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PAPER

Dirty two-band superconductivity with interband pairing order

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Abstract

We study theoretically the effects of random nonmagnetic impurities on the superconducting transition temperature T_c in a two-band superconductor characterized by an equal-time *s*-wave interband pairing order parameter. Because of the two-band degree of freedom, it is possible to define a spin-triplet *s*-wave pairing order parameter as well as a spin-singlet *s*-wave order parameter. The former belongs to odd-band-parity symmetry class, whereas the latter belongs to even-band-parity symmetry class. In a spin-singlet superconductor, T_c is insensitive to the impurity concentration when we estimate the self-energy due to the random impurity potential within the Born approximation. On the other hand in a spin-triplet superconductor, T_c decreases with the increase of the impurity concentration. We conclude that Cooper pairs belonging to odd-band-parity symmetry class are fragile under the random impurity potential even though they have *s*-wave pairing symmetry.

1. Introduction

Conventional wisdom suggests that the dependence of superconducting transition temperature T_c on the concentration of nonmagnetic impurities is closely related to the momentum-symmetry of the pair potential. It is well known that T_c of an *s*-wave superconductor is insensitive to the impurity concentration [1–3]. On the other hand, unconventional superconductivity such as *p*- and *d*-wave symmetry is fragile in the presence of impurities. The robustness of an *s*-wave Cooper pair under potential disorder, however, may be weakened in a two-band superconductor as discussed in previous literature [4–10]. In these papers, the *intraband* pairing order is assumed in each conduction band. Namely, two electrons at the first (second) band form the pair potential Δ_1 (Δ_2). Such theoretical model would describe the superconducting states in MgB_2 [11, 12] and iron pnictides [13, 14]. The suppression of T_c by the interband impurity scatterings is a common conclusion of all the theoretical studies.

In addition to the *intraband* pair potentials, the *interband* (or interorbital) Cooper pairing order has been discussed in a topological superconductor $Cu_xBi_2Se_3$ [15–17]. Various types of multiband superconductivity would be expected in topological-material based superconductors because the band-crossing plays an essential role in realizing the topologically nontrivial states. Moreover, a possibility of interband/interorbital Cooper pairing is pointed out also in a heavy fermionic superconductor UPt_3 [18, 19] and an antiperovskite superconductor $Sr_{3-x}SnO$ [20]. In addition to the spin-singlet order parameter, the spin–orbit coupling may make the spin-triplet order parameter possible. Thus, a superconductor with the interband pairing order can be a superconductor of a novel class. So far, however, little attention has been paid to physical phenomena unique to an interband superconductor.

In this paper, we theoretically study the effects of nonmagnetic random impurities on T_c in a two-band superconductor characterized by an equal-time *s*-wave interband pairing order. The pair potential is defined by the product of two annihilation operators of an electron. Therefore, the pair potential must be antisymmetric under the commutation of the two annihilation operators, which is the requirement from the Fermi–Dirac statistics of electrons. Due to the two-band degree of freedom, a spin-triplet *s*-wave pair potential is allowed as well as a spin-singlet s-wave one. The latter is symmetric under the permutation of the two-band indices (even-band-parity), whereas the former is antisymmetric (odd-band-parity). The effects of impurity potential are considered through the self-energy estimated within the Born approximation. The transition temperature is calculated from the linearized gap equation. We find that T_c is insensitive to the impurity concentration in a spin-singlet s-wave interband superconductor. However, T_c in a spin-triplet s-wave case decreases with the increase of the impurity concentration. We conclude that odd-band parity Cooper pairs are fragile under the potential disorder even though they belong to s-wave symmetry class.

This paper is organized as follows. In section 2, we explain the normal state that makes possible spatially uniform interband Cooper pairing orders. The gap equation in the clean limit is derived for both a spin-singlet superconductor and a spin-triplet superconductor. The effects of random impurities on the superconducting transition temperature are studied in section 3. The conclusion is given in section 4. Throughout this paper, we use the units of $k_{\mathrm{B}} = c = \hbar = 1$, where k_{B} is the Boltzmann constant and c is the speed of light.

2. Interband pairing order

The interband s-wave pair potential is defined by

$$\Delta_{1,\sigma;2,\sigma'}(\boldsymbol{r}) = g\,\langle\psi_{1,\sigma}(\boldsymbol{r})\psi_{2,\sigma'}(\boldsymbol{r})\rangle, \tag{1}$$

where $\psi^{\dagger}_{\lambda,\sigma}(\boldsymbol{r})$ ($\psi_{\lambda,\sigma}(\boldsymbol{r})$) is the creation (annihilation) operator of an electron with spin σ ($=\uparrow$ or $\downarrow$) at the λth conduction band and $g > 0$ represents the interband attractive interaction. By applying the Fourier transformation,

$$\psi_{\lambda,\sigma}(\boldsymbol{r}) = \frac{1}{\sqrt{V_{\mathrm{vol}}}}\sum_{\boldsymbol{k}}\psi_{\lambda,\sigma}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{2}$$

the pair potential becomes

$$\Delta_{1,\sigma;2,\sigma'}(\boldsymbol{r}) = \frac{g}{V_{\mathrm{vol}}}\sum_{\boldsymbol{k},\boldsymbol{k}'}\langle\psi_{1,\sigma}(\boldsymbol{k})\psi_{2,\sigma'}(\boldsymbol{k}')\rangle\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}+\boldsymbol{k}')\cdot\boldsymbol{r}} \tag{3}$$

$$= \frac{g}{V_{\mathrm{vol}}}\sum_{\boldsymbol{k}}\langle\psi_{1,\sigma}(\boldsymbol{k})\psi_{2,\sigma'}(-\boldsymbol{k})\rangle. \tag{4}$$

In the second line, we assume the spatially uniform order parameter which is realized at $\boldsymbol{k} + \boldsymbol{k}' = 0$. To apply the weak coupling mean-field theory, the state at $\boldsymbol{k}$ with spin σ in the first band and the state at $-\boldsymbol{k}$ with spin σ' in the second band must be degenerate at the Fermi level. Otherwise interband Cooper pairs have the center-of-mass momenta and their order parameter oscillates in real space [21–23]. Thus, the interband pair potential requires a characteristic band structure. In this paper, we consider a normal state described by the Hamiltonian,

$$\check{\mathcal{H}}_{\mathrm{N}} = \int d\boldsymbol{r}\,[\psi^{\dagger}_{1,\uparrow}(\boldsymbol{r}),\ \psi^{\dagger}_{1,\downarrow}(\boldsymbol{r}),\ \psi^{\dagger}_{2,\uparrow}(\boldsymbol{r}),\ \psi^{\dagger}_{2,\downarrow}(\boldsymbol{r})]\check{H}_{\mathrm{N}}(\boldsymbol{r})\begin{bmatrix}\psi_{1,\uparrow}(\boldsymbol{r})\\ \psi_{1,\downarrow}(\boldsymbol{r})\\ \psi_{2,\uparrow}(\boldsymbol{r})\\ \psi_{2,\downarrow}(\boldsymbol{r})\end{bmatrix}, \tag{5}$$

$$\check{H}_{\mathrm{N}}(\boldsymbol{r}) = \begin{bmatrix}\xi(\boldsymbol{r})\,\hat{\sigma}_0 & v\,\mathrm{e}^{\mathrm{i}\theta}\,\hat{\sigma}_0\\ v\,\mathrm{e}^{-\mathrm{i}\theta}\,\hat{\sigma}_0 & \xi(\boldsymbol{r})\hat{\sigma}_0\end{bmatrix}, \tag{6}$$

$$\xi(\boldsymbol{r}) = -\frac{\nabla^2}{2m} - \mu, \tag{7}$$

where m is the mass of an electron, μ is the chemical potential, and v represents the hybridization between the two conduction bands. Generally speaking, the hybridization potential is a complex number characterized by a phase θ. We will show that observable values in a superconductor are independent of θ although the expression of the Green function depends on it. Throughout this paper, Pauli matrices in spin, two-band, particle-hole spaces are denoted by $\hat{\sigma}_j$, $\hat{\rho}_j$, and $\hat{\tau}_j$ for $j = 1$–3, respectively. In addition, $\hat{\sigma}_0$, $\hat{\rho}_0$, and $\hat{\tau}_0$ are the unit matrices in these spaces. Since the two bands are identical to each other, the Hamiltonian preserves the symmetry described by

$$\check{\Gamma}\,\check{H}_{\mathrm{N}}(\boldsymbol{r})\,\check{\Gamma}^{-1} = \check{H}_{\mathrm{N}}(\boldsymbol{r}), \tag{8}$$

$$\check{\Gamma} = \mathcal{T}\hat{\rho}_1, \quad \mathcal{T} = \mathrm{i}\,\hat{\sigma}_2\,\mathcal{K}, \tag{9}$$

where $\mathcal{T}$ is the time-reversal operator, $\mathcal{K}$ means the complex conjugation. Thus, Γ represents the combined operation of the time-reversal and the exchange between the two bands. The normal state Hamiltonian in equation (6) is simplest model which satisfies equation (8). The conclusions of this paper are insensitive to the normal state Hamiltonian. We will explain the reasons after reaching the main results. The electronic structure given in equation (6) may poses both the interband and the intraband s-wave order parameters in its superconducting phase. The effects of potential disorder on T_c for intraband superconductivity have been already studied theoretically in previous papers [5–10]. In our model, the amplitudes of two intraband pair potentials are expected be equal to each other because of the symmetry in the two conduction bands. It has been well established that T_c of intraband superconductivity in such symmetric case is insensitive to the impurity scatterings [5, 7, 10]. Thus, we focus only on interband superconductivity in this paper.

According to equation (4), we define the spatially uniform superconducting order parameter explicitly as

$$\Delta \equiv \frac{g}{V_{\rm vol}} \sum_{\boldsymbol{k}} \langle \psi_{1,\uparrow}(\boldsymbol{k})\, \psi_{2,\downarrow}(-\boldsymbol{k}) \rangle. \tag{10}$$

In the two-band model, it is possible to define two types of interband pairing order: spin-singlet and spin-triplet. In spin-singlet symmetry, the pair potential in equation (10) is symmetric (antisymmetric) under the permutation of band (spin) indices

$$\Delta = -\frac{g}{V_{\rm vol}} \sum_{\boldsymbol{k}} \langle \psi_{1,\downarrow}(\boldsymbol{k})\, \psi_{2,\uparrow}(-\boldsymbol{k}) \rangle = \frac{g}{V_{\rm vol}} \sum_{\boldsymbol{k}} \langle \psi_{2,\uparrow}(\boldsymbol{k})\, \psi_{1,\downarrow}(-\boldsymbol{k}) \rangle. \tag{11}$$

On the other hand in spin-triplet symmetry, the pair potential in equation (10) is antisymmetric (symmetric) under the permutation of band (spin) indices

$$\Delta = \frac{g}{V_{\rm vol}} \sum_{\boldsymbol{k}} \langle \psi_{1,\downarrow}(\boldsymbol{k})\, \psi_{2,\uparrow}(-\boldsymbol{k}) \rangle = -\frac{g}{V_{\rm vol}} \sum_{\boldsymbol{k}} \langle \psi_{2,\uparrow}(\boldsymbol{k})\, \psi_{1,\downarrow}(-\boldsymbol{k}) \rangle. \tag{12}$$

In what follows, we consider opposite-spin-triplet pairing order. The Bogoliubov–de Gennes (BdG) Hamiltonian in momentum space is represented by

$$\bar{H}_{\rm S(T)}(\boldsymbol{k}) = \begin{bmatrix} \check{H}_N(\boldsymbol{k}) & \check{\Delta}_{\rm S(T)} \\ -\check{\Delta}_{\rm S(T)} & -\check{H}_N^*(-\boldsymbol{k}) \end{bmatrix}, \tag{13}$$

$$\check{\Delta}_{\rm S} = \Delta\, \hat{\rho}_1\, {\rm i}\, \hat{\sigma}_2, \qquad \check{\Delta}_{\rm T} = \Delta\, {\rm i}\, \hat{\rho}_2\, \hat{\sigma}_1, \tag{14}$$

where $\check{\Delta}_{\rm S}$ and $\check{\Delta}_{\rm T}$ represent the spin-singlet pair potential and the spin-triplet one, respectively. Hereafter we fix the superconducting phase at zero for simplicity. The BdG Hamiltonian can be described in reduced 4×4 matrix form

$$\check{H}_0(\boldsymbol{k}) = \begin{bmatrix} \xi(\boldsymbol{k}) & v\, {\rm e}^{{\rm i}\theta} & 0 & \Delta \\ v\, {\rm e}^{-{\rm i}\theta} & \xi(\boldsymbol{k}) & -s_s\Delta & 0 \\ 0 & -s_s\Delta & -\xi(\boldsymbol{k}) & -v\, {\rm e}^{-{\rm i}\theta} \\ \Delta & 0 & -v\, {\rm e}^{{\rm i}\theta} & -\xi(\boldsymbol{k}) \end{bmatrix}, \tag{15}$$

by choosing spin of an electron as $\uparrow$ and that of a hole as $\downarrow$, where $s_s = 1$ for a spin-triplet superconductor and $s_s = -1$ for a spin-singlet superconductor. We note in the normal state that $\xi^*(-\boldsymbol{k}) = \xi(\boldsymbol{k})$ holds true in the presence of time-reversal symmetry.

The Green function is obtained by solving the Gor'kov equation,

$$[{\rm i}\omega_n \check{1} - \check{H}_0(\boldsymbol{k})]\, \check{G}_0(\boldsymbol{k}, {\rm i}\omega_n) = \check{1}, \tag{16}$$

$$\check{G}_0(\boldsymbol{k}, {\rm i}\omega_n) = \begin{bmatrix} \hat{\mathcal{G}}_0(\boldsymbol{k}, {\rm i}\omega_n) & \hat{\mathcal{F}}_0(\boldsymbol{k}, {\rm i}\omega_n) \\ -s_s\hat{\mathcal{F}}_0^*(-\boldsymbol{k}, {\rm i}\omega_n) & -\hat{\mathcal{G}}_0^*(-\boldsymbol{k}, {\rm i}\omega_n) \end{bmatrix}, \tag{17}$$

where $\omega_n = (2n + 1)\pi T$ is a fermionic Matsubara frequency with T being a temperature. The solution of the normal Green function within the first order of Δ is represented as

$$\hat{\mathcal{G}}_0(\boldsymbol{k}, \omega_n) = \frac{\xi(\xi + 2{\rm i}\omega_n) - \omega_n^2 - v^2}{Z_0}[({\rm i}\omega_n - \xi)\hat{\rho}_0 + v\cos\theta\hat{\rho}_1 - v\sin\theta\hat{\rho}_2], \tag{18}$$

$$Z_0 = \xi^4 + 2\xi^2(\omega_n^2 - v^2) + (\omega_n^2 + v^2)^2, \tag{19}$$

where we omit $\boldsymbol{k}$ from $\xi(\boldsymbol{k})$ for simplicity. The results are common in both spin-singlet and spin-triplet cases because the normal Green function does not include the pair potential in the lowest order of Δ. The anomalous Green functions for a spin-singlet superconductor within the first order of Δ is calculated as

$$\hat{\mathcal{F}}_0(\boldsymbol{k}, \omega_n) = \frac{\Delta}{Z_0}[2v\cos\theta\ \xi\ \hat{\rho}_0 - (\omega_n^2 + v^2 + \xi^2)\hat{\rho}_1 + 2{\rm i}v\sin\theta\ \xi\ \hat{\rho}_3]. \tag{20}$$

The $\hat{\rho}_1$ component in equation (20) is linked to the pair potential through the gap equation

$$\Delta = -gT\sum_{\omega_n} \frac{1}{V_{\rm vol}} \sum_{\boldsymbol{k}} \frac{1}{2} {\rm Tr}[\hat{\mathcal{F}}_0(\boldsymbol{k}, \omega_n)\, \hat{\rho}_1] \tag{21}$$

$$= \pi g N_0 T \sum_{\omega_n} \frac{\Delta}{|\omega_n|}, \tag{22}$$

where N_0 is the density of states at the Fermi level per spin. We have used the relation

$$\frac{1}{V_{\mathrm{vol}}} \sum_{\boldsymbol{k}} \frac{a + b\, \xi^2}{Z_0} = \frac{\pi N_0 [a + b(\omega_n^2 + \nu^2)]}{2\, |\omega_n| (\omega_n^2 + \nu^2)}, \tag{23}$$

where a and b are constants. The last equation in equation (22) is identical to the gap equation in the BCS theory. The hybridization generates the $\hat{\rho}_0$ and $\hat{\rho}_3$ components in equation (20) which belong to even-frequency spin-singlet even-momentum-parity even-band-parity (ESEE) symmetry class.

In the case of a spin-triplet superconductor, the anomalous Green function becomes

$$\hat{\mathcal{F}}_0(\boldsymbol{k}, \omega_n) = \frac{\Delta}{Z_0} [2\omega_n\, \nu \sin\theta\; \hat{\rho}_0 - (\omega_n^2 - \nu^2 + \xi^2) \mathrm{i}\, \hat{\rho}_2 - 2\mathrm{i}\omega_n\, \nu \cos\theta \hat{\rho}_3]. \tag{24}$$

The $\hat{\rho}_2$ component is linked to the pair potential. The gap equation is represented by equation (21) with replacing $\hat{\rho}_1$ by $-\mathrm{i}\hat{\rho}_2$. The results of the gap equation in the linear regime,

$$\Delta = \pi g N_0 T \sum_{\omega_n} \frac{\Delta\, |\omega_n|}{\omega_n^2 + \nu^2} \tag{25}$$

deviate from equation (22). In equation (24), the hybridization generates the $\hat{\rho}_0$ and $\hat{\rho}_3$ components which belong to odd-frequency spin-triplet even-momentum-parity even-band-parity (OTEE) symmetry class [24–27]. We attribute this suppression of T_c to the presence of odd-frequency pairs which typically have a detrimental effect on thermodynamic stability [25, 28, 29]. At $\nu = 0$, the gap equation in equation (25) is identical to equation (22) because the odd-frequency pairing correlations are absent.

3. Effects of impurities

Let us consider the nonmagnetic random impurities described by

$$\check{H}_{\mathrm{imp}} = V_{\mathrm{imp}}(\boldsymbol{r}) \begin{bmatrix} 1 & \mathrm{e}^{\mathrm{i}\theta} & 0 & 0 \\ \mathrm{e}^{-\mathrm{i}\theta} & 1 & 0 & 0 \\ 0 & 0 & -1 & -\mathrm{e}^{-\mathrm{i}\theta} \\ 0 & 0 & -\mathrm{e}^{\mathrm{i}\theta} & -1 \end{bmatrix} \tag{26}$$

$$= V_{\mathrm{imp}}(\boldsymbol{r}) \hat{\tau}_3\, \hat{\rho}_0 + V_{\mathrm{imp}}(\boldsymbol{r}) \check{A}, \tag{27}$$

$$\check{A} = \hat{\tau}_3\, \hat{\rho}_1 \cos\theta - \hat{\rho}_2 \sin\theta. \tag{28}$$

The first and the second terms in equation (27) cause the intraband and the interband scatterings, respectively. We assume that the impurity potential satisfies the following properties,

$$\overline{V_{\mathrm{imp}}(\boldsymbol{r})} = 0, \tag{29}$$

$$\overline{V_{\mathrm{imp}}(\boldsymbol{r})\, V_{\mathrm{imp}}(\boldsymbol{r}')} = n_{\mathrm{imp}} v_{\mathrm{imp}}^2 \delta(\boldsymbol{r} - \boldsymbol{r}'), \tag{30}$$

where $\overline{\cdots}$ means the ensemble average, n_{imp} is the impurity concentration, and v_{imp} represents the strength of the impurity potential. We also assume that the attractive electron–electron interactions are insensitive to the impurity potentials [3]. To discuss the effects of impurities with equations (29) and (30), Hamiltonian in real space is necessary. The impurity Hamiltonian in equation (27) is described in real space as well as the kinetic part and the hybridization in equations (5) and (6). In the real space representation with the basis shown in equation (5), the random potential $V_{\mathrm{imp}}(\boldsymbol{r})$ should be independent of band indices. The phase of random potential generating the interband scattering must be equal to that of the hybridization. Otherwise, time-reversal symmetry is broken. The effects of the impurity scatterings are taken into account through the self-energy estimated within the Born approximation. The Green function in the presence of the impurity potential is calculated within the second order perturbation expansion with respect to the impurity potential,

$$\begin{aligned} \check{G}(\boldsymbol{r} - \boldsymbol{r}', \omega_n) &\approx \check{G}_0(\boldsymbol{r} - \boldsymbol{r}', \omega_n) + \int \mathrm{d}\boldsymbol{r}_1 \check{G}_0(\boldsymbol{r} - \boldsymbol{r}_1, \omega_n)\, \overline{\check{H}_{\mathrm{imp}}(\boldsymbol{r}_1)}\, \check{G}(\boldsymbol{r}_1 - \boldsymbol{r}', \omega_n) \\ &+ \int \mathrm{d}\boldsymbol{r}_1 \int \mathrm{d}\boldsymbol{r}_2\, \check{G}_0(\boldsymbol{r} - \boldsymbol{r}_1, \omega_n)\, \overline{\check{H}_{\mathrm{imp}}(\boldsymbol{r}_1)\, \check{G}_0(\boldsymbol{r}_1 - \boldsymbol{r}_2, \omega_n)\, \check{H}_{\mathrm{imp}}(\boldsymbol{r}_2)} \\ &\times \check{G}(\boldsymbol{r}_2 - \boldsymbol{r}', \omega_n), \end{aligned} \tag{31}$$

where 0 in the subscript indicates unperturbed Green function. By considering equations (29) and (30), we obtain

$$\check{G}(\boldsymbol{r}-\boldsymbol{r}', \omega_n) = \check{G}_0(\boldsymbol{r}-\boldsymbol{r}', \omega_n) + n_{\text{imp}}\, v_{\text{imp}}^2 \int \mathrm{d}\boldsymbol{r}_1 \check{G}_0(\boldsymbol{r}-\boldsymbol{r}_1, \omega_n)\, \hat{\tau}_3\, \check{G}_0(0, \omega_n)\, \hat{\tau}_3\, \check{G}(\boldsymbol{r}_1-\boldsymbol{r}', \omega_n) + n_{\text{imp}}\, v_{\text{imp}}^2 \int \mathrm{d}\boldsymbol{r}_1 \check{G}_0(\boldsymbol{r}-\boldsymbol{r}_1, \omega_n)\, \check{A}\, \check{G}_0(0, \omega_n)\, \check{A}\, \check{G}(\boldsymbol{r}_1-\boldsymbol{r}', \omega_n). \tag{32}$$

The second and the third terms are derived from the intraband impurity scatterings and the interband impurity scatterings, respectively. By applying the Fourier transformation, the Green function becomes

$$\check{G}(\boldsymbol{k}, \omega_n) = \check{G}_0(\boldsymbol{k}, \omega_n) + \check{G}_0(\boldsymbol{k}, \omega_n)\, \check{\Sigma}_{\text{imp}}(\omega_n)\, \check{G}(\boldsymbol{k}, \omega_n), \tag{33}$$

$$\check{\Sigma}_{\text{imp}} = \check{\Sigma}_{\text{intra}} + \check{\Sigma}_{\text{inter}}, \tag{34}$$

where $\check{\Sigma}_{\text{intra}}$ and $\check{\Sigma}_{\text{inter}}$ are the self-energy due to the intraband impurity scatterings and that of interband impurity scatterings, respectively. The details of the derivation are given in appendix. In the Born approximation, the self-energies are represented as

$$\check{\Sigma}_{\text{intra}} = n_{\text{imp}} v_{\text{imp}}^2 \hat{\tau}_3\, \hat{\rho}_0 \frac{1}{V_{\text{vol}}} \sum_{\boldsymbol{k}} \check{G}_0(\boldsymbol{k}, \omega_n) \hat{\tau}_3\, \hat{\rho}_0, \tag{35}$$

$$\check{\Sigma}_{\text{inter}} = n_{\text{imp}} v_{\text{imp}}^2\, \check{A}\, \frac{1}{V_{\text{vol}}} \sum_{\boldsymbol{k}} \check{G}_0(\boldsymbol{k}, \omega_n)\, \check{A}. \tag{36}$$

The total self-energy is calculated as

$$\check{\Sigma}_{\text{imp}} = \begin{bmatrix} \hat{\Sigma}_G & \hat{\Sigma}_F \\ -s_s \hat{\Sigma}_F^* & -\hat{\Sigma}_G^* \end{bmatrix}, \tag{37}$$

with

$$\hat{\Sigma}_G = 2n_{\text{imp}} v_{\text{imp}}^2 [\langle g_0 \rangle \hat{\rho}_0 + \cos\theta\, S_g\, \hat{\rho}_1 - \sin\theta\, S_g\, \hat{\rho}_2], \tag{38}$$

$$\hat{\Sigma}_F = -2n_{\text{imp}} v_{\text{imp}}^2 [\cos\theta\, S_f\, \hat{\rho}_0 + \langle f_1 \rangle \hat{\rho}_1 + \mathrm{i} \sin\theta\, S_f\, \hat{\rho}_3], \tag{39}$$

$$S_g = \langle g_1 \rangle \cos\theta - \langle g_2 \rangle \sin\theta, \tag{40}$$

$$S_f = \langle f_0 \rangle \cos\theta - \mathrm{i} \langle f_3 \rangle \sin\theta. \tag{41}$$

Here the Green function after carrying out the summation of $\boldsymbol{k}$ is indicated by $\langle \cdots \rangle$ as,

$$\langle \hat{\mathcal{G}}_0(\omega_n) \rangle \equiv \frac{1}{V_{\text{vol}}} \sum_{\boldsymbol{k}} \hat{\mathcal{G}}_0(\boldsymbol{k}, \omega_n) = \sum_{\nu=0}^{3} \langle g_\nu \rangle \hat{\rho}_\nu, \tag{42}$$

$$\langle \hat{\mathcal{F}}_0(\omega_n) \rangle \equiv \frac{1}{V_{\text{vol}}} \sum_{\boldsymbol{k}} \hat{\mathcal{F}}_0(\boldsymbol{k}, \omega_n) = \sum_{\nu=0}^{3} \langle f_\nu \rangle \hat{\rho}_\nu, \tag{43}$$

where $\hat{\rho}_\nu$ with $\nu = 0 - 3$ are the Pauli matrices in band space. The Gor'kov equation in the presence of impurities is expressed by

$$[\mathrm{i}\omega_n \check{1} - \check{H}_0(\boldsymbol{k}) - \check{\Sigma}_{\text{imp}}] \check{G}(\boldsymbol{k}, \mathrm{i}\omega_n) = \check{1}, \tag{44}$$

$$\check{G}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{bmatrix} \hat{\mathcal{G}}(\boldsymbol{k}, \mathrm{i}\omega_n) & \hat{\mathcal{F}}(\boldsymbol{k}, \mathrm{i}\omega_n) \\ -s_s \hat{\mathcal{F}}^*(-\boldsymbol{k}, \mathrm{i}\omega_n) & -\hat{\mathcal{G}}^*(-\boldsymbol{k}, \mathrm{i}\omega_n) \end{bmatrix}, \tag{45}$$

equation (37) with equations (38)–(43) give the general expression self-energy due to impurity scattering within the Born approximation. The properties in the normal state and those in the superconducting state are mainly embedded in the normal Green function in equation (42) and in the anomalous Green function in equation (43), respectively. Therefore the results can be applied to various two-band superconductors. Here we briefly mention a general feature of the self-energy. In equation (39), $\langle f_1 \rangle \hat{\rho}_1$ is present but $\langle f_2 \rangle \hat{\rho}_2$ is absent in $\hat{\Sigma}_F$ because of the anticommutation relations among $\hat{\rho}_\nu$. This feature is independent of the normal state Hamiltonian. As shown in the remaining part of this section, the effects of random nonmagnetic impurity scatterings on the transition temperature T_c depends on spin symmetry of the pair potential. The difference comes from such general property of $\hat{\Sigma}_F$. We will explain details of the difference in the following subsections.

3.1. Spin-singlet

The normal part of the self-energy is calculated as

$$\hat{\Sigma}_G = \frac{-\mathrm{i}\omega_n}{2\tau_{\text{imp}}|\omega_n|} \hat{\rho}_0, \tag{46}$$

$$\frac{1}{\tau_{\rm imp}} = 2 \times 2\pi N_0 n_{\rm imp} v_{\rm imp}^2, \tag{47}$$

where $\tau_{\rm imp}$ represents the life time due to impurity scatterings. The factor 2 in equation (47) stems from the two contributions of different scattering processes: the intraband impurity scatterings and the interband impurity scatterings. In a spin-singlet superconductor, the self-energy of the anomalous part results in

$$\hat{\Sigma}_F = \frac{\Delta}{2\tau_{\rm imp}|\omega_n|}\hat{\rho}_1, \tag{48}$$

because equation (39) includes $\langle f_1 \rangle \hat{\rho}_1$. As a consequence, the Gor'kov equation in the presence of impurities becomes,

$$\begin{bmatrix} (\mathrm{i}\tilde{\omega}_n - \xi)\hat{\rho}_0 - \hat{V} & -\tilde{\Delta}\hat{\rho}_1 \\ -\tilde{\Delta}\hat{\rho}_1 & (\mathrm{i}\tilde{\omega}_n + \xi)\hat{\rho}_0 + \hat{V}^* \end{bmatrix} \check{G}(\boldsymbol{k}, \mathrm{i}\omega_n) = \check{1}, \tag{49}$$

$$\hat{V} = v\cos\theta\ \hat{\rho}_1 - v\sin\theta\ \hat{\rho}_2, \tag{50}$$

$$\tilde{\omega}_n = \omega_n\,\eta_n, \quad \tilde{\Delta} = \Delta\,\eta_n, \quad \eta_n = 1 + \frac{1}{2\tau_{\rm imp}|\omega_n|}. \tag{51}$$

The self-energy renormalizes the frequency and the pair potential exactly in the same manner as $\omega_n \to \tilde{\omega}_n$ and $\Delta \to \tilde{\Delta}$. As a consequence, the anomalous Green function can be calculated as

$$\hat{\mathcal{F}}(\boldsymbol{k}, \omega_n) = \hat{\mathcal{F}}_0(\boldsymbol{k}, \tilde{\omega}_n)|_{\Delta\to\tilde{\Delta}}, \tag{52}$$

where $\hat{\mathcal{F}}_0$ on the right hand side is shown in equation (20). The gap equation in the presence of impurities is given by equation (21) with $\hat{\mathcal{F}}_0(\boldsymbol{k}, \omega_n) \to \hat{\mathcal{F}}(\boldsymbol{k}, \omega_n)$. The resulting gap equation

$$\Delta = \pi g N_0 T \sum_{\omega_n} \frac{\tilde{\Delta}}{|\tilde{\omega}_n|} = \pi g N_0 T \sum_{\omega_n} \frac{\Delta}{|\omega_n|}, \tag{53}$$

remains unchanged from that in the clean limit. Thus, the impurity scatterings do not change T_c in a spin-singlet superconductor. The argument here is exactly the same as that in [1] for a single-band spin-singlet s-wave superconductor and is consistent with the Anderson's theorem [3].

3.2. Spin-triplet

In a spin-triplet superconductor, the Green function in equation (43) with equation (24) is calculated as

$$\langle \hat{\mathcal{F}}_0 \rangle = \frac{N_0 \pi \Delta}{|\omega_n|(\omega_n^2 + v^2)} [\omega_n\, v\sin\theta\ \hat{\rho}_0 - \mathrm{i}\omega_n^2 \hat{\rho}_2 - \mathrm{i}\omega_n\, v\cos\theta\ \hat{\rho}_3]. \tag{54}$$

By substituting the results into equation (39), we find

$$\hat{\Sigma}_F = 0, \tag{55}$$

because equation (39) does not include $\langle f_2 \rangle \hat{\rho}_2$. The resulting Gor'kov equation becomes,

$$\begin{bmatrix} (\mathrm{i}\tilde{\omega}_n - \xi)\hat{\rho}_0 - \hat{V} & -\Delta\mathrm{i}\hat{\rho}_2 \\ \Delta\mathrm{i}\hat{\rho}_2 & (\mathrm{i}\tilde{\omega}_n + \xi)\hat{\rho}_0 + \hat{V}^* \end{bmatrix} \check{G}(\boldsymbol{k}, \mathrm{i}\omega_n) = \check{1}. \tag{56}$$

The impurity self-energy renormalizes the frequency as $\omega_n \to \tilde{\omega}_n$ but leaves the pair potential as it is. Thus, the anomalous Green function in the presence of impurities becomes

$$\hat{\mathcal{F}}(\boldsymbol{k}, \omega_n) = \hat{\mathcal{F}}_0(\boldsymbol{k}, \tilde{\omega}_n), \tag{57}$$

where $\hat{\mathcal{F}}_0$ on the right hand side is given in equation (24). The gap equation (21) with $\hat{\mathcal{F}}_0(\boldsymbol{k}, \omega_n) \to \hat{\mathcal{F}}(\boldsymbol{k}, \omega_n)$ and $\hat{\rho}_1 \to -\mathrm{i}\hat{\rho}_2$ results in

$$\Delta = \pi g N_0 T \sum_{\omega_n} \frac{\Delta\,(|\omega_n| + 1/2\tau_{\rm imp})}{(|\omega_n| + 1/2\tau_{\rm imp})^2 + v^2}. \tag{58}$$

The results suggest that the impurity scatterings decrease T_c for a spin-triplet superconductor.

In figure 1, we show T_c of a spin-triplet interband superconductor as a function of ξ_0/ℓ, where T_0 is the transition temperature in the clean limit in the absence of the hybridization (i.e., $v = 0$), $\xi_0 = v_F/2\pi T_0$ is the coherence length, $v_F = k_F/m$ is the Fermi velocity, and $\ell = v_F\ \tau_{\rm imp}$ is the mean free path due to the impurity scatterings. We numerically solve equation (58) with $\omega_c/2\pi T_0 = 10^3$. The results show that T_c decreases with the increase of ξ_0/ℓ. In the clean limit, T_c decreases with the increase of the hybridization v as indicated in equation (25). The superconducting phase vanishes when the amplitude of hybridization goes over its critical value of $v_c \approx 2\pi T_0/C$, where $C = 4e^{\gamma_E}$ and $\gamma_E = 0.577$ is the Euler's constant. In the presence of impurities, the

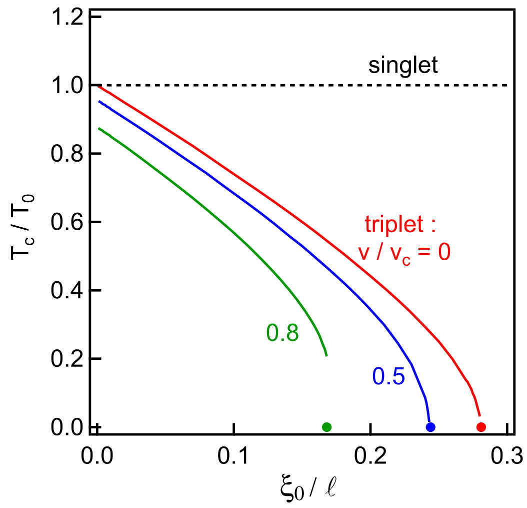

Figure 1. The transition temperature T_c versus ξ_0/ℓ. The impurity concentration is proportional to ξ_0/ℓ, where ξ_0 is the coherence length and ℓ is the elastic mean free path. In a spin-singlet case, T_c is independent of ξ_0/ℓ within the Born approximation as shown with a broken line, which is consistent with the Anderson's theorem. The results for a spin-triplet interband superconductor at $v = 0$ are identical to those for a single-band unconventional superconductor characterized such symmetry as spin-singlet d-wave or spin-triplet p-wave.

interband spin-triplet superconductivity vanishes at $\xi_0/\ell \approx 2/C = 0.281$ at $v = 0$, $\xi_0/\ell \approx 0.244$ at $v = 0.5\ v_c$, and $\xi_0/\ell \approx 0.168$ at $v = 0.8\ v_c$.

The suppression of T_c by impurities in a spin-triplet case can be interpreted as follows. The interband impurity scatterings hybridize the electronic states in the two bands and average the pair potential over the two-band degree of freedom. As shown in equation (15), the sign of pair potential in one sector is opposite to that in the other where we set $s_s = 1$ for a triplet superconductor. Thus, the pair potentials in the two sectors cancel each other when the interband impurity potential hybridizes the two sectors. As a result, the anomalous part of the self-energy vanishes as shown in equation (55). Namely, the impurity self-energy does not renormalize the pair potential, which leads to the suppression of T_c. The absence of $\langle f_2 \rangle \hat{\rho}_2$ in equation (39) can be understood by such physical interpretation. It would be worth mentioning that the gap equation in equation (58) with $v = 0$ is identical to that for a single-band unconventional superconductor under the potential disorder. In a p-wave or d-wave superconductor, the anomalous Green function $\langle \mathcal{F}_0(\omega_n) \rangle$ vanishes due to their unconventional pairing symmetries, which leads to $\Sigma_F = 0$ and the suppression of T_c. We conclude that the odd-band-parity pairing correlation is fragile under impurity potential even though it belongs to s-wave momentum parity symmetry class. Therefore, a clean enough sample is necessary to observe spin-triplet interband superconductivity in experiments.

Mathematically, the robustness of a spin-singlet s-wave interband superconducting state is described by the anomalous part of the self-energy $\hat{\Sigma}_F = \hat{\Delta}/2\tau_{\text{imp}}|\omega_n|$ in equation (48). The suppression of T_c in a spin-triplet superconductor is described by $\hat{\Sigma}_F = 0$ in equation (55). As we already explained below equation (44), these features are derived from the general expression of the self-energy in equation (39) and are independent of the normal state Hamiltonian. Therefore, our conclusions are valid for various interband superconductors.

4. Conclusion

We studied the effects of random nonmagnetic impurities on the superconducting transition temperature T_c in a two-band superconductor characterized by an equal-time s-wave interband pair potential. Due to the two-band degree of freedom, both spin-singlet and spin-triplet pairing order parameters satisfy the requirement from the Fermi–Dirac statistics of electrons. The effects of impurity potential is considered through the self-energy obtained within the Born approximation. The transition temperature is calculated from the linearized gap equation. In a spin-singlet superconductor, the random potential does not change T_c. On the other hand in a spin-triplet superconductor, T_c decreases with the increase of the impurity concentration. We conclude that Cooper pairs belonging to odd-band-parity symmetry class are fragile under the random impurity potential even though they belong to s-wave momentum symmetry.

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Appendix

We show the details of the derivation of the impurity self-energy in equation (37). The Fourier representation of the Green function is defined by

$$\check{G}(\boldsymbol{r}-\boldsymbol{r}', \omega_n) = \frac{1}{V_{\mathrm{vol}}} \sum_{\boldsymbol{k}} \check{G}(\boldsymbol{k}, \omega_n) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}. \tag{A.1}$$

The Green function $\check{G}_0(0, \omega_n)$ in equation (32) is obtained by putting $\boldsymbol{r} = \boldsymbol{r}'$. When we substitute equation (A.1) into (32) and carrying out the integration over $\boldsymbol{r}_1$, we find equation (33). Since $\check{G}_0(\boldsymbol{k}, \omega_n)$ satisfies equation (16), we obtain equation (44) with the self-energy in equation (34). To proceed the calculation, the Green function integrated over the momenta is necessary. The general expression of them are defined by equations (42) and (43). By substituting equations (42) and (43) into (35) and (36), we find

$$\check{\Sigma}_{\mathrm{intra}} = n_{\mathrm{imp}}\, v_{\mathrm{imp}}^2 \sum_{\nu=0}^{3} \begin{bmatrix} \langle g_\nu \rangle \hat{\rho}_\nu & -\langle f_\nu \rangle \hat{\rho}_\nu \\ s_s [\langle f_\nu \rangle \hat{\rho}_\nu]^* & -[\langle g_\nu \rangle \hat{\rho}_\nu]^* \end{bmatrix}, \tag{A.2}$$

$$\check{\Sigma}_{\mathrm{inter}} = n_{\mathrm{imp}}\, v_{\mathrm{imp}}^2 \sum_{\nu=0}^{3} \begin{bmatrix} \hat{A}_- \langle g_\nu \rangle \hat{\rho}_\nu \hat{A}_- & -\hat{A}_- \langle f_\nu \rangle \hat{\rho}_\nu \hat{A}_+ \\ s_s \hat{A}_+ [\langle f_\nu \rangle \hat{\rho}_\nu]^* \hat{A}_- & -\hat{A}_+ [\langle g_\nu \rangle \hat{\rho}_\nu]^* \hat{A}_+ \end{bmatrix}, \tag{A.3}$$

$$\hat{A}_\pm = \hat{\rho}_1 \cos\theta \pm \hat{\rho}_2 \sin\theta. \tag{A.4}$$

Here we focus on the anomalous part of the self-energy because its general expression is important to justify the main conclusion. We find the relation,

$$\begin{aligned} \hat{A}_- \sum_\nu \langle f_\nu \rangle \hat{\rho}_\nu \hat{A}_+ = {} & \langle f_1 \rangle \hat{\rho}_1 - \langle f_2 \rangle \hat{\rho}_2 + (\cos 2\theta \langle f_0 \rangle - \mathrm{i}\sin 2\theta \langle f_3 \rangle) \hat{\rho}_0 \\ & - (\cos 2\theta \langle f_3 \rangle - \mathrm{i}\sin 2\theta \langle f_0 \rangle) \hat{\rho}_3. \end{aligned} \tag{A.5}$$

The most important feature is that $\langle f_2 \rangle \hat{\rho}_2$ component changes its sign due to the anticomutation relations among $\hat{\rho}_j$. Together with the intraband contribution $\sum_\nu \langle f_\nu \rangle \hat{\rho}_\nu$, we obtain the general expression of the anomalous part in equation (39).

References

[1] Abrikosov A A, Gor'kov L P and Dzyaloshinski I E 1975 *Methods of Quantum Field Theory in Statistical Physics* (New York: Dover)
[2] Abrikosov A A and Gor'kov L P 1959 *Sov. Phys.—JETP* **9** 220
[3] Anderson P 1959 *J. Phys. Chem. Solids* **11** 26 – 30
[4] Allen P B and Mitrovic B 1983 Theory of superconducting T_c *Solid State Physics* ed H Ehrenreich *et al* vol 37 (New York: Academic) pp 1–92
[5] Golubov A A and Mazin I I 1997 *Phys. Rev.* B **55** 15146–52
[6] Onari S and Kontani H 2009 *Phys. Rev. Lett.* **103** 177001
[7] Efremov D V, Korshunov M M, Dolgov O V, Golubov A A and Hirschfeld P J 2011 *Phys. Rev.* B **84** 180512
[8] Korshunov M M, Efremov D V, Golubov A A and Dolgov O V 2014 *Phys. Rev.* B **90** 134517
[9] Hoyer M, Scheurer M S, Syzranov S V and Schmalian J 2015 *Phys. Rev.* B **91** 054501
[10] Asano Y and Golubov A A 2017 arXiv:710.04348
[11] Nagamatsu J, Nakagawa N, Muranaka T, Zenitani Y and Akimitsu J 2001 *Nature* **410** 63
[12] Choi H J, Roundy D, Sun H, Cohen M L and Louie S G 2002 *Nature* **418** 758
[13] Kamihara Y, Watanabe T, Hirano M and Hosono H 2008 *J. Am. Chem. Soc.* **130** 3296
[14] Hosono H and Kuroki K 2015 *Physica* C **514** 399–422
[15] Hor Y S, Williams A J, Checkelsky J G, Roushan P, Seo J, Xu Q, Zandbergen H W, Yazdani A, Ong N P and Cava R J 2010 *Phys. Rev. Lett.* **104** 057001
[16] Fu L and Berg E 2010 *Phys. Rev. Lett.* **105** 097001
[17] Sato M and Ando Y 2017 *Rep. Prog. Phys.* **80** 076501
[18] Joynt R and Taillefer L 2002 *Rev. Mod. Phys.* **74** 235–94

[19] Yanase Y 2016 *Phys. Rev.* B **94** 174502
[20] Oudah M, Ikeda A, Hausmann N J, Yonezawa S, Fukumoto T, Kobayashi S, Sato M and Maeno Y 2016 *Nat. Commun* **7** 13617
[21] Caldas H and Continentino M A 2012 *Phys. Rev.* B **86** 144503
[22] Fulde P and Ferrell R A 1964 *Phys. Rev.* **135** A550–63
[23] Larkin A I and Ovchinnikov Y N 1965 *Sov. Phys.—JETP* **20** 762
[24] Black-Schaffer A M and Balatsky A V 2013 *Phys. Rev.* B **88** 104514
[25] Asano Y and Sasaki A 2015 *Phys. Rev.* B **92** 224508
[26] Vasenko A S, Golubov A A, Silkin V M and Chulkov E V 2017 *J. Phys.: Condens. Matter* **29** 295502
[27] Vasenko A S, Golubov A A, Silkin V M and Chulkov E V 2017 *JETP Lett.* **105** 497–501
[28] Asano Y, Golubov A A, Fominov Y V and Tanaka Y 2011 *Phys. Rev. Lett.* **107** 087001
[29] Suzuki S I and Asano Y 2015 *Phys. Rev.* B **91** 214510