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SOBOLEV INEQUALITIES WITH SYMMETRY

YONGGEUN CHO AND TOHRU OZAWA

Abstract. In this paper we derive some Sobolev inequalities for radially symmetric functions in \( \dot{H}^s \) with \( \frac{1}{2} < s < \frac{n}{2} \). We show the end point case \( s = \frac{1}{2} \) on the homogeneous Besov space \( \dot{B}^{\frac{1}{2}}_{2,1} \). These results are extensions of the well-known Strauss’ inequality [11]. Also non-radial extensions are given, which show that compact embeddings are possible in some \( L^p \) spaces if a suitable angular regularity is imposed.

1. Introduction

In this paper we derive Sobolev inequalities with symmetry. We first consider several Sobolev inequalities for radially symmetric functions in \( \dot{H}^s(\mathbb{R}^n) \) with \( \frac{1}{2} < s < \frac{n}{2} \). There is a sharp result by Sickel and Skrzypczak [8], although the argument below is much simpler and direct and a constant in the inequality in Proposition 1 below is given explicitly in terms of \( s \).

Definition 1.

\[ \dot{H}^s_{\text{rad}} = \{ u \in \dot{H}^s(\mathbb{R}^n) : u \text{ is radially symmetric } \}, \ s \geq 0. \]
\[ \dot{B}^{s}_{p,q,\text{rad}} = \{ u \in \dot{B}^{s}_{p,q}(\mathbb{R}^n) : u \text{ is radially symmetric} \}, \ s \geq 0, \ 1 \leq p, q \leq \infty. \]

The inhomogeneous spaces of radially symmetric functions are defined by the same way with spaces \( H^s \) and \( B^s_{p,q} \).

Proposition 1. Let \( n \geq 2 \) and let \( s \) satisfy \( 1/2 < s < n/2 \).

Then

\[ \sup_{x \in \mathbb{R}^n \setminus \{0\}} |x|^{n/2-s}|u(x)| \leq C(n, s) \|(-\Delta)^{s/2}u\|_{L^2} \]  

for all \( u \in \dot{H}^s_{\text{rad}} \), where

\[ C(n, s) = \left( \frac{\Gamma(2s - 1) \Gamma(n/2 - s) \Gamma(n/2)}{2^{2s-n/2} \Gamma(s)^2 \Gamma(n/2 - 1 + s)} \right)^{1/2} \]

and \( \Gamma \) is the gamma function.

Remark 1. For \( s = 1 \) with \( n \geq 3 \), the inequality (1) reduces to Ni’s inequality [6, 7].

Remark 2. The restriction \( 1/2 < s < n/2 \) is necessary for \( C(n, s) \) to be finite.

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Remark 3. The inequality (1) fails for \( s = n/2 \). Indeed, \( u(x) = \mathcal{F}^{-1} \left( \frac{1}{(1+|\xi|)^{n/2}+\log(1+|\xi|)} \right) \) satisfies \( u \in H^{n/2}_\text{rad} \), and \( u \notin L^\infty \) where \( \mathcal{F} \) is the Fourier transform [12] and \( \mathcal{F}^{-1} \) is its inverse.

Remark 4. The inequality (1) fails if \( 0 \leq s < 1/2 \) and \( n \geq 3 \). Indeed, \( u(x) = \mathcal{F}^{-1}(J_{n/2-1}(|\xi|)|\xi|^{-n/2}) \) satisfies \( u \in \dot{H}^s_\text{rad} \) and \( u(x) = \infty \) for all \( x \in S^{n-1} \), where we note that \( u \in \dot{H}^s_\text{rad} \) if and only if \( 1-n/2 < s < 1/2 \), since

\[
\|(-\Delta)^{s/2}u\|_{L^2}^2 = c_n \int_0^\infty |J_{n/2-1}(\rho)|^2 \rho^{2s-1} \, d\rho
\]

and that by the asymptotic behavior of Bessel function (10)

\[
u(x) = \int_0^\infty |J_{n/2-1}(\rho)|^2 \, d\rho = \infty, \quad x \in S^{n-1}.
\]

See also the proof of Proposition 1 below.

In the endpoint case \( s = 1/2 \), we have the following proposition.

**Proposition 2.** Let \( n \geq 2 \). Then there exists a constant \( C \) such that

\[
\sup_{x \in \mathbb{R}^n \setminus \{0\}} |x|^{(n-1)/2} |u(x)| \leq C \|u\|_{\dot{B}^{1/2}_{2,1}}
\]

for all \( u \in \dot{B}^{1/2}_{2,1,\text{rad}} \).

Remark 5. The inhomogeneous version of (2) has been given in [8] whose proof is based on the atomic decomposition.

**Proposition 3.** Let \( n \geq 2 \) and let \( s \) satisfy \( 1/2 \leq s < 1 \). Then there exists \( C \) such that

\[
\sup_{x \in \mathbb{R}^n \setminus \{0\}} |x|^{n/2-s} |u(x)| \leq C(n, s) \|u\|_{L^2}^{1-s} \|\nabla u\|_{L^2}^s.
\]

**Remark 6.** For \( s = 1/2 \), the inequality (3) reduces to Strauss’ inequality [11].

**Remark 7.** For \( s = 0 \), the inequality (3) holds for nonincreasing functions in \(|x|\) [2]. For \( s = 1 \), the inequality (3) holds for \( n \geq 3 \) and fails for \( n = 2 \). See Proposition 1 and Remark 2.

Now we extend the results above on radial functions to the non-radial functions with additional angular regularity. For details, let us define function spaces \( H^{s,m}_\omega \) and \( B^{s,m}_{2,1,\omega} \), \( s \geq 0, m \geq 0 \) as follows.
Definition 2.

\[ H^{s,m}_\omega = \left\{ u \in H^s : \| u \|_{H^{s,m}_\omega} = \| (1 - \Delta_\omega)^{m/2} u \|_{H^s} < \infty \right\}, \]

\[ B^{s,m}_{2,1,\omega} = \left\{ u \in B^{\frac{3}{2}}_{2,1} : \| u \|_{B^{s,m}_{2,1,\omega}} = \| (1 - \Delta_\omega)^{m/2} u \|_{B^{s}_{2,1}} < \infty \right\}, \]

where \( \Delta_\omega \) is the Laplace-Beltrami operator on \( S^{n-1} \).

The homogeneous spaces \( \hat{H}^{s,m}_\omega \) and \( \hat{B}^{s,m}_{2,1,\omega} \) is similarly defined by the definition of \( \hat{H}^s \) and \( \hat{B}^s_{2,1} \). Then we have the following.

Proposition 4.  
(1) If \( 1/2 < s < n/2 \) and \( m > n - 1 - s \), then there exists a constant \( C \) such that for any \( u \in H^{s,m}_\omega \)

\[ \sup_{\mathbb{R}^n \setminus \{0\}} |x|^{n/2 - s} |u(x)| \leq C \| u \|_{H^{s,m}_\omega}. \]  

(2) If \( m > n - \frac{3}{2} \), then there exists a constant \( C \) such that for any \( u \in B^{\frac{1}{2},m}_{2,1,\omega} \)

\[ \sup_{\mathbb{R}^n \setminus \{0\}} |x|^{(n-1)/2} |u(x)| \leq C \| u \|_{B^{\frac{1}{2},m}_{2,1,\omega}}. \]

Remark 8. \( H^{s,m}_\omega \) and \( B^{\frac{1}{2},m}_{2,1,\omega} \) are closed subspaces of \( H^s \) and \( B^{\frac{1}{2}}_{2,1} \), respectively and they contain \( H^s_{\text{rad}} \) and \( B^{\frac{1}{2}}_{2,1,\text{rad}} \) naturally, respectively. We can identify the spaces \( H^s \) with \( (1 - \Delta_\omega)^{m/2} H^{s,m}_\omega \) and also \( B^{\frac{1}{2}}_{2,1} \) with \( (1 - \Delta_\omega)^{m/2} B^{\frac{1}{2},m}_{2,1,\omega} \).

Remark 9. \( H^{s,m}_\omega \) is a Hilbert space with the same inner product as \( L^2 \) space and its dual space is given by \( H^{-s,-m}_\omega \).

From the decay at infinity we deduce compact embeddings of \( H^{s,m}_\omega \) and \( B^{\frac{1}{2},m}_{2,1,\omega} \) into some \( L^p \) spaces as follows. See [2, 3, 4, 8, 9] for the radial case.

Corollary 1. The embedding \( H^{s,m}_\omega \hookrightarrow L^p \) is compact for \( 1/2 < s < n/2, m > n - 1 - s \) and \( 2 < p < 2n/(n-2s) \).

Corollary 2. The embedding \( B^{\frac{1}{2},m}_{2,1,\omega} \hookrightarrow L^p \) is compact for \( m > n - 3/2 \) and \( 2 < p < 2n/(n-1) \).

2. Proofs

2.1. Proof of Proposition 1. We use the following Fourier representation for radially symmetric functions as

\[ u(x) = |x|^{1-n/2} \int_0^\infty J_{\frac{n}{2}-1}(|x|\rho)\hat{u}(\rho)\rho^{\frac{n}{2}} d\rho, \quad (6) \]

where \( J_\nu \) is the Bessel function of order \( \nu \), \( \hat{u} \) is the Fourier transform normalized as

\[ \hat{u}(\xi) = (2\pi)^{-n/2} \int e^{-ix\cdot\xi} u(x) dx, \]
and we have identified radially symmetric functions on $\mathbb{R}^n$ with the corresponding functions on $(0,\infty)$.

By the Cauchy-Schwarz inequality and the Plancherel formula, we have
\[
| |x|^{n/2-s}|u(x)| \leq |x|^{1-s} \left( \int_0^\infty |J_{n/2-1}(|x|\rho)|^2 \rho^{1-2s}d\rho \right)^{1/2} \left( \int_0^\infty |\hat{u}(\rho)|^2 \rho^{2s+n-1}d\rho \right)^{1/2}
\]
\[
= \left( \int_0^\infty |J_{n/2-1}(\rho)|^2 \rho^{1-2s}d\rho \right)^{1/2} \left( \frac{\Gamma(n/2)}{2\pi^{n/2}} \int |\xi|^{2s}|\hat{u}(\xi)|^2d\xi \right)^{1/2}
\]
\[
= C(n, s)\|(-\Delta)^{s/2}u\|_{L^2},
\]
as required.

2.2. **Proof of Proposition 2.** We use the following estimates on Bessel functions:
\[
\sup_{r\geq 0} |J_{n/2-1}(r)| \leq 1. \quad (8)
\]
\[
\sup_{r\geq 0} r^{1/2} |J_{n/2-1}(r)| \leq C. \quad (9)
\]
The first inequality (8) follows from the integral representation (see [13])
\[
J_{n/2-1}(r)^2 = \frac{2}{\pi} \int_0^{\pi/2} J_{n-2}(2r \cos \theta)d\theta,
\]
\[
J_m(t) = \frac{1}{2\pi} \int_0^{2\pi} \cos(m\theta - t \sin \theta)d\theta, \quad m \in \mathbb{Z}.
\]
The second inequality (9) follows from the first and the well-known asymptotics (see [10])
\[
J_\nu(r) \sim \sqrt{\frac{2}{\pi r}} \cos \left( r - \frac{(2\nu + 1)\pi}{4} \right) \quad \text{as} \quad r \to \infty. \quad (10)
\]
We apply the Littlewood-Paley decomposition $\{\varphi_j\}_{j \in \mathbb{Z}}$ on $\mathbb{R}^n \setminus \{0\}$ to (4) to obtain
\[
u(x) = |x|^{n/2} \sum_{j \in \mathbb{Z}} \int_0^\infty J_{n/2-1}(|x|\rho)\varphi_j(\rho)\hat{u}(\rho)\rho^n d\rho,
\]
where $\psi_j = \varphi_{j-1} + \varphi_j + \varphi_{j+1}$ and supp $\varphi_j \subset \{2^j \leq \rho \leq 2^{j+1}\}$.

As in (7), we have
\[
|x|^{(n-1)/2}|u(x)| \leq |x|^{1/2} \sup_{j \in \mathbb{Z}} \left( \int_0^\infty |J_{n/2-1}(|x|\rho)|^2 \psi_j(\rho)^2 d\rho \right)^{1/2}
\]
\[
\cdot \sum_{j \in \mathbb{Z}} \left( \int_0^\infty |\varphi_j(\rho)\hat{u}(\rho)|^2 \rho^n d\rho \right)^{1/2}.
\]
By (9), we estimate
\[ |x|^{1/2} \sup_{j \in \mathbb{Z}} \left( \int_0^\infty |J_{\frac{n}{2} - 1}(|x|\rho)|^2 \psi_j(\rho)^2 d\rho \right)^{1/2} \]
\[ \leq C \sup_{j \in \mathbb{Z}} \left( \int_0^\infty \frac{1}{\rho} \psi_j(\rho)^2 d\rho \right)^{1/2} \]
\[ \leq C \sup_{j \in \mathbb{Z}} \left( \int_{2^j}^{2^{j+2}} \frac{1}{\rho} d\rho \right)^{1/2} \leq C. \]

This proves (2) since
\[ \sum_{j \in \mathbb{Z}} \left( \int_0^\infty |\varphi_j(\rho) \hat{u}(\rho)|^2 \rho^n d\rho \right)^{1/2} \]
is equivalent to the seminorm on $\dot{B}^{1/2}_{2,1,rad}$.

2.3. **Proof of Proposition 3.** If we use Cauchy-Schwartz inequality as in (7), we have
\[ \left| |x|^{n/2-s}|u(x)| \right| \]
\[ \leq |x|^{-s} \left( \int_0^{M|x|} |J_{\frac{n}{2} - 1}(|x|\rho)|^2 \rho d\rho \right)^{1/2} \left( \int_0^{M|x|} |\hat{u}(\rho)|^2 \rho^{n-1} d\rho \right)^{1/2} \]
\[ + |x|^{-s} \left( \int_{M|x|}^\infty |J_{\frac{n}{2} - 1}(|x|\rho)|^2 \rho^{n-1} d\rho \right)^{1/2} \left( \int_{M|x|}^\infty |\hat{u}(\rho)|^2 \rho^{n+1} d\rho \right)^{1/2} \]
\[ \leq |x|^{-s} \left( \int_0^M |J_{\frac{n}{2} - 1}(r)|^2 r dr \right)^{1/2} \|u\|_{L^2} \]
\[ + |x|^{-s} \left( \int_M^\infty |J_{\frac{n}{2} - 1}(r)|^2 r^{-1} dr \right)^{1/2} \|\nabla u\|_{L^2}. \]

From (8) and (9) we deduce that $\sup_{r \geq 0} |r^{1-s}J_{\frac{n}{2} - 1}(r)| \leq C$ for any $\frac{1}{2} \leq s < 1$. Hence we have for any $M > 0$
\[ |x|^{n/2-s}|u(x)| \]
\[ \leq |x|^{-s} \left( \int_0^M |J_{\frac{n}{2} - 1}(r)|^2 r dr \right)^{1/2} \|u\|_{L^2} \]
\[ + |x|^{1-s} \left( \int_M^\infty |J_{\frac{n}{2} - 1}(r)|^2 r^{-1} dr \right)^{1/2} \|\nabla u\|_{L^2} \]
\[ \leq C |x|^{-s} M^n \|u\|_{L^2} + |x|^{1-s} M^{-1-s} \|\nabla u\|_{L^2}. \]

The minimization of the RHS of the last inequality with respect to $M$ yields Proposition 3.
2.4. Proofs of Proposition 4 and Corollaries. The proof for (4) follows from the one of Proposition 1 and the spherical harmonic expansion of functions in $H^s_w$ [5, 10]. In fact, if we write $u(r\omega) = \sum_{k\geq 0} \sum_{1 \leq l \leq d(k)} f_{k,l}(r)Y_{k,l}(\omega)$, where $d(k)$ is the dimension of space of spherical harmonic functions of degree $k$ and

$$d(k) \leq Ck^{n-2} \text{ for large } k.$$  \hfill (13)

Then we have

$$|x|^{\frac{n}{2}-s}|u(x|\omega|) = c_n \sum_{k,l} |x|^{1-s} \int_0^\infty J_{\nu(k)}(|x|\rho)|\rho^2 g_{k,l}(\rho)\rho Y_{k,l}(\omega),$$ \hfill (14)

where $\omega \in S^{n-1}$, $\nu(k) = \frac{n+2k-2}{2}$ and $\int_{\partial S^d} Y_{k,l}(\rho\omega) = g_{k,l}(\rho)Y_{k,l}(\omega)$. Here

$$g_{k,l}(\rho) = c_{n,k} \int_0^\infty f_{k,l}(r)|r^{n-1}(r\rho)^{-\frac{n-2}{2}}J_{\nu(k)}(r\rho)\rho dr.$$ \hfill \text{(see for instance)} \hfill \text{[10]} \text{.}

The absolute value of $c_{n,k}$ is bounded by a constant depending only on $n$. See [10] for this.

Using the Cauchy-Schwarz inequality as in (7), we have

$$|x|^{\frac{n}{2}-s}|u(x|\omega|)$$

$$\leq C \sum_{k,l} \|Y_{k,l}\|_{L^\infty(S^{n-1})} \left( \int_0^\infty |J_{\nu(k)}(|x|\rho)|^2 \rho^{1-2s} \left( \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1}d\rho \right)^{\frac{1}{2}} \right)$$

$$\leq C \sum_{k,l} k^{\frac{n-2}{2}} \left( \frac{\Gamma(\nu(k) + 1 - s)}{\Gamma(\nu(k) + s)} \right)^{\frac{1}{2}} \left( \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1}d\rho \right)^{\frac{1}{2}} \|Y_{k,l}\|_{L^2(S^{n-1})}.$$ \hfill \text{(see for instance)} \hfill \text{[10]} \text{.

Here we used the inequality that $\|Y_{k,l}\|_{L^\infty} \leq Ck^{-\frac{n-2}{2}}\|Y_{k,l}\|_{L^2}$ (see for instance [10]). Using the Stirling’s formula for gamma function that $\Gamma(t) \approx t^{-\frac{1}{2}} e^{-(t-1)}$ for large $t$ (for instance, see [1]) and the fact $-\Delta_\omega Y_{k,l} = k(k + n - 2)Y_{k,l}$, we have from (13)

$$|x|^{\frac{n}{2}-s}|u(x|\omega|)$$

$$\leq C \sum_k k^{\frac{n-2}{2}} d(k)^{\frac{1}{2}} \left( \frac{\Gamma(\nu(k) + 1 - s)}{\Gamma(\nu(k) + s)} \right)^{\frac{1}{2}}$$

$$\cdot \left( \sum_{1 \leq l \leq d(k)} \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1}d\rho \|Y_{k,l}\|_{L^2(S^{n-1})}^2 \right)^{\frac{1}{2}}$$

$$\leq C \left( \sum_k k^{2(n-\frac{2}{2}-s-m)} \right)^{\frac{1}{2}} \left( \sum_{k,l} k^{2m} \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1}d\rho \|Y_{k,l}\|_{L^2(S^{n-1})}^2 \right)^{\frac{1}{2}}$$

$$\leq C \left( \sum_{k,l} k^{2m} \int_0^\infty \int_{S^{n-1}} |F(f_{k,l}Y_{k,l})(\rho\omega)|^2 \rho^{2s+n-1}d\rho d\omega \right)^{\frac{1}{2}}$$

$$\leq C \left( \sum_{k,l} \int_0^\infty \int_{S^{n-1}} |F((1 - \Delta_\omega)\frac{m}{2}(f_{k,l}Y_{k,l}))(\rho\omega)|^2 \rho^{2s+n-1}d\rho d\omega \right)^{\frac{1}{2}}$$

$$\leq C \|u\|_{H^s_w},$$
where $F$ is the Fourier transform. This proves part (1).

For the part (2), if we use the Littlewood-Paley decomposition \{\phi_j\}_{j \in \mathbb{Z}} as in the proof of Proposition 2, the we have

$$|x|^{\frac{n-1}{2}} u(|x|) = c_n \sum_{j \in \mathbb{Z}} \sum_{k,l} |x|^{\frac{1}{2}} \int_0^\infty \psi_j(\rho) \phi_j(\rho) g_{k,l}(\rho) d\rho Y_{k,l}(\omega),$$  \hspace{1cm} (15)

Since $m > n - 3/2$, by (12) we deduce that

$$|x|^{\frac{n-1}{2}} |u(|x|)| \leq C \sum_{j \in \mathbb{Z}} \sum_{k,l} \left[ \int_0^\infty |\phi_j(\rho) g_{k,l}(\rho)|^2 \rho^n d\rho \right]^{\frac{1}{2}} \|Y_{k,l}\|_{L^\infty(S^{n-1})}$$

$$\leq C \sum_{j \in \mathbb{Z}} \left( \sum_k k^{2(n-2-m)} \right)^{\frac{1}{2}} \left( \sum_{k,l} \int_0^\infty |\phi_j(\rho) g_{k,l}(\rho)|^2 \rho^n d\rho \right)^{\frac{1}{2}}$$

$$\leq C \sum_{j \in \mathbb{Z}} 2^{\frac{j}{2}} \|\phi_j F((1 - \Delta_\omega) \frac{u}{R})\|_{L^2} = C\|u\|_{B^{\frac{3}{2}}_{2,1,\omega}}.$$  

To show Corollary 1 we use the fact that $H^{s,m}_\omega$ is a Hilbert space. Hence any bounded sequence \{\{u_j\}\} in $H^{s,m}_\omega$ satisfies $u_j(x) \to 0$ as $|x| \to \infty$ uniformly and has a subsequence converges to $u$ in $H^{s,m}_\omega$ weakly. Let us denote the subsequence by $u_{j_k}$.

Now choose a smooth function $\phi$ supported in the ball of radius $R + 1$ and with value 1 in the ball of radius $R$. By the standard argument one can easily show that for each $R$ the mapping $u \mapsto \phi u$ is compact from $H^t$ to $H^{t'}$ if $t' < t$. By the compactness above and Sobolev embedding we may assume that the sequence $\phi u_{j_k}$ satisfies that for $2 \leq q < \frac{2n}{n-2m}$,

$$\|\phi u_{j_k} - \phi u\|_{L^q} \to 0 \text{ as } k \to \infty.$$  \hspace{1cm} (16)

Thus we have

$$\|u_{j_k} - u\|_{L^p} \leq \|\phi (u_{j_k} - u)\|_{L^p} + \|\phi (u_{j_k} - u)\|_{L^p} \equiv I_k + II_k$$

with $I_k \to 0$ as $k \to \infty$ by (16) since $2 < p < \frac{2n}{n-2m}$. From the uniform convergence that $|u_{j_k}(x)| + |u(x)| \to 0$ as $|x| \to \infty$ it follows that

$$\limsup_{k \to \infty} II_k \leq \sup_k \|u_{j_k} - u\|_{L^\infty(|x| > R)} \to 0 \text{ as } R \to \infty.$$ 

This proves the compactness of the embedding $H^{s,m}_\omega \hookrightarrow L^p$.

Since $B^{\frac{3}{2}}_{2,1,\omega} \hookrightarrow H^{s,m}_\omega$, one can adapt the same arguments (compactness of cut-off mapping and uniform convergence at infinity) as above except for weak-* convergence of $u_{j_k}$ to $u$ in $B^{\frac{3}{2}}_{2,1,\omega}$ for the compactness of embedding $B^{\frac{3}{2}}_{2,1,\omega}$ to $L^p$. This completes the proof.
References


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