The bed instability of stratified open channel flow and turbidity currents is investigated in this study. The first setting is free surface flow in a channel with dominant suspended sediment load, and the second is a turbidity current on the ocean floor the movement of which is driven by suspended sediment. A linear stability analysis is performed in order to investigate the bed instability of open channel flow. The bed is assumed to be covered with sediment. The sediment is assumed to be so fine as to completely follow the flow except it sinks at a settling velocity in the vertical direction. The governing equations are the two-dimensional Reynolds-averaged Navier-Stokes equations (RANS) and the dispersion/diffusion equation of suspended sediment. In addition, as a turbulent closure model, we employ the standard $k-\varepsilon$ model which includes the transport equations of the turbulent kinetic energy and the dissipation rate.

The open channel flow is assumed to be in the upper flow regime, and dominated by suspended sediment with bedload being negligible. As far as suspended load is concerned, density stratification is an important factor as it affects internal structures of turbulence, and changes the flow velocity and the suspended sediment concentration. Therefore, density stratification is expected to affect the stability of the bed under free surface flow with dominant suspended sediment load. The base state is assumed to be in an equilibrium condition. Under this condition, all the variables are uniform in the streamwise direction and the vertical component of velocity vanishes. The equations in the base state are solved numerically by the use of a finite control volume method. It is found that the density stratification increases the flow velocity in the upper depth region, and increases the suspended sediment concentration near the bottom.

The mixing capacity due to turbulence is reduced under the density stratification as reflected in the decrease of the eddy viscosity. In the perturbation problem, the asymptotic expansion of the variables are then introduced into the governing equations to obtain the perturbation equations. Because the perturbation equations cannot be solved analytically, they are solved by a numerical scheme. The spectral collocation method incorporated with the Chebyshev polynomials are used to solve the perturbation equations. Substituting the solutions into the Exner equation, the growth rate of perturbation is obtained.

The results of the analysis are illustrated in instability diagrams. The antidunes are found to form in the upper flow regime. The instability regions predicted by the analysis are fairly reasonable as the experimental results fall almost all in the instability regions. Under the density stratification effect, the instability region of antidune is shifted to the range of smaller wavenumbers, which is corresponding to the range of longer wavelengths, around the critical Froude number. In addition, this analysis also shed...
light on the migration mechanism of the antidunes. The model predicts that antidunes could migrate in both the upstream and downstream directions. The mechanism of the migration is explained by the phase shift between the bed elevation and the net erosion rate. For the upstream migrating antidunes, the net erosion rate reaches maximum (minimum) slightly upstream of the trough (crest) of the bed waves. This process implies that the wave amplitude increases and the waves migrate in the upstream direction. In the case of downstream migrating antidunes, increases in the amplitude of bed waves is governed by the same process as that of the upstream migrating antidunes, however, the maximum (minimum) of the erosion rate takes place slightly downstream of the trough (crest) of the bed waves. Thus, the bed waves migrate in the downstream direction.

Instability generated under turbidity currents is studied. A turbidity current is a density flow the driving force of which is a density increase due to suspended sediment contained in water. In the case of saline or thermal density flows, salt concentration or temperature as driving force is diluted due to diffusion as it flows down, and therefore, it cannot move long distance. In the case of turbidity currents however, it has been found that the lower layer with high sediment concentration has an equilibrium state because the diffusion is balanced with the settling of suspended sediment.

In the analysis of turbidity currents, the two-dimensional Reynolds-averaged Navier-Stokes equations are used with the Boussinesq approximation. Because the suspended sediment is the dominant driving force of turbidity currents, the suspended sediment concentration appears in the momentum equations. In addition, the dispersion/diffusion equation of suspended sediment is employed. As a turbulent closure, the mixing length hypothesis is used to evaluate the eddy viscosity. In the base state condition, the flow velocity is affected by the suspended sediment concentration. As the sediment settling velocity increases, the suspended sediment concentration is reduced in the upper depth region, and emphasized in the lower depth region, resulting in a decrease and an increase in the flow velocity in the upper and lower depth regions, respectively. Meanwhile, as the sediment settling velocity decreases, the suspended sediment concentration becomes relatively uniform, resulting in the velocity profile similar to that in open channel flow.

As a result of the linear stability analysis of bed instability under turbidity currents, it is found that the flat bed becomes unstable to evolve into a bed covered with bed waves in the range of densimetric Froude number larger than approximately 0.4. In the condition of small settling velocities, the instability region in the case of turbidity currents resembles that in open channel flow. In addition, the instability region is affected also by the settling velocity non-dimensionalized by the friction velocity in the base state. For the non-dimensional settling velocity larger than 0.08, the instability region shows a strange shape. It is suggested that turbidity currents do not have normal flow conditions under the condition of sufficiently coarse suspended sediment.