Experimental verification of a real-time tuning method of a model-based controller by perturbations to its poles

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Abstract
Model-based controllers with adaptive design variables are often used to control an object with time-dependent characteristics. However, the controller’s performance is influenced by many factors such as modeling accuracy and fluctuations in the object’s characteristics. One method to overcome these negative factors is to tune model-based controllers. Herein we propose an online tuning method to maintain control performance for an object that exhibits time-dependent variations. The proposed method employs the poles of the controller as design variables because the poles significantly impact performance. Specifically, we use the simultaneous perturbation stochastic approximation (SPSA) to optimize a model-based controller with multiple design variables. Moreover, a vibration control experiment of an object with time-dependent characteristics as the temperature is varied demonstrates that the proposed method allows adaptive control and stably maintains the closed-loop characteristics.

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1. Introduction

Improved control performance is required to realize smaller and more accurate mechanical systems. Consequently, numerous methods have been developed to actively control performance. One such method is PID control, which is based on accumulated knowledge and model-based control. An advantage of model-based control methods is that they efficiently control performance, but they have limited applicability. This limitation is due to multiple reasons, including model construction, complex theories used to derive the model, modeling accuracy, and practical factors such as reasonable calculation costs and time-dependent characteristics of the controlled object. Over time, these limitations degrade the control property, inhibiting successful deployment of model-based controllers.

Control system tuning theories have been proposed to overcome the aforementioned limitations [1]. These theories are often complex. For example, self-tuning regulators (STRs) automatically classify and tune unknown parameters of an object to refine the control parameters. STRs have employed the minimum variance criterion [2], closed-loop poles [3], and the model reference adaptive control (MRAC). MRAC, which logs errors relative to a reference to adjust the control parameters, has been used to estimate non-modeled hysteresis [4]. The above studies assume that the optimization problem is a parameterization issue between the closed-loop system and the reference model. Although a few studies on simple concepts like PID control do not employ a mathematical model or a complicated theory, simple models are rare for model-based
controllers. In addition, controller-tuning approaches have been actively investigated in the field of nonlinear system control. Both a neural network controller [5] and a fuzzy controller [6] have been applied to adaptively suppress the vibrations of a flexible robotic manipulator system. An adaptive fuzzy output-feedback control method was presented by combining backstepping design together with fuzzy systems’ universal approximation capability [7]. Boundary control has been employed to control the vibrations in nonlinear systems, including an industrial moving strip system [8], a nonuniform gantry crane [9], and flexible wings of a robotic aircraft [10]. Furthermore, control strategies for nonlinear systems have been developed based on a near-optimal control scheme [11] and a novel iterative two-stage dual heuristic programming [12]. However, from the viewpoint of an adaptive algorithm with a high calculation efficiency implemented in a real-time controller mounted in an actual system, the practicability of the adaptive control approach for time-varying systems has yet to be experimentally validated.

The simultaneous perturbation stochastic approximation (SPSA) algorithm has been employed as an adaptive control method [13–15]. It is applicable to complex optimization problems. The SPSA algorithm is based on a highly efficient gradient approximation, which measures the loss function twice regardless of the number of parameters. Consequently, its algorithm is independent of the number of parameters. SPSA has been used to estimate the parameters in system identification [16–18]. Self-tuning of the PID control parameters [19–21], a neural network [22], and online optimization of NOx soft sensors for the aftertreatment of diesel engines [23]. To implement the adaptive algorithm based on SPSA in a real online control system, the calculation time must be taken into account because SPSA uses an iterative optimization process, even though the calculation efficiency of SPSA is high.

We previously applied the SPSA algorithm to an online adaptive optimization of PID parameters of a PID controller [20], diesel engine control with an adaptive PID controller [21], and an online parameter tuning of a NOx soft sensor used in a diesel engine [23]. Currently, mass production uses gain scheduling of map-based PID control, where each gain is tuned at various operational points. Map calibration has many drawbacks, including time-consuming tuning, difficulty tuning during transient operations, and problems adapting to individual variations in engine characteristics. In [21,23], the effectiveness of SPSA was demonstrated by comparing the results from SPSA to those obtained by a traditional adaptive approach based on map-based gain scheduling with respect to various operational points of the diesel engine. The SPSA adaptive approach is suitable for systems like an automotive engine where the controlled object is a time-varying system with gradual and continuous characteristic variations. Furthermore, other factors, including aging of the mechanical system and environmental condition (e.g., temperature) changes, cause gradual and continuous characteristic variations in the system as well as influence the mechanical properties such as rigidity and elasticity. Consequently, it is important that the controller maintains control performance and stability against these kinds of characteristic variations.

Previously, we simulated vibration control using the finite element method (FEM) to assess the effectiveness of the proposed tuning method for an object with time-dependent characteristics [24]. In this study, we propose an intuitive tuning method where the poles of a model-based controller are refined to maintain the performance. The poles dominate the dynamic characteristics of the controller and deeply influence the performance and the stability of the closed-loop control system [16,25]. The controller is designed using a model of the object. The time-dependent variations are compensated by perturbing the controller’s poles to tune the gain and the frequency of the controller. Then SPSA updates the adaptive parameters in real-time. This study experimentally evaluates the fundamental features of the proposed method because experimental verification is important to confirm the practical applicability of the present adaptive control strategy. Specifically, the characteristics of a vibration-controlled object are examined as a function of temperature fluctuations. The proposed method allows adaptive control and stably maintains the closed-loop characteristics.

2. Controller tuning system

2.1. Tuning method

The controller’s poles are used as design variables because they influence both the stability and the performance. The real (imaginary) part of the poles governs the gain (frequency) of the controller. The state-space equation used to represent a model-based controller is written as

\[
\begin{align*}
\dot{x}_c &= A_c x_c + B_c y_2 \\
u &= C_c x_c + D_c y_2
\end{align*}
\]

where \(y_2\) is the observed output, \(x_c\) is the controller’s state variable, and \(\nu\) is the control input. The poles of the transfer function are represented by the corresponding eigenvalues of system matrix \(A_c\). Perturbing the elements of matrix \(A_c\) tunes the poles. In an \(n\)th order system, \(A_c\) has \(n \times n\) elements. To decrease the calculation costs, the controller state equation is transformed into the diagonal canonical form. In this form, the poles, which are the design variables, are denoted as the diagonal elements of system matrix \(A_c\).
The real (imaginary) part of the poles denotes the gain (frequency). Because complex conjugate relationship remains intact. Matrix the conjugate complex and real numbers represent the poles. Thus, matrix factors in Eq.(8), the complex numbers cancel.

Similarly, we transform the other system matrices in Eq.(1) as

\[
\tilde{B}_c = T^{-1}B_c
\]
\[
\tilde{C}_c = C_cT
\]
\[
\tilde{D}_c = D_c
\]

Transfer function \(G(s)\) is unaffected by this transformation.

\[
G(s) = \tilde{C}_c(sI - \tilde{A}_c)^{-1}\tilde{B}_c + \tilde{D}_c
\]
\[
= C_cT(sI - T^{-1}A_cT)^{-1}T^{-1}B_c + D_c
\]
\[
= C(sI - A_c)^{-1}B_c + D_c
\]

Perturbations of the controller’s poles alter the gain and the frequency of the controller to account for modeling errors. The real (imaginary) part of the poles denotes the gain (frequency). Because \(\lambda_j = \alpha_j + i\beta_j\) represents the \(j\)th pole, perturbed pole \(\lambda_j'\) is expressed as

\[
\lambda_j' = k_v\alpha_j + ik_m\beta_j
\]

where \(k_v\) and \(k_m\) are positive perturbations provided to each part of the pole. This realizes a stable perturbed controller. If each pair of complex conjugate poles is given identical perturbations, the complex conjugate relationship remains intact. Matrix \(A'_c\), which includes the perturbations, is expressed as Eq. (8). Both the conjugate complex and real numbers represent the poles. Thus, matrix \(A'_c\) includes complex numbers in the diagonal elements. However, the proposed method yields a real system matrix even after the inverse transformation of Eq. (2), which is written as

\[
A'_c = TA'cT^{-1}
\]
\[
= \sum \nu_j' \lambda_j' \nu_j
\]

where \(T^{-1}\) is the inverse matrix of \(T\) and \(\nu_i\) is the \(i\)th row vector of \(T^{-1}\). If \(\lambda_j' = \lambda_k'\) (\(\lambda_j'\) is the complex conjugate number of \(\lambda_j\)), the corresponding elements of \(T\) and \(T^{-1}\) become \(\nu_j = \nu_k\) and \(\nu_k' = \nu_k\). As shown in Eq. (9), which describes the \(j\)th and \(k\)th factors in Eq. (8), the complex numbers cancel.

\[
\nu_j' \lambda_j' \nu_k' + \nu_j' \lambda_k' \nu_k' = \nu_j' \lambda_j' \nu_k' + (\nu_k' \lambda_j' \nu_k')'
\]

Consequently, system matrix \(A'_c\) is comprised of real number elements.

2.2. SPSA algorithm

SPSA is a gradient method that uses the stochastic approximation algorithm [13]. It is well suited for complex optimization problems. We employ SPSA to efficiently tune the controller poles.

In general, loss function \(L(\theta)\) in the gradient method is minimized using gradient \(g(\theta)\). The gradient is written as \(g(\theta) = \partial L(\theta)/\partial \theta\) where \(\theta\) is the design variable vector of a \(p\)-dimension multivariate system. Because directly determining the gradient is challenging, the stochastic gradient method is often used. The stochastic gradient method employs estimated gradient \(g\), and design variable \(\theta\). Their relationship is expressed as
\[ \theta_{t+1} = \theta_t - \alpha_t \hat{g}_t(\theta_t) \]  

where \( t \) is the iteration number, \( \alpha_t \) is the scalar gain coefficient, and \( \hat{g}_t(\theta_t) \) is the stochastic approximation of the unknown gradient of the loss function.

One popular stochastic method is the finite difference stochastic approximation (FDSA). In FDSA, each parameter undergoes small positive and negative perturbations. Then the gradient is estimated by the difference of the loss function due to the positive and negative perturbations. The gradient is written as

\[ \hat{g}_{t,i}(\theta_t) = \frac{L(\theta_t + c_t e_i) - L(\theta_t - c_t e_i)}{2c_t} \quad (i = 1, 2, \ldots, p) \]

where \( e_i \) is the index vectors and \( c_t \) is a coefficient that determines the amount of the given perturbation. To update all the parameters in FDSA, the loss function must be calculated \( 2p \) times. Hence, as the number of parameters increases, the calculation costs greatly increase.

On the other hand, the loss function only has to be calculated twice in SPSA. Because SPSA is based on a highly efficient gradient approximation, it is independent of the number of parameters. \( \hat{g}_t(\theta_t) \), which denotes a two-measurement approximation, is given as

\[ \hat{g}_{t,i}(\theta_t) = \frac{L(\theta_t + c_t \Delta t_i) - L(\theta_t - c_t \Delta t_i)}{2c_t \Delta t_i} \]

where \( \Delta t_i \) is the vector of \( p \) mutually independent mean-zero random variables. Thus, all the design variables are perturbed simultaneously. Unlike FDSA, which has \( 2p \) calculation costs, SPSA has \( 1/p \) calculation costs. Fig. 1 overview the adaptive procedure of the controller with SPSA. Two coefficients \( \alpha_t \) and \( c_t \) are defined as constants determined by the degree of the change and the noise level of the control system.

\[ \alpha_t = \frac{a}{(A + t + \gamma)^2} \]

\[ c_t = \frac{c}{(t + 1)^\gamma} \]

where \( a, c, A, \alpha, \) and \( \gamma \) are non-negative coefficients. These coefficients decrease exponentially and converge to zero, terminating the adaptation process. As iteration number \( t \) increases, \( \alpha_t \) and \( c_t \) become smaller.

An object with time-dependent characteristics changes constantly. Consequently, the controller must be tuned continuously. In the proposed method, \( \alpha_t \) and \( c_t \) are defined as constants determined by the degree of the change and the noise level of the control system.

2.3. Online tuning of the control system

We evaluate loss function \( L(\theta_t) \) in real-time. The loss function, which is the sum of the mean square of the observed output, can be written as

\[ L(\theta_t) = \frac{1}{f_s} \sum_{k=0}^{m} y_2^2(k) \]

where \( m \) is the number of evaluation points, \( f_s \) is the sampling frequency, and \( y_2(k) \) is the observed output at sampling point \( k \). The results are used in the online tuning of the controller. The advantage of SPSA is that regardless of the number of variables, calculating the loss function twice updates all design variables. Updating period \( T_e \) is written as

\[ T_e = \frac{2m}{f_s} \]
During the first and second calculations, the design variables are kept at $\theta_t = \theta_t + c_i \Delta_i$ and $\theta_t = \theta_t - c_i \Delta_i$, respectively. In this manner, the loss function is determined after each perturbation and the gradient is estimated. Consequently, SPSA is used to adaptively tune the controller with respect to the time-dependent characteristics.

Below we experimentally investigate the capability of the proposed tuning method.

3. Experimental system

3.1. Controlled object with changing dynamic characteristics

To evaluate the efficacy of the proposed method, vibration control experiments were conducted. In these experiments, the vibration control characteristics and the stability are maintained despite the changing dynamic characteristics of the

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**Fig. 1.** Flow chart of the SPSA algorithm.

**Fig. 2.** Overview of the controlled object.
controlled object by tuning the controller parameters designed for the controlled object in the nominal state. Fig. 2 overviews the controlled object, which is a flat plate (200 mm × 150 mm). The controlled object is composed of a 1-mm-thick flat aluminum plate and a 3-mm-thick flat polyethylene plate.

Typically, the temperature greatly affects the dynamic characteristics of plastic materials. To investigate the efficacy of the proposed method, the temperature of the composite plates was controlled as the dynamic characteristics change. A flat heater and a thermocouple were affixed on the reverse side of the aluminum plate to control the temperature. Two piezoelectric (PZT) actuators (40 mm × 5 mm) were attached near the fixed end of the flat plate to conduct the vibration control experiments; one served as control input $u$ and the other as disturbance input $w$. Additionally, an accelerometer was mounted near the tip of the flat plate to measure the response to the input vibration. The output from the accelerometer shown in Fig. 2 is observed as output $y_2$ in Eq. (1).

The frequency response was measured with an accelerometer as the temperature was varied between 20 °C and 40 °C. Fig. 3 compares the measurement results of the frequency responses of the controlled object at different temperatures, where the acceleration responses are detected at the measurement point in 5 °C increments. The changes in the resonant frequencies and the magnitude of the responses indicate that the characteristics depend on the temperature. In this study, the controller is tuned to adapt to the changes in the dynamic characteristics of the controlled object to evaluate the vibration control characteristics and stability. The characteristic variations are sufficiently large that a time-invariant controller designed for a fixed temperature will easily destabilize the closed-loop system as the temperature fluctuates.

3.2. Design of the vibration control system

The state of the flat plate with one fixed end introduced in 3.1 at 20 °C is defined as the nominal state. The nominal state characteristics were obtained by system identification to generate a model for the controlled object. Moreover, a controller was designed for the identified model, which was subsequently adopted as the nominal controller.

First, the frequency response of the accelerometer output for the PZT actuator input was measured, and then a curve fitting was performed. The upper frequency for the vibration control was 1 kHz. All the mode characteristics at or below 1 kHz can be identified. Fig. 4 shows the measured and identified frequency responses (compliance responses) of the controlled object. The identified controlled object model includes eight modes. The good agreement between the measured and the identified values demonstrates a satisfactory curve fitting. The model of the controlled object is described in the state-space form with the identified modal parameters [26].

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**Fig. 3.** Comparison of the open-loop FRF for the controlled object between the disturbance and the acceleration output at different temperatures.
Next, the $H_1$ control theory was applied to the identified controlled object model to design a controller. Fig. 5 shows a block diagram of the vibration control system for the identified model. $P(s)$ is the identified controlled object and $K(s)$ is the controller. During the control system design, unknown disturbance characteristics are assumed to be added to the control input. $W_1$, $W_2$, and $W_3$ are the frequency weighting functions for controlled response $z_1$, control input $u$, and observed output $y_2$, respectively. Furthermore, $Q_1$ and $R_1$ are weighting matrices. Controlled variable $y_1$ is defined as

$$y_1 = \begin{bmatrix} Q_1^{1/2} z_1 \\ R_1^{1/2} u \end{bmatrix}$$

(18)

In this study, the acceleration responses at the measurement point were evaluated. Controller $K(s)$ was designed to minimize the $H_\infty$ norm of the transfer function from disturbance $w$ to controlled variable $y_1$. Furthermore, a high shelf filter was defined for frequency weighting function $W_2$ for the control input in order to prevent spillover at frequencies outside the

![Fig. 4. Identified compliance model of the controlled object.](image)

![Fig. 5. Diagram of the control system.](image)

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<th>Parameters used to design the nominal controllers.</th>
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<td>$Q$</td>
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<td>$R$</td>
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<td>$W_1$</td>
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control frequency range. Other frequency weighting functions $W_1$ and $W_3$ have constant values. Table 1 shows the parameters to design the control system using the Control System Toolbox from MATLAB. Herein the control system is adopted as the nominal controllers.

4. Control results and discussion

4.1. Vibration control experiments with nominal controllers

Vibration control experiments were conducted to evaluate the vibration control characteristics of the nominal controllers. Fig. 6 overviews the experimental system with the SPSA adaptive controller. White noise was fed into the PZT actuator from a signal generator to apply a disturbance. A PID control system was used to adjust the temperature of the controlled object. The digital controller incorporated an SPSA-based learning rule in addition to the controller designed herein to provide realtime tuning.

First, the temperature of the controlled object was linearly increased from 20°C to 40°C over 200 s to evaluate the changes in the vibration control characteristics of the nominal controller. The loss function in Eq. (16) was used to evaluate the vibration control characteristics. Fig. 7 shows the chronological relationships between the loss function and the temperature of the controlled object. The loss function value is roughly constant up to about 30°C and the vibration control characteristics are maintained at the nominal controller. However, the vibration control characteristics gradually deteriorate at higher temperatures, and the control system becomes unstable near 38°C.

Next, the frequency response of the closed-loop system was measured. Fig. 8 shows the frequency responses of the controlled object at its nominal state (20°C) and a state where the dynamic characteristics change (35°C). Acceleration responses occur at the measurement point, demonstrating that the nominal controller provides good vibration control characteristics at major peaks at or below 1 kHz. However, the vibration control characteristics deteriorate when the dynamic characteristics change. In particular, the large response peak near 500 Hz indicates a major instability in the system. Moreover, further increasing the temperature of the controlled object causes an oscillation, confirming that the controlled system is unstable. Next, the efficacy of the proposed method is evaluated by tuning the controller to adapt to the dynamic characteristic changes.
Fig. 7. Loss function and temperature obtained from the nominal controller over time (20°C to 40°C over 200 s).

Fig. 8. FRFs of the closed-loop system with the nominal controller at 20°C and 35°C.
4.2. Controller tuning experiments

Tuning was applied to the proposed controller, and the vibration control characteristics of the controlled object against dynamic characteristic changes were evaluated. Similar to the above experiments, the temperature of the controlled object was linearly increased from 20°C to 40°C over 200 s. However, these experiments evaluated the changes in the vibration control characteristics when tuning was applied.

Table 2 shows the parameters used in this experiment. Because the degree of the controller is 18, the number of poles is 18. All the poles were used as design variables. Fig. 9 shows the chronological response of the loss function values and the temperatures when tuning is applied to the control system. Red and blue lines denote the results with tuning and the nominal controller, respectively. As previously discussed, the vibration control characteristics gradually deteriorate with the nominal controller and become unstable near 38°C. In contrast, the loss function values are roughly constant with the tuned controller, indicating that the vibration control characteristics are maintained. Fig. 10 shows the frequency responses from the vibration control experiments with the tuned controller at 40°C as well as the results of the vibration control experiments on the controlled object at 35°C using the nominal controller for comparison. As seen from these frequency responses, tuning provides good vibration control characteristics for major peaks at or below 1 kHz. The results are comparable to those in the nominal state, demonstrating that the proposed method provides vibration control and stability against dynamic characteristic changes to the controlled object.

Detailed analysis of the control characteristics reveals that the response is slightly larger near 450 Hz. This difference is attributed to the fact that the tuning lags behind the changes in the dynamic characteristics of the controlled objects, resulting in deteriorated vibration control characteristics. Fig. 11 shows the chronological response of the loss function when the

<table>
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<th>Parameters for the tuning experiment.</th>
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<tr>
<td>Number of design variables, ( p )</td>
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<td></td>
<td></td>
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<tr>
<td>Number of evaluation points, ( m )</td>
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<tr>
<td>Sampling frequency, ( f_s )</td>
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<td></td>
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<tr>
<td>Correction gain, ( \kappa_c )</td>
<td>( 7.0 \times 10^{-5} )</td>
<td></td>
<td></td>
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<tr>
<td>Perturbation gain, ( c_t )</td>
<td>( 6.0 \times 10^{-3} )</td>
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Fig. 9. Loss function and temperature obtained from the nominal and tuned controllers over time (20°C to 40°C over 200 s).
**Fig. 10.** FRFs of the closed-loop system with the tuned controller at 40 °C and the nominal controller at 35 °C.

**Fig. 11.** Loss function and temperature obtained from the nominal and tuned controllers over time (20 °C to 40 °C in about 100 s).
The rate of change in the dynamic characteristics of the controlled objects is increased from 20 °C to 40 °C in about 100 s. The loss function slightly increases near 100 s even with tuning, indicating that the vibration control characteristics deteriorate slightly. Because the proposed method requires a certain period of time to measure the loss function, the updating frequency of the design variables may be limited. The vibration control characteristics deteriorate when the change in the dynamic characteristics of the controlled object does not allow sufficient learning time. However, the results shown in Fig. 11 demonstrate that the loss function values decrease to the nominal state after the controlled object is held at the maximum temperature, indicating the vibration control characteristics are recovered.

Fig. 12 shows the measurement results of the frequency response when the temperature of the controlled object is kept at 40 °C using the tuned controller shown in Fig. 11 and learning is continued for another 200 s. Continued tuning successfully suppresses the peak near 450 Hz, confirming the improved vibration control characteristics. Repeating the experiment under the same conditions confirms the reproducibility of the results.

Although the proposed method has a limit in the magnitude of the dynamic characteristic change in the controlled objects, sufficient learning can maintain the vibration control characteristics and the stability. Continued learning can improve the vibration control characteristics. Additionally, optimizing the adaptive algorithm of the proposed method and increasing the updating frequency for the design variables should minimize the deterioration of the vibration control characteristics. Using a loss function that overlaps with the learning time on the time axis can reduce the updating time for design variables, resulting in an improved tracking performance against dynamic characteristic changes [20,21].

5. Conclusions

This study proposes a model-based tuning method for controllers and experimentally demonstrates its efficacy. The proposed method tunes the design variables using SPSA to adapt to the changes in the dynamic characteristics of the controlled objects when the number of poles is the design variable. The nominal controller deteriorates the closed-loop performance and exhibits insufficient vibration control characteristics for a system whose dynamic characteristics change over time. However, applying the proposed controller to adaptively control this system maintains both the vibration control characteristics and the stability.

Future research involves four distinct tasks. First, we plan to improve the stability of the controlled object against large dynamic characteristic changes by devising a method to increase the updating frequency of the design variables. Second, we should conduct stability analysis of the closed-loop system against the characteristic variations. Third, we intend to compare
the proposed method to other adaptive methods to evaluate its efficacy. Finally, we will apply the present approach to various mechanical systems.

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