$) is a standard face morphism (see, e.g., [D3, (5.1.1)]) and $d_{\mathcal{K}} : \mathcal{K}^{ij} \longrightarrow \mathcal{K}^{i+1,j}$ is the boundary morphism arising from the boundary morphism of the filtered complex $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X_{\bullet}/S}), P_k)$. Our convention on the turn of the degrees is the same as that in [NA, (2.3)] and different from that in [D3, (5.1.9) (IV)] and in [CT, (3.9)]; our convention on the signs of boundary morphisms of $\mathbf{s}\mathcal{K}$ is superior to that in [D3, (5.1.9.2)] and in [CT, (3.9)]; if we follow the convention in [D3, (5.1.9.2)], we have to consider distinct signs before $(-1)^s \delta_s^*$ with respect to the degrees arising from the complex $(\mathcal{C}_{\text{crys}}(\mathcal{O}_{X_{\bullet}/S}), P_k)$; we should eliminate signs $(-1)^p$ in [D3, (5.1.9.2)] by following our convention because these morphisms with various signs are not induced by morphisms of algebro-geometric objects. If we follow the convention in [loc. cit.], it seems to me that it is impossible to give a description of the boundary morphisms in [D3, p. 35]; the diagram in [D3, p. 35] is not a part of a double complex; since it is commutative, it is mistaken. Furthermore, ‘‘Gysin’’ in the diagram in [loc. cit.] is not clear. See [Nak2] for a correction and the explicit formula of ‘‘Gysin’’.

Let L be the stupid filtration of $\mathbf{s}\mathcal{K}$ with respect to the first degree:

$$(3.0.2) \quad L^i(\mathbf{s}\mathcal{K}) = \bigoplus_{i' \geq i} \mathcal{K}^{i' \bullet}.$$

Let (S, \mathcal{I}, γ) and S_0 be as in §2. Now, let us construct the *preweight spectral sequence* of $(X, D)/(S, \mathcal{I}, \gamma)$. Let $\delta(P, L)$ be the diagonal filtration of P and L ([D3, (7.1.6.1)]):

$$(3.0.3) \quad \delta(P, L)_k(\mathbf{s}\mathcal{K}) = \bigoplus_{i,j \geq 0} P_{i+k} \mathcal{K}^{ij} = \sum_{i \geq 0} (\mathbf{s}P_{i+k} \mathcal{K}) \cap L^i(\mathbf{s}\mathcal{K}).$$

(Note that the formula [D3, (7.1.6.1)] should be replaced by $\bigoplus_{p,q} W_{n+p}(K^{q,p})$.) Then we have $\text{gr}_k^{\delta(P,L)}(\mathbf{s}\mathcal{K}) = \bigoplus_{i \geq 0} \text{gr}_{i+k}^P \mathcal{K}^{i \bullet}[-i]$. Hence we have the following spectral sequence by the Convention (3) (cf. [D3, (8.1.15)]):

$$(3.0.4) \quad E_1^{-k,h+k}((X_{\bullet}, D_{\bullet})/S) = \bigoplus_{i \geq 0} \mathcal{H}^{h-i}(\text{gr}_{i+k}^P \mathcal{K}^{i \bullet}) \implies \mathcal{H}^h(\mathbf{s}\mathcal{K}).$$

By (1; c) and (4; c), the spectral sequence (3.0.4) is equal to the following spectral sequence

$$(3.0.5) \quad E_1^{-k,h+k}((X_{\bullet}, D_{\bullet})/S) = \bigoplus_{i \geq 0} R^{h-2i-k} f_{D_i^{(i+k)}/S^*}(\mathcal{O}_{D_i^{(i+k)}/S} \otimes_{\mathbb{Z}} \varpi_{\text{crys}}^{(i+k)}(D_i/S)) \implies R^h f_{X_{\bullet}/S^*}(\mathcal{O}_{X_{\bullet}/S}).$$

Definition 3.1. We call (3.0.5) the *preweight spectral sequence* of $(X, D)/(S, \mathcal{I}, \gamma)$. If S is a p -adic formal V -scheme in the sense of [O, §1], we call (3.0.5) the *p -adic weight spectral sequence* of $(X, D)/(S, \mathcal{I}, \gamma)$. We denote by $\{P_k\}_{k \in \mathbb{Z}}$ the induced filtration on $R^h f_{X_{\bullet}/S^*}(\mathcal{O}_{X_{\bullet}/S})$. We call $\{P_k\}_{k \in \mathbb{Z}}$ the *weight filtration* on $R^h f_{X_{\bullet}/S^*}(\mathcal{O}_{X_{\bullet}/S})$.

Remark 3.2. In [Nak2], we have given an explicit description of the boundary morphism between E_1 -terms of (3.0.5).

Theorem 3.3. ([NS], [Nak2]) If S is a p -adic formal V -scheme in the sense of [O, §1] and if $S_0 := \underline{\text{Spec}}_S(\mathcal{O}_S/p)$, then (3.0.5) degenerates at E_2 modulo torsion.

Corollary 3.4. ([NS], [Nak2]) There exists the following spectral sequence of convergent F -isocrystals:

$$(3.4.1) \quad E_1^{-k, h+k}((X_\bullet, D_\bullet)/K) = \bigoplus_{i \geq 0} R^{h-2i-k} f_*(\mathcal{O}_{D_i^{(i+k)}/K} \otimes_{\mathbb{Z}} \varpi^{(i+k)}(D_i/K)) \implies R^h f_*(\mathcal{O}_{X_\bullet/K}).$$

The spectral sequence (3.4.1) degenerates at E_2 . Here $R^r f_*(\mathcal{O}_{D_i^{(i+k)}/K} \otimes_{\mathbb{Z}} \varpi^{(i+k)}(D_i/K))$ ($r \in \mathbb{Z}$) is a convergent F -isocrystal on S/V whose value at a p -adic enlargement T of S/V is $R^r f_{(D_i^{(i+k)})_{T_1}/T^*}(\mathcal{O}_{(D_i^{(i+k)})_{T_1}/T} \otimes_{\mathbb{Z}} \varpi_{\text{crys}}^{(i+k)}((D_i)_{T_1}/T))$.

4 Weight filtration on rigid cohomologies.

Let \mathcal{V} be a complete discrete valuation ring of mixed characteristics with perfect residue field κ of characteristic $p > 0$. Let \mathcal{W} be the Witt ring of κ . Let K (resp. K_0) be the fraction field of \mathcal{V} (resp. \mathcal{W}). Let U be a separated scheme of finite type over κ , and let $\iota: U \xrightarrow{\subset} \bar{U}$ be an open immersion into a proper scheme over κ [Nag]. Let (X_\bullet, D_\bullet) be a simplicial proper smooth scheme with a simplicial SNCD over κ . Put $U_\bullet := X_\bullet \setminus D_\bullet$.

First we recall the following:

Definition 4.1. ([T], (cf. [D3, (5.3.8)])) The pair (U_\bullet, X_\bullet) is called a *proper hypercovering* of (U, \bar{U}) if the following conditions are satisfied:

- (1) (U_\bullet, X_\bullet) is augmented to (U, \bar{U}) over κ ,
- (2) The natural morphism $U_{n+1} \longrightarrow \text{cosk}_n^U(U_{\bullet \leq n})_{n+1}$ is proper and surjective for any $n \in \mathbb{N}$,
- (3) $U_n = U \times_{\bar{U}} X_n$ for any $n \in \mathbb{N}$.

The following is one of main results of this report:

Theorem 4.2. ([Nak2]) *Let the notations be as above. Then the following hold:*

- (1) *If (U_\bullet, X_\bullet) is a split proper hypercovering of (U, \bar{U}) , then there exists a canonical isomorphism*

$$(4.2.1) \quad R\Gamma_{\text{rig}}(U/K) \xrightarrow{\sim} R\Gamma((X_\bullet, D_\bullet)/\mathcal{W}) \otimes_{\mathcal{W}} K.$$

- (2) *Let c be an integer such that $H_{\text{rig}}^h(U/K) = 0$ for all $h \geq c$. (We can show the existence of c .) Let N be an integer such that there exists a positive integer r satisfying the following two inequalities: $c \leq 2^{-1}r(r-1)$ and $N \geq 2^{-1}r(r+1)$.*

Assume that there exists a closed immersion $(X_N, D_N) \xrightarrow{\subset} (\mathcal{P}, \mathcal{M})$ into a fine log smooth scheme over $\mathrm{Spf} \mathcal{W}$ such that the underlying formal scheme \mathcal{P} is also formally smooth over $\mathrm{Spf} \mathcal{W}$.

Then there exists a canonical isomorphism

$$(4.2.2) \quad R\Gamma_{\mathrm{rig}}(U/K) \xrightarrow{\sim} \tau_N R\Gamma((X_{\bullet}, D_{\bullet})/\mathcal{W}) \otimes_{\mathcal{W}} K.$$

Remark 4.3. (1) Over the complex number field, we have an equality

$$(4.3.1) \quad R\Gamma(U^{\mathrm{an}}, \mathbb{Q}) = R\Gamma(U_{\bullet}^{\mathrm{an}}, \mathbb{Q})$$

[D3]. In (4.2), the reader should note that we have proved the coincidence of cohomologies in two different cohomology theories; (4.2.1) is harder than (4.3.1).

(2) In the constant simplicial case, the isomorphism (4.2.1) immediately follows from Shiho's comparison theorem $H_{\mathrm{rig}}^h(U/K) \xrightarrow{\sim} H^h((X, D)/\mathcal{W}) \otimes_{\mathcal{W}} K$ [S, Cor. 2.4.13, Thm. 3.1.1].

(3) The proof of (4.2) essentially uses an argument of (a generalization of) the proof in [T] of the spectral sequence

$$(4.3.2) \quad E_1^{i, h-i} = H_{\mathrm{rig}}^{h-i}(U_i/K) \implies H_{\mathrm{rig}}^h(U/K)$$

and uses arguments of the proofs for [S, Thm. 2.4.4, Thm. 3.1.1].

(4) The right hand side of (4.2.1) depends only on U and K ; this solves a problem raised in [dJ, Intro.] for the split case. In fact, (4.4) below tells us that the weight filtration on $H^h((X_{\bullet}, D_{\bullet})/\mathcal{W}) \otimes_{\mathcal{W}} K$ depends only on U and K .

(5) I think that the assumption on the embedding in (4.2) (2) is mild.

(6) By (4.2) and a standard argument in [Bl, III (3.2), (3.4)] (cf. [I2, II (3.2), (3.5)]), we obtain

$$(4.3.3) \quad H_{\mathrm{rig}}^h(U/K)_{[i, i+1]} = H^{h-i}(X_{\bullet}, \mathcal{W}\Omega_{X_{\bullet}}^i(\log D_{\bullet})) \otimes_{\mathcal{W}} K.$$

In particular, $H^{h-i}(X_{\bullet}, \mathcal{W}\Omega_{X_{\bullet}}^i(\log D_{\bullet})) \otimes_{\mathcal{W}} K$ depends only on U and K . Furthermore, we can endow $H^{h-i}(X_{\bullet}, \mathcal{W}\Omega_{X_{\bullet}}^i(\log D_{\bullet})) \otimes_{\mathcal{W}} K$ with the weight filtration P [Nak2] (cf. [Nak1, §5]).

(7) We have generalized (4.2) for certain overconvergent F -isocrystals when $\mathcal{V} = \mathcal{W}$ (cf. [S, Cor. 2.4.13, Thm. 3.1.1]).

By (3.0.5) and (4.2), we have the following spectral sequence:

$$(4.3.4) \quad E_1^{-k, h+k} = \bigoplus_{i \geq 0} H^{h-2i-k}(\widetilde{(D_i^{(i+k)})/\mathcal{W}})_{\mathrm{crys}}, \mathcal{O}_{D_i^{(i+k)}/\mathcal{W}} \otimes_{\mathbb{Z}} \varpi_{\mathrm{crys}}^{(i+k)}(D_i/\mathcal{W}) \otimes_{\mathcal{W}} K \implies H_{\mathrm{rig}}^h(U/K).$$

Theorem-Definition 4.4. ([Nak2]) There exists a well-defined finite increasing filtration $\{P_k\}_{k \in \mathbb{Z}}$ on $H_{\mathrm{rig}}^h(U/K)$ which is calculated by the spectral sequence (4.3.4). We call this filtration the *weight filtration* on $H_{\mathrm{rig}}^h(U/K)$.

Though we do not know a p -adic analogue of the theorem for a morphism in (MHS/ \mathbb{Q}) in (1.0.1), we can prove the following by using the specialization argument of Deligne-Illusie [I1, 3.10] (cf. [Nak1, §3]).

Theorem 4.5. ([Nak2]) *Let $f: U \rightarrow V$ be a morphism of separated schemes of finite type over κ . Then the induced morphism $f^*: H_{\text{rig}}^h(V/K) \rightarrow H_{\text{rig}}^h(U/K)$ is strictly compatible with the weight filtration.*

Remark 4.6. (1) We have proved p -adic analogues of theorems in [D3], e.g., [D3, (8.2.4) (ii) (iii), (8.2.5) \sim (8.2.11)].

(2) Assume that κ is algebraically closed. Assume, also, that U and X_0 are connected (for simplicity). In [Nak2], as in [KH], by using dga's, the bar construction and the Thom-Whitney functor, we have defined a unipotent rigid fundamental group scheme $\pi_1^{\text{rig}}(U/K, *)$, a simplicial unipotent crystalline fundamental group scheme $\pi_1^{\text{log-crys}}((X_\bullet, D_\bullet)/K, *_0)$ and a simplicial unipotent de Rham-Witt fundamental group scheme $\pi_1^{\text{dRW}}((X_\bullet, D_\bullet)/K, *_0)$. We have proved

$$\pi_1^{\text{rig}}(U/K, *) = \pi_1^{\text{log-crys}}((X_\bullet, D_\bullet)/K, *_0) = \pi_1^{\text{dRW}}((X_\bullet, D_\bullet)/K, *_0)$$

if $(X_\bullet, D_\bullet, *_0)$ is a pointed split proper hypercovering of $(U, \bar{U}, *)$ (cf. [H, §6]).

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