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**HOKKAIDO UNIVERSITY**
SOBOLEV INEQUALITIES WITH SYMMETRY

YONGGEUN CHO AND TOHRU OZAWA

Abstract. In this paper we derive some Sobolev inequalities for radially symmetric functions in \( \dot{H}^s \) with \( \frac{1}{2} < s < \frac{n}{2} \). We show the end point case \( s = \frac{1}{2} \) on the homogeneous Besov space \( \dot{B}^\frac{n}{2}_2 \). These results are extensions of the well-known Strauss’ inequality [11]. Also non-radial extensions are given, which show that compact embeddings are possible in some \( L^p \) spaces if a suitable angular regularity is imposed.

1. Introduction

In this paper we derive Sobolev inequalities with symmetry. We first consider several Sobolev inequalities for radially symmetric functions in \( \dot{H}^s(\mathbb{R}^n) \) with \( \frac{1}{2} < s < \frac{n}{2} \). There is a sharp result by Sickel and Skrzypczak [8], although the argument below is much simpler and direct and a constant in the inequality in Proposition 1 below is given explicitly in terms of \( s \).

Definition 1.

\[ \dot{H}^s_{\text{rad}} = \{ u \in \dot{H}^s(\mathbb{R}^n) : u \text{ is radially symmetric } \}, \hspace{1cm} s \geq 0. \]

\[ \dot{B}^s_{p,q,\text{rad}} = \{ u \in \dot{B}^s_{p,q}(\mathbb{R}^n) : u \text{ is radially symmetric} \}, \hspace{1cm} s \geq 0, \hspace{0.5cm} 1 \leq p, q \leq \infty. \]

The inhomogeneous spaces of radially symmetric functions are defined by the same way with spaces \( H^s \) and \( B^s_{p,q} \).

Proposition 1. Let \( n \geq 2 \) and let \( s \) satisfy \( 1/2 < s < n/2 \).

Then

\[ \sup_{x \in \mathbb{R}^n \setminus \{0\}} |x|^{n/2-s}|u(x)| \leq C(n, s)(-\Delta)^{s/2}u \|L^2\] (1)

for all \( u \in \dot{H}^s_{\text{rad}} \), where

\[ C(n, s) = \left( \frac{\Gamma(2s - 1)\Gamma(\frac{n}{2} - s)\Gamma(\frac{n}{2})}{2^{2s-n/2}\Gamma(s)^2\Gamma(\frac{n}{2} - 1 + s)} \right)^{1/2} \]

and \( \Gamma \) is the gamma function.

Remark 1. For \( s = 1 \) with \( n \geq 3 \), the inequality (1) reduces to Ni’s inequality [6, 7].

Remark 2. The restriction \( 1/2 < s < n/2 \) is necessary for \( C(n, s) \) to be finite.

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Remark 3. The inequality (1) fails for $s = n/2$. Indeed, $u(x) = \mathcal{F}^{-1}\left(\frac{1}{(1+|\xi|)^n(1+\log(1+|\xi|))}\right)$ satisfies $u \in H^{n/2}_{\text{rad}}$, and $u \notin L^\infty$ where $\mathcal{F}$ is the Fourier transform [12] and $\mathcal{F}^{-1}$ is its inverse.

Remark 4. The inequality (1) fails if $0 \leq s < 1/2$ and $n \geq 3$. Indeed, $u = \mathcal{F}^{-1}(J_{n/2-1}(\xi)|\xi|^{-n/2})$ satisfies $u \in \dot{H}^s_{\text{rad}}$ and $u(x) = \infty$ for all $x \in S^{n-1}$, where we note that $u \in \dot{H}^s_{\text{rad}}$ if and only if $1 - n/2 < s < 1/2$, since
\[
\|(-\Delta)^{s/2}u\|^2_{L^2} = c_n \int_0^\infty |J_{n/2-1}(\rho)|^2 \rho^{2s-1}d\rho
\]
and that by the asymptotic behavior of Bessel function (10)
\[
u(x) = \int_0^\infty |J_{n/2-1}(\rho)|^2d\rho = \infty, \quad x \in S^{n-1}.
\]
See also the proof of Proposition 1 below.

In the endpoint case $s = 1/2$, we have the following proposition.

Proposition 2. Let $n \geq 2$. Then there exists a constant $C$ such that
\[
\sup_{x \in \mathbb{R}^n \backslash \{0\}} |x|^{(n-1)/2}|u(x)| \leq C\|u\|_{\dot{B}^{1/2}_{2,1}}
\]
for all $u \in \dot{B}^{1/2}_{2,1,\text{rad}}$.

Remark 5. The inhomogeneous version of (2) has been given in [8] whose proof is based on the atomic decomposition.

Proposition 3. Let $n \geq 2$ and let $s$ satisfy $1/2 \leq s < 1$. Then there exists $C$ such that for all $u \in H^1_{\text{rad}}$
\[
\sup_{x \in \mathbb{R}^n \backslash \{0\}} |x|^{n/2-s}|u(x)| \leq C(n, s)\|u\|_{L^2}^{1-s}\|\nabla u\|_{L^2}^s.
\]

Remark 6. For $s = 1/2$, the inequality (3) reduces to Strauss’ inequality [11].

Remark 7. For $s = 0$, the inequality (3) holds for nonincreasing functions in $|x|$ [2]. For $s = 1$, the inequality (3) holds for $n \geq 3$ and fails for $n = 2$. See Proposition 1 and Remark 2.

Now we extend the results above on radial functions to the non-radial functions with additional angular regularity. For details, let us define function spaces $H^{s,m}_\omega$ and $B^{s,m}_{2,1,\omega}$, $s \geq 0, m \geq 0$ as follows.
Definition 2.

\[ H_{\omega}^{s,m} = \left\{ u \in H^{s} : \|u\|_{H_{\omega}^{s,m}} = \|(1 - \Delta_{\omega})^{\frac{s}{2}} u\|_{H^{s}} < \infty \right\}, \]

\[ B_{2,1,\omega}^{s,m} = \left\{ u \in B_{2,1}^{s} : \|u\|_{B_{2,1,\omega}^{s,m}} = \|(1 - \Delta_{\omega})^{\frac{s}{2}} u\|_{B_{2,1}^{s}} < \infty \right\}, \]

where \( \Delta_{\omega} \) is the Laplace-Beltrami operator on \( S^{n-1} \).

The homogeneous spaces \( \hat{H}_{\omega}^{s,m} \) and \( \hat{B}_{2,1,\omega}^{s,m} \) is similarly defined by the definition of \( \hat{H}^{s} \) and \( \hat{B}_{2,1}^{s} \). Then we have the following.

**Proposition 4.**

1. If \( 1/2 < s < n/2 \) and \( m > n - 1 - s \), then there exists a constant \( C \) such that for any \( u \in H_{\omega}^{s,m} \)

\[
\sup_{\mathbb{R}^{n}\setminus\{0\}} |x|^{n/2-s}|u(x)| \leq C\|u\|_{\hat{H}_{\omega}^{s,m}}.
\]

2. If \( m > n - \frac{3}{2} \), then there exists a constant \( C \) such that for any \( u \in B_{2,1,\omega}^{s,m} \)

\[
\sup_{\mathbb{R}^{n}\setminus\{0\}} |x|^{(n-1)/2}|u(x)| \leq C\|u\|_{\hat{B}_{2,1,\omega}^{s,m}}.
\]

**Remark 8.** \( H_{\omega}^{s,m} \) and \( B_{2,1,\omega}^{s,m} \) are closed subspaces of \( H^{s} \) and \( B_{2,1}^{s} \), respectively and they contain \( H_{\text{rad}}^{s} \) and \( B_{2,1,\text{rad}}^{s} \) naturally, respectively. We can identify the spaces \( H^{s} \) with \( (1 - \Delta_{\omega})^{m/2}H_{\omega}^{s,m} \) and also \( B_{2,1}^{s} \) with \( (1 - \Delta_{\omega})^{m/2}B_{2,1,\omega}^{s,m} \).

**Remark 9.** \( H_{\omega}^{s,m} \) is a Hilbert space with the same inner product as \( L^{2} \) space and its dual space is given by \( H_{\omega}^{-s,-m} \).

From the decay at infinity we deduce compact embeddings of \( H_{\omega}^{s,m} \) and \( B_{2,1,\omega}^{s,m} \) into some \( L^{p} \) spaces as follows. See \([2, 3, 4, 8, 9]\) for the radial case.

**Corollary 1.** The embedding \( H_{\omega}^{s,m} \hookrightarrow L^{p} \) is compact for \( 1/2 < s < n/2, \ m > n - 1 - s \) and \( 2 < p < 2n/(n-2s) \).

**Corollary 2.** The embedding \( B_{2,1,\omega}^{s,m} \hookrightarrow L^{p} \) is compact for \( m > n - 3/2 \) and \( 2 < p < 2n/(n-1) \).

2. Proofs

2.1. **Proof of Proposition 1.** We use the following Fourier representation for radially symmetric functions as

\[
u(x) = |x|^{1-\nu} \int_{0}^{\infty} J_{\nu}^{-1}(\rho)\hat{u}(\rho)\rho^{\frac{\nu}{2}}d\rho, \]

where \( J_{\nu} \) is the Bessel function of order \( \nu \), \( \hat{u} \) is the Fourier transform normalized as

\[
\hat{u}(\xi) = (2\pi)^{-n/2} \int e^{-ix\xi}u(x)dx,
\]
and we have identified radially symmetric functions on $\mathbb{R}^n$ with the corresponding functions on $(0, \infty)$.

By the Cauchy-Schwarz inequality and the Plancherel formula, we have

$$|x|^{\frac{n}{2} - s}|u(x)| \leq |x|^{1 - s} \left( \int_0^\infty |J_{\frac{n}{2} - 1}(|x|\rho)|^2 \rho^{1 - 2s} d\rho \right)^{1/2} \left( \int_0^\infty |\hat{u}(\rho)|^2 \rho^{2s + n - 1} d\rho \right)^{1/2}$$

$$= \left( \int_0^\infty |J_{\frac{n}{2} - 1}(\rho)|^2 \rho^{1 - 2s} d\rho \right)^{1/2} \left( \frac{\Gamma(\frac{n}{2})}{2\pi^{n/2}} \int |\xi|^{2s} |\hat{u}(\xi)|^2 d\xi \right)^{1/2}$$

$$= C(n, s)\|(-\Delta)^{s/2} u\|_{L^2}$$

as required.

2.2. Proof of Proposition 2. We use the following estimates on Bessel functions:

$$\sup_{r \geq 0} |J_{\frac{n}{2} - 1}(r)| \leq 1. \quad (8)$$

$$\sup_{r \geq 0} r^{1/2} |J_{\frac{n}{2} - 1}(r)| \leq C. \quad (9)$$

The first inequality (8) follows from the integral representation (see [13])

$$J_{\frac{n}{2} - 1}(r)^2 = \frac{2}{\pi} \int_0^{\pi/2} J_{n-2}(2r \cos \theta) d\theta,$$

$$J_m(t) = \frac{1}{2\pi} \int_0^{2\pi} \cos(m\theta - t \sin \theta) d\theta, \ m \in \mathbb{Z}.$$

The second inequality (9) follows from the first and the well-known asymptotics (see [10])

$$J_\nu(r) \sim \sqrt{\frac{2}{\pi r}} \cos \left( r - \frac{(2\nu + 1)\pi}{4} \right) \quad \text{as} \quad r \to \infty. \quad (10)$$

We apply the Littlewood-Paley decomposition $\{\varphi_j\}_{j \in \mathbb{Z}}$ on $\mathbb{R}^n \setminus \{0\}$ to (4) to obtain

$$u(x) = |x|^{1 - \frac{n}{2}} \sum_{j \in \mathbb{Z}} \int_0^\infty J_{\frac{n}{2} - 1}(|x|\rho) \varphi_j(\rho) \tilde{u}(|\rho|) \rho^n d\rho,$$

where $\psi_j = \varphi_{j-1} + \varphi_j + \varphi_{j+1}$ and supp $\varphi_j \subset \{2^j - 1 \leq \rho \leq 2^{j+1}\}$.

As in (7), we have

$$|x|^{(n-1)/2}|u(x)| \leq |x|^{1/2} \sup_{j \in \mathbb{Z}} \left( \int_0^\infty |J_{\frac{n}{2} - 1}(|x|\rho)|^2 \psi_j(\rho)^2 d\rho \right)^{1/2} \cdot \sum_{j \in \mathbb{Z}} \left( \int_0^\infty |\varphi_j(\rho)\tilde{u}(\rho)|^2 \rho^n d\rho \right)^{1/2}.$$
By (9), we estimate
\[ |x|^{1/2} \sup_{j \in \mathbb{Z}} \left( \int_0^\infty |J_{2^{-1}}(|x|)\rho|^2 \psi_j(\rho)^2 \, d\rho \right)^{1/2} \]
\[ \leq C \sup_{j \in \mathbb{Z}} \left( \int_0^\infty \frac{1}{\rho} \psi_j(\rho)^2 \, d\rho \right)^{1/2} \]
\[ \leq C \sup_{j \in \mathbb{Z}} \left( \int_{2^{j+2}}^{2^{j+3}} \frac{1}{\rho} \, d\rho \right)^{1/2} \leq C. \]  

(12)

This proves (2) since
\[ \sum_{j \in \mathbb{Z}} \left( \int_0^\infty |\varphi_j(\rho)\hat{u}(\rho)|^2 \rho^n \, d\rho \right)^{1/2} \]
is equivalent to the seminorm on \( \dot{B}_{2,1}^{1/2, \text{rad}} \).

2.3. **Proof of Proposition 3.** If we use Cauchy-Schwartz inequality as in (7), we have for any \( M > 0 \)
\[ |x|^{n/2-s} |u(x)| \]
\[ \leq |x|^{1-s} \left( \int_0^M |J_{2^{-1}}(|x|)\rho|^2 \rho d\rho \right)^{1/2} \left( \int_0^M |\hat{u}(\rho)|^2 \rho^{n-1} d\rho \right)^{1/2} \]  
\[ + |x|^{1-s} \left( \int_M^\infty |J_{2^{-1}}(|x|)\rho|^2 \rho^{-1} d\rho \right)^{1/2} \left( \int_M^\infty |\hat{u}(\rho)|^2 \rho^{n+1} d\rho \right)^{1/2} \]
\[ \leq |x|^{-s} \left( \int_0^M |J_{2^{-1}}(r)|^2 r \, dr \right)^{1/2} \|u\|_{L^2} \]
\[ + |x|^{1-s} \left( \int_M^\infty |J_{2^{-1}}(r)|^2 r^{-1} \, dr \right)^{1/2} \|\nabla u\|_{L^2}. \]

From (8) and (9) we deduce that \( \sup_{r \geq 0} |r^{1-s} J_{2^{-1}}(r)| \leq C \) for any \( \frac{1}{2} \leq s < 1 \). Hence we have for any \( M > 0 \)
\[ |x|^{n/2-s} |u(x)| \]
\[ \leq |x|^{-s} \left( \int_0^M |J_{2^{-1}}(r)|^2 r \, dr \right)^{1/2} \|u\|_{L^2} \]  
\[ + |x|^{1-s} \left( \int_M^\infty |J_{2^{-1}}(r)|^2 r^{-1} \, dr \right)^{1/2} \|\nabla u\|_{L^2} \]
\[ \leq C |x|^{-s} M^n \|u\|_{L^2} + |x|^{1-s} M^{-(1-s)} \|\nabla u\|_{L^2}. \]

The minimization of the RHS of the last inequality with respect to \( M \) yields Proposition 3.
2.4. Proofs of Proposition 4 and Corollaries. The proof for (4) follows from the one of Proposition 1 and the spherical harmonic expansion of functions in $H^{s,m}_n$ [5, 10]. In fact, if we write $u(r\omega) = \sum_{k,l} a_{k,l}(r)Y_{k,l}(\omega)$, where $d(k)$ is the dimension of space of spherical harmonic functions of degree $k$ and

$$d(k) \leq Ck^{n-2} \quad \text{for large } k. \quad (13)$$

Then we have

$$|x|^s \frac{1}{x^{n-s}} u(|x|\omega) = c_n \sum_{k,l} |x|^{1-s} \int_0^\infty J_{\nu(k)}(|x|\rho)\rho^{\frac{n}{2}} g_{k,l}(\rho)d\rho Y_{k,l}(\omega), \quad (14)$$

where $\omega \in S^{n-1}$, $\nu(k) = \frac{n+2k-2}{2}$ and $J_{\nu(k)}(r\rho) = g_{k,l}(\rho)Y_{k,l}(\omega)$. Here $g_{k,l}(\rho) = c_{n,k} \int_0^\infty f_{k,l}(r) r^{n-1}(r\rho)^{-\frac{n+2}{2}} J_{\nu(k)}(r\rho) dr$. The absolute value of $c_{n,k}$ is bounded by a constant depending only on $n$. See [10] for this.

Using the Cauchy-Schwarz inequality as in (7), we have

$$|x|^{\frac{n}{2}-s} |u(|x|\omega)| \leq C \sum_{k,l} \|Y_{k,l}\|_{L^\infty(S^{n-1})} \left( \int_0^\infty |J_{\nu(k)}(|x|\rho)|^2 \rho^{1-2s} \right)^{\frac{1}{2}} \left( \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1} d\rho \right)^{\frac{1}{2}} \leq C \sum_{k,l} k \frac{n-2}{2} \left( \frac{\Gamma(\nu(k) + 1 - s)}{\Gamma(\nu(k) + s)} \right)^{\frac{1}{2}} \left( \frac{1}{\Gamma(\nu(k) + s)} \right)^{\frac{1}{2}} \left( \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1} d\rho \right)^{\frac{1}{2}} \|Y_{k,l}\|_{L^2(S^{n-1})}.$$

Here we used the inequality that $\|Y_{k,l}\|_{L^\infty} \leq Ck^{\frac{n-2}{2}} \|Y_{k,l}\|_{L^2}$ (see for instance [10]). Using the Stirling’s formula for gamma function that $\Gamma(t) \approx t^{\frac{1}{2}} e^{-t}$ for large $t$ (for instance, see [1]) and the fact $-\Delta Y_{k,l} = (k+n-2)Y_{k,l}$, we have from (13)

$$|x|^s \frac{1}{x^{n-s}} |u(|x|\omega)| \leq C \sum_{k} k^{\frac{n-2}{2}} d(k)^{\frac{1}{2}} \left( \Gamma(\nu(k) + 1 - s) \Gamma(\nu(k) + s) \right)^{\frac{1}{2}} \left( \frac{1}{\Gamma(\nu(k) + s)} \right)^{\frac{1}{2}} \left( \sum_{1 \leq l \leq d(k)} \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1} d\rho \|Y_{k,l}\|_{L^2(S^{n-1})}^2 \right)^{\frac{1}{2}} \leq C \left( \sum_{k} k^{2(\nu(k) + 1 - s - m)} \Gamma(\nu(k) + 1 - s) \Gamma(\nu(k) + s) \right)^{\frac{1}{2}} \left( \sum_{k} k^{2m} \int_0^\infty |g_{k,l}(\rho)|^2 \rho^{2s+n-1} d\rho \|Y_{k,l}\|_{L^2(S^{n-1})}^2 \right)^{\frac{1}{2}} \leq C \left( \sum_{k,l} k^{2m} \int_0^\infty \int_{S^{n-1}} |f_{k,l}(r\rho)\rho^{\frac{n}{2}}| \rho^{2s+n-1} d\rho d\omega \right)^{\frac{1}{2}} \leq C \left( \sum_{k,l} \int_0^\infty \int_{S^{n-1}} \left| f_{k,l}(r\rho) \rho^{\frac{n}{2}} \right| \rho^{2s+n-1} d\rho d\omega \right)^{\frac{1}{2}} \leq C \|u\|_{H^{s,m}_n}.
where $\mathcal{F}$ is the Fourier transform. This proves part (1).

For the part (2), if we use the Littlewood-Paley decomposition $\{\varphi_j\}_{j \in \mathbb{Z}}$ as in the proof of Proposition 2, the we have

$$|x|^{n-1}u(|x|\omega) = c_n \sum_{j \in \mathbb{Z}} \sum_{k,l} |x|^k \int_0^\infty J_{n(k)}(|x|\rho) \rho \varphi_j(\rho)g_{k,l}(\rho) d\rho Y_{k,l}(\omega),$$

(15)

Since $m > n - 3/2$, by (12) we deduce that

$$|x|^{n-1}u(|x|\omega) \leq C \sum_{j \in \mathbb{Z}} \sum_{k,l} \left( \int_0^\infty |\varphi_j(\rho)g_{k,l}(\rho)|^2 \rho^m d\rho \right)^{\frac{1}{2}} \|Y_{k,l}\|_{L^\infty(S^{n-1})}$$

$$\leq C \sum_{j \in \mathbb{Z}} \left( \sum_k k^{2(n-2-m)} \right)^{\frac{1}{2}} \left( \sum_{k,l} \int_0^\infty |\varphi_j(\rho)g_{k,l}(\rho)|^2 \rho^m d\rho \|Y_{k,l}\|_{L^2(S^{n-1})}^2 \right)^{\frac{1}{2}}$$

$$\leq C \sum_{j \in \mathbb{Z}} 2^j \|\varphi_j \mathcal{F}((1 - \Delta) \frac{\rho^n}{p} u)\|_{L^2} = C\|u\|_{B^{\frac{1}{2},1,\omega}}.$$  

To show Corollary 1 we use the fact that $H^{s,m}_\omega$ is a Hilbert space. Hence any bounded sequence $\{u_j\}$ in $H^{s,m}_\omega$ satisfies $u_j(x) \to 0$ as $|x| \to \infty$ uniformly and has a subsequence converges to $u$ in $H^{s,m}_\omega$ weakly. Let us denote the subsequence by $u_{j_k}$.

Now choose a smooth function $\varphi$ supported in the ball of radius $R + 1$ and with value 1 in the ball of radius $R$. By the standard argument one can easily show that for each $R$ the mapping $u \mapsto \varphi u$ is compact from $H^t$ to $H^{t'}$ if $t' < t$. By the compactness above and Sobolev embedding we may assume that the sequence $\varphi u_{j_k}$ satisfies that for $2 < q < \frac{2n}{n-2s}$

$$\|\varphi u_{j_k} - \varphi u\|_{L^q} \to 0 \text{ as } k \to \infty.$$  

(16)

Thus we have

$$\|u_{j_k} - u\|_{L^p} \leq \|\varphi(u_{j_k} - u)\|_{L^p} + \|(1 - \varphi)(u_{j_k} - u)\|_{L^p} \equiv I_k + II_k$$

with $I_k \to 0$ as $k \to \infty$ by (16) since $2 < p < \frac{2n}{n-2s}$. From the uniform convergence that $|u_{j_k}(x)| + |u(x)| \to 0$ as $|x| \to \infty$ it follows that

$$\limsup_{k \to \infty} II_k \leq \sup_k \|u_{j_k} - u\|_{L^\infty(|x| > R)} \to 0 \text{ as } R \to \infty.$$  

This proves the compactness of the embedding $H^{s,m}_\omega \hookrightarrow L^p$.

Since $B^{s,m}_{2,1,\omega} \hookrightarrow H^{s,m}_\omega$, one can adapt the same arguments (compactness of cut-off mapping and uniform convergence at infinity) as above except for weak-* convergence of $u_{j_k}$ to $u$ in $B^{s,m}_{2,1,\omega}$ for the compactness of embedding $B^{s,m}_{2,1,\omega}$ to $L^p$. This completes the proof.
References


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