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Hodge Theory and Algebraic Geometry

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\rho(X) = 1, f \le 2 のQ-Fano 3-fold Fanoの分類

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Let $F$ be an algebraic number field. Let $\Sigma_F = \text{Hom}(F, \mathbb{C})$ and $X_F$ the free abelian group generated by $\Sigma_F$. Let $\iota$ denote the complex conjugation on $\Sigma_F$ and $X_F$. Then one can define a map called regulator

$$\rho : K_{2m-1}(F) \to (X_F \otimes \mathbb{R}(m-1))^{\tau = \text{id}}.$$ 

In [9], Zagier conjectured that the image of $\rho$ can be expressed in terms of the $m$-th polylogarithm function

$$\text{Li}_m(z) = \sum_{k \geq 1} \frac{z^k}{k^m}, \quad |z| < 1.$$ 

More precisely, he conjectured that $K_{2m-1}(F)$ is isomorphic modulo torsion to a certain subquotient $B_m(F)$ of $\mathbb{Z}[F^\times - \{1\}]$ called the $m$-th Bloch group of $F$, and that the composite of the isomorphism with the regulator $\rho$ can be expressed by values of $\text{Li}_m(z)$ at algebraic numbers. The conjecture was solved affirmatively for $m \leq 3$, but it is still open when $m \geq 4$.

As for the construction of the map $B_m(F) \to K_{2m-1}(F)\mathbb{Q}$, two methods are well-known. Beilinson and Deligne showed that the above map can be constructed under the existence of an appropriate category of mixed Tate motives [1]. On the other hand, de Jeu also constructed the map by using the wedge complexes developed by himself [5]. In this note, we will present an alternative approach. The main tool here is higher Bott-Chern forms developed by Burgos and Wang [4].

1. The Bloch group

First we introduce a one-valued version of the polylogarithm function:

$$P_m(z) = \mathcal{R}_m \left( \sum_{k=0}^{m-1} \frac{z^k B_k}{k!} (\log |z|)^k \text{Li}_{m-k}(z) \right),$$

where $\mathcal{R}_m$ is $\text{Re}$ or $\text{Im}$ according as $m$ is odd or even and $B_k$ is the $k$-th Bernoulli number. This is a one-valued real analytic function on \( \mathbb{P}^1_{\mathbb{C}} - \{0, 1, \infty\} \).
In this note, we are interested in \( K \)-theory in rational coefficients, therefore we will define Bloch group with rational coefficients. Let \( F^\times_Q = F^\times \otimes_\mathbb{Z} \mathbb{Q} \) and define a map

\[
\beta_2 : \mathbb{Q}[F^\times - \{1\}] \rightarrow F^\times_Q \wedge F^\times_Q
\]

by \([x] \mapsto x \wedge (1 - x)\). Set \( A_2(F) = \text{Ker} \beta_2 \) and

\[
C_2(F) = \left\{ \sum_i n_i [x_i] \in A_2(F) ; \sum_i n_i P_2(x_i^2) = 0 \text{ for any } \sigma \in \Sigma_F \right\}.
\]

It turns out that \( C_2(F) \) is generated by

\([x] + [y] + \left[ \frac{1-x}{1-xy} \right] + [1 - xy] + \left[ \frac{1-y}{1-xy} \right]\)

for all \( x, y \in F^\times - \{1\} \) with \( xy \neq 1 \). The quotient group \( B_2(F) = A_2(F)/C_2(F) \) is called Bloch group of \( F \).

Suppose \( m \geq 3 \) and subgroups \( C_{m-1}(F) \subset A_{m-1}(F) \subset \mathbb{Q}[F^\times - \{1\}] \) are given. Define a map

\[
\beta_m : \mathbb{Q}[F^\times - \{1\}] \rightarrow F^\times_Q \otimes (\mathbb{Q}[F^\times - \{1\}]/C_{m-1}(F))
\]

by \( \beta_m([x]) = x \otimes [x] \). Set \( A_m(F) = \text{Ker} \beta_m \) and

\[
C_m(F) = \left\{ \sum_i n_i [x_i] \in A_m(F) ; \sum_i n_i P_m(x_i^\sigma) = 0 \text{ for any } \sigma \in \Sigma_F \right\}.
\]

The quotient group \( B_m(F) = A_m(F)/C_m(F) \) is called \( m \)-th Bloch group of \( F \).

**Zagier Conjecture** ([9]). For \( m \geq 2 \), the rational algebraic \( K \)-theory \( K_{2m-1}(F)_\mathbb{Q} \) is isomorphic to \( B_m(F) \), and the composite

\[
B_m(F) \cong K_{2m-1}(F)_\mathbb{Q} \xrightarrow{\rho} (X_F \otimes \mathbb{R}(m-1))^\tau = \text{id}
\]

is written as

\[
\sum_i n_i [x_i] \mapsto \left( (-1)^{\alpha_m} \sum_i n_i P_m(x_i^\sigma) \right)_{\sigma \in \Sigma_F},
\]

where \( \alpha_m \) is 0 or 1 according as \( m \) is odd or even.

## 2. The Complex of Exact Hermitian Cubes

Let \( < -1, 0, 1 >^n \) be the ordered set consisting of three elements and \( < -1, 0, 1 >^n \) its \( n \)-th power. For a small exact category \( \mathfrak{A} \) with a fixed zero object 0, a functor \( \mathcal{F} : < -1, 0, 1 >^n \rightarrow \mathfrak{A} \) is called an \( n \)-cube of \( \mathfrak{A} \). Let \( \mathcal{F}_{\alpha_1, \ldots, \alpha_n} \) denote the image of an object \((\alpha_1, \ldots, \alpha_n)\) of \( < -1, 0, 1 >^n \). For integers \( 1 \leq i \leq n \) and \( -1 \leq j \leq 1 \), an \((n-1)\)-cube \( \partial_i \mathcal{F} \) is defined by \((\partial_i \mathcal{F})_{\alpha_1, \ldots, \alpha_{i-1}, \alpha_i, \alpha_{i+1}, \ldots, \alpha_n} = \mathcal{F}_{\alpha_1, \ldots, \alpha_{i-1}, j, \alpha_i, \alpha_{i+1}, \ldots, \alpha_n} \). It is called a face of \( \mathcal{F} \). For
an object $\alpha$ of $<-1,0,1>^{n-1}$ and an integer $1 \leq i \leq n$, a 1-cube $\partial^i_\alpha F$ called an edge of $F$ is defined by

$$F_{\alpha_1,\ldots,\alpha_{i-1},-1,\alpha_i,\ldots,\alpha_{n-1}} \to F_{\alpha_1,\ldots,\alpha_{i-1},0,\alpha_i,\ldots,\alpha_{n-1}} \to F_{\alpha_1,\ldots,\alpha_{i-1},1,\alpha_i,\ldots,\alpha_{n-1}}.$$  

An $n$-cube $F$ is said to be exact if all edges of $F$ are short exact sequences.

Let $C_n A$ denote the set of all exact $n$-cubes of $A$. If $F$ is an exact $n$-cube, then any face $\partial^j_i F$ is also exact. Hence $\partial^j_i$ induces a map

$$\partial^j_i : C_n A \to C_{n-1} A.$$ 

Let $F$ be an exact $n$-cube of $A$. For an integer $1 \leq i \leq n+1$, let $s^1_i F$ be an exact $(n+1)$-cube such that its edge $\partial^i_s(s^1_i F)$ is $F_\alpha \overset{id}{\to} F_\alpha \to 0$. Similarly, let $s^{-1}_i F$ be an exact $(n+1)$-cube such that $\partial^i_s(s^{-1}_i F)$ is $0 \to F_\alpha \overset{id}{\to} F_\alpha$. An exact cube written as $s^1_i F$ is said to be degenerate.

Let $QC_n A$ be the free $Q$-module generated by $C_n A$ and $D_n \subset QC_n A$ the submodule generated by all degenerate exact $n$-cubes. Let $\tilde{Q}C_n A = QC_n A/D_n$ and

$$\partial = \sum_{i=1}^n \sum_{j=-1}^1 (-1)^{i+j+1} \partial^j_i : \tilde{Q}C_n A \to \tilde{Q}C_{n-1} A.$$ 

Then $\tilde{Q}C_n A = (\tilde{Q}C_n A, \partial)$ becomes a homological complex.

**Theorem 2.1** ([6]). The homology of $(\tilde{Q}C_n A, \partial)$ is isomorphic to the rational algebraic $K$-theory of $A$:

$$H_n(\tilde{Q}C_n A, \partial) \simeq K_n(A)_{Q}.$$ 

This isomorphism preserves products on the both sides if $A$ is equipped with a strictly associative tensor product.

### 3. The higher Bott-Chern forms

Let $M$ be a compact complex algebraic manifold, namely, the analytic space consisting of all $\mathbb{C}$-valued points of a smooth proper algebraic variety over $\mathbb{C}$. Let $\mathcal{E}^p_{\mathbb{R}}(M)$ be the space of real smooth differential forms of degree $p$ on $M$ and $\mathcal{E}_{\mathbb{R}}^p(M) = \mathcal{E}^p_{\mathbb{R}}(M) \otimes_{\mathbb{R}} \mathbb{C}$. Let $\mathcal{E}^{p,q}(M)$ be the space of complex differential forms of type $(p,q)$ on $M$. Set

$$\mathcal{D}^n(M,p) = \begin{cases} 
\mathcal{E}^{p-1}_{\mathbb{R}}(M)(p-1) \cap \bigoplus_{p'+q'=n-1} \mathcal{E}^{p'+q'}(M), & n < 2p, \\
\mathcal{E}^{2p}_{\mathbb{R}}(M)(p) \cap \mathcal{E}^{p,p}(M) \cap \text{Ker } d, & n = 2p, \\
0, & n > 2p.
\end{cases}$$
and define a differential \( d_D : \mathcal{D}^n(M, p) \to \mathcal{D}^{n+1}(M, p) \) by
\[
d_D(\omega) = \begin{cases} 
-\pi(d\omega), & n < 2p - 1, \\
-2\partial\bar{\partial}\omega, & n = 2p - 1, \\
0, & n > 2p - 1,
\end{cases}
\]
where \( \pi : \mathcal{E}^n(M) \to \mathcal{D}^n(M, p) \) is the canonical projection. Then it is shown in [3, Thm.2.6] that \((\mathcal{D}^*(M, p), d_D)\) becomes a complex of \(\mathbb{R}\)-vector spaces computing the real Deligne cohomology, that is, for \( n \leq 2p \) we have
\[
H^n(\mathcal{D}^*(M, p), d_D) \simeq H^n_\mathbb{R}(M, \mathbb{R}(p)).
\]

By a hermitian vector bundle \( \mathcal{E} = (E, h) \) on \( M \) we mean an algebraic vector bundle \( E \) on \( M \) with a smooth hermitian metric \( h \). Let \( K_\mathcal{E} \) denote the curvature form of the unique connection on \( \mathcal{E} \) that is compatible with both the metric and the complex structure. The Chern form of \( \mathcal{E} \) is defined as
\[
\text{ch}_0(\mathcal{E}) = \text{Tr}(\exp(-K_\mathcal{E})) \in \oplus_p \mathcal{D}^{2p}(M, p).
\]

An exact hermitian \( n \)-cube on \( M \) is an exact \( n \)-cube made of hermitian vector bundles on \( M \). Let \( \mathcal{F} = \{ \mathcal{E}_\alpha \} \) be an exact hermitian \( n \)-cube on \( M \). We call \( \mathcal{F} \) an emi-\( n \)-cube if the metric on any \( \mathcal{E}_\alpha \) with \( \alpha_i = 1 \) coincides with the metric induced from \( \mathcal{E}_{\alpha_1, \cdots, \alpha_{i-1}, 0, \alpha_{i+1}, \cdots, \alpha_n} \) by the surjection \( \mathcal{E}_{\alpha_1, \cdots, \alpha_{i-1}, 0, \alpha_{i+1}, \cdots, \alpha_n} \to \mathcal{E}_{\alpha_i} \).

For an emi-1-cube \( \mathcal{E} : \mathcal{E}_{-1} \to \mathcal{E}_0 \to \mathcal{E}_1 \), a canonical way of constructing a hermitian vector bundle \( \text{tr}_1 \mathcal{E} \) on \( M \times \mathbb{P}^1 \) connecting \( \mathcal{E}_0 \) with \( \mathcal{E}_{-1} \oplus \mathcal{E}_1 \) is given in [4]. If \( (x : y) \) denotes the homogeneous coordinate of \( \mathbb{P}^1 \) and \( z = x/y \), then \( \text{tr}_1 \mathcal{E} \) fulfills the conditions \( (\text{tr}_1 \mathcal{E})|_{z=0} \simeq \mathcal{E}_0 \) and \( (\text{tr}_1 \mathcal{E})|_{z=\infty} \simeq \mathcal{E}_{-1} \oplus \mathcal{E}_1 \). For an emi-\( n \)-cube \( \mathcal{F} \), let \( \text{tr}_1(\mathcal{F}) \) be an emi-(\( n-1 \))-cube on \( M \times \mathbb{P}^1 \) defined by \( \text{tr}_1(\mathcal{F})_\alpha = \text{tr}_1(\partial_{n-1}\mathcal{F}) \) for any object \( \alpha \) of \( \sim -1, 0, 1 >^{-1} \), and \( \text{tr}_n(\mathcal{F}) \) a hermitian vector bundle on \( M \times (\mathbb{P}^1)^n \) given by taking \( \text{tr}_1 \mathcal{n} \) times.

Let \( \pi_i : (\mathbb{P}^1)^n \to \mathbb{P}^1 \) be the \( i \)-th projection and \( z_i = \pi_i^* z \). Let \( \mathfrak{S}_n \) be the symmetric group of \( n \)-letters. For an integer \( 1 \leq i \leq n \), a differential form with logarithmic poles \( S^i_n \) on \( (\mathbb{P}^1)^n \) is defined as
\[
S^i_n = \sum_{\sigma \in \mathfrak{S}_n} (1)^{\sigma} \log |z_\sigma(1)|^2 \frac{dz_\sigma(2)}{z_\sigma(2)} \wedge \cdots \wedge \frac{dz_\sigma(i)}{z_\sigma(i)} \wedge \frac{d\bar{z}_\sigma(i+1)}{\bar{z}_\sigma(i+1)} \wedge \cdots \wedge \frac{d\bar{z}_\sigma(n)}{\bar{z}_\sigma(n)},
\]
and \( T_n \) is defined as
\[
T_n = \frac{(-1)^n}{2n!} \sum_{i=1}^{n} (-1)^i S^i_n.
\]
Let us define the Bott-Chern form of an emi-\( n \)-cube \( \mathcal{F} \) as
\[
\text{ch}_n(\mathcal{F}) = \frac{1}{(2\pi \sqrt{-1})^n} \int_{(\mathbb{P}^1)^n} \text{ch}_0(\text{tr}_n(\mathcal{F})) \wedge T_n \in \oplus_p \mathcal{D}^{2p-n}(M, p).
\]
A process to make an emi-$n$-cube $\lambda F$ from an arbitrary exact hermitian $n$-cube $F$ has been given in [4]. By virtue of this process, we can extend the definition of the Bott-Chern form to an arbitrary exact hermitian $n$-cube.

**Definition 3.1.** The Bott-Chern form of an exact hermitian $n$-cube $F$ is an element of $\bigoplus_p D^{2p-n}(M, p)$ defined as

$$
\operatorname{ch}_n(F) = \frac{1}{(2\pi i)^n} \int_{(p1)^n} \operatorname{ch}_0(\operatorname{tr}_n(\lambda F)) \wedge T_n.
$$

**Theorem 3.2 ([4]).** Let $\hat{\mathcal{P}}(M)$ denote the category of hermitian vector bundles on $M$ and let $\hat{\mathcal{Q}}\hat{\mathcal{C}}_\ast(M) = \hat{\mathcal{Q}}C_\ast\hat{\mathcal{P}}(M)$. Then the higher Bott-Chern forms induce a homomorphism of complexes

$$
\operatorname{ch} : \hat{\mathcal{Q}}\hat{\mathcal{C}}_\ast(M) \to \bigoplus_p D^\ast(M, p)[2p].
$$

Moreover, the following map

$$
K_n(M)_Q \simeq H_n(\hat{\mathcal{Q}}\hat{\mathcal{C}}_\ast(M)) \operatorname{ch} \bigoplus_p H^{2p-n}(M, \mathbb{R}(p))
$$

agrees with the higher Chern character with values in the real Deligne cohomology.

Let $X$ be a smooth proper variety defined over $\mathbb{Q}$. By a hermitian vector bundle $E = (E, h)$ on $X$, we mean a vector bundle $E$ on $X$ with an $\iota$-invariant smooth hermitian metric $h$ on the holomorphic vector bundle $E(\mathbb{C})$. In the same way as above, one can consider an exact hermitian $n$-cube $F$ on $X$ and define its Bott-Chern form $\operatorname{ch}_n(F)$. Let $\hat{\mathcal{Q}}\hat{\mathcal{C}}_\ast(X)$ denote the complex of exact hermitian cubes on $X$. Then we have an isomorphism preserving products

$$
K_\ast(X)_Q \simeq H_\ast(\hat{\mathcal{Q}}\hat{\mathcal{C}}_\ast(X))
$$

and the Bott-Chern forms leads to the regulator map for $X$:

$$
K_n(X)_Q \xrightarrow{\operatorname{ch}} \bigoplus_p H^{2p-n}(X(\mathbb{C}), \mathbb{R}(p)) := \bigoplus_p H^{2p-n}(X(\mathbb{C}), \mathbb{R}(p))^{\tau = \text{id}}
$$

4. The main theorem

We introduce a new real analytic function on $\mathbb{P}^1_\mathbb{C} - \{0, 1, \infty\}$ coming from the polylogarithm function. Let

$$
I_m(z) = \sum_{j=0}^{m-1} \frac{(-\log |z|)^j}{j!} Li_{m-j}(z),
$$
and
\[ L_m(z) = R_m \left( \sum_{0 \leq 2r < m} \frac{(-1)^r}{2^r r!} \frac{(\log |z|)^{2r}}{(2m-3)(2m-5) \cdots (2m-2r-1)} I_{m-2r}(z) \right). \]

When \( m \leq 3 \), \( L_m(z) \) is equal to \( P_m(z) \), but is not so when \( m \geq 4 \). However, for \( \sum_i n_i [x_i] \in A_m(F) \), we have
\[ \sum_i n_i L_m(x_i^r) = \sum_i n_i P_m(x_i^r). \]

The function \( L_m(z) \) satisfies the following differential equation, which is obtained by a direct calculation.

**Theorem 4.1.** [8, Thm.5.4] If \( m \geq 2 \), then
\[ (-1)^m dL_m(z) = Im \left( \frac{dz}{z} \right) L_{m-1}(z) - \frac{\sqrt{-1}}{2m-3} \log |z| (\overline{\partial L_{m-1}(z)} - \partial L_{m-1}(z)). \]

Let \( X = \mathbb{P}^1 - \{0, 1, \infty\} \) over \( \mathbb{Q} \) and let \( z \) be the absolute coordinate of \( X \). Hence we can write \( X = \text{Spec} \mathbb{Q}[z, 1/z, 1/(1-z)] \). We want to apply the theory of Bott-Chern forms to \( X \). But since \( X \) is not proper over \( \mathbb{Q} \), we can not apply directly the results mentioned in the preceding section.

For an exact hermitian \( n \)-cube \( F \) on \( X \), one can define \( ch_n(F) \) as a differential form on \( X(\mathbb{C}) \) by the same integral expression. Moreover, for \( n \geq 2 \), we have \( dch_n(F) = -ch_{n-1}(\partial F) \). Hence when \( n \geq 2 \), \( ch_n(F) \) induces a map from the rational \( K \)-theory of \( X \) to the de Rham cohomology of \( X(\mathbb{C}) \).

For \( f \in \mathcal{O}_X^\infty \), let \( \langle f \rangle \) be an exact hermitian 1-cube on \( X \) given as
\[ 0 \to \mathcal{O}_X \xrightarrow{f} \hat{\mathcal{O}}_X. \]

**Proposition 4.2.** [8, Prop.6.1] There exists an element \( h_n(z) \in \hat{\mathcal{O}}C_{2n-1}(X) \) for each \( n \geq 1 \) satisfying the following conditions:

1. \( h_1(z) = \langle z \rangle. \)
2. \( \partial h_n(z) = \sum_{i=1}^{n-1} h_i(z) \otimes h_{n-i}(z). \)
3. \( ch_{2n-1}(h_n(z)) = 0 \) for \( n \geq 2 \).

**Theorem 4.3.** [8, Thm.6.2] For each \( m \geq 1 \), there exists \( L_m(z) \in \hat{\mathcal{O}}C_{2m-1}(X) \) satisfying the following conditions:

1. \( L_1(z) = -2(1 - z). \)
2. \( \partial L_m(z) = \sum_{i=1}^{m-1} 2^i h_i(z) \otimes L_{m-i}(z) \) for \( m \geq 2 \).
3. If \( ch_{2m-1}(L_m(z))^{(0)} \) denotes the part of degree 0 of \( ch_{2m-1}(L_m(z)) \), then
\[ ch_{2m-1}(L_m(z))^{(0)} = (\sqrt{-1})^m L_m(z), \]
where $\alpha_m$ is 0 or 1 according as $m$ is odd or even.

Outline of the proof: We will prove the theorem by induction on $m$. Assume that $L_1(z), \ldots, L_{m-1}(z)$ exist. By the product formula for Bott-Chern forms [7, Prop. 4.2],

$$
\text{ch}_{2m-2} \left( \sum_{i=1}^{m-1} 2^i h_i(z) \otimes L_{m-i}(z) \right) \\
= \left( \frac{-1}{\sqrt{-1}} \right)^{\alpha_m} \left( \text{Im} \left( \frac{dz}{z} \right) L_{m-1}(z) - \frac{1}{2m-3} \log |z| (\partial L_{m-1}(z) - \partial L_{m-1}(z)) \right) \\
= \left( \frac{-1}{\sqrt{-1}} \right)^{\alpha_m} dL_m(z).
$$

It can be shown that the map

$$
\text{ch}_{2m-2} : K_{2m-2}(X)_{\mathbb{Q}} \to H^1_{dR}(X(\mathbb{C}), \mathbb{R}(m-1)) = \text{id}
$$

is injective. Hence there exists $L_m(z) \in \hat{Q}\hat{C}_{2m-1}(X)$ satisfying the equation (2). Then

$$
d\text{ch}_{2m-1}(L_m(z))^{(0)} = -\text{ch}_{2m-2}(\partial L_m(z)) = \left( \frac{-1}{\sqrt{-1}} \right)^{\alpha_m} dL_m(z),
$$

therefore

$$
\text{ch}_{2m-1}(L_m(z))^{(0)} = \left( \frac{-1}{\sqrt{-1}} \right)^{\alpha_m} L_m(z) + a_m
$$

for some constant $a_m$. We can eliminate this constant term by using the Borel’s theorem for the regulator map of $\mathbb{Q}$ [2].

Theorem 4.4. [8, Thm. 7.2, Thm. 7.5] There exists a homomorphism

$$
\mathcal{P}_m : \mathcal{A}_m(F) \to \hat{\mathbb{Q}}\hat{C}_{2m-1}(F)
$$

satisfying the following conditions:

1. $\text{ch}_{2m-1}(\mathcal{P}_m(\xi)) = 0$ for any $\xi \in \mathcal{A}_m(F)$.
2. $\partial(L_m(\xi) + \mathcal{P}_m(\xi)) = 0$ for any $\xi \in \mathcal{A}_m(F)$.

By virtue of the above theorems, one can define a map

$$
\mathcal{B}_m(F) = \mathcal{A}_m(F)/\mathcal{C}_m(F) \to K_{2m-1}(F)_{\mathbb{Q}}
$$

by $[\xi] \mapsto L_m(\xi) + \mathcal{P}_m(\xi)$. It is easy to see that the composite of this map with the regulator satisfies the condition of the Zagier conjecture.

References


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