



Title	Instability in the spectral and the Fredholm properties of an infinite dimensional Dirac operator on the abstract Boson-Fermion Fock space
Author(s)	Arai, A.
Citation	Hokkaido University Preprint Series in Mathematics, 507, 1-6
Issue Date	2000-12-1
DOI	10.14943/83653
Doc URL	<a href="http://hdl.handle.net/2115/69257">http://hdl.handle.net/2115/69257</a>
Type	bulletin (article)
File Information	pre507.pdf



[Instructions for use](#)

**Instability in the Spectral and the Fredholm  
Properties of an Infinite Dimensional Dirac  
Operator on the Abstract Boson-Fermion Fock  
Space**

Asao Arai

Series #507. December 2000

**HOKKAIDO UNIVERSITY**  
**PREPRINT SERIES IN MATHEMATICS**

- #482 N. H. Bingham and A. Inoue, Tauberian and Mercerian theorems for systems of kernels, 16 pages. 2000.
- #483 N. H. Bingham and A. Inoue, Abelian, Tauberian and Mercerian theorems for arithmetic sums, 29 pages. 2000.
- #484 I. A. Bogaevski and G. Ishikawa, Lagrange mappings of the first open Whitney umbrella, 22 pages. 2000.
- #485 A. Arai and H. Kawano, A class of deformations of the Schrödinger representation of the Heisenberg commutation relation and exact solution to a Heisenberg equation and a Schrödinger equation, 22 pages. 2000.
- #486 T. Nakazi, Functions in  $N_+$  with the positive real parts on the boundary, 21 pages. 2000.
- #487 Y. Shibukawa, Classification of the  $R$ -operator, 36 pages. 2000.
- #488 A. Inoue, Asymptotic behaviour for partial autocorrelation functions of fractional ARIMA processes, 20 pages. 2000.
- #489 S. Ohtani, Construction of unramified Galois extensions over maximal abelian extensions of algebraic number fields, 14 pages. 2000.
- #490 T. Nakazi and T. Yamamoto, The real part of an outer function and a Helson-Szegö weight, 13 pages. 2000.
- #491 A. Yamagami, On Gouvêas conjecture on controlling the conductor, 11 pages. 2000.
- #492 I. Tsuda and M. Hatakeyama, Making sense of internal logic: Theory and a case study, 10 pages. 2000.
- #493 I. Tsuda, Towards an interpretation of dynamic neural activity in terms of chaotic dynamical systems, 73 pages. 2000.
- #494 T. Mikami, Optimal control for absolutely continuous stochastic processes and the mass transportation problem, 17 pages. 2000.
- #495 M. Arisawa and Y. Giga, Anisotropic curvature flow in a very thin domain, 21 pages. 2000.
- #496 T. Nakazi, Backward shift invariant subspaces in the bidisc, 9 pages. 2000.
- #497 Y. Giga, K. Inui, J. Kato and S. Matsui, Remarks on the uniqueness of bounded solutions of the Navier-Stokes equations, 4 pages. 2000.
- #498 Y. Giga, M. Paolini and P. Rybka, On the motion by singular interfacial energy, 21 pages. 2000.
- #499 J. Escher and Y. Giga, On a limiting motion and self-interactions of curves moved by the intermediate surface diffusion flow, 12 pages. 2000.
- #500 I. Tsuda and S. Kuroda, Cantor coding in the hippocampus, 20 pages. 2000.
- #501 M. Tsujii, Fat solenoidal attractors, 20 pages. 2000.
- #502 A. Arai, Ground state of the massless Nelson model without infrared cutoff in a non-Fock representation, 19 pages. 2000.
- #503 Y. Giga, S. Matsui and O. Sawada, Global existence of two-dimensional Navier-Stokes flow with nondecaying initial velocity, 19 pages. 2000.
- #504 A. Inoue and Y. Kasahara, Partial autocorrelation functions of the fractional ARIMA processes with negative degree of differencing, 14 pages. 2000.
- #505 T. Nakazi, Interpolation problem for  $\ell^1$  and a uniform algebra, 12 pages. 2000.
- #506 R. Kobayashi and Y. Giga, On anisotropy and curvature effects for growing crystals, 38 pages. 2000.

# Instability in the Spectral and the Fredholm Properties of an Infinite Dimensional Dirac Operator on the Abstract Boson-Fermion Fock Space

Asao Arai \*

Department of Mathematics, Hokkaido University  
Sapporo 060-0810, Japan

Email: arai@math.sci.hokudai.ac.jp

November 23, 2000

to be published in the Proceedings of the Conference PARTIAL DIFFERENTIAL EQUATIONS 2000 held in Clausthal, Germany (July 24–July 28, 2000)

## Abstract

A perturbed Dirac operator  $Q(\alpha)$  on the abstract Boson-Fermion Fock space is considered, where  $\alpha \in \mathbb{C}$  is a perturbation (coupling) parameter and the unperturbed operator  $Q(0)$  is taken to be a free infinite dimensional Dirac operator introduced by the author (A. Arai, J. Funct. Anal. **105**(1992), 342–408). The following results are reported: (i) Under some conditions, the kernel of  $Q(\alpha)$  is one dimensional for all  $\alpha \neq \alpha_0$  with some  $\alpha_0 \neq 0$  and degenerate at  $\alpha = \alpha_0$ , while, under another condition, the kernel of  $Q(\alpha)$  is one dimensional for all  $\alpha \in \mathbb{C}$ . (ii) There are cases where, for all sufficiently large  $|\alpha|$  with  $\alpha < 0$ ,  $Q(\alpha)$  has infinitely many non-zero eigenvalues even if  $Q(0)$  has no non-zero eigenvalues. This is a strong coupling effect. (iii) Fredholm property of  $Q(\alpha)$  also depends on the coupling parameter  $\alpha$ .

**Key words:** Boson-Fermion Fock space, supersymmetric quantum field, infinite dimensional Dirac operator, non-regular perturbation, kernel, spectrum, Fredholm property, strong coupling effect

---

\*Supported by the Grant-In-Aid No.11440036 for Scientific Research from the Ministry of Education, Science, Sports and Culture, Japan.

# 1 Introduction

Let  $\mathcal{H}$  and  $\mathcal{K}$  be separable complex Hilbert spaces. Then the abstract Boson-Fermion Fock space  $\mathcal{F}(\mathcal{H}, \mathcal{K})$  associated with the pair  $(\mathcal{H}, \mathcal{K})$  is defined by

$$\mathcal{F}(\mathcal{H}, \mathcal{K}) := \mathcal{F}_b(\mathcal{H}) \otimes \mathcal{F}_f(\mathcal{K}), \quad (1.1)$$

where  $\mathcal{F}_b(\mathcal{H}) := \bigoplus_{n=0}^{\infty} \bigotimes_s^n \mathcal{H}$  is the Boson Fock space over  $\mathcal{H}$  ( $\bigotimes_s^n \mathcal{H}$  denotes the  $n$ -fold symmetric tensor product Hilbert space of  $\mathcal{H}$ ;  $\bigotimes_s^0 \mathcal{H} := \mathbb{C}$ ) and  $\mathcal{F}_f(\mathcal{K}) := \bigoplus_{p=0}^{\infty} \bigwedge^p \mathcal{K}$  is the Fermion Fock space over  $\mathcal{K}$  ( $\bigwedge^p \mathcal{K}$  denotes the  $p$ -fold anti-symmetric tensor product Hilbert space of  $\mathcal{K}$ ;  $\bigwedge^0 \mathcal{K} := \mathbb{C}$ ).

Let  $\mathcal{C}(\mathcal{H}, \mathcal{K})$  be the set of densely defined closed linear operators from  $\mathcal{H}$  to  $\mathcal{K}$ . Then, for each  $A \in \mathcal{C}(\mathcal{H}, \mathcal{K})$ , one can define a Dirac-type operator  $Q_A$  on  $\mathcal{F}(\mathcal{H}, \mathcal{K})$  [Ar92] (for the definition of  $Q_A$ , Section 2 below). The operator  $Q_A$  is an infinite dimensional version of free Dirac operators on finite dimensional spaces.

In [Ar92] some fundamental properties of  $Q_A$  were established. Moreover, a perturbed Dirac operator of the form  $Q_A(V) := Q_A + V$  was considered in view of index theory, where  $V$  is a symmetric operator on  $\mathcal{F}(\mathcal{H}, \mathcal{K})$ , and a functional integral representation for the index of  $Q_A(V)$  restricted to a subspace, called the ‘‘bosonic subspace’’, was derived (for related aspects and further developments, see [Ar89], [Ar91], [Ar93], [Ar93], [Ar94], [Ar96], [Ar97], [AM91], [AM93]). It still remains, however, as an important problem, to investigate spectral properties of  $Q_A(V)$ . In this paper, as a first step towards this direction, we present a perturbation of  $Q_A$  which is not of the form  $V$  considered in [Ar92] and rather simple, but, gives rise to interesting *nonperturbative instability phenomena* in spectral and Fredholm properties. For proofs on the results reported in this paper, see [Ar00].

## 2 A Class of Perturbed Dirac Operators

We denote by  $a(f)$  ( $f \in \mathcal{H}$ ) and  $b(u)$  ( $u \in \mathcal{K}$ ) the annihilation operators on  $\mathcal{F}_b(\mathcal{H})$  and on  $\mathcal{F}_f(\mathcal{K})$  respectively (e.g., [BR97, §5.2]). Let  $\Omega_b := \{1, 0, 0, \dots\} \in \mathcal{F}_b(\mathcal{H})$  (resp.  $\Omega_f := \{1, 0, 0, \dots\} \in \mathcal{F}_f(\mathcal{K})$ ) be the Fock vacuum in  $\mathcal{F}_b(\mathcal{H})$  (resp.  $\mathcal{F}_f(\mathcal{K})$ ). Let  $A \in \mathcal{C}(\mathcal{H}, \mathcal{K})$  and

$$\mathcal{D}_A^\infty := \mathcal{L} \left\{ a(f_1)^* \cdots a(f_n)^* \Omega_b \otimes b(u_1)^* \cdots b(u_p)^* \Omega_f \mid n, p \geq 0, f_j \in C^\infty(A^*A), \right. \\ \left. j = 1, \dots, n, u_k \in C^\infty(AA^*), k = 1, \dots, p \right\}, \quad (2.1)$$

where  $\mathcal{L}\{\dots\}$  means the subspace algebraically spanned by the vectors in the set  $\{\dots\}$  and  $C^\infty(T) := \bigcap_{n=1}^{\infty} D(T^n)$  for a linear operator  $T$  on a Hilbert space ( $D(T^n)$  denotes the domain of  $T^n$ ). It follows that  $\mathcal{D}_A^\infty$  is dense in  $\mathcal{F}(\mathcal{H}, \mathcal{K})$ .

Let  $\{e_n\}_{n=1}^{\infty}$  be a complete orthonormal system of  $\mathcal{K}$  such that  $e_n \in D(A^*)$ ,  $n \in \mathbb{N}$ . Then it is shown that there is a unique densely defined closed linear operator  $d_A$  on  $\mathcal{F}(\mathcal{H}, \mathcal{K})$  such that (i) for all  $\Psi \in \mathcal{D}_A^\infty$

$$d_A \Psi = \sum_{n=1}^{\infty} a(A^* e_n) b(e_n)^* \Psi \quad (2.2)$$

independently of the choice of  $\{e_n\}_{n=1}^\infty$  and (ii)  $\mathcal{D}_A^\infty$  is a core of  $d_A$ . It follows that  $d_A^2 = 0$ . The operator may be regarded as an infinite dimensional version of finite dimensional exterior differential operators. A free Dirac operator on  $\mathcal{F}(\mathcal{H}, \mathcal{K})$  is defined by

$$Q_A = d_A + d_A^*. \quad (2.3)$$

We have an orthogonal decomposition

$$\mathcal{F}(\mathcal{H}, \mathcal{K}) = \mathcal{F}_+(\mathcal{H}, \mathcal{K}) \oplus \mathcal{F}_-(\mathcal{H}, \mathcal{K}) \quad (2.4)$$

with

$$\mathcal{F}_+(\mathcal{H}, \mathcal{K}) := \bigoplus_{p=0}^\infty \mathcal{F}_b(\mathcal{H}) \otimes \wedge^{2p}(\mathcal{K}), \quad \mathcal{F}_-(\mathcal{H}, \mathcal{K}) := \bigoplus_{p=0}^\infty \mathcal{F}_b(\mathcal{H}) \otimes \wedge^{2p+1}(\mathcal{K}). \quad (2.5)$$

Let  $P_+$  and  $P_-$  be the orthogonal projections onto  $\mathcal{F}_+(\mathcal{H}, \mathcal{K})$  and  $\mathcal{F}_-(\mathcal{H}, \mathcal{K})$  respectively and define

$$\Gamma := P_+ - P_-. \quad (2.6)$$

For a self-adjoint operator  $S$  on  $\mathcal{H}$  (resp.  $\mathcal{K}$ ),  $d\Gamma_b(S)$  (resp.  $d\Gamma_f(S)$ ) denotes the second quantization of  $S$  in the Boson Fock space  $\mathcal{F}_b(\mathcal{H})$  (resp. the Fermion Fock space  $\mathcal{F}_f(\mathcal{H})$ ) [BR97, §5.2].

Basic properties of  $Q_A$  are as follows [Ar92]:

- (i) The operator  $Q_A$  is self-adjoint and essentially self-adjoint on  $\mathcal{D}_A^\infty$ .
- (ii) The operator  $\Gamma$  leaves  $D(Q_A)$  invariant and  $\Gamma Q_A + Q_A \Gamma = 0$  on  $D(Q_A)$ .
- (iii) The following operator equations hold :

$$Q_A^2 = d_A^* d_A + d_A d_A^* = d\Gamma_b(A^* A) \otimes I + I \otimes d\Gamma_f(AA^*), \quad (2.7)$$

where  $I$  denotes identity.

We perturb  $Q_A$  through a perturbation of  $d_A$ . Let  $g \in D(A)$ ,  $g \neq 0$ , and  $v \in D(A^*)$ ,  $v \neq 0$  and define

$$d(\alpha) := d_A + \alpha a(g) \otimes b(v)^*. \quad (2.8)$$

where  $\alpha \in \mathbb{C}$  is a coupling constant. It is easy to see that  $D(d(\alpha)) \supset \mathcal{D}_A^\infty$  and  $d(\alpha)|_{\mathcal{D}_A^\infty}$  is closable. Let

$$\bar{d}(\alpha) := \overline{d(\alpha)|_{\mathcal{D}_A^\infty}}, \quad (2.9)$$

the closure of  $d(\alpha)|_{\mathcal{D}_A^\infty}$ , and

$$Q(\alpha) := \bar{d}(\alpha) + \bar{d}(\alpha)^*. \quad (2.10)$$

This is the perturbed Dirac operator considered in this paper.

### 3 Results

**Theorem 3.1** (i) For all  $\alpha \in \mathbb{C}$ ,  $Q(\alpha)$  is self-adjoint, and essentially self-adjoint on  $\mathcal{D}_A^\infty$  with  $Q(\alpha) = \overline{Q_A + V_{g,v}}$ , where  $V_{g,v} := \alpha a(g) \otimes b(v)^* + \alpha^* a(g)^* \otimes b(v)$ . Moreover,  $\Gamma$  leaves  $D(Q(\alpha))$  invariant and  $\Gamma Q(\alpha) + Q(\alpha)\Gamma = 0$  on  $D(Q(\alpha))$ .

(ii) Suppose that  $A$  is injective and  $g \in D(|A|^{-1})$ . Then, for all  $|\alpha| < (\|v\| \| |A|^{-1}g \|)^{-1}$ ,  $D(Q(\alpha)) = D(Q_A)$ .

(iii) For all  $z \in \mathbb{C} \setminus \mathbb{R}$ ,  $(Q(\alpha) - z)^{-1}$  is strongly continuous in  $\alpha \in \mathbb{C}$ .

To describe the spectral properties of  $Q(\alpha)$ , we introduce a bounded linear operator  $T_{g,v}$  from  $\mathcal{H}$  to  $\mathcal{K}$  by

$$T_{g,v}f := (g, f)v, \quad f \in \mathcal{H}, \quad (3.1)$$

where  $(\cdot, \cdot)$  denotes inner product, and define

$$A(\alpha) := A + \alpha T_{g,v}. \quad (3.2)$$

For a linear operator  $T$  on a Hilbert space, we denote by  $\sigma(T)$  (resp.  $\sigma_p(T)$ ) the (resp. point) spectrum of  $T$ . A general feature of the spectra of  $Q(\alpha)$  is given in the following theorem:

**Theorem 3.2** For all  $\alpha \in \mathbb{C}$ ,  $\sigma(Q(\alpha))$  and  $\sigma_p(Q(\alpha))$  are symmetric with respect to the origin and

$$\sigma(Q(\alpha)) = \{0\} \cup \overline{\left( \bigcup_{n=1}^{\infty} \left\{ \pm \sqrt{\sum_{j=1}^n \lambda_j} \mid \lambda_j \in \sigma(A(\alpha)^* A(\alpha)), j = 1, \dots, n \right\} \right)}, \quad (3.3)$$

$$\sigma_p(Q(\alpha)) = \{0\} \cup \left( \bigcup_{n=1}^{\infty} \left\{ \pm \sqrt{\sum_{j=1}^n \lambda_j} \mid \lambda_j \in \sigma_p(A(\alpha)^* A(\alpha)), j = 1, \dots, n \right\} \right), \quad (3.4)$$

with

$$\dim \ker(Q(\alpha) - \lambda) = \dim \ker(Q(\alpha) + \lambda), \quad \lambda \in \sigma_p(Q(\alpha)). \quad (3.5)$$

This theorem shows that the spectrum and the point spectrum of  $Q(\alpha)$  are completely determined from those of  $A(\alpha)^* A(\alpha)$ .

To state properties of the kernel of  $Q(\alpha)$ , we introduce the following conditions on  $\{A, g, v\}$ :

(C.1)  $A$  is injective and  $v \in D(A^{-1})$  with  $(g, A^{-1}v) \neq 0$ . In this case we introduce a constant

$$\alpha_0 := -\frac{1}{(g, A^{-1}v)}. \quad (3.6)$$

(C.2)  $A^*$  is injective and  $g \in D(A^{*-1})$  with  $(A^{*-1}g, v) \neq 0$ . In this case we introduce a constant

$$\beta_0 := -\frac{1}{(A^{*-1}g, v)}. \quad (3.7)$$

(C.3)  $A$  is injective, and  $v \notin D(A^{-1})$  or  $v \in D(A^{-1})$  with  $(g, A^{-1}v) = 0$ .

(C.4)  $A^*$  is injective, and  $g \notin D(A^{*-1})$  or  $g \in D(A^{*-1})$  with  $(A^{*-1}g, v) = 0$ .

For a linear operator  $T$  on a Hilbert space, we set  $\text{nul } T := \dim \ker T$ .

**Theorem 3.3** (i) *Suppose that (C.1) and (C.2) hold. Let*

$$\Psi_{n,j} := a(A^{-1}v)^{*n}\Omega_b \otimes b(A^{*-1}g)^{*j}\Omega_f, \quad n = 0, 1, 2, \dots, \quad j = 0, 1. \quad (3.8)$$

*Then  $\text{nul } Q(\alpha_0) = \infty$  with  $\ker Q(\alpha_0) = \overline{\mathcal{L}\{\Psi_{n,j} | n \geq 0, j = 0, 1\}}$ . Moreover, for all  $\alpha \neq \alpha_0$ ,*

$$\text{nul } Q(\alpha) = 1, \quad \ker Q(\alpha) = \{c\Omega_b \otimes \Omega_f | c \in \mathbb{C}\}. \quad (3.9)$$

(ii) *Suppose that (C.1) and (C.4) hold. Then  $\text{nul } Q(\alpha_0) = \infty$  with*

$$\ker Q(\alpha_0) = \overline{\mathcal{L}\{\Psi_{n,0} | n \geq 0\}}.$$

*Moreover, for all  $\alpha \neq \alpha_0$ , (3.9) holds.*

(iii) *Suppose that (C.2) and (C.3) hold. Then  $\text{nul } Q(\beta_0) = 2$  with  $\ker Q(\beta_0) = \mathcal{L}\{\Psi_{0,j} | j = 0, 1\}$ .*

(iv) *Suppose that (C.3) and (C.4) hold. Then, for all  $\alpha \in \mathbb{C}$ , (3.9) holds.*

As for non-zero eigenvalues of  $Q(\alpha)$ , we have the following result:

**Theorem 3.4** *Consider the case where  $\mathcal{H} = \mathcal{K}$ ,  $A$  is a nonnegative self-adjoint operator with  $\ker A = \{0\}$  and  $g = v \in D(A^{-1})$  (then  $\alpha_0 = -1/(v, A^{-1}v) < 0$ ). Let  $\alpha < \alpha_0$ . Then, there exists a constant  $x_0(\alpha) < 0$  such that  $\alpha(v, (x_0(\alpha) - A)^{-1}v) = 1$ , and for all  $n \in \{0\} \cup \mathbb{N}$ ,*

$$\pm \sqrt{n}|x_0(\alpha)| \in \sigma_p(Q(\alpha)). \quad (3.10)$$

Note that this theorem holds even if  $Q_A$  has no non-zero eigenvalues. As the condition that  $\alpha < \alpha_0$  shows, this is a *strong coupling effect*.

By Theorem 3.1 (i)-(ii), the operator  $Q_+(\alpha)$  defined by

$$D(Q_+(\alpha)) := D(Q(\alpha)) \cap \mathcal{F}_+(\mathcal{H}, \mathcal{K}), \quad Q_+(\alpha)\Psi := Q(\alpha)\Psi, \quad \Psi \in D(Q_+(\alpha)) \quad (3.11)$$

is a densely defined closed linear operator from  $\mathcal{F}_+(\mathcal{H}, \mathcal{K})$  to  $\mathcal{F}_-(\mathcal{H}, \mathcal{K})$ . We define an index of  $Q(\alpha)$  by

$$\text{ind}_\Gamma(Q(\alpha)) := \text{nul } Q_+(\alpha) - \text{nul } Q_+(\alpha)^*, \quad (3.12)$$

the index of  $Q_+(\alpha)$ , provided that  $\text{nul } Q_+(\alpha) < \infty$  or  $\text{nul } Q_+(\alpha)^* < \infty$ .

Results on the (semi-)Fredholm property and the index  $\text{ind}_\Gamma Q(\alpha)$  are as follows:

**Theorem 3.5** (i) *Suppose that (C.1) and (C.2) hold. Then  $Q(\alpha_0)$  is not semi-Fredholm. Moreover, for all  $\alpha \neq \alpha_0$ ,  $Q(\alpha)$  is Fredholm with  $\text{ind}_\Gamma Q(\alpha) = 1$ .*



- (ii) Suppose that (C.1) and (C.4) hold. Then  $Q(\alpha_0)$  is semi-Fredholm with  $\text{ind}_\Gamma Q(\alpha) = \infty$ . Moreover, for all  $\alpha \neq \alpha_0$ ,  $Q(\alpha)$  is Fredholm with  $\text{ind}_\Gamma Q(\alpha) = 1$ .
- (iii) Suppose that (C.2) and (C.3) hold. Then  $Q(\beta_0)$  is Fredholm with  $\text{ind}_\Gamma Q(\beta_0) = 0$ . Moreover, for all  $\alpha \neq \beta_0$ ,  $Q(\alpha)$  is Fredholm with  $\text{ind}_\Gamma Q(\alpha) = 1$ .
- (iv) Suppose that (C.3) and (C.4) hold. Then, for all  $\alpha \in \mathbb{C}$ ,  $Q(\alpha)$  is Fredholm with  $\text{ind}_\Gamma Q(\alpha) = 1$ .

## References

- [Ar89] A. Arai, Path integral representation of the index of Kähler-Dirac operators on an infinite dimensional manifold, *J. Funct. Anal.* **82**(1989), 330-369.
- [Ar91] A. Arai, A general class of infinite dimensional Dirac operators and related aspects, *Functional Analysis & Related Topics* (Ed. S. Koshi), pages 85–98, World Scientific, Singapore, 1991.
- [Ar92] A. Arai, A general class of infinite dimensional Dirac operators and path integral representation of their index, *J. Funct. Anal.* **105**(1992), 342–408.
- [Ar93] A. Arai, Dirac operators in Boson-Fermion Fock spaces and supersymmetric quantum field theory, *J. Geom. Phys.* **11**(1993), 465–490.
- [Ar93] A. Arai, Supersymmetric extension of quantum scalar field theories, *Quantum and Non-Commutative Analysis* (Ed. H. Araki et al), pages 73–90, Kluwer Academic Publishers, Dordrecht 1993.
- [Ar94] A. Arai, On self-adjointness of Dirac operators in boson-fermion Fock spaces, *Hokkaido Math. Jour.* **23**(1994), 319-353.
- [Ar96] A. Arai, Supersymmetric quantum field theory and infinite dimensional analysis, *Sugaku Expositions* **9**(1996), 87-98.
- [Ar97] A. Arai, Strong anticommutativity of Dirac operators on Boson-Fermion Fock spaces and representations of a supersymmetry algebra, *Math. Nachr.* **207** (1999), 61–77.
- [Ar00] A. Arai, Spectral properties of a Dirac operator on the abstract Boson-Fermion Fock space, in preparation.
- [AM91] A. Arai and I. Mitoma, De Rham-Hodge-Kodaira decomposition in  $\infty$  - dimensions, *Math. Ann.* **291** (1991), 51–73.
- [AM93] A. Arai and I. Mitoma, Comparison and nuclearity of spaces of differential forms on topological vector spaces, *J. Funct. Anal.* **111**(1993), 278–294.
- [BR97] O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 2*, Second Edition, Springer, Berlin, Heidelberg, 1997.