Théorie de Hodge I, II, III pour cohomologies p-adiques.

Yukiyoshi Nakkajima *

Resumé: Dans ce rapport, nous présentons quelques résultats démontrés dans [NS] et [Nak2]. Nous avons construit deux objets filtrés fondamentaux: *un complexe cristallin filtré par le prépoids* et *un complexe zariskian par le prépoids.* Nous avons construit aussi la filtration par le poids sur la cohomologie rigide d'un schéma séparé de type fini sur un corps parfait de caractéristique $p > 0$.

Mots-clefs: cohomologies rigides, cohomologies log-cristallines, suites p-adiques spectrales des poids.

1 Introduction.

Motivated by a conjectural theory of motives due to A. Grothendieck, P. Deligne has given a translation of concepts on mixed Hodge structures into those on l-adic cohomologies and vice versa in [D1], and he has proved his conjectures on mixed Hodge structures in [D2] and [D3].

Let U be a separated scheme of finite type over a field κ. Then there is the following translation [D1], [D2], [D3]:

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(1.0.1)

l-adic objects/$\mathbb{F}_q$ (for simplicity), $(l,q)=1$	objects/$\mathbb{C}$
$H^h_{\rm et}(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l) \quad (h \in \mathbb{Z})$	$H^h(U^{\rm an}, \mathbb{Q}) \quad (h \in \mathbb{Z})$
F: geometric Frobenius $\curvearrowright H^h_{\rm et}(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$; there exists a unique finite increasing filtration $\{P_k\}_{k \in \mathbb{Z}}$ characterized by the following: $P_k H^h_{\rm et}(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$ is the principal subspace of $H^h_{\rm et}(U_{\overline{\mathbb{F}}_q}, \mathbb{Q}_l)$ where the eigenvalues α of F satisfy the following: $\lvert\sigma(\alpha)\rvert \leq q^{k/2}$ $(\forall \sigma \colon \overline{\mathbb{Q}} \overset{\subset}{\longrightarrow} \mathbb{C})$ (cf. [dJ]).	(weight filtrations)+ (Hodge ones) =:(mixed Hodge structures).
A morphism commuting F's is strictly compatible with P_k's.	A morphism in the category (MHS/$\mathbb{Q}$) is strictly compatible with the weight filtration and the Hodge filtration.

Let us consider a more special case as in [D2]. Let (X, D) be a smooth scheme over κ with a SNCD(=simple normal crossing divisor) over κ. Put $U := X \setminus D$ and let $j \colon U \overset{\subset}{\longrightarrow} X$ be the natural open immersion. Put $M_D := \{g \in \mathcal{O}_X \mid g$ is invertible outside $D\}$. Let $\epsilon \colon \widetilde{X}^{\rm log}_{\rm et} \longrightarrow \widetilde{X}_{\rm et}$ be the forgetting log morphism induced by a natural morphism $(X, M_D) \longrightarrow (X, \mathcal{O}^*_X)$ of log schemes in the sense of Fontaine-Illusie-Kato [K]. Let Y be an analytic variety over $\mathbb{C}$, and let us also denote by ϵ the real blow up $Y^{\rm log} \longrightarrow Y$ [KN] which is denoted by τ in [loc. cit.]. Then we have the following translation:

(1.0.2)

objects/$\mathbb{C}$	l-adic objects
$U^{\rm an}$, $(X^{\rm an})^{\rm log}$	$\widetilde{U}_{\rm et}$, $\widetilde{X}^{\rm log}_{\rm et}$
$X^{\rm an}$	$\widetilde{X}_{\rm et}$
$j^{\rm an} \colon U^{\rm an} \overset{\subset}{\longrightarrow} X^{\rm an}, \quad \epsilon \colon (X^{\rm an})^{\rm log} \longrightarrow X^{\rm an}$	$j_{\rm et} \colon \widetilde{U}_{\rm et} \longrightarrow \widetilde{X}_{\rm et}, \quad \epsilon \colon \widetilde{X}^{\rm log}_{\rm et} \longrightarrow \widetilde{X}_{\rm et}$
$Rj^{\rm an}_* \mathbb{Z} = R\epsilon_* \mathbb{Z}$ [KN]	$Rj_{{\rm et}*}(\mathbb{Z}/l^n) = R\epsilon_*(\mathbb{Z}/l^n)$ [Fu], [FK]
$X^{\rm an} \longrightarrow X$	${\rm id} \colon \widetilde{X}_{\rm et} \longrightarrow \widetilde{X}_{\rm et}$
$(\Omega^\bullet_{X/\mathbb{C}}(\log D), P_k)$	?
$(\Omega^\bullet_{X^{\rm an}/\mathbb{C}}(\log D^{\rm an}), \tau_k) = (\Omega^\bullet_{X^{\rm an}/\mathbb{C}}(\log D^{\rm an}), P_k)$	?

The purpose of this report is to inform the reader that we have succeeded in making two translations (1.0.1) and (1.0.2) for p-adic cohomologies.

This report came out of two preprints [NS], [Nak2] and my lecture entitled with *Théorie de Hodge* III *pour cohomologies p-adiques* at a symposium *Hodge theory and algebraic geometry* at Hokkaido-University in October in 2002. We give no proof in this report; we have given proofs in [NS] and [Nak2].

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Conventions. Let $\mathcal{A}$ be an abelian category.

(1) For a complex $(E^\bullet, d^\bullet)$ of objects in $\mathcal{A}$ and for an integer n, $(E^{\bullet+n}, d^{\bullet+n})$ or $(E^\bullet\{n\}, d^\bullet\{n\})$ denotes the following complex: $\cdots \longrightarrow \underset{q}{E^{q+n}} \overset{d^{q+n}}{\longrightarrow} \underset{q+1}{E^{q+1+n}} \overset{d^{q+1+n}}{\longrightarrow} \cdots$. Here the numbers under the objects above in $\mathcal{A}$ mean the degrees.

(2) For a complex $(E^\bullet, d^\bullet)$ of objects in $\mathcal{A}$ and for an integer n, $(E^\bullet[n], d^\bullet[n])$ denotes the following complex as usual: $(E^\bullet[n])^q := E^{q+n}$ with boundary morphisms $d^\bullet[n] = (-1)^n d^{\bullet+n}$.

(3) For a complex $(E^\bullet, d^\bullet)$ of objects in $\mathcal{A}$, the identity $\mathrm{id}\colon E^q \longrightarrow E^q$ $(\forall q \in \mathbb{Z})$ induces an isomorphism $\mathcal{H}^q((E^\bullet, -d^\bullet)) \overset{\sim}{\longrightarrow} \mathcal{H}^q((E^\bullet, d^\bullet))$ $(\forall q \in \mathbb{Z})$ of cohomologies.

2 Constant simplicial open schemes.

In this section we state some results which Shiho and I have obtained in [NS].

Let $(S, \mathcal{I}, \gamma)$ be a PD-scheme with quasi-coherent ideal $\mathcal{I} \subset \mathcal{O}_S$. Put $S_0 := \underline{\mathrm{Spec}}_S(\mathcal{O}_S/\mathcal{I})$. Let $f\colon (X, D) \longrightarrow S_0$ be a smooth scheme with a relative SNCD(=simple normal crossing divisor) over S_0. By abuse of notation, we denote the composite morphism $(X, D) \longrightarrow S_0 \overset{\subset}{\longrightarrow} S$ also by f. Then (X, D) gives an fs log structure M_D on X [NS] (cf. [K, p. 222–223], [Fa, §2 (c)]). Let $j\colon X \setminus D \overset{\subset}{\longrightarrow} X$ be the natural open immersion. Note that $M_D \subsetneqq \mathcal{O}_X \cap j_*(\mathcal{O}_U^*)$ in general; indeed, the sections of $(\mathcal{O}_X \cap j_*(\mathcal{O}_U^*))/\mathcal{O}_X^*$ over an affine open subscheme of X is not even finitely generated in general [NS].

Let $\Delta := \{D_\lambda\}_{\lambda \in \Lambda}$ be a decomposition of D by smooth components of D, where Λ is a set: $D = \bigcup_{\lambda \in \Lambda} D_\lambda$ and each D_λ is smooth over S_0. Let k be a positive integer. Put $D^{(k)} := \coprod_{\{\lambda_1, \ldots, \lambda_k \,|\, \lambda_i \neq \lambda_j\ (i \neq j)\}} D_{\lambda_1} \cap \cdots \cap D_{\lambda_k}$. Then $D^{(k)}$ has been shown to be independent of the choice of the decomposition of D by smooth components of D [NS]. Put $D^{(0)} := X$.

For simplicity of notation, we denote the log crystalline topos $((\widetilde{X, M_D})/S)^{\log}_{\mathrm{crys}}$ simply by $(\widetilde{X/S})^{\log}_{\mathrm{crys}}$. Let $(\widetilde{X/S})_{\mathrm{crys}}$ be the usual crystalline topos. Let $\mathcal{O}_{X/S}$ (resp. $\overset{\circ}{\mathcal{O}}_{X/S}$) be the structure sheaf in $(\widetilde{X/S})^{\log}_{\mathrm{crys}}$ (resp. $(\widetilde{X/S})_{\mathrm{crys}}$).

Definition 2.1. ([NS]) Let $\iota\colon (X, D) \overset{\subset}{\longrightarrow} (\mathcal{X}, \mathcal{D})$ be an exact closed immersion into a smooth scheme with a relative SNCD over S. Then we call ι an *admissible immersion with respect to* Δ if $\mathcal{D}$ has a decomposition $\widetilde{\Delta} = \{\mathcal{D}_\lambda\}_{\lambda \in \Lambda}$ by smooth components of $\mathcal{D}$ and if ι induces an isomorphism $D_\lambda \overset{\sim}{\longrightarrow} X \times_{\mathcal{X}} \mathcal{D}_\lambda$ of schemes for all $\lambda \in \Lambda$.

Next let us recall and define orientation sheaves for $D^{(k)}/S_0$ and $D^{(k)}/S$ ($k \in \mathbb{N}$) (cf. [D2, (3.1.4)]).

Let E be a non-empty finite set with cardinality k. Put $\varpi_E := \wedge^k \mathbb{Z}^E$. Let k be a positive integer. Let P be a point of $D^{(k)}$. Let $\{D_\lambda\}_{\lambda\in\Lambda}$ be a decomposition of D by smooth components of D. Let $D_{\lambda_0}, \dots, D_{\lambda_k}$ be distinct smooth components of D such that $D_{\lambda_0} \cap \cdots \cap D_{\lambda_k}$ contains P. Then, the set $\{D_{\lambda_0}, \dots, D_{\lambda_k}\}$ gives an orientation sheaf $\varpi_{\lambda_0\cdots\lambda_k,\mathrm{zar}}(D/S_0)$ on a local neighborhood of P in $D^{(k)}$. The sheaf $\varpi_{\lambda_0\cdots\lambda_k,\mathrm{zar}}(D/S_0)$ is globalized, and we denote this globalized sheaf by the same symbol $\varpi_{\lambda_0\cdots\lambda_k,\mathrm{zar}}(D/S_0)$. Put $\varpi^{(k)}_{\mathrm{zar}}(D/S_0) := \oplus_{\{\lambda_0,\dots\lambda_k\}} \varpi_{\lambda_0\cdots\lambda_k,\mathrm{zar}}(D/S_0)$. Put $\varpi^{(k)}_{\mathrm{zar}}(D/S_0) := \mathbb{Z}_X$ for $k = 0$. The sheaf $\varpi^{(k)}_{\mathrm{zar}}(D/S_0)$ is extended to a sheaf $\varpi^{(k)}_{\mathrm{crys}}(D/S)$ in the usual crystalline topos $(\widetilde{D^{(k)}/S})_{\mathrm{crys}}$.

Definition 2.2. We call $\varpi^{(k)}_{\mathrm{zar}}(D/S_0)$ (resp. $\varpi^{(k)}_{\mathrm{crys}}(D/S)$) the *zariskian orientation sheaf* (resp. *crystalline orientation sheaf*) of $D^{(k)}/S_0$ (resp. $D^{(k)}/(S, \mathcal{I}, \gamma)$).

The sheaves $\varpi^{(k)}_{\mathrm{zar}}(D/S_0)$, $\varpi^{(k)}_{\mathrm{crys}}(D/S)$ are defined by the local nature of D; they are independent of the choice of the decomposition of D by smooth components of D.

If S_0 is of characteristic $p > 0$, we define the Frobenius action on the orientations sheaves above. Let $F\colon (X, D) \longrightarrow (X', D')$ be the relative Frobenius morphism over S_0. The morphism F induces the relative Frobenius morphism $F^{(k)}\colon D^{(k)} \longrightarrow D'^{(k)} = D^{(k)\prime}$. Let $a^{(k)}\colon D^{(k)} \longrightarrow X$ and $a^{(k)\prime}\colon D^{(k)\prime} \longrightarrow X'$ be the natural morphisms. Then we define the following two Frobenius morphisms

$$\Phi^{(k)}\colon a^{(k)\prime}_{\mathrm{crys}*}\varpi^{(k)}_{\mathrm{crys}}(D'/S) \longrightarrow F_{\mathrm{crys}*}a^{(k)}_{\mathrm{crys}*}\varpi^{(k)}_{\mathrm{crys}}(D/S) \tag{2.2.1}$$

and

$$\Phi^{(k)}\colon a^{(k)\prime}_{*}\varpi^{(k)}_{\mathrm{zar}}(D'/S_0) \longrightarrow F_{*}a^{(k)}_{*}\varpi^{(k)}_{\mathrm{zar}}(D/S_0) \tag{2.2.2}$$

by the multiplication by p^k under the natural identifications

$$\varpi^{(k)}_{\mathrm{crys}}(D'/S) \xrightarrow{\sim} F^{(k)}_{\mathrm{crys}*}\varpi^{(k)}_{\mathrm{crys}}(D/S), \quad \varpi^{(k)}_{\mathrm{zar}}(D'/S) \xrightarrow{\sim} F^{(k)}_{*}\varpi^{(k)}_{\mathrm{zar}}(D/S).$$

In [NS] we have proved the following:

Theorem-Definition 2.3. ([NS]) Let $(S, \mathcal{I}, \gamma)$ and $(X, D)/S_0$ be as in the beginning of this section. Then there exist two filtered objects

$$(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k) = (\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}))_{k\in\mathbb{Z}}$$

and

$$(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k) = (\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}))_{k\in\mathbb{Z}}$$

in the filtered derived categories $\mathrm{D}^+\mathrm{F}(\overset{\circ}{\mathcal{O}}_{X/S})$ of bounded below complexes of $\overset{\circ}{\mathcal{O}}_{X/S}$-modules and $\mathrm{D}^+\mathrm{F}(f^{-1}(\mathcal{O}_S))$ of bounded below complexes of $f^{-1}(\mathcal{O}_S)$-modules, respectively. We have called $(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$ and $(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k)$ the *preweight-filtered crystalline complex* of $(X,D)/(S,\mathcal{I},\gamma)$ and the *preweight-filtered zariskian complex* of $(X,D)/(S,\mathcal{I},\gamma)$, respectively. They enjoy the following properties:

(1; c): $\{P_k\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S})\}_{k\in\mathbb{Z}}$ is an increasing "filtration" on $\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S})$ which is finite locally on $(X/S)_{\mathrm{crys}}$ such that $P_{-1}\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}) = 0$, $P_0\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}) \overset{\sim}{\longleftarrow} \overset{\circ}{\mathcal{O}}_{X/S}$, and $\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}) \overset{\sim}{\longleftarrow} R\epsilon_{X/S*}(\mathcal{O}_{X/S})$.

(2; c): If (X,D) has an admissible immersion $(X,D) \overset{\subset}{\longrightarrow} (\mathcal{X},\mathcal{D})$ over S with respect to Δ, then

$$(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k) \simeq (\overset{\circ}{L}(\Omega^{\bullet}_{\mathcal{X}/S}(\log\mathcal{D})), \overset{\circ}{L}(P_k\Omega^{\bullet}_{\mathcal{X}/S}(\log\mathcal{D}))),$$

where $\overset{\circ}{L}$ is the classical linearization functor for $\mathcal{O}_{\mathcal{X}}$-modules [Be, Chap. IV, 3].

(3): $(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k) = R\overset{\circ}{u}_{X/S*}(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$.

In particular,

(1; z): $\{P_k\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S})\}_{k\in\mathbb{Z}}$ is an increasing "filtration" on $\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S})$ which is finite locally on X such that $P_{-1}\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}) = 0$, $P_0\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}) \overset{\sim}{\longleftarrow} R\overset{\circ}{u}_{X/S*}\overset{\circ}{\mathcal{O}}_{X/S}$, and $\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}) \overset{\sim}{\longleftarrow} Ru_{X/S*}(\mathcal{O}_{X/S})$,

and

(2; z): If (X,D) has an admissible immersion $(X,D) \overset{\subset}{\longrightarrow} (\mathcal{X},\mathcal{D})$ over S with respect to Δ,

$$(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k) \simeq (\mathcal{O}_{\mathcal{Y}} \otimes_{\mathcal{O}_{\mathcal{X}}} \Omega^{\bullet}_{\mathcal{X}/S}(\log\mathcal{D}), \mathcal{O}_{\mathcal{Y}} \otimes_{\mathcal{O}_{\mathcal{X}}} P_k\Omega^{\bullet}_{\mathcal{X}/S}(\log\mathcal{D})),$$

where $\mathcal{Y}$ is the PD-envelope of the closed immersion $X \overset{\subset}{\longrightarrow} \mathcal{X}$.

(4; c): Let $\mathrm{gr}_k^P\colon \mathrm{D}^+\mathrm{F}(\overset{\circ}{\mathcal{O}}_{X/S}) \longrightarrow \mathrm{D}^+(\overset{\circ}{\mathcal{O}}_{X/S})$ ($k \in \mathbb{Z}$) be the gr-functor [NS]. Then $\mathrm{gr}_k^P((\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_l)) = a^{(k)}_{\mathrm{crys}*}(\mathcal{O}_{D^{(k)}/S} \otimes_{\mathbb{Z}} \varpi^{(k)}_{\mathrm{crys}}(D/S))\{-k\}$.

(4; z): Let $\mathrm{gr}_k^P\colon \mathrm{D}^+\mathrm{F}(f^{-1}(\mathcal{O}_S)) \longrightarrow \mathrm{D}^+(f^{-1}(\mathcal{O}_S))$ ($k \in \mathbb{Z}$) be the gr-functor. Then $\mathrm{gr}_k^P((\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_l)) = a^{(k)}_{*}(Ru_{D^{(k)}/S*}(\mathcal{O}_{D^{(k)}/S}) \otimes_{\mathbb{Z}} \varpi^{(k)}_{\mathrm{zar}}(D/S_0))\{-k\}$.

(5): $(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), \tau_k) \overset{\sim}{\longrightarrow} (\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$, where τ_k is the canonical filtration.

(6): If S_0 is the spectrum of a perfect field κ of characteristic p and if S is the spectrum of the Witt ring W_n of κ of finite length $n > 0$, then $(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k)$ is canonically isomorphic to the filtered object $(W_n\Omega^{\bullet}_X(\log D), P_kW_n\Omega^{\bullet}_X(\log D))$ in [M1] and [M2].

We give important remarks on (2.3).

(5) is equivalent to the following *p-adic purity*:

$$R^k\epsilon_{X/S*}\mathcal{O}_{X/S} = a^{(k)}_{\mathrm{crys}*}(\mathcal{O}_{D^{(k)}/S}\otimes_{\mathbb{Z}}\varpi^{(k)}_{\mathrm{crys}}(D/S)). \tag{2.3.1}$$

Logically speaking, in [NS], we have first proved (2.3.1) by using $(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$ and then we have proved (5).

By (5), $(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$ and $(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k)$ are functorial; that is, for another smooth scheme Y with a relative SNCD E over S_0 and for a morphism $g\colon (X,D)\longrightarrow (Y,E)$ of log schemes in the sense of Fontaine-Illusie-Kato, we have natural morphisms

$$g^*_{\mathrm{crys}}\colon (\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{Y/S}), P_k)\longrightarrow Rg_{\mathrm{crys}*}(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$$

and

$$g^*\colon (\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{Y/S}), P_k)\longrightarrow Rg_*(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k).$$

By (1; c) and (4; c), we have the following spectral sequence:

Corollary-Definition 2.4. ([NS]) There exists the following functorial spectral sequence

$$E_1^{-k,h+k} = R^{h-k}f_{D^{(k)}/S*}(\mathcal{O}_{D^{(k)}/S}\otimes_{\mathbb{Z}}\varpi^{(k)}_{\mathrm{crys}}(D/S)) \Longrightarrow R^hf_{X/S*}(\mathcal{O}_{X/S}). \tag{2.4.1}$$

We call the spectral sequence (2.4.1) the *preweight spectral sequence* of $(X,D)/(S,\mathcal{I},\gamma)$.

Remark 2.5. Though we have obtained deep other theorems in [NS], we can only mention them because of lack of spaces: (pre)weight-filtered base change formula, (pre)weight-filtered Künneth formula, weight-filtered Poincaré duality, a weight-filtered Berthelot-Ogus isomorphism, theory for compact support cohomology, the convergence of weight filtrations. See [NS] for details.

If S is a p-adic formal V-scheme in the sense of [O, §1], we obtain the following translation as a conclusion:

(2.5.1)

$/\mathbb{C}$	crystal
U^{an}, $(X^{\mathrm{an}})^{\log}$	$\widetilde{(X/S)}^{\log}_{\mathrm{crys}}$
X^{an}	$\widetilde{(X/S)}_{\mathrm{crys}}$
$j^{\mathrm{an}}\colon U^{\mathrm{an}}\overset{\subset}{\longrightarrow} X^{\mathrm{an}}$, $\epsilon\colon (X^{\mathrm{an}})^{\log}\longrightarrow X^{\mathrm{an}}$	$\epsilon_{X/S}\colon \widetilde{(X/S)}^{\log}_{\mathrm{crys}}\longrightarrow \widetilde{(X/S)}_{\mathrm{crys}}$
$Rj^{\mathrm{an}}_*\mathbb{Z} = R\epsilon_*\mathbb{Z}$	$R\epsilon_{X/S*}(\mathcal{O}_{X/S})$
$X^{\mathrm{an}}\longrightarrow X$	$\overset{\circ}{u}_{X/S}\colon \widetilde{(X/S)}_{\mathrm{crys}}\longrightarrow \widetilde{X}_{\mathrm{zar}}$
$(\Omega^\bullet_{X/\mathbb{C}}(\log D), P_k)$	$(\mathcal{C}_{\mathrm{zar}}(\mathcal{O}_{X/S}), P_k)$
$(\Omega^\bullet_{X^{\mathrm{an}}/\mathbb{C}}(\log D^{\mathrm{an}}), \tau_k) = (\Omega^\bullet_{X^{\mathrm{an}}/\mathbb{C}}(\log D^{\mathrm{an}}), P_k)$	$(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), \tau_k) = (\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X/S}), P_k)$

Remark 2.6. Let $\mathcal{V}$ be a complete discrete valuation ring of mixed characteristics $(0,p)$. Let κ (resp. K) be the (not necessarily perfect) residue (resp. fraction) field of V. Let $\sigma \in \operatorname{Aut}\mathcal{V}$ be a fixed lift of the p-th power endomorphism of κ. Let (X,D) be a smooth scheme with a SNCD over κ. Fix a decomposition $\{D_\lambda\}_{\lambda\in\Lambda}$ of D by smooth components of D and fix a total order on Λ. Assume that there exists a closed immersion $X \overset{\subset}{\longrightarrow} \mathcal{P}$ into a formal V-scheme such that $\mathcal{P}/\operatorname{Spf} V$ is formally smooth around X. In [CL, (3.8)], Chiarellotto and Le Stum have constructed the following p-adic weight spectral sequence by the method of local rigid cohomologies:

$$E_{1,\mathrm{rig}}^{-k,h+k} = H^{h-k}_{\mathrm{rig}}(D^{(k)}/K)(-k) \Longrightarrow H^h_{\mathrm{rig}}(U/K). \tag{2.6.1}$$

More generally, in (4.3.4) below, we shall construct a p-adic weight spectral sequence for the rigid cohomology of a separated scheme of finite type over κ; our method is a complicated p-adic version of that in [D3], and it is different from theirs.

3 Simplicial open schemes.

Let $(S,\mathcal{I},\gamma)$ and S_0 be as in §2. Let $f\colon (X_\bullet,D_\bullet)_{\bullet\in\mathbb{N}} \longrightarrow S_0$ be a simplicial smooth scheme with a simplicial relative SNCD over S_0. The morphism f induces a morphism $f_\bullet\colon (X_\bullet,D_\bullet)_{\bullet\in\mathbb{N}} \longrightarrow S_{0\bullet}$, where $S_{0\bullet}$ is the constant simplicial scheme defined by S_0. By abuse of notation, we denote by f (resp. $f_\bullet$) the composite morphism $(X_\bullet,D_\bullet) \longrightarrow S_0 \overset{\subset}{\longrightarrow} S$ (resp. $(X_\bullet,D_\bullet) \longrightarrow S_{0\bullet} \overset{\subset}{\longrightarrow} S_\bullet$). Furthermore, for simplicity of notation, we sometimes denote simply by f the morphism $f_t\colon (X_t,D_t) \longrightarrow S_0$ ($t\in\mathbb{N}$) and also by f the composite morphism $(X_t,D_t) \overset{f_t}{\longrightarrow} S_0 \overset{\subset}{\longrightarrow} S$. We have a log crystalline topos $(\widetilde{X_\bullet/S})^{\log}_{\mathrm{crys}} := (\widetilde{(X_\bullet,M_{D_\bullet})/S})^{\log}_{\mathrm{crys}}$ and a usual crystalline topos $(\widetilde{X_\bullet/S})_{\mathrm{crys}}$. Let $\mathcal{O}_{X_\bullet/S}$ (resp. $\overset{\circ}{\mathcal{O}}_{X_\bullet/S}$) be the structure sheaf in $(\widetilde{X_\bullet/S})^{\log}_{\mathrm{crys}}$ (resp. $(\widetilde{X_\bullet/S})_{\mathrm{crys}}$). The morphisms f and $f_\bullet$ induce morphisms of topoi

$$f_{X_\bullet/S}\colon (\widetilde{X_\bullet/S})^{\log}_{\mathrm{crys}} \longrightarrow \widetilde{S}_{\mathrm{zar}} \quad\text{and}\quad f_{X_\bullet/S_\bullet}\colon (\widetilde{X_\bullet/S})^{\log}_{\mathrm{crys}} \longrightarrow \widetilde{S}_{\bullet\mathrm{zar}},$$

$$\overset{\circ}{f}_{X_\bullet/S}\colon (\widetilde{X_\bullet/S})_{\mathrm{crys}} \longrightarrow \widetilde{S}_{\mathrm{zar}} \quad\text{and}\quad \overset{\circ}{f}_{X_\bullet/S_\bullet}\colon (\widetilde{X_\bullet/S})_{\mathrm{crys}} \longrightarrow \widetilde{S}_{\bullet\mathrm{zar}}.$$

Then we have a cosimplicial filtered object $R\overset{\circ}{f}_{X_\bullet/S_\bullet}(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X_\bullet/S}),P_k) \in \mathrm{D}^+\mathrm{F}(f_\bullet^{-1}(\mathcal{O}_S))$ by the functoriality of $(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X_\bullet/S}),P_k)$. Let $(\mathcal{K},P_k)$ be a representative of the filtered object $R\overset{\circ}{f}_{X_\bullet/S_\bullet}(\mathcal{C}_{\mathrm{crys}}(\mathcal{O}_{X_\bullet/S}),P_k)$. The filtered complex $(\mathcal{K},P_k)$ defines a filtered double complex $(\mathcal{K}^{\bullet\bullet},P_k)$ (cf. [D3, (5.1.9) (IV)]). Here the first degree is the cosimplicial degree. We make the following convention on the signs of boundary morphisms of the single complex $\mathbf{s}\mathcal{K}$ of $\mathcal{K}^{\bullet\bullet}$:

$$(\mathbf{s}\mathcal{K})^n = \bigoplus_{i+j=n}\mathcal{K}^{ij};\quad d(x^{ij}) = \sum_{s=0}^{i+1}(-1)^s\delta^*_s(x^{ij}) + (-1)^i d_{\mathcal{K}}(x^{ij}), \tag{3.0.1}$$

where $\delta_s\colon (X_{i+1},D_{i+1}) \longrightarrow (X_i,D_i)$ ($0\leq s\leq i{+}1$) is a standard face morphism (see, e.g., [D3, (5.1.1)]) and $d_{\mathcal{K}}\colon \mathcal{K}^{ij} \longrightarrow \mathcal{K}^{i+1,j}$ is the boundary morphism arising from the boundary morphism of the filtered complex $(\mathcal{C}_{\rm crys}(\mathcal{O}_{X_\bullet/S}),P_k)$. Our convention on the turn of the degrees is the same as that in [NA, (2.3)] and different from that in [D3, (5.1.9) (IV)] and in [CT, (3.9)]; our convention on the signs of boundary morphisms of $\mathbf{s}\mathcal{K}$ is superior to that in [D3, (5.1.9.2)] and in [CT, (3.9)]; if we follow the convention in [D3, (5.1.9.2)], we have to consider distinct signs before $(-1)^s\delta_s^*$ with respect to the degrees arising from the complex $(\mathcal{C}_{\rm crys}(\mathcal{O}_{X_\bullet/S}),P_k)$; we should eliminate signs $(-1)^p$ in [D3, (5.1.9.2)] by following our convention because these morphisms with various signs are not induced by morphisms of algebro-geometric objects. If we follow the convention in [loc. cit.], it seems to me that it is impossible to give a description of the boundary morphisms in [D3, p. 35]; the diagram in [D3, p. 35] is not a part of a double complex; since it is commutative, it is mistaken. Furthermore, "Gysin" in the diagram in [loc. cit.] is not clear. See [Nak2] for a correction and the explicit formula of "Gysin".

Let L be the stupid filtration of $\mathbf{s}\mathcal{K}$ with respect to the first degree:

(3.0.2)
$$L^i(\mathbf{s}\mathcal{K}) = \bigoplus_{i'\geq i}\mathcal{K}^{i'\bullet}.$$

Let $(S,\mathcal{I},\gamma)$ and S_0 be as in §2. Now, let us construct the *preweight spectral sequence* of $(X,D)/(S,\mathcal{I},\gamma)$. Let $\delta(P,L)$ be the diagonal filtration of P and L ([D3, (7.1.6.1)]):

(3.0.3)
$$\delta(P,L)_k(\mathbf{s}\mathcal{K}) = \bigoplus_{i,j\geq 0}P_{i+k}\mathcal{K}^{ij} = \sum_{i\geq 0}(\mathbf{s}P_{i+k}\mathcal{K})\cap L^i(\mathbf{s}\mathcal{K}).$$

(Note that the formula [D3, (7.1.6.1)] should be replaced by $\bigoplus_{p,q}W_{n+p}(K^{q,p})$.) Then we have $\mathrm{gr}_k^{\delta(P,L)}(\mathbf{s}\mathcal{K}) = \bigoplus_{i\geq 0}\mathrm{gr}^P_{i+k}\mathcal{K}^{i\bullet}[-i]$. Hence we have the following spectral sequence by the Convention (3) (cf. [D3, (8.1.15)]):

(3.0.4)
$$E_1^{-k,h+k}((X_\bullet,D_\bullet)/S) = \bigoplus_{i\geq 0}\mathcal{H}^{h-i}(\mathrm{gr}^P_{i+k}\mathcal{K}^{i\bullet}) \Longrightarrow \mathcal{H}^h(\mathbf{s}\mathcal{K}).$$

By (1; c) and (4; c), the spectral sequence (3.0.4) is equal to the following spectral sequence

(3.0.5)
$$E_1^{-k,h+k}((X_\bullet,D_\bullet)/S) = \bigoplus_{i\geq 0}R^{h-2i-k}f_{D_i^{(i+k)}/S*}(\mathcal{O}_{D_i^{(i+k)}/S}\otimes_{\mathbb{Z}}\varpi^{(i+k)}_{\rm crys}(D_i/S)) \Longrightarrow R^hf_{X_\bullet/S*}(\mathcal{O}_{X_\bullet/S}).$$

Definition 3.1. We call (3.0.5) the *preweight spectral sequence* of $(X,D)/(S,\mathcal{I},\gamma)$. If S is a p-adic formal V-scheme in the sense of [O, §1], we call (3.0.5) the *p-adic weight spectral sequence* of $(X,D)/(S,\mathcal{I},\gamma)$. We denote by $\{P_k\}_{k\in\mathbb{Z}}$ the induced filtration on $R^hf_{X_\bullet/S*}(\mathcal{O}_{X_\bullet/S})$. We call $\{P_k\}_{k\in\mathbb{Z}}$ the *weight filtration* on $R^hf_{X_\bullet/S*}(\mathcal{O}_{X_\bullet/S})$.

Remark 3.2. In [Nak2], we have given an explicit description of the boundary morphism between E_1-terms of (3.0.5).

Theorem 3.3. ([NS], [Nak2]) If S is a p-adic formal V-scheme in the sense of [O, §1] and if $S_0 := \underline{\mathrm{Spec}}_S(\mathcal{O}_S/p)$, then (3.0.5) degenerates at E_2 modulo torsion.

Corollary 3.4. ([NS], [Nak2]) There exists the following spectral sequence of convergent F-isocrystals:
(3.4.1)

$$E_1^{-k,h+k}((X_\bullet, D_\bullet)/K) = \bigoplus_{i\geq 0} R^{h-2i-k} f_*(\mathcal{O}_{D_i^{(i+k)}/K} \otimes_{\mathbb{Z}} \varpi^{(i+k)}(D_i/K)) \Longrightarrow R^h f_*(\mathcal{O}_{X_\bullet/K}).$$

The spectral sequence (3.4.1) degenerates at E_2. Here $R^r f_*(\mathcal{O}_{D_i^{(i+k)}/K} \otimes_{\mathbb{Z}} \varpi^{(i+k)}(D_i/K))$ ($r \in \mathbb{Z}$) is a convergent F-isocrystal on S/V whose value at a p-adic enlargement T of S/V is $R^r f_{(D_i^{(i+k)})_{T_1}/T*}(\mathcal{O}_{(D_i^{(i+k)})_{T_1}/T} \otimes_{\mathbb{Z}} \varpi^{(i+k)}_{\mathrm{crys}}((D_i)_{T_1}/T))$.

4 Weight filtration on rigid cohomologies.

Let $\mathcal{V}$ be a complete discrete valuation ring of mixed characteristics with perfect residue field κ of characteristic $p > 0$. Let $\mathcal{W}$ be the Witt ring of κ. Let K (resp. K_0) be the fraction field of $\mathcal{V}$ (resp. $\mathcal{W}$). Let U be a separated scheme of finite type over κ, and let $\iota\colon U \overset{\subset}{\longrightarrow} \overline{U}$ be an open immersion into a proper scheme over κ [Nag]. Let $(X_\bullet, D_\bullet)$ be a simplicial proper smooth scheme with a simplicial SNCD over κ. Put $U_\bullet := X_\bullet \setminus D_\bullet$.

First we recall the following:

Definition 4.1. ([T], (cf. [D3, (5.3.8)])) The pair $(U_\bullet, X_\bullet)$ is called a *proper hypercovering* of $(U, \overline{U})$ if the following conditions are satisfied:

(1) $(U_\bullet, X_\bullet)$ is augmented to $(U, \overline{U})$ over κ,

(2) The natural morphism $U_{n+1} \longrightarrow \mathrm{cosk}_n^U(U_{\bullet\leq n})_{n+1}$ is proper and surjective for any $n \in \mathbb{N}$,

(3) $U_n = U \times_{\overline{U}} X_n$ for any $n \in \mathbb{N}$.

The following is one of main results of this report:

Theorem 4.2. ([Nak2]) *Let the notations be as above. Then the following hold*:

(1) *If* $(U_\bullet, X_\bullet)$ *is a split proper hypercovering of* $(U, \overline{U})$*, then there exists a canonical isomorphism*

(4.2.1) $$R\Gamma_{\mathrm{rig}}(U/K) \overset{\sim}{\longrightarrow} R\Gamma((X_\bullet, D_\bullet)/\mathcal{W}) \otimes_{\mathcal{W}} K.$$

(2) *Let c be an integer such that $H^h_{\mathrm{rig}}(U/K) = 0$ for all $h \geq c$.* (*We can show the existence of c.*)*. Let N be an integer such that there exists a positive integer r satisfying the following two inequalities*: $c \leq 2^{-1}r(r-1)$ *and* $N \geq 2^{-1}r(r+1)$.

Assume that there exists a closed immersion $(X_N, D_N) \overset{\subset}{\longrightarrow} (\mathcal{P}, \mathcal{M})$ *into a fine log smooth scheme over* $\mathrm{Spf}\,\mathcal{W}$ *such that the underlying formal scheme* $\mathcal{P}$ *is also formally smooth over* $\mathrm{Spf}\,\mathcal{W}$.

Then there exists a canonical isomorphism

$$R\Gamma_{\mathrm{rig}}(U/K) \overset{\sim}{\longrightarrow} \tau_N R\Gamma((X_\bullet, D_\bullet)/\mathcal{W}) \otimes_{\mathcal{W}} K. \tag{4.2.2}$$

Remark 4.3. (1) Over the complex number field, we have an equality

$$R\Gamma(U^{\mathrm{an}}, \mathbb{Q}) = R\Gamma(U_\bullet^{\mathrm{an}}, \mathbb{Q}) \tag{4.3.1}$$

[D3]. In (4.2), the reader should note that we have proved the coincidence of cohomologies in two different cohomology theories; (4.2.1) is harder than (4.3.1).

(2) In the constant simplicial case, the isomorphism (4.2.1) immediately follows from Shiho's comparison theorem $H^h_{\mathrm{rig}}(U/K) \overset{\sim}{\longrightarrow} H^h((X, D)/\mathcal{W}) \otimes_{\mathcal{W}} K$ [S, Cor. 2.4.13, Thm. 3.1.1].

(3) The proof of (4.2) essentially uses an argument of (a generalization of) the proof in [T] of the spectral sequence

$$E_1^{i,h-i} = H^{h-i}_{\mathrm{rig}}(U_i/K) \Longrightarrow H^h_{\mathrm{rig}}(U/K) \tag{4.3.2}$$

and uses arguments of the proofs for [S, Thm. 2.4.4, Thm. 3.1.1].

(4) The right hand side of (4.2.1) depends only on U and K; this solves a problem raised in [dJ, Intro.] for the split case. In fact, (4.4) below tells us that the weight filtration on $H^h((X_\bullet, D_\bullet)/\mathcal{W}) \otimes_{\mathcal{W}} K$ depends only on U and K.

(5) I think that the assumption on the embedding in (4.2) (2) is mild.

(6) By (4.2) and a standard argument in [Bl, III (3.2), (3.4)] (cf. [I2, II (3.2), (3.5)]), we obtain

$$H^h_{\mathrm{rig}}(U/K)_{[i,i+1)} = H^{h-i}(X_\bullet, \mathcal{W}\Omega^i_{X_\bullet}(\log D_\bullet)) \otimes_{\mathcal{W}} K. \tag{4.3.3}$$

In particular, $H^{h-i}(X_\bullet, \mathcal{W}\Omega^i_{X_\bullet}(\log D_\bullet)) \otimes_{\mathcal{W}} K$ depends only on U and K. Furthermore, we can endow $H^{h-i}(X_\bullet, \mathcal{W}\Omega^i_{X_\bullet}(\log D_\bullet)) \otimes_{\mathcal{W}} K$ with the weight filtration P [Nak2] (cf. [Nak1, §5]).

(7) We have generalized (4.2) for certain overconvergent F-isocrystals when $\mathcal{V} = \mathcal{W}$ (cf. [S, Cor. 2.4.13, Thm. 3.1.1]).

By (3.0.5) and (4.2), we have the following spectral sequence:

(4.3.4)
$$E_1^{-k,h+k} = \bigoplus_{i \geq 0} H^{h-2i-k}((\widetilde{D_i^{(i+k)}/\mathcal{W}})_{\mathrm{crys}}, \mathcal{O}_{D_i^{(i+k)}/\mathcal{W}} \otimes_{\mathbb{Z}} \varpi^{(i+k)}_{\mathrm{crys}}(D_i/\mathcal{W})) \otimes_{\mathcal{W}} K \Longrightarrow H^h_{\mathrm{rig}}(U/K).$$

Theorem-Definition 4.4. ([Nak2]) There exists a well-defined finite increasing filtration $\{P_k\}_{k \in \mathbb{Z}}$ on $H^h_{\mathrm{rig}}(U/K)$ which is calculated by the spectral sequence (4.3.4). We call this filtration the *weight filtration* on $H^h_{\mathrm{rig}}(U/K)$.

Though we do not know a p-adic analogue of the theorem for a morphism in (MHS/$\mathbb{Q}$) in (1.0.1), we can prove the following by using the specialization argument of Deligne-Illusie [I1, 3.10] (cf. [Nak1, §3]).

Theorem 4.5. ([Nak2]) *Let $f\colon U \longrightarrow V$ be a morphism of separated schemes of finite type over κ. Then the induced morphism $f^*\colon H^h_{\rm rig}(V/K) \longrightarrow H^h_{\rm rig}(U/K)$ is strictly compatible with the weight filtration.*

Remark 4.6. (1) We have proved p-adic analogues of theorems in [D3], e.g., [D3, (8.2.4) (ii) (iii), (8.2.5) $\sim$ (8.2.11)].

(2) Assume that κ is algebraically closed. Assume, also, that U and X_0 are connected (for simplicity). In [Nak2], as in [KH], by using dga's, the bar construction and the Thom-Whitney functor, we have defined a unipotent rigid fundamental group scheme $\pi_1^{\rm rig}(U/K, *)$, a simplicial unipotent crystalline fundamental group scheme $\pi_1^{\text{log-crys}}((X_\bullet, D_\bullet)/K, *_0)$ and a simplicial unipotent de Rham-Witt fundamental group scheme $\pi_1^{\rm dRW}((X_\bullet, D_\bullet)/K, *_0)$. We have proved

$$\pi_1^{\rm rig}(U/K, *) = \pi_1^{\text{log-crys}}((X_\bullet, D_\bullet)/K, *_0) = \pi_1^{\rm dRW}((X_\bullet, D_\bullet)/K, *_0)$$

if $(X_\bullet, D_\bullet, *_0)$ is a pointed split proper hypercovering of $(U, \overline{U}, *)$ (cf. [H, §6]).

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Department of Mathematics, Tokyo Denki University, 2–2 Kanda Nishiki-cho Chiyoda-ku, Tokyo 101–8457, Japan.
E-mail address: nakayuki@cck.dendai.ac.jp