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Abstract

This paper shows the existence of gains from trade in a dynamic world free trade economy over a discrete-time infinite horizon with using Grandmont-McFadden's(1972) domestic income transfer policy which makes each consumer benefit from world free trade. For this purpose, this paper employs l_∞, the space of all bounded sequences, as the underlying commodity space and l_1, the space of all summable sequences, as the price space, with following the general equilibrium analysis of infinite dimensional commodity spaces developed by Bewley(1972), Mas-Colell(1986), and Zame(1987). Moreover, since gains from trade requires the comparison of two consumption bundles, one under free trade and the other under autarky, but transitivity of preferences assumed the comparison of three consumption bundles, this paper also drops transitive preferences to establish gains from trade in this dynamic world economy. Furthermore, since this paper uses general consumption sets instead of the positive orthant ${l_\infty}^+$, the argument finding an equilibrium price in l_1, requires the exclusion condition to consumption sets and the mixture condition to the production set. This paper also drops the cheaper point assumption used in Grandmont–McFadden(1972) and replaces it with a variant of McKenzie's(1959, 1981, 2002) irreducibility assumption and the strong irreducibility assumption of Boy-McKenzie(1993), requiring the interrelatedness of consumers in the world economy.

1 Introduction

In a traditional world economy with a finite number of commodities, Kemp(1962) and Samuelson(1939,1962) establish the existence of gains from world free trade, and consider a hypothetical domestic income redistribution which can make free trade policy beneficial for every consumer. Although they mention only the possibility of some form of domestic income redistribution which can make free trade policy beneficial for every consumer, and do not formulate explicitly the form of their domestic income redistribution for making every consumer better off under free trade, Grandmont–McFadden(1972) formulates an explicit form of the domestic income distribution policy which makes every consumer better off under free trade in a traditional world economy with a finite number of commodities. Its fundamental idea is following. Based on the fact that, as shown in Kemp(1962) and Samuelson(1939,1962), the aggregate consumption bundle in a country under autarky is feasible and affordable under free trade owing to profit maximizing behavior of the production side of the country, Grandmont–McFadden(1972) considers an explicit form of the domestic income distribution policy with which every consumer can afford his autarky consumption bundle at any world price under free trade. Then, the argument similar to the one used in the theory of revealed preferences implies the consumption bundles chosen by each consumer under free trade is preferred to, generally not worse than, the one under autarky. Hence the existence of actual gains from free trade is established in a traditional world economy with a finite number of commodities.

Although this idea is robust, no generalization of this Grandmont–McFadden's(1972) result is, however, attempted in dynamic world free trade economy over an infinite horizon, which is one of a simple generalized world economy of the one used in Kemp(1962), Samuelson(1939,1962), and Grandmont–McFadden(1972), since the appropriate method to analyze this issue becomes available only after Bewley(1972)' seminar paper on the general equilibrium theory of infinite dimensional commodity spaces. Thus, the purpose of this paper is to establish Grandmont–McFadden's(1972) result of gains from trade in a simple dynamic world free trade economy over a discrete-time infinite horizon with employing the general equilibrium theory on infinite dimensional commodity spaces of Bewley(1972), Mas-Colell(1986), and Zame(1987).[1] For this purpose, l_∞, this paper uses the space of all bounded sequences, as the underlying commodity spaces of this simple dynamic world free trade economy over a discrete-time infinite horizon.[2] This result implies that world free trade with domestic income transfers is beneficial for a country of any size. This is an extension of the result of Grandmont–McFadden(1972) on gains from free trade in traditional world economies with a finite number of commodities to the dynamic world economy with countably many commodities considered in this paper. Note that the set of feasible outputs is, thus, $||\cdot||_\infty$-bounded, and the paths of feasible allocations are uniformly bounded in this dynamic world economy.

The argument on dynamic gains from free trade discusses typically the possibility that the steady state (per capita) consumption level of a small country may be smaller under world free trade than under autarky in a simple capital accumulation model of international trade.[3] Since there is a constant loss in consumption at the free trade steady state than at the autarky steady state, in contrast to the static case, this observation may lead to the conclusion that free trade is not a beneficial policy to a small country and dynamic gains from trade may not exist in the dynamic infinite horizon case. In order to analyze the welfare aspect of this phenomenon, it is necessary to take into account the welfare change accruing over the transition from the autarky steady state to the free trade steady state. In particular, it is necessary to specify an appropriate welfare criterion. It is insufficient, as is well-known, to compare only two different steady states, one under free trade and one under autarky.[4] Kemp(1976), Smith(1979), and Dixit(1981) among neoclassical trade

[1] Mas-Colell–Zame(1989) is a comprehensive and excellent survey on infinite dimensional commodity spaces, and Aliplantis et al.(1989) is a more concrete treatment on this topic. Note that Debreu(1954) is the classical paper on Pareto optimality and competitive equilibrium in an economy with an infinite dimensional commodity space.

[2] See Appendix A:

[3] Smith(1984) for a survey on this issue. See also Bhagwati, Srinivasan, and Panagariya(1998, Chap. 36).

[4] Burmeister(1980, Chap. 4, 5) for the issues on comparative statics of steady state.

theorists particularly mentioned this point explicitly.[5] Since the simple dynamic world free trade economy over a discrete-time infinite horizon is an example of economies with infinite dimensional commodity spaces, the general equilibrium analysis of infinite dimensional commodity spaces is necessary to treat this kind of welfare analysis. Since it was, however, not developed enough well by that period so that it was unavailable to this literature, no one has tried to do this task in such a way so far. Kemp(1976), Smith(1979), and Dixit(1981), thus, did not analyze this issue from the viewpoint of the general equilibrium analysis of infinite dimensional commodity spaces.

This paper shows that the value of a consumption path of a consumer under world free trade is never smaller than the one under autarky in this dynamic world economy when a corresponding equilibrium price under world free trade is used to evaluate these two different consumption paths. Then the similar result follows for aggregate consumption paths of a country under world free trade and under autarky. The latter is obtained by Smith(1979) as his result on dynamic gains from trade. Thus, the result established in this paper is an extension of the result on dynamic gains from trade obtained by Smith(1979) to the dynamic world free trade economy of this paper with an explicit introduction of consumer's preferences. Since consumers' preferences are employed in this argument, the comparison of the two entire consumption paths under autarky and under free trade is possible.[6] The choice of welfare criteria, in particular, its continuity requirement, turns out to be important in this issue. Since the infinite horizon model of world economy over an infinite horizon in Smith(1979) and Dixit(1981) is considered as a special case of the general world economy treated in this paper and they do not introduce welfare criteria explicitly in their arguments, this treatment within the framework of the general equilibrium analysis of infinite dimensional commodity spaces in this paper, thus, complements their works considered in this simple capital accumulation model.

Since in this dynamic world free trade economy, preferences of consumers are introduced with explicit continuity requirements, two different entire consumption paths can be compared. This enables to interpret from the viewpoint of consumer preferences the phenomenon where a small country has a smaller amount of steady state consumption under world free trade than under autarky. In particular, we can take into account the effect of the transition of consumption while moving from the steady state under autarky to the one under world free trade. Since continuity of preferences with respect to the topology employed in l_∞ yields a strong form of myopia or impatience in the dynamic context, the result on dynamic gains from trade implies that the gains accruing over the transition periods dominates the losses incurred at the steady state under world free trade. [7] The continuity requirement, interpreted as myopia typically, thus, has important implications for the existence of dynamic gains from trade. It is necessary for the existence of competitive equilibrium in a dynamic world economy over an infinite horizon.

Note also that since gains from free trade is based on the comparison of two competitive equilibria, the one under world free trade and the other under autarky, so only two states are compared in these arguments. On the other hand, transitivity of preferences is based on the comparison of three states. Thus transitivity of preferences has no relevance in the arguments on gains from free trade where only two states are compared, and hence, it is desirable to dispense with transitivity

[5] Kemp(1976, Introduction to Part II, p. 87 - 8) suggests that we should appeal the general equilibrium analysis of infinite dimensional commodity spaces as the appropriate tool in analyzing this issue. This paper, thus, follows his suggestion and completes the argument in the direction he suggested. Srinivasan-Bhagwati(1980) shows dynamic gains from trade in two sector model of wotld economy over continuous-time infinite horizon with employing the convergence argument of turnpike theorm in an optimal economic growth model. Their paper uses welfare criteiron expressed as integral of discounted life time utility. Kemp(1995, Chap.10) and Kemp–Wong(1995) considers gains from trade in a dynamic overlapping generations model. Although the existence of competitive equilibrium under world free trade is proved with a uniformly bounded feasible set, once a competitive equilibrium exist under world free trade as well as under autarky, the result on dynamic gains from trade can hold even without the uniformly bounded set of feasible outputs. In particular, dynamic gains from trade still exist even in a case with growing economies.

[6] Although the existence of competitive equilibrium under world free trade is proved with a uniformly bounded feasible set, once a competitive equilibrium exist under world free trade as well as under autarky, the result on dynamic gains from trade can hold even without the uniformly bounded set of feasible outputs. In particular, dynamic gains from trade still exist even in a case with growing economies.

[7] See Appendix A:

of preferences in this argument. From this viewpoint, this paper proves the existence of dynamic gains from free trade with Grandmont–McFadden's(1972) domestic income transfer policy without transitivity of preferences. The irrelevancy of transitivity of preferences in the arguments on gains from free trade has not been mentioned so far in the literature on the theory of international trade.[8] This paper do not assume transitivity of preferences in the dynamic world economy from this viewpoint.

Moreover, since this paper also employs general consumption sets instead of the positive orthant l_∞^+, a form of the exclusion assumption is imposed on consumption sets and a form of the mixture condition is imposed on the production set of the world economy, respectively. These assumptions are crucial in the proof of the existence of competitive equilibrium with domestic income transfers under world free trade in this paper, where an equilibrium price at world free trade belongs to l_1^+, i.e., summable.

In a finite dimensional commodity space case of Debreu(1959), every consumer is assumed to have the initial endowment in the (relative) interior of his consumption set to make his income is always beyond the subsistence level, called cheaper point condition or minimum wealth constraint, regardless of prices. Then as the result, the excess demand correspondence is upper hemi-continuous over the entire price set. When non-traded goods are allowed, however, in the world economy as in Samuelson(1939), it is inappropriate to make this assumption to consumers in the world economy.[9] Grandmont–McFadden(1972) assumes that their domestic income transfer policy always leads every consumer to have his income above the subsistence level regardless of prices. Thus, the individual demand correspondences, and hence the world excess demand correspondence, become upper hemi-continuous over the entire price set. This paper drops this cheaper point condition used by Grandmont–McFadden(1972), and replaces it with a variant of McKenzie's(1959, 1981, 2002) irreducibility assumption, requiring the interrelatedness of consumers in the world economy. The irreducibility assumption used in this paper is, however, a bit difference from the original one of McKenzie's(1959, 1981, 2002).[10] Then this irreducibility assumption can guarantee that every consumer in the world economy has enough income at a world quasi-equilibrium with domestic income transfers. Then the expenditure minimization of consumers in the world economy becomes the preference maximization, and hence this world quasi-equilibrium with Grandmont–McFadden's(1972) domestic income transfers becomes a world competitive equilibrium. Kemp–Wan(1993) take into account the existence of non-traded goods as well and try to make the irreducibility assumption, they, however, do not define the irreducibility condition explicitly.[11]

This paper is organized as follows. The next section formulates the form of Grandmont-McFadden's(1972) domestic income transfers in a dynamic world free trade economy in l_∞. Section 3 lists the assumptions of the dynamic world free trade economy with Grandmont-McFadden's(1972) domestic income transfers. Section 4 discusses the existence of quasi-equilibrium with price systems in l_1 in a dynamic world free trade economy. All proofs of the theorems are established in Appendix B. Final section contains some concluding remarks.

[8] The same argument holds for the law of comparative advantage and terms of trade improvement as well as long as only two states are compared, although this paper do not treat the law of comparative advantage and terms of trade improvement as done in, for examples, Krueger-Sonnenschein(1967), Ohyama(1972), Deardoroff(198), and Dixit-Norman(1980).

[9] By applying Arrow's(1951) anomalous corner case, Chipman(1965, Part 2, Figure 2.5, p. 708.) illustrates the non-existence of competitive equilibrium in a two country world with two goods. One country under autarky does not have one of the goods. It becomes available under free trade but the other country is satiated in it under free trade.

[10] This is discussed in detai in section 3.

[11] Kemp–Wan(1972) also takes into consider the existence of non-traded goods. They modify this assumption in such a way that a similar condition holds only for the goods available to the country under consideration which are also strongly desirable for each consumer of the country. This condition, however, can not exclude a situation similar to Arrow's(1951) corner at the consumption bundles of consumers under autarky, unless there are some common goods which are available to every country under autarky. Cordella–Minelli–Polemarchakis(1993) construct this kind of example numerically. Kemp–Wan(1986) uses Debreu's(1959) assumption for the initial consumption bundle of each consumer to rule out this case from the beginning. Kemp(1995, Chap.24) modifies Debreu's assumption to make the condition fit to the context of gains from trade with Grandmond-McFadden's(1972) domestic income transfers

2 Formulation of Grandmont–McFadden's(1972) Domestic Income Transfer Policy

This section gives a formulation of Grandmont–McFadden's(1972) domestic income transfer policy in dynamic world free trade over a discrete-time infinite horizon. In this dynamic world free trade economy over an infinite horizon, we use the same domestic income transfer policy to establish that this domestic income transfer policy makes every consumers in world economy actually not worse off under world free trade than under autarky. We do not however assume transitivity of preferences since it has no relevance to gains from free trade. Note that Grandmont–McFadden(1972) establishes that a domestic income transfer policy exists in the traditional world economy with a finite number of commodities and transitivity of preferences.

We first remind the argument based on the second fundamental theorem of welfare economics in finding a world free trade equilibrium which makes every consumer in the world economy as well off than under autarky. The allocations of autarky competitive equilibrium of countries constitute a feasible allocation in the world economy. It is however not necessarily Pareto optimal in the world economy despite that the allocation of autarky competitive equilibrium of each country is Pareto optimal in the country. When the world allocation is not Pareto optimal, we may find an world allocation which is Pareto optimal and Pareto improves over the allocations of autarky competitive equilibrium of countries. Then from the second fundamental theorem of welfare economics, we can find an associated quasi-equilibrium price under world free trade, where every consumer in the world economy is as well off than under autarky.

This argument is based on the following two facts. The first is that the world economy can find a Pareto optimal allocation which also Pareto improves over the autarky allocations. The second is that between countries, they can agree with the allocation in the world economy which is supported as a quasi-equilibrium under world free trade. The first requires that the world economy as whole have to know and can collect the information about preferences, initial endowments, consumption sets, and autarky consumption bundles of consumers, and production sets and autarky production plans of countries. Collecting the information on these usually costs a lot. The second requires an arrangement of the bargaining between countries. Since the world economy has no authority like the one in each country, the countries face a difficulty in arranging the bargaining process.

In these two aspects, Grandmont–McFadden's(1972) domestic income transfer policy has advantage over the second fundamental theorem of welfare economics. The world economy as a whole need not try to find a Pareto optimal allocation which also Pareto improves over the allocations of autarky competitive equilibrium of countries. The world economy as a whole do not need to collect the information on the underlying economic data of countries. Moreover the world economy can avoid the bargaining problem. In order to carry out the Grandmont–McFadden type domestic income transfer policy, each country only needs to know its aggregate endowment and the commodity bundles of consumers under autarky. The world economy as a whole do not need to collect the information on these over the countries.

Under laissez-faire world free trade, income of some consumer of a country may be short to buy his autarky consumption bundle. Such a consumer may be worse off under laissez-faire world free trade. On the other hand, Grandmont–McFadden's domestic income transfer policy makes consumers affordable to buy their autarky consumption bundles under world free trade. Thus, the consumption bundles they choose under world free trade are as desirable as their autarky consumption bundles. Therefore, world free trade becomes actually preferable to every consumer in the world economy when each country employs Grandmont–McFadden's domestic income transfer policy. Once we prove the existence of competitive equilibrium under world free trade with Grandmont–McFadden's domestic income transfer policy, we can also prove the actual preferability of world free trade over autarky. Thus, our aim is to prove the existence of competitive equilibrium under world free trade with this domestic income transfer policy. We prove the existence of such a competitive equilibrium without transitivity of preferences. Since autarky consumption bundle of every consumer is affordable under such world free trade, his demand point under world free trade is actually preferable to his autarky consumption bundle. W can prove the actual preferability of world free trade even without transitivity of preferences. Transitivity of preferences is irrelevant in this actual preferability of world free trade.

We set up a dynamic world free trade over a discrete-time infinite horizon with Grandmont–McFadden's(1972) domestic income transfer policy. In each period, N number of goods is available in the world economy. As before, N is assumed constant over time. In this world economy, there may exist some non-traded goods. If a country has some non-traded goods, then they are unavailable to the rest of countries in the world economy. When non-traded goods exist in the world economy, then some goods are unavailable to a county in the world economy. Let $N_{\mathrm{k}}^{\mathrm{T}}$ be the set of the indices of goods available to k-th country under world free trade. The economic data of k-th country must satisfy the condition that the coordinates of $N \backslash N_{\mathrm{k}}^{\mathrm{T}}$ associated with goods unavailable to them are zero.[12] Although non-traded goods may exist in the world economy, the world aggregate adequacy condition, defined below, implies that all goods including these non-traded goods can be in excess in the entire world economy as a whole.

Let $\mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ and $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}(\subset l_{\infty}^{+})$ be production set and consumption set of the i-th consumer in the k-th country which are actually available under dynamic world free trade economy.[13] The number of goods available to the k-th country is usually larger under world free trade than under autarky. We assume $\mathbf{Y}_{\mathrm{k}}^{\mathrm{A}} \subset \mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ and $\mathbf{C}_{\mathrm{ik}}^{\mathrm{A}} \subset \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$. Also let $\mathbf{P}_{\mathrm{ik}}(\cdot)$ be the preference of the i-th consumer in the k-th country defined over $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$. Note that the restriction of $\mathbf{P}_{\mathrm{ik}}$ over $\mathbf{C}_{\mathrm{ik}}^{\mathrm{A}}$ becomes his preference under autarky. Thus a regime switch from autarky to free trade does not affect preferences of consumers in k-th country. Denote $\mathrm{I} = \mathrm{I}_{\mathrm{k}} \times \mathrm{J}$, the set of indices of consumers in the world economy. A vector $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{(\mathrm{i,k}) \in \mathrm{I}}$ is called an allocation under world free trade if $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ holds for $(\mathrm{i,k}) \in \mathrm{I}$ and $\mathbf{y}_{\mathrm{k}} \in \mathbf{Y}_{\mathrm{k}}$ holds for $\mathrm{k} \in \mathrm{J}$. It is also called feasible if further $\sum_{(\mathrm{i,k}) \in \mathrm{I}} \mathbf{x}_{\mathrm{ik}} = \sum_{\mathrm{k} \in \mathrm{J}} (\mathbf{y}_{\mathrm{k}} + \omega_{\mathrm{k}})$ holds.

In a traditional finite dimensional world economy, $\mathbf{p} \cdot (\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} - \omega_{\mathrm{ik}}) + \theta_{\mathrm{ik}}\{\mathbf{p} \cdot \sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})\}$ is i-th consumer's amount of domestic income transfer in Grandmont–McFadden's(1972) domestic income transfer policy used in the k-th country, where $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ is i-th consumer's autarky consumption bundle and θ_{ik} is his share in the total gains of the country with $\theta_{\mathrm{ik}} \geq 0$ and $\sum_{\mathrm{i} \in \mathrm{I}_{\mathrm{k}}} \theta_{\mathrm{ik}} = 1$ and $\mathbf{p}$ is taken in $(\mathbf{Y}^{\mathrm{T}})^{*}$, the dual cone of the aggregate world production set $\mathbf{Y}^{\mathrm{T}}$. Then his total disposal income $\tau_{\mathrm{ik}}(\mathbf{p})$ becomes $\mathbf{p} \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}} \mathbf{p} \cdot (\omega_{\mathrm{k}} - \sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$. Since the profit maximization of the production side and $(\sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}} - \omega_{\mathrm{k}}) \in \mathbf{Y}_{\mathrm{k}}^{\mathrm{A}} \subset \mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ imply $\mathbf{p} \cdot (\sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}} - \omega_{\mathrm{k}}) \leq 0$, the second term in $\tau_{\mathrm{ik}}(\mathbf{p})$ is always non-negative and so $\tau_{\mathrm{ik}}(\mathbf{p}) \geq \mathbf{p} \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ holds for $\mathbf{p} \in (\mathbf{Y}^{\mathrm{T}})^{*}$, although $\tau_{\mathrm{ik}}(\mathbf{p}) < \mathbf{p} \cdot \omega_{\mathrm{ik}}$ may, however, occur for some $\mathrm{i} \in \mathrm{I}_{\mathrm{k}}$ at some $\mathbf{p}$.

In the dynamic world economy over an infinite horizon of this paper, the same domestic income transfer policy is used and the amount of the disposable income to the i-th consumer in the k-th country becomes

$$\tau_{\mathrm{ik}}(\mathbf{p}) = \pi \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}} \mathbf{p} \cdot (\omega_{\mathrm{k}} - \sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$$

for $\mathbf{p} \in \{\mathbf{p} \in \mathbf{ba}^{+} \backslash \{\mathbf{0}\} : ||\mathbf{p}|| \leq 1\}$, where θ_{ik} is his share again.[14] Since $\sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} (\mathbf{x}_{\mathrm{hk}}^{\mathrm{A}} - \omega_{\mathrm{hk}}) \in \mathbf{Y}_{\mathrm{k}}^{\mathrm{A}} \subset \mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ holds from the feasibility of the allocation $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}})_{\mathrm{i} \in \mathrm{I}_{\mathrm{k}}}$ under autarky, $\mathbf{p}^{T} \in \{\mathbf{p} \in \mathbf{ba}^{+} \backslash \{\mathbf{0}\} : ||\mathbf{p}|| \leq 1\} \cap (\mathbf{Y}^{\mathrm{T}})^{*}$ and $\sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} (\mathbf{x}_{\mathrm{hk}} - \omega_{\mathrm{hk}}) \in l_{\infty}$ imply $\mathbf{p}^{T} \cdot \sum_{\mathrm{h} \in \mathrm{I}_{\mathrm{k}}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}}) \geq 0$ from the profit maximization of the production side. [15] Thus $\tau_{\mathrm{ik}}(\mathbf{p}^{T}) \geq \mathbf{p}^{T} \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ follows at a competitive

[12] As in the autarky case, orders $\geq$ and $>$ are restricted asscording to satisfying this constraint. If there is no non-traded goods in the world economy, then it becomes a general dynamic economy.

[13] We may extend preferences and consumption sets of consumers in each country such that new consumption sets include the goods unavailable to them and new preferences are independent of these goods. This choice may cause some country break the balance condition of non-traded goods in the country. Thus, we do not use approach here.

[14] Theorem A in Appendix gives rise to the existence of competitive equilibrium under autarky $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}}, \pi_{\mathrm{k}}^{\mathrm{A}})_{\mathrm{i} \in \mathrm{I}_{\mathrm{k}}}$. Since $\mathbf{Y}_{\mathrm{k}}^{\mathrm{A}}$ and $\mathbf{C}_{\mathrm{ik}}^{\mathrm{A}}$ are obtained by restricting $\mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$, $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ over $\mathbf{L}_{\infty}^{\mathrm{k}\prime}$, the subspace defined by the commodities available under autarky, then all except (A–7) and (A–11) follow if (T–1)′, (T–2)′, (T–3)′, (T–4), (T–5), (T–6), (T–8), (T–9), and (T–10)′ are assumed in finding a competitive equilibrium under world free trade with domestic income transfers shown in following theorem 1.

[15] Since the world production set, $\mathbf{Y}^{\mathrm{T}}$ is assumed to be a convex cone, profit maximization condition of the production side implies that the condidate prices for equilibrium must be restricted in $(\mathbf{Y}^{\mathrm{T}})^{*}$, the dual cone of the world

equilibrium under world free trade with domestic income transfers. When the demand condition holds with a disposable income equal to $\tau_{\mathrm{ik}}(\pi)$ in the dynamic world free trade economy, since each consumer can afford the autarky consumption bundle $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ by the profit maximization of the production side in the dynamic world free trade economy, the preference maximization with this disposable income implies that each consumer can afford to buy it and is as well off under free trade than under autarky with Grandmont–McFadden's domestic income transfer policy. Then the demand condition of consumers implies that his demand point with income equal to $\tau_{\mathrm{ik}}(\mathbf{p})$ becomes as desirable as $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ for him by the argument similar to the one in revealed preference theory. Since this state is realized as a competitive equilibrium under world free trade with domestic income transfer, this welfare property is not hypothetical, but *actually* attainable. Every consumer in the world economy is *actually* as well off than under autarky in world free trade with Grandmont–McFadden's domestic income transfers.[16] This is a basis for the result on the existence of dynamic gains from trade in this paper.[17]

When the cheaper point condition holds for every consumer in the world economy, preference maximization condition with domestic income transfers holds. Since $\tau_{\mathrm{ik}}(\pi^D) < \pi^D \cdot \omega_{\mathrm{ik}}$ may occur for some consumer at a quasi-equilibrium price $\pi^D (\in \mathbf{ba})$ in the dynamic world economy with domestic income transfers, the cheaper point condition obtained with income $\pi^D \cdot \omega_{\mathrm{ik}}$ does not yield the cheaper point condition with $\tau_{\mathrm{ik}}(\pi^D)$ as hid income level. Since $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ is in the budget set with $\tau_{\mathrm{ik}}(\pi^D)$ from the profit maximization of the production side of the world economy, the role of ω_{ik} played in the following standard irreducibility condition in the laissez-faire world free trade economy is replaced by $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ as in the case of the traditional world free trade economy with finitely many commodities considered in Kubota(1997a).

Note that since the disposable income $\tau_{\mathrm{ik}}(\mathbf{p})$ is equal to $\mathbf{p}\cdot[\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}+\theta_{\mathrm{ik}}(\omega_{\mathrm{k}}-\sum_{\mathrm{h}\in \mathrm{I_k}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})]$, although $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{k}} - \sum_{\mathrm{h}\in \mathrm{I_k}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ may be regarded as his new initial endowment, $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{k}} - \sum_{\mathrm{h}\in \mathrm{I_k}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ may not be in $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$. Thus, $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{k}} - \sum_{\mathrm{h}\in \mathrm{I_k}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ may not be regarded as his new initial endowment. Since the existence of a competitive equilibrium under laissez-faire world free trade uses that the initial endowment of a consumer is in his consumption set, we can not treat this case as a special case of a laissez-faire world free trade economy with $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}} \sum_{\mathrm{h}\in \mathrm{I_k}}(\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ regarded as initial endowments. We can not directly apply the theorem on the existence of competitive equilibrium in the dynamic laissez-faire world free trade economy to prove the theorem on the existence of competitive equilibrium in the dynamic world free trade economy with domestic income transfers considered here. We need to prove the existence of competitive equilibrium under world free trade with Grandmont–McFadden's(1972) domestic income transfers independently. In this paper, we follows Bewley's(1972) method of approximating the original economy with a family

production set from.

[16]Trom this welfare property, the weak law of comparative advantage in the sence of Deardoroff(1980) and Dixit-Norman(1980) holds even when each country in the world economy has many heterogeneous consumers.

[17]In the case where $\mathbf{s}^n$ is the underlying commodity space of dynamic world economy over an infinite horizon, there is a slight difficulty in well-defining Grandmont-McFadden's domestic income transfer policy. When $\mathbf{p} \cdot (\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} - \omega_{\mathrm{ik}}) + \theta_{\mathrm{ik}} \mathbf{p} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ is well-defined and finite for $\mathbf{p} \in \mathbf{s}$,, this amount is transferred to him as income transfer. Again $\theta_{\mathrm{ik}} (\geq 0)$ is his given share of $\mathbf{p} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$, and $\sum_{\mathrm{h}\in \mathrm{I_k}} \theta_{\mathrm{hk}} = 1$ holds. Note that $\mathbf{p} \cdot (\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} - \omega_{\mathrm{ik}}) + \theta_{\mathrm{ik}} \mathbf{p} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ may not be well-defined for some $\mathbf{p} \in \mathbf{s} \backslash_+ \{\mathbf{0}\}$. But since the set of feasible allocations is uniformly bounded and allocation $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}})_{(\mathrm{i,k})\in \mathrm{I_k}}$ is also feasible in the world economy. Thus, ω_{ik} and $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ are indeed in l_∞. Moreover, we can also prove that a competitive equilibrium price $\mathbf{p}^{\mathrm{T}}$ exists in l_1^+ in the world free trade economy with Grandmont-MvFadden's domestic income transfers. Thus, at this world competitive equilibrium, $\mathbf{p}^{\mathrm{T}} \cdot (\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} - \omega_{\mathrm{ik}}) + \theta_{\mathrm{ik}} \mathbf{p}^{\mathrm{T}} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$ becomes well-defined and finite. Since in this situation, $\sum_{\mathrm{i}\in \mathrm{I_k}} [\mathbf{p}^{\mathrm{T}} \cdot (\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} - \omega_{\mathrm{ik}}) + \theta_{\mathrm{ik}} \mathbf{p}^{\mathrm{T}} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})]$ is 0, these domestic income transfers are feasible in the k-th country. Note that disposal income $\tau_{\mathrm{ik}}(\mathbf{p}^{\mathrm{T}})$ of i-th consumer is equal to $\mathbf{p}^{\mathrm{T}} \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}} \mathbf{p}^{\mathrm{T}} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}})$. Since $\sum_{\mathrm{h}\in \mathrm{I_k}} (\mathbf{x}_{\mathrm{hk}}^{\mathrm{A}} - \omega_{\mathrm{hk}}) \in \mathbf{Y}_{\mathrm{k}}^{\mathrm{A}} \subset \mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ holds from the feasibility of the allocation $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}})_{\mathrm{i}\in \mathrm{I_k}}$ under autarky, $\mathbf{p}^{\mathrm{T}} \in l_1$ and $\sum_{\mathrm{h}\in \mathrm{I_k}} (\mathbf{x}_{\mathrm{hk}} - \omega_{\mathrm{hk}}) \in l_\infty$ imply $\mathbf{p}^{\mathrm{T}} \cdot \sum_{\mathrm{h}\in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}}) \geq 0$. Thus $\tau_{\mathrm{ik}}(\mathbf{p}^{\mathrm{T}}) \geq \mathbf{p}^{\mathrm{T}} \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ follows at a competitive equilibrium under world free trade with domestic income transfers. Then the demand condition of consumers implies that his demand point with income equal to $\tau_{\mathrm{ik}}(\mathbf{p})$ becomes as desirable as $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ for him. Therefore, in world free trade with Grandmont–McFadden's(1972) domestic income transfers, every consumer in the world economy is *actually* as well off than under autarky.

of finite dimensional subeconomies.[18]

We define a *competitive equilibrium with domestic income transfers under world free trade* as a vector $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D_{\mathrm{k}}, \mathbf{p}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ with $\mathbf{p}^D \in \mathbf{ba}\backslash\{\mathbf{0}\}$ satisfying

(*i*) $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D_{\mathrm{k}})_{(\mathrm{i,k})\in \mathrm{I}}$ is a feasible allocation under world free trade.

(*ii*) $\mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{ik}} \leq \tau_{\mathrm{ik}}(\mathbf{p}^D)$ holds, and $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}^D_{\mathrm{ik}})$ implies $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D)$.

(*iii*) $\mathbf{p}^D \cdot \mathbf{y}^D_{\mathrm{k}} = 0$ holds, and $\mathbf{y}_{\mathrm{k}} \in \mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ implies $\mathbf{p}^D \cdot \mathbf{y}_{\mathrm{k}} \leq 0$.

When only (*ii*) is replaced with following (*ii*)′, then we call $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D_{\mathrm{k}}, \mathbf{p}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ a *quasi-equilibrium with domestic income transfers under world free trade.*

(*ii*)′ $\mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{ik}} \leq \tau_{\mathrm{ik}}(\mathbf{p}^D)$ holds, and $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}^D_{\mathrm{ik}})$ implies $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} \geq \tau_{\mathrm{ik}}(\mathbf{p}^D)$.

Note that when $\mathbf{y}^D$ is $\sum_{\mathrm{k}\in \mathrm{J}} \mathbf{y}^D_{\mathrm{k}}$, (*iii*) is equivalent to following (*iii*)′:

(*iii*)′ $\mathbf{p}^D \cdot \mathbf{y}^D = 0$ holds, and $\mathbf{y} \in \mathbf{Y}^{\mathrm{T}}$ implies $\mathbf{p}^D \cdot \mathbf{y} \leq 0$.

3 Assumptions of Dynamic World Free Trade Economy with Grandmont–McFadden(1972)'s Domestic Income Transfers

We make the following assumptions to get the existence of competitive equilibrium under dynamic world free trade economy with Grandmont–McFadden's domestic income transfers. We use the world aggregate production set $\mathbf{Y}^{\mathrm{T}} \equiv \sum_{\mathrm{k}\in \mathrm{J}} \mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ as a primitive concept.

(T–1) $\mathbf{Y}^{\mathrm{T}}$ is a non-empty convex cone with vertex at $\mathbf{0}$. $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ contains $\mathbf{0}$.

(T–2) $\mathbf{Y}^{\mathrm{T}}$ is closed with respect to the weak* $\sigma(l_\infty, l_1)$ topology.[19]

(T–3) For $\mathbf{y}, \mathbf{y}' \in \mathbf{Y}^{\mathrm{T}}$, there are T and M such that for $t > T$ there is $\overline{\mathbf{y}^t} \in \{\mathbf{z} \in \mathbf{c}_0 : ||\mathbf{z}||_\infty \leq M\}$ satisfying $\widetilde{\mathbf{y}^t} = \mathbf{y}(t) + \widehat{\mathbf{y}'}(t) + \widehat{\overline{\mathbf{y}^t}}(t) = (\mathbf{y}_1, \mathbf{y}_2, \cdot\cdot, \mathbf{y}_t, \mathbf{y}'_{t+1} + \overline{\mathbf{y}^t}_{t+1}, \cdot\cdot) \in \mathbf{Y}^{\mathrm{T}}$.

(T–4) $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ is a non-empty convex subset of $\mathbf{s}^+$ and closed with respect to the weak* $\sigma(l_\infty, l_1)$ topology. ω_{ik} is in $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$.

(T–5) $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$, there are T and M such that for $t > T$, there is $\overline{\mathbf{x}^t}_{\mathrm{ik}} \in \{\mathbf{z} \in \mathbf{c}^+_0 : ||\mathbf{z}||_\infty \leq M\}$ satisfying $\widetilde{\mathbf{x}^t}_{\mathrm{ik}} = \mathbf{x}_{\mathrm{ik}}(t) + \widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t) = (\mathbf{x}_{\mathrm{ik}_1}, \cdot\cdot, \overline{\mathbf{x}^t}_{\mathrm{ik}_{t+1}}, \cdot\cdot) \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$.

(T–6) For $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$, $\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$ and $\mathbf{P}^{-1}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$ are open in $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ with respect to the weak* $\sigma(l_\infty, l_1)$ topology, and $\mathbf{x}_{\mathrm{ik}} \notin co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})]$ holds.

(T–7) $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ and $\mathbf{x}'_{\mathrm{ik}} > \mathbf{x}_{\mathrm{ik}} \Rightarrow \mathbf{x}'_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$.[20]

(T–8) $(\mathbf{Y}^{\mathrm{T}} + \omega) \cap l^+_\infty$ is $||\cdot||_\infty$-bounded, where ω is the world aggregate initial endowment $\sum_{\mathrm{k}\in \mathrm{J}} \omega_{\mathrm{k}}$.

(T–9) $(\mathbf{Y}^{\mathrm{T}} + \omega - \mathbf{C}^{\mathrm{T}}) \cap int_{||\cdot||_\infty}(l^+_\infty)(= \{\mathbf{z} \in l^+_\infty : \mathbf{z} \geq r\mathbf{e}\ \exists\, r > 0\}) \neq \emptyset$, where $\mathbf{C}^{\mathrm{T}}$ is $\sum_{(\mathrm{i,k})\in \mathrm{I}} \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$.

[18] The existence of competitive equilibrium has not proved yet in economies with infinite dimensional commodity space, even although existence of competitive equilibrium is already in economies with finite dimensional commodity space as in Moore(1975) and McKenzie(1981). A simple argument using Bewley(1972)'s approximation method turns out not to work. Thus, since this issue is quite interesing, it is left for a further research topic.

[19] See Appendix A:

[20] Here $>$ is resticted over the goods available in the k-th country.

(T–10) Whenever $\{\mathrm{I}^1,\mathrm{I}^2\}$ is a non-trivial partition of I and $(\mathbf{x}_{\mathrm{ik}},\mathbf{y}_{\mathrm{k}})_{(\mathrm{i,k})\in\mathrm{I}}$ is a feasible allocation under world free trade, there are $(\widetilde{\mathbf{x}}_{\mathrm{ik}})_{(\mathrm{i,k})\in\mathrm{I}^1}$, $\widetilde{\mathbf{y}}_{\mathrm{k}}\in\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$, and $\widetilde{\mathbf{z}}\in\sum_{(\mathrm{h,j})\in\mathrm{I}^2}\mathbf{C}^{\mathrm{T}}_{\mathrm{hj}}$ satisfying

$$(\mathbf{x}_{\mathrm{ik}}+\widetilde{\mathbf{x}}_{\mathrm{ik}})\in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})] \text{ for (i,k)} \in \mathrm{I}^1 \text{ and}$$
$$\sum_{(\mathrm{i,k})\in\mathrm{I}^1}\widetilde{\mathbf{x}}_{\mathrm{ik}}=\sum_{\mathrm{k}\in\mathrm{J}}(\widetilde{\mathbf{y}}_{\mathrm{k}}-\mathbf{y}_{\mathrm{k}})+\alpha\sum_{(\mathrm{h,j})\in\mathrm{I}^2}(\mathbf{x}^{\mathrm{A}}_{\mathrm{hj}}-\widetilde{\mathbf{z}}_{\mathrm{hj}}) \text{ for some } \alpha>0.$$

We do not impose convexity and closedness to each $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ in (T–1). We only impose $\mathbf{0}\in\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$. Since $\mathbf{Y}^{\mathrm{T}}$ is assumed to be a cone and $\mathbf{0}$ is in production set $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ of each country, the amount of profit from $\mathbf{y}^{\mathrm{T}}_{\mathrm{k}}$ is equal to 0. Indeed $\mathbf{Y}^{\mathrm{T}}$ is a cone and satisfies $\mathbf{0}\in\mathbf{Y}^{\mathrm{T}}$ in (T–1), aggregate profit, $\mathbf{p}^{\mathrm{T}}\cdot\mathbf{y}^{\mathrm{T}}$, is zero at a competitive equilibrium under dynamic world free trade. Then $\mathbf{0}\in\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ implies profit $\mathbf{p}^{\mathrm{T}}\cdot\mathbf{y}^{\mathrm{T}}_{\mathrm{k}}$ accruing from $\mathbf{y}^{\mathrm{T}}_{\mathrm{k}}$ becomes 0 as well. The source of income of a consumer in the world economy is only his initial endowment. The position of ownerships of firms is irrelevant to decisions of consumers in the dynamic world economy. Thus, in this situation, consumers are indifferent between they have ownerships of foreign firms or not. If each $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ satisfies the similar exclusion assumption, then $\mathbf{Y}^{\mathrm{T}}$ also satisfies (T–3). Note that $\mathbf{0}\in\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ implies $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}\subset\mathbf{Y}^{\mathrm{T}}$, but (T–3) does not necessarily imply the exclusion assumption to each $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$. Note that (T-2) and (T-4) imply that $\mathbf{Y}^{\mathrm{T}}$ and $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ are Mackey $\tau(l_\infty,l_1)$-closed.

In the discrete-time infinite horizon case of l_∞, we can characterize weak* $\sigma(l_\infty,l_1)$-upper semicontinuity of preferences, i.e., weak* $\sigma(l_\infty,l_1)$-open lower section of preference and weak* $\sigma(l_\infty,l_1)$-lower semicontinuity of preferences, i.e., weak* $\sigma(l_\infty,l_1)$-open valuedness of preference in (T–6) as follows. Let $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ be l^+_∞ for simplicity. Suppose $\mathbf{x}'\in\mathbf{P}_{\mathrm{ik}}(\mathbf{x})$ occurs. This occurs for $\mathbf{x}'=\mathbf{x}+\mathbf{e}_1$, for example, from monotonicity of preferences in (T-5) and $\mathbf{x}+\mathbf{e}_1>\mathbf{x}$.[21] Since $\widehat{\mathbf{e}}(t)=(0,\cdots,0,1,1,\cdots)$ $\to 0$ in the weak* $\sigma(l_\infty,l_1)$-topology for $\mathbf{e}=(1,1,\cdots)$, weak* $\sigma(l_\infty,l_1)$-upper semicontinuity of preferences implies $\mathbf{x}+\mathbf{e}_1\in\mathbf{P}_{\mathrm{ik}}(\mathbf{x}+\widehat{\mathbf{e}}(t))$ for t, sufficiently large. Thus, an additional gain in the far future occurred by the constant rise over far future consumption level has only a very small effect. Similarly, weak* $\sigma(l_\infty,l_1)$-lower semicontinuity of preferences implies $(\mathbf{x}+\mathbf{e}_1)-\widehat{\mathbf{e}}(t)\in\mathbf{P}_{\mathrm{ik}}(\mathbf{x})$ for t, sufficiently large. Thus, an additional loss in the far future occurred by the constant fall over far future consumption has a very large effect. [22] These property exhibits a from of myopia or impatience. when preference $\mathbf{P}_{\mathrm{ik}}$ is, however, $||\cdot||_\infty$-open valued, $\mathbf{x}'\in\mathbf{P}_{\mathrm{ik}}(\mathbf{x})$ implies that $\mathbf{z}\in\mathbf{P}_{\mathrm{ik}}(\mathbf{x})$ as long as $\mathbf{z}$ is uniformly close to $\mathbf{x}'$. In this case, consumers may not discount the future and treats the future as equally important as the present, and the future is not discounted.

We can interpret the paradoxical case with a larger current consumption but a smaller steady state consumption level under world free trade from this viewpoint of weak* $\sigma(l_\infty,l_1)$-upper semicontinuity of preferences.[23] We show later $\mathbf{x}^F\in\mathbf{P}_{\mathrm{ik}}(\mathbf{x}^A)$, where $\mathbf{x}^F$ and $\mathbf{x}^A$ are consumption under free trade and under autarky, respectively. Since this form of continuity of preference implies that a gain in the near future can not overcome by a loss in the sufficiently far future, when the reduction of the steady state consumption level under free trade from the one under autarky occurs in the far future, the consumption path under free trade is still preferable to the one under autarky.

[21] Here e_1 is $(1,0,\cdots)\in l_\infty$, so only first coordinate of e_1 is 0 and all other coordinates after second coordinate of e_1 is 0.

[22] See Appendix A:

[23] Note that in this situation, this country must be forced still to continue to engage in free trade policy as *precommitment* even after the period when the consumption level of this country under free trade becomes smaller than under autarky. Every country agrees to all future contacts and carries out them sequentially. Since, however, this country can consume more than under free trade, by abandoning the free trade policy and cancelling the trade with other countries after its consumption level becomes smaller under world free trade than under autarky, it has a strong incentive to move back to the state of autarky after the free trade consumption level of the country becomes smaller than its autarky consumption level. Unless this country is a small open economy, all other countries faces the default risk of this country that makes them unable to enjoy their free trade consumptions. Thus, in order for the world economy to share the dynamic gains from trade, it is necessary for every country to follow the precommitment that forces it to continue to engage in free trade once it joins into world free trade. Since there is no international authority such as the domestic authrity in each country, the mutual agreement of this precommitment is crucial to sustain the free trade system in world economy over an infinite horizon.

Thus, when consumers discounts the future and the transition periods are explicitly taken into account, then there still exists dynamic gains from trade, despite that the paradoxical phenomenon is occurred under free trade where the steady state consumption level (or generally the consumption level in the sufficiently far future) is smaller under free trade than under autarky.[24] This corresponds exactly to the belief of the neoclassical trade theorists.[25]

An implication of weak* $\sigma(l_\infty, l_1)$-upper semicontinuity of preferences to dynamic gains from trade is following. The consumption under world free trade falls for a while and then rises above the corresponding consumption under autarky. If the current consumption fall continues for sufficiently many periods, then this pattern of the consumption variation under world free trade can not occur even if the future consumption rise is large enough. For, otherwise, the consumption path under world free trade becomes inferior to the consumption path under autarky. Thus, weak* $\sigma(l_\infty, l_1)$-upper semi-continuity of the preference implies that losses in near future can not be compensated by a gain in the sufficiently far future, if these loss continue up to a sufficiently far period. Even if there is a large constant gain in each period in the far future, the gains in the far future can not overwhelm the current losses. Note that when the current consumption fall under free trade ends soon and then the free trade consumption path stay above the autarky consumption path, then this pattern may be consistent with dynamic gains from trade.

From this argument, continuity of preferences, particularly, weak* $\sigma(l_\infty, l_1)$-lower semicontinuity of preferences, expressed as impatient behavior are important in the existence of dynamic gains from trade. Moreover, as seen in the next section, weak* $\sigma(l_\infty, l_1)$-lower semi-continuity of preferences has a further important implication in finding an equilibrium price in l_1 since once the future is not discounted, for example, when preferences are $||\cdot||_\infty$-norm continuous, then the existence of a competitive equilibrium may not guaranteed.[26]

In the first step of the following proof, we show that a quasi-equilibrium price exists in *ba* in the dynamic world economy with domestic income transfers. For this purpose, we appeal the finite dimensional approximation method owing to Bewley(1972). When $\mathbf{Y}^{\mathrm{T}}$ is weak* $\sigma(l_\infty, l_1)$-closed, the existence of a quasi-equilibrium in finite dimensional spaces can apply to each finite dimensional subeconomy without any changes. When, however, only each $\mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ is weak* $\sigma(l_\infty, l_1)$-closed, then the weak* $\sigma(l_\infty, l_1)$-closedness of $\mathbf{Y}^{\mathrm{T}}$ does not necessarily follow. Although the weak* $\sigma(l_\infty, l_1)$-compactness of $\widehat{\mathbf{Y}^{\mathrm{T}}}(\equiv \mathbf{Y}^{\mathrm{T}} \cap \sum_{(\mathrm{i,k})\in \mathrm{I}} \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}})$ also follows from the uniform boundedness condition (T–8) in this case, it is not enough itself. weak* $\sigma(l_\infty, l_1)$-compactness of $\widehat{\mathbf{Y}^{\mathrm{T}}}$ yields compactness of the set of feasible world outputs in finite dimensional subeconomies. In the finite dimensional subeconomies, the compactness of the set of feasible world outputs is not enough for the existence of a quasi-equilibrium with domestic income transfers. When, however, the irreversibility assumption holds for the aggregate world production set in each finite dimensional subeconomy, the existence of quasi-equilibrium with domestic income transfers follows in finite dimensional subeconomies . Then, we can show that the limit of the sequence of such quasi-equilibria is a quasi-equilibrium

[24]Note that in this situation, this country must be forced still to continue to engage in free trade policy as *precommitment* even after the period when the consumption level of this country under free trade becomes smaller than under autarky. Every country agrees to all future contacts and carries out them sequentially. Since, however, this country can consume more than under free trade, by abandoning the free trade policy and cancelling the trade with other countries after its consumption level becomes smaller under world free trade than under autarky, it has a strong incentive to move back to the state of autarky after the free trade consumption level of the country becomes smaller than its autarky consumption level. Unless this country is a small open economy, all other countries faces the default risk of this country that makes them unable to enjoy their free trade consumptions. Thus, in order for the world economy to share the dynamic gains from trade, it is necessary for every country to follow the precommitment that forces it to continue to engage in free trade once it joins into world free trade. Since there is no international authority such as the domestic authrity in each country, the mutual agreement of this precommitment is crucial to sustain the free trade system in world economy over an infinite horizon.

[25]This argument is analogous to the relation between the modified golden rule and the golden rule in the one sector optimal growth model of a closed economy.

[26]For this point, see Alaujo(1985). Also note that the typical time-additively separable form of intertemporal welfare criteria in discrete-time infinite horizon model, which is often used in the literature on this argument, is weak $\sigma(l_\infty, l_1)$-continuous. Bewley(1972, Appendix II, p. 535) shows that in a continuous-time model, time additive preferences may be only Mackey continious and not weak*-continuous.

with domestic income transfers in the original world free trade economy. Thus in this case, we add the irreversibility assumption, $\mathbf{Y}^{\mathrm{T}} \cap (-\mathbf{Y}^{\mathrm{T}}) = \{\mathbf{0}\}$. Note that $\mathbf{Y}^{\mathrm{T}} \cap (-\mathbf{Y}^{\mathrm{T}}) = \{\mathbf{0}\}$ rule out that $\mathbf{Y}^{\mathrm{T}}$ has non-trivial straight lines in it.[27] Thus when we use $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ as a primitive concept, then we replace (T–1) and (T–2) with following (T–1)′ and (T–2)′.

(T–1)′ $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ is a non-empty convex cone with vertex at $\mathbf{0}$.

(T–2)′ $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ is closed with respect to the weak* $\sigma(l_\infty, l_1)$-topology. Also $\mathbf{Y}^{\mathrm{T}} \cap (-\mathbf{Y}^{\mathrm{T}}) = \{\mathbf{0}\}$ holds.

Similarly we use the following version of the uniform boundedness condition for this case. It indeed gives rise to the uniform boundedness condition (T–8) and weak* $\sigma(l_\infty, l_1)$-compactness of $\widehat{\mathbf{Y}^{\mathrm{T}\prime}}$ owing to (T–2)′.

(T–8)′ For $\mathrm{k} \in \mathrm{J}$, $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}} \cap [l^+_\infty - \sum_{\mathrm{i}\neq\mathrm{k}} \mathbf{Y}^{\mathrm{T}}_{\mathrm{j}} - \omega]$ is $||\cdot||_\infty$-bounded.

(T–8)′ implies (T–8). Suppose $\mathbf{y} + \omega = \mathbf{z} \geq \mathbf{0}$ holds for some $\mathbf{y} = \sum_{\mathrm{k}\in\mathrm{J}} \mathbf{y}_{\mathrm{k}} \in \sum_{\mathrm{k}\in\mathrm{J}} \mathbf{Y}^{\mathrm{T}}_{\mathrm{k}} = \mathbf{Y}^{\mathrm{T}}$. For $\mathrm{k} \in \mathrm{J}$, $\mathbf{y}_{\mathrm{k}} = \mathbf{z} - \sum_{\mathrm{j}\neq\mathrm{k}} \mathbf{y}_{\mathrm{j}} - \omega$ holds. (T–8)′ then implies that there is some $\beta > 0$, such $\mathbf{y}_{\mathrm{k}}$ satisfies $||\mathbf{y}_{\mathrm{k}}||_\infty \leq \beta$ for $\mathrm{k} \in \mathrm{J}$. Then $||\mathbf{y} + \omega||_\infty \leq ||\mathbf{y}||_\infty + ||\omega||_\infty \leq \sum_{\mathrm{k}\in\mathrm{J}} ||\mathbf{y}_{\mathrm{k}}||_\infty + ||\omega||_\infty \leq \mathrm{K}\alpha + ||\omega||_\infty = M < +\infty$ holds. Thus, (T–8) follows. Also we can show the weak* $\sigma(l_\infty, l_1)$-compactness of $\widehat{\mathbf{Y}^{\mathrm{T}}}$.[28]

Also in the second step of the following proof, we show that the l_1-part of a quasi-equilibrium price found in *ba* is a quasi-equilibrium price in the dynamic world free trade economy with domestic income transfers. The proof requires $\tau_{\mathrm{ik}}(\pi) = \tau_{\mathrm{ik}}(\pi_c)$ for consumers in the world economy, where π is a quasi-equilibrium price in *ba* and π_c is its l_1-part.[29]

For this purpose, we make some modifications to some assumptions on consumption sets and production sets, in particular, on the following exclusion assumptions (T–3)′ and (T–5)′ to production set and consumption sets[30]. They are not enough for this purpose. We need to employ the stronger mixture assumption (T–3), which implies the exclusion assumption (T–3)′, to production set. Also since $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{i}} - \sum_{\mathrm{h}\in\mathrm{I}_k} \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}})$ may not be in $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ for some consumer, the condition $\overline{\mathbf{x}^t}_{\mathrm{ik}} \leq \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{k}} - \sum_{\mathrm{h}\in\mathrm{I}_{\mathrm{k}}} \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}})$ as in (T–5)′ is inappropriate. We modify exclusion assumption (T–5)′ for consumption sets as exclusion assumption (T–3)′ for production set. According to the modifications to the exclusion assumptions for production set and consumption sets, we also modify the corresponding exclusion assumptions to production set and consumption sets in the dynamic world free trade economy. The exclusion assumption to production set is following.

(T–3)′ For $\mathbf{y} \in \mathbf{Y}^{\mathrm{T}}$, there are T and M such that for $t > T$ there is $\overline{\mathbf{y}^t} \in \{\mathbf{z} \in \mathbf{c}_0 : ||\mathbf{z}||_\infty \leq M\}$ satisfying $\widetilde{\mathbf{y}^t} = \mathbf{y}(t) + \widehat{\overline{\mathbf{y}^t}}(t) = (\mathbf{y}_1, \cdot\cdot, \mathbf{y}_t, \overline{\mathbf{y}^t}_{t+1}, \cdot\cdot) \in \mathbf{Y}^{\mathrm{T}}$.

[27] No straight lines condition is used in Boyd–McKenzie(1993) with s^n as the underlying commodity space to establish that $\mathbf{G} - \mathbf{Y}$ is closed in the coordinatewise convergence topology by Choquet's theorem, where $\mathbf{G}$ is the convex hull of $\cup_{\mathrm{i}\in\mathrm{I}} \mathbf{R}_{\mathrm{i}}(\mathbf{x}_{\mathrm{i}})$ and closed with respect to the coordinatewise convergence topology.

[28] See, Kubota(1998a, Lemma 2 – 1). It is estabilshed in the more general $\mathbf{L}_\infty$ case.

[29] An element ν in $\mathbf{ba}^+$ is called purely finitely addtive if $\lambda = \mathbf{0}$ holds for any $\lambda \in \mathbf{ca}$ with $\mathbf{0} \leq \lambda \leq \nu$. This theorem says that any element π in $\mathbf{ba}^+$ can be decomposed uniquely into the countably additive part $\pi_c \in \mathbf{ba}^+$ and the purely finitely additeve part $\pi_p \in \mathbf{ba}^+$ (Bhaskara Rao–Bhaskara Rao(1983, Theorem 10.2.1, p. 241), Yosida–Hewitt(1952, Theorem 1.23, p. 52), or Dunford–Schwartz(1958, III 7.10, p. 163)). The Radon–Nikodym theorem implies that this countably additive part is indeed as an integrable function in this case. Moreover the Banach limit mentioned before is an example of a purely finitely additive measure in the case of l_∞.

[30] These are used in Kubota(1998)

From the definition of (T–3)$'$, $\widetilde{\mathbf{y}^t} = \mathbf{y}(t) + \widehat{\mathbf{y}'}(t) + \widehat{\overline{\mathbf{y}}^t}(t)$ is in l_∞.[31] Since, a substitution of $\mathbf{0}$ for $\mathbf{y}'$ in (T–3) implies the exclusion assumption (T–3)$'$, (T–3) is stronger than (T–3)$'$.[32] (T–3) says that the tail of a uniformly bounded production plan $\mathbf{y}'$ can approximate for the tail of any production plan $\mathbf{y}$ in $\mathbf{Y}$. Since $\overline{\mathbf{y}}$ is not necessarily $\mathbf{0}$, (T–3)$'$ does not implies $\mathbf{y}(t) + \widehat{\mathbf{y}'}(t) \in \mathbf{Y}^{\mathrm{T}}$. We allow an adjustment for switching the tail of $\mathbf{y}$ with that of $\mathbf{y}'$, and $(\overline{\mathbf{y}^t})_{t \geq T}$ expresses this adjustment. Note that in the dynamic world free trade economy, (T–3) implies following (G–3):

(G–3) For $\mathbf{y}, \mathbf{y}' \in \mathbf{Y}^{\mathrm{D}}$ and $\pi \in \mathbf{ba}^+$, there is $\{\mathbf{y}^n\}_{n=1}^\infty \subset \mathbf{Y}^{\mathrm{D}}$ such that $\mathbf{y}^n \to \mathbf{y}$ in the weak* $\sigma(l_\infty, l_1)$ topology and $\pi_p \cdot \mathbf{y}^n \to \pi_p \cdot \mathbf{y}'$ hold as $n \to \infty$, where π_p is the purely finitely additive part of π in the Yosida–Hewitt decomposition.

Since $\widehat{\mathbf{y}'}(t) + \widehat{\overline{\mathbf{y}^t}}(t) = \widehat{(\mathbf{y}' + \overline{\mathbf{y}^t})}(t) \to \mathbf{0}$ in the Mackey $\tau(l_\infty, l_1)$ topology, and hence, in the weak* $\sigma(l_\infty, l_1)$ topology, $\mathbf{y}(t) + \widehat{\mathbf{y}'}(t) + \widehat{\overline{\mathbf{y}^t}}(t) (\in \mathbf{Y}^{\mathrm{T}}) \to \mathbf{y}$ holds as $n \to \infty$. Also the property of purely finitely additive measures implies $\pi_p \cdot \mathbf{y}(t) = 0$ for $t \geq 1$ since $\mathbf{y}(t)$ has only a finite number of non-zero coordinates. Moreover, $\overline{\mathbf{y}^t} \in \mathbf{c}_0$ implies $\pi_p \cdot \widehat{\overline{\mathbf{y}^t}}(t) = 0$ for $t \geq 1$. Thus, $\pi_p \cdot (\mathbf{y}(t) + \widehat{\mathbf{y}'}(t) + \widehat{\overline{\mathbf{y}^t}}(t)) = \pi_p \cdot \widehat{\mathbf{y}'}(t) = \pi_p \cdot \mathbf{y}'$ holds for $t \geq T$. Thus, (G–3) holds.

The standard exclusion assumption to consumption sets is following.

(T–5)$'$ For $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$, there is T such that for $t > T$ there is $\overline{\mathbf{x}^t}_{\mathrm{ik}} \in \{\mathbf{z} \in \mathbf{s}^+ : \mathbf{z} \leq \omega_{\mathrm{ik}}\}$ satisfying $\widetilde{\mathbf{x}^t}_{\mathrm{ik}} = \mathbf{x}_{\mathrm{ik}}(t) + \widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t) = (\mathbf{x}_{\mathrm{ik}_1}, \cdot \cdot, \mathbf{x}_{\mathrm{ik}_t}, \overline{\mathbf{x}^t}_{\mathrm{ik}t+1}, \cdot \cdot) \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$.

Since $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} + \theta_{\mathrm{ik}} \sum_{\mathrm{h} \in \mathrm{I_k}} (\omega_{\mathrm{hk}} - \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}})$ is not necessarily in $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ for some consumer, $\overline{\mathbf{x}^t}_{\mathrm{ik}}(t) \leq \omega_{\mathrm{ik}}(t)$ in the following exclusion assumption(T–5)$'$ does not necessarily yield $\overline{\mathbf{x}^t}_{\mathrm{ik}}(t) \leq [\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{k}} - \sum_{\mathrm{h} \in \mathrm{I_k}} \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}})](t)$.[33] Thus we replace a constraint $\overline{\mathbf{x}^t}_{\mathrm{ik}}(t) \leq \omega_{\mathrm{ik}}(t)$ in the standard exclusion (T-5)$'$ to $\overline{\mathbf{x}^t}_{\mathrm{ik}}$ in (T–5). We, however, still assume that $\overline{\mathbf{x}^t}_{\mathrm{ik}}$ is in $\mathbf{c}_0^+$The constraint in (T-5) is like the one to $\overline{\mathbf{y}^t}$ in exclusion assumption (T–3)$'$. Consumers can survive even the tail of consumption plan $\mathbf{x}_{\mathrm{ik}}$ is decreased sufficiently small in far future. Let $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}_t}$ and $\mathbf{b}_{\mathrm{ik}_t} (\geq \mathbf{0})$ be the projection of $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ over the coordinates in period t and the lower bound of $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}_t}$. Then (T–5) implies $\mathbf{b}_{\mathrm{ik}_t} \to 0$ as $t \to \infty$. Note that $\mathbf{b}_{\mathrm{ik}_t}$ does not necessarily belong to $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}_t}$. If a consumption set is the positive orthant l^+_∞, then (T–5) holds. Prescott–Lucas(1972) use $\mathbf{0}$ for $\overline{\mathbf{x}^t}_{\mathrm{ik}}$ in an economy in l_∞. (T–5) is, in particular, important in establishing the l_1-part of a quasi-equilibrium price found in *ba* in the dynamic world free trade economy with domestic income transfers is a quasi-equilibrium price. Note that (T–5) implies following (G–5):

(G–5) For $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{D}}_{\mathrm{ik}}$ and $\pi \in \mathbf{ba}^+$, there is $\{\mathbf{x}^n_{\mathrm{ik}}\}_{n=1}^\infty \subset \mathbf{C}^{\mathrm{D}}_{\mathrm{ik}}$ such that $\mathbf{x}^n_{\mathrm{ik}} \to \mathbf{x}_{\mathrm{ik}}$ in the weak* $\sigma(l_\infty, l_1)$ topology and $\pi_p \cdot \mathbf{x}^n_{\mathrm{ik}} \to 0$ hold as $n \to \infty$.

Since $\widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t) \to 0$ holds in the Mackey $\tau(l_\infty, l_1)$ topology, and hence, in the weak* $\sigma(l_\infty, l_1)$ topology as $t \to \infty$, $\widetilde{\mathbf{x}^t}_{\mathrm{ik}} = \mathbf{x}_{\mathrm{ik}}(t) + \widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t) = (\mathbf{x}_{\mathrm{ik}_1}, \cdot \cdot, \overline{\mathbf{x}^t}_{\mathrm{ik}_{t+1}}, \cdot \cdot) \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}} \to \mathbf{x}_{\mathrm{ik}}$ as $t \to \infty$ holds as well. Also the property of purely finitely additive measures implies $\pi_p \cdot \mathbf{x}_{\mathrm{ik}}(t) = 0$ for $t \geq 1$ since $\mathbf{x}_{\mathrm{ik}}(t)$ has only a finite number of non-zero coordinates. Moreover, $\overline{\mathbf{x}^t}_{\mathrm{ik}} \in \{\mathbf{z} \in \mathbf{c}_0^+ : ||\mathbf{z}||_\infty \leq M\}$ implies $\pi_p \cdot \widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t) = 0$ for $t \geq 1$. Thus, $\pi_p \cdot \widetilde{\mathbf{x}^t}_{\mathrm{ik}} = \pi_p \cdot (\mathbf{x}_{\mathrm{ik}}(t) + \widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t)) = 0$ holds for $t \geq T$. Thus,

[31] In the Malinvaud model of capital accumulation, the mixture property of production set on l_∞ holds as far as $\{\mathbf{v}_t\}_{t=1}^\infty$ is restricted in l^+_∞. Choose $\mathbf{y}$ and $\mathbf{y}'$ from $\mathbf{Y} \cap l_\infty$. Then $\mathbf{y}^{(\prime)}_t$ is expressed as $u^{(\prime)}_t + v^{(\prime)}_t$ for $(u^{(\prime)}_t, v^{(\prime)}_{t+1}) \in \mathbf{Y}(t)$ for $t \geq 1$. Let M be $\max\{||v||_\infty, ||v'||_\infty\}$. Define $\overline{\mathbf{y}^t} = (0, \cdot\cdot, 0, v'_t - v_t, 0, \cdot\cdot)$ for $t \geq 1$. Then For $t \geq 1$, $\mathbf{y}(t) + \widehat{\mathbf{y}'}(t) + \widehat{\overline{\mathbf{y}^t}}(t) = \mathbf{y}(t) + \widehat{\mathbf{y}'}(t) + \overline{\mathbf{y}^t}$ is in $\mathbf{Y}$ and $||\overline{\mathbf{y}^t}||_\infty \leq M$ holds. Note that $\{\mathbf{v}_t\}_{t=1}^\infty \subset l_\infty$ is necessary to get $||\overline{\mathbf{y}^t}||_\infty \leq M$ besides $(\mathbf{Y} \cap l_\infty)$. Note that Back(1984) also imposes this requirement to show the production set of the Malinvaud model satisfies his mixture conditiont.

[32] This is a special l_∞ case of the mixture assumption considered in the $\mathbf{L}_\infty$ case of Kubota(1998a).

[33] If $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ happens to be equal to ω_{ik} for $\mathrm{i} \in \mathrm{I_k}$, $\omega_{\mathrm{ik}} = \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} + \theta_{\mathrm{ik}}(\omega_{\mathrm{k}} - \sum_{\mathrm{h} \in \mathrm{I_k}} \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}})$ holds as in the laissez-faire world free trade case of Kubota(1998b), and $\overline{\mathbf{x}^t}_{\mathrm{ik}}(t) \leq \omega_{\mathrm{ik}}(t)$ is enough.

(G–5) holds. Although (G-3) and (G-5) are more general than (T-3) and (T-5), this paper employs (T-3) and (T-5) since (T-3) and (T-5) use only production and consumption points but (G-3) and (G-5) use $\pi \in \mathbf{ba}^+$ besides production and consumption points.

Non-traded goods may exist in the dynamic world economy. But (T–9) implies that regardless of the existence of non-traded goods in the dynamic world economy, all goods including non-trade goods can be in excess in the dynamic world economy as a whole.

In the last step of converting this quasi-equilibrium into a competitive equilibrium, once we show that a quasi-equilibrium with prices in l_1 in the dynamic world economy with domestic income transfers, we have to derive preference maximization from expenditure minimization with income $\tau_{\mathrm{ik}}(\mathbf{p})$, where $\mathbf{p}$ is a quasi-equilibrium price found in l_1.

To make this conversion possible, we need to establish that every consumer has a cheaper point than income $\tau_{\mathrm{ik}}(\mathbf{p})$. Since the profit maximization at quasi-equilibrium with $\mathbf{p}$ implies $\mathbf{p} \cdot (\sum_{\mathrm{h}\in\mathrm{I_k}} \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}} - \omega_{\mathrm{k}}) \leq 0$, $\mathbf{p} \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} \leq \mathbf{p} \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} + \theta_{\mathrm{ik}}\mathbf{p} \cdot \sum_{\mathrm{h}\in\mathrm{I_k}}(\omega_{\mathrm{hk}} - \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}}) = \tau_{\mathrm{ik}}(\mathbf{p})$ follows. Thus, when $\mathbf{p} \cdot \sum_{\mathrm{h}\in\mathrm{I_k}}(\omega_{\mathrm{hk}} - \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}}) > 0$ holds, the consumers with $\theta_{\mathrm{ik}} > 0$ in the k-th country have cheaper point $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ than $\tau_{\mathrm{ik}}(\mathbf{p})$.

If, however, $\mathbf{p} \cdot \sum_{\mathrm{h}\in\mathrm{I_k}}(\omega_{\mathrm{hk}} - \mathbf{x}^{\mathrm{A}}_{\mathrm{hk}}) = 0$ holds, in particular, in a pure exchange economy, $\tau_{\mathrm{ik}}(\mathbf{p}) = \mathbf{p} \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ holds, Some consumers may not have cheaper points than $\tau_{\mathrm{ik}}(\mathbf{p})$ in this situation. We need to guarantee that such consumers also have cheaper points with $\mathbf{p}$ than $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ in order to handle this situation. In laissez-faire world free trade case, we employ the following irreducibility assumption (T–10)′ to make consumers have a cheaper point with $\mathbf{p}$ than ω_{ik}. Here we give to $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ the role of ω_{ik} in the irreducibility assumption (T–10)′. Once, $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ takes place the role of ω_{ik} in the modified irreducibility assumption, then, we can show that every consumer has a cheaper point than $\tau_{\mathrm{ik}}(\mathbf{p})$. Then we can translate the quasi-equilibrium with domestic income transfers in the original dynamic world economy into a corresponding competitive equilibrium. This condition means that any group of consumers in the world economy evaluate, in particular, the additional commodity bundles from the autarky consumption bundles of the other group of consumers.[34]

The standard irreducibility assumption is following.

(T–10)′ Let $\{\mathrm{I}^1,\mathrm{I}^2\}$ and $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{(\mathrm{i,k})\in\mathrm{I}}$ be a non-trivial partition of I and a feasible allocation in the world free trade economy. Then there exist $(\widetilde{\mathbf{x}}_{\mathrm{ik}})_{(\mathrm{i,k})\in\mathrm{I}^1}$, $\widetilde{\mathbf{y}} \in \mathbf{Y}^{\mathrm{T}}$, and $\widetilde{\mathbf{z}} \in \sum_{(\mathrm{h,j})\in\mathrm{I}^2} \mathbf{C}^{\mathrm{T}}_{\mathrm{hj}}$ satisfying

$$(\mathbf{x}_{\mathrm{ik}} + \widetilde{\mathbf{x}}_{\mathrm{ik}}) \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})] \text{ for all } (\mathrm{i,k}) \in \mathrm{I}^1 \text{ and}$$
$$\sum_{(\mathrm{i,k})\in\mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}} = \sum_{\mathrm{k}\in\mathrm{J}}(\widetilde{\mathbf{y}}_{\mathrm{k}} - \mathbf{y}_{\mathrm{k}}) + \alpha \sum_{(\mathrm{h,j})\in\mathrm{I}^2} (\omega_{\mathrm{hj}} - \widetilde{\mathbf{z}}_{\mathrm{hj}}) \text{ for some } \alpha > 0.$$

It replaces the role of $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ in modified irreducibility assumption (T–10) with that of ω_{ik}. (T–10), however, is not necessarily obtained from this original irreducibility assumption (T–10)′.

Although $((\mathbf{x}_{\mathrm{ik}} + \widetilde{\mathbf{x}}_{\mathrm{ik}})_{(i,k)\in I^1}, \widetilde{\mathbf{z}}, \widetilde{\mathbf{y}})$ in (T–10) and the one in (T–10)′ are not necesarily feasible allocations, when $((\widetilde{\mathbf{x}}_{\mathrm{ik}})_{(i,k)\in I^1}, \widetilde{\mathbf{z}}, \widetilde{\mathbf{y}})$, corresponding to $((\mathbf{x}_{\mathrm{ik}} + \widetilde{\mathbf{x}}_{\mathrm{ik}})_{(i,k)\in I^1}, \widetilde{\mathbf{z}}, \widetilde{\mathbf{y}})$ in (T–10)′, is feasible in the world economy, it is called following stongly irreducible assumption.[35]

(T–11): Let $\{\mathrm{I}^1,\mathrm{I}^2\}$ and $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{(\mathrm{i,k})\in\mathrm{I}}$ be a non-trivial partition of I and a feasible allocation in the world free trade economy. Then there exist $(\widetilde{\mathbf{x}}_{\mathrm{ik}})_{(i,k)\in I^1} \in \sum_{(i,k)\in I^1} \mathbf{C}^T_{ik}$, $\widetilde{\mathbf{y}} \in \mathbf{Y}^{\mathrm{T}}$, and $\widetilde{\mathbf{z}}$

[34] Grandmont–McFadden(1972) has such an example in a pure exchange economy with two goods. Note also that (T-10) holds when each consumer has a monotonic preference, the positive orthant as his consumption sets, and an initial endowment in the interior of positive orthant, and there are no non-traded goods, positive. A nice set of sufficient condtion for (T-10) is, hiwever, not established yet.

[35] This is introduced in Boyd-McKenzie(1993) to make equal treatment core non-empty in their mode. See also McKenzie(2002, Ch.5).

$\in \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{C}^{\mathrm{T}}_{\mathrm{hj}}$ satisfying

$$\widetilde{\mathbf{x}}_{\mathrm{ik}} \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})] \text{ for all (i,k)} \in \mathrm{I}^1 \text{ and}$$
$$\sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}} = \sum_{\mathrm{k}\in \mathrm{J}} \widetilde{\mathbf{y}}_{\mathrm{k}} - \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \widetilde{\mathbf{z}}_{\mathrm{hj}}.$$

The modified irreducibility condition (T–10) focuses on the usefulness of $(\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}})_{(i,k)\in I}$ from the first, but the stongly irreducible assumption does not so. Of course, when $(\mathbf{x}_{\mathrm{hj}}) \in \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{C}^{\mathrm{T}}_{\mathrm{hj}}$ is at their worst consumption points, it may difficult for this assumption to hold.

4 Existence of Equilibrium in World Free Trade Economy with Domestic Income Transfers

In this section, we prove the existence of competitive equilibrium under free trade with Grandmont-McFadden's domestic income transfers in the dynamic world economy over discrete-time infinite horizon. We first prove the existence of quasi-equilibrium with domestic income transfers under world free trade with prices in *ba*. At this stage, we do not use either the mixture condition (T–3) to the production set and (T–5) to consumption sets. They are necessary later in making the l_1-part of a quasi-equilibrium price found in *ba* a quasi-equilibrium price. We employ the finite dimensional approximation of Bewley(1972). Remember $\mathbf{e}^{\mathrm{k}} = (\mathbf{e}^{\mathrm{k}}_t)_{t=1}^{\infty}$ is defined by $\mathbf{e}^{\mathrm{k}}_{ti} = 1$ i $\in N^T_{\mathrm{k}}$ and $= 0$ otherwise for $t \geq 1$ and $\mathbf{e} = \sum_{\mathrm{k}\in \mathrm{J}} \mathbf{e}^{\mathrm{k}}$ holds. Since there exist non-traded goods in the world economy, these $\mathbf{e}^{\mathrm{k}}$ are used instead of $\mathbf{e}$ in the proof of the existence of quasi-equilibrium with domestic income transfers with prices in *ba* here.

Theorem 1 : *Under* (T–1), (T–2), (T–4), (T–6), (T–7), *and* (T–8), *the dynamic world economy over an discrete-time infinite horizon has a quasi-equilibrium with domestic income transfers with prices in* $ba^+\backslash\{\mathbf{0}\}$. *The same conclusion also holds when* (T–2) *is replaced with* (T–2)$'$.

Proof) See Appendix B.

As in mentioned in the proof, (T–1)$'$, (T–2)$'$, (T–8)$'$, and (T–4) gives rise to the weak* $\sigma(l_\infty, l_1)$ -compactness of $\widehat{\mathbf{Y}^{\mathrm{T}}}$. Thus, when (T–1)$'$,(T–2)$'$ and (T–8)$'$ replace (T–1),(T–2) and (T–8), the world free trade economy still has a quasi-equilibrium with domestic income transfers under world free trade. In the above theorem, we appeal weak* $\sigma(l_\infty, l_1)$-upper semi-continuity of preference even without convexity of preferences. But we do not use weak* $\sigma(l_\infty, l_1)$-lower semi-continuity of preferences in (T–6) in the above proof. In finding a quasi-equilibrium with domestic income transfers with prices in *ba*, weak* $\sigma(l_\infty, l_1)$-upper semi-continuity of preference is sufficient. Weak* $\sigma(l_\infty, l_1)$-lower semi-continuity of preference becomes relevant below in establishing that the l_1-part π_c of quasi-equilibrium price π found above in *ba*. Also in the above proof, we do not use mixture condition (T–3) and exclusion condition (T–5). These also become crucial below in establishing that π_c is a quasi-equilibrium price. Moreover, we need to establish $\pi_c \neq \mathbf{0}$ to make π_c non-trivial. (T–9) comes in for this purpose. (T–9) makes some consumer have a cheaper point with π. Then we can show that such a consumer has a cheaper point with π_c as well. Indeed we prove that π_c is also a quasi-equilibrium price.

We next show that the l_1-part π_c of quasi-equilibrium price π found above in *ba* is also a quasi-equilibrium price. We can prove the following result on this issue. (T-3) and (T-5), the conditions not used in the theorem 1, are used here.

Theorem 2 : *Under* (T–1), $\cdots$, (T–8), *and* (T–9), *the general world economy in* l_∞ *has a quasi-equilibrium with domestic income transfers with prices in* $l_1^+\backslash\{\mathbf{0}\}$. *The same conclusion also holds when* (T–2) *is replaced with* (T–2)$'$.

Proof) See Appendix B.

Since periodwise monotonicity of preference is assumed, $\pi_{ct}^D > \mathbf{0}$ holds for any $t \geq 1$. For above consumer (i′,k′), his expenditure minimization with π_c^D becomes preference maximization since he has a cheaper point than $\tau_{\mathrm{i'k'}}(\pi_c^D)$, as is shown below. Some of other consumers in the world may not satisfy the cheaper point condition. Thus, above quasi-equilibrium with domestic income transfers $(\mathbf{x}_{\mathrm{ik}}^D, \mathbf{y}_{\mathrm{k}}^D, \pi^D)_{(\mathrm{i,k})\in \mathrm{I}}$ is not necessarily a competitive equilibrium with domestic income transfers under world free trade. Also (T–2)′ and (T–8)′ can replace (T–2) and (T–8) in above theorem 2.

We finally prove that a quasi-equilibrium with domestic income transfers in the dynamic world free trade economy is a competitive equilibrium with domestic income transfers in the dynamic world free trade economy over an infinite horizon. The irreducibility condition (T–10) finally enters in translating the quasi-equilibrium with domestic income transfers in the dynamic world free trade economy into a competitive equilibrium under domestic income transfers in the dynamic world economy.

We need following lemma saying that expenditure minimization becomes indeed preference maximization for consumers with cheaper points with income level equal to $\tau_{\mathrm{ik}}(\mathbf{p}^D)$.

Lemma 1 : *Suppose that* $\tau_{\mathrm{ik}}(\mathbf{p}^D) > \inf\{\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} : \mathbf{x}_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}\}$ *holds and* $\mathbf{x}_{\mathrm{ik}} \in \mathbf{R}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ *implies* $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} \geq \tau_{\mathrm{ik}}(\mathbf{p}^D)$. *Then, under* (T–6), $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ *implies* $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D)$.

Proof) Suppose $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} = \tau_{\mathrm{ik}}(\mathbf{p}^D)$ happens for some $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$. By the assumption, there is $\widetilde{\mathbf{x}}_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ with $\mathbf{p}^D \cdot \widetilde{\mathbf{x}}_{\mathrm{ik}} < \mathbf{p}_{\mathrm{ik}}(\pi^D)$. Let $\mathbf{x}(\alpha)$ be the convex combination between $\mathbf{x}_{\mathrm{ik}}$ and $\widetilde{\mathbf{x}}_{\mathrm{ik}}$, $(1-\alpha)\mathbf{x}_{\mathrm{ik}} + \alpha\widetilde{\mathbf{x}}_{\mathrm{ik}}$ for $\alpha \in (0,1)$. By (T–6), $\alpha^n \to 0 \Longrightarrow (1-\alpha)\mathbf{x}_{\mathrm{ik}} + \alpha\widetilde{\mathbf{x}}_{\mathrm{ik}} \to \mathbf{x}_{\mathrm{ik}}$ in the weak* $\sigma(l_\infty, l_1)$ topology, there is some $\widetilde{\alpha}$, close to 0, satisfying $\mathbf{x}(\widetilde{\alpha}) \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$. Then, $\mathbf{p}^D \cdot \mathbf{x}(\widetilde{\alpha}) \geq \tau_{\mathrm{ik}}(\mathbf{p}^D)$ occurs by the hypothesis. But this is a contradiction since $\mathbf{p}^D \cdot \mathbf{x}(\widetilde{\alpha}) = \widetilde{\alpha}\pi^D \cdot \widetilde{\mathbf{x}}_{\mathrm{ik}} + (1-\widetilde{\alpha})\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} < \tau_{\mathrm{ik}}(\mathbf{p}^D)$ holds from the definition.

We now prove a quasi-equilibrium with domestic income transfers in the dynamic world free trade economy is indeed a competitive equilibrium with domestic income transfers in the dynamic world free trade economy owing to (T-9) and (T-10).

Theorem 3 : *Under assumptions* (T–1), (T–2), (T–3), (T–4), (T–5), (T–6), · ·, (T–9), *and* (T–10), *the dynamic world free trade economy over a discrete-time infinite horizon has a competitive equilibrium with domestic income transfers with prices in* l_1. *Moreover, no one in the world economy is worse off under world free trade than under autarky. The same conclusion holds when* (T–1), (T–2), *and* (T–8) *are replaced with* (T–1)′, (T–2)′, *and* (T–8)′.

Proof) See Appendix B.

Theorem 4 : *Under assumptions* (T–1), (T–2), (T–3), (T–4), (T–5), (T–6), · ·, (T–9), *and* (T–11), *the dynamic world free trade economy over a discrete-time infinite horizon has a competitive equilibrium with domestic income transfers with prices in* l_1. *Moreover, no one in the world economy is worse off under world free trade than under autarky. The same conclusion holds when* (T–1), (T–2), *and* (T–8) *are replaced with* (T–1)′, (T–2)′, *and* (T–8)′.

Proof) See Appendix B.

This result is an extensions of Grandmont–McFadden's(1972) result on gains from trade in the traditional world free trade economy with a finite number of commodities to a dynamic world free trade economy over a discrete-time infinite horizon. We show that their conclusion on gains from trade also holds in this dynamic world free trade economy over a discrete-time infinite horizon. Since $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ is affordable with $\tau_{\mathrm{ik}}(\mathbf{p}^D)$ and $\mathbf{x}_{\mathrm{ik}}^D$ is a demand point of consumer (i,k) with $\tau_{\mathrm{ik}}(\mathbf{p}^D)$, $\mathbf{x}_{\mathrm{ik}}^D$ is as desirable as $\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ even without transitivity of preferences. Thus, we can prove that world free trade with Grandmont–McFadden's income transfer policy is actually preferable, generally not worse off, to autarky even without transitivity of preferences. Transitivity of preferences is irrelevant in this result on the existence of gains from trade. Note that since this paper introduces preferences of consumers explicitly, $\mathbf{p}^D \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} \leqq \tau_{\mathrm{ik}}(\mathbf{p}^D) = \mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{ik}}$ holds $\forall$(i,k)$\in$I. Then $\mathbf{p}^D \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{k}} = \mathbf{p}^D \cdot \sum_{\mathrm{i}\in\mathrm{I_k}} \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} \leqq \mathbf{p}^D \cdot \sum_{\mathrm{i}\in\mathrm{I_k}} \mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{ik}} = \mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{k}}$ follows for the aggregate consumptions under free trade and under autarky as well. The latter is obtained as the result on dynamic gains from trade obtained by Smith(1979). The former is, however, not shown in Smith(1979) since it does not introduce preferences of consumers. Thus, this paper extend the result on dynamic gains from trade over an infinite horizon obtained by Smith(1979) in a world economy without preferences of consumers to a world economy with preferences of consumers.

When (T–10) is replaced with (T–11), the similar result follows.

Theorem 4 : *Under assumptions* (T–1), (T–2), (T–3), (T–4), (T–5), (T–6), · ·, (T–9), *and* (T–11), *the dynamic world free trade economy over a discrete-time infinite horizon has a competitive equilibrium with domestic income transfers with prices in* l_1. *Moreover, no one in the world economy is worse off under world free trade than under autarky. The same conclusion holds when* (T–1), (T–2), *and* (T–8) *are replaced with* (T–1)′, (T–2)′, *and* (T–8)′.

Proof) See Appendix B.

Note that the strong irreducibility assumption (T-12) establishes the existence of a free trade equilibrium with GM type domestic income trasfersthe as well as the one of a Laissez-faire free trade equilibrium without GM type domestic income trasfers. Note also that the usefulness of autarky consumption bundles within each country in the modified irreducibilty (T-11) or the usefulness of feasible consumption bundles in the world economy in the strong irreducibility (A-12) is important to the existence of gains from trade, and that the standard irreducibility assumption (T-11)′ is not enough for the same purpose.

5 Concluding Remarks

When the set of feasible outputs in the economy is uniformly (or $|| \cdot ||_\infty$-) bounded, the feasibility of allocations required by market equilibrium implies that any unbounded program can never be realized as an optimal choice of agents in the economy. In this circumstance, only bounded programs are feasible choices for agents. This is a reason for using l_∞ as the underlying commodity space and the choice of l_∞ as the underlying commodity space is considered to be a reasonable approximation to express such a world economy. From this viewpoint, this paper establishes this Grandmont–McFadden's(1972) result on gains from trade in a world free trade economy with l_∞ as the underlying commodity space. Although the existence of competitive equilibrium under world free trade is proved with a uniformly bounded feasible set, once a competitive equilibrium exist under world free trade as well as under autarky the result on dynamic gains from trade, indeed, can hold even without the uniformly bounded set of feasible outputs. Thus, it is also interesting to establish Grandmont–McFadden's(1972) result on gains from trade in a world free trade economy with $l_\infty(\beta)$ as the underlying commodity space after proving the existence of competitive equilibrium under free trade and the one under autarky in a world free trade economy with $l_\infty(\beta)$ as the underlying commodity space. We may prove the existence of competitive equilibrium in the economy in $l_\infty(\beta)$ by transforming this economy into an economy in l_∞ as often used in the growth theory. The method of Araujo–Monteiro(1994) may work for this purpose.[36] Becker–Boyd(1996, Chap. 8) prove the existence of competitive equilibrium in an economy in $l_\infty(\beta)$ with the condition that $\{\mathbf{y} \in \mathbf{Y} : \mathbf{y} \geq \overline{\mathbf{y}}\}$ is $|| \cdot ||^\beta_\infty$-bounded for $\overline{\mathbf{y}} \in l_\infty$ through Edgeworth equilibrium approach as Boyd–McKenzie(1993) do in the economy in $\mathbf{s}$ with the condition that $\{\mathbf{y} \in \mathbf{Y} : \mathbf{y} \geq \overline{\mathbf{y}}\}$ is order bound for $\overline{\mathbf{y}} \in l_\infty$. [37] Note that although the choice of l_∞ as the commodity space is reasonable

[36] Bewley(1972) mentioned this device of changing the unit of commodities in each period in the case of pure exchange economy with connocting his result in l_∞ to that of Peleg–Yaari(1970) in $\mathbf{s}$, although he did not proof this result formally.

[37] Note that their results on existence theorems use transitivity of preferences crucially to apply Scarf's theorem for non-emptiness of core and to get the equal treatment property of core allocations. It is also an interesting questing whether we can still dispense with transitivity of preferences in the economies they treat.

under the condition of uniformly bounded feasible outputs, this choice restricts the choices of agents in the economy to be bounded from the beginning. It imposes an additional constraint on agents in the economy besides their behavioral constraints. In a competitive economy, consumers try to choose the consumption bundles in their budget sets which are best according to their preferences. Firms similarly try to choose production plans in their technology sets from the viewpoint of profit maximization. They never take into account the feasibility of the plans they made in the entire economy.[38]

Since the argument of the proof of the theorem on the existence of competitive equilibrium in l_∞ with prices in ba indeed works for any dual pair (X, X^*) where X is the norm dual of a norm space and X^* is the norm dual of X, the existence of equilibrium prices in X^* holds, and hence the existence of equilibrium prices in $l_\infty(\beta)^*$ holds as well. Since the argument to establish the L_1-part of an equilibrium price found in ba^*, the norm dual of l_∞, uses Yosida-Hewitt decomposition theorem following Bewley's(1972) approach, the existence of equilibrium prices in $l_1(\beta)$ holds as well as long as the similar decomposition result holds in the $l_\infty(\beta)^*$ case. Since Lucas-Prescott(1972) also establish that the L_1-part of an equilibrium price found in ba^* is still a equilibrium price without using Yosida-Hewitt decomposition theorem explicitly, we may establish that the $l_1(\beta)$ -part of an equilibrium price found in $l_\infty(\beta)^*$ is still an equilibrium price. Then, we may establish Grandmont–McFadden's(1972) result on gains from trade in a world free trade economy with $l_\infty(\beta)$ as the underlying commodity space.[39] It may be also interesting to prove Grandmont–McFadden's(1972) result on gains from trade in a world free trade economy in general vector lattice as in Richard(1989) and Aliprantis-Brown-Burkinshaw(1989).

In this paper, we also assume that the markets are complete and the economies have a finite number of infinitely lived consumers. It is, thus, also an interesting question to consider the issues treated in this paper in a world free trade economy with incomplete markets over an infinite horizon and that in an OLG model of world free trade as in Kemp–Wong(1995) and Kemp(1995).

Based on the existence of competitive equilibrium prices in l_1^N both under autarky and under world free trade, the weak law of comparative advantage developed by Deardorff(1980) and Dixit–Norman(1980) is proved in a dynamic world trade economy where each country has heterogeneous consumers. In this argument, the autarky competitive equilibrium price is used to evaluate the path of net exports of the competitive equilibrium under world free trade. The general weak law of comparative advantage holds in the dynamic world free trade economy. This is an extension of the weak law of comparative advantage obtained in the traditional finite dimensional world economy to the dynamic world economy over infinite horizon in this paper. It can be interpreted similarly as in the traditional finite dimensional case. The validity of periodwise comparative advantage, however, may be weaker in the far future. When the set of feasible allocations under world free trade is uniformly bounded, the set of feasible allocations of a country under autarky is also uniformly bounded. Then we can find an autarky competitive equilibrium price belonging to l_1^N. In this situation, the value of the net export path under world free trade is well-defined and finite, even when it is evaluated with the autarky equilibrium price. Then even if the periodwise weak law of comparative advantage does not hold after a sufficiently far period, the weak law of comparative advantage can hold. In this sense, the weak law of comparative advantage turns out to be weaker in the dynamic context than in static context.

Kubota(1997b) establishes Kemp-Wan-Grinols' argument on gains from forming a customs union in the $\mathbf{L}_\infty$ case of world economy with applying Grandmont–McFadden's result on gains from trade. In the $\mathbf{L}_\infty$ case of world economy, a world price before forming a customs union is assumed to be in **ba**, not necessarily in $\mathbf{L}_1$, and applying Grandmont–McFadden's result on gains from trade with prices in **ba** gives rise to the desired result of Kemp-Wan-Grinols' argument on gains from forming a customs union. Since there may exist tariffs in a world economy before forming a customs union so that a re-union world price can be found only in **ba** and can not be found in $\mathbf{L}_1$. Note that since Kemp-Wan-Grinols' argument on gains from forming a customs union is

[38] See Kubota(1998) for this issue.

[39] This issue is left for a topic of futher research.

mainly concerned with its welfare consequence and not its implication to prices, in the $\mathbf{L}_\infty$ case a re-union world price is assumed to be in general **ba**, although the existence of such a re-union world price in **ba** is not treated explicitly.

Appendix A: Mathematical Note

Let $\mathbf{s}$ be the Cartesian product $\Pi_{t=1}^{\infty}\mathbf{R}(t)$ endowed with the product topology with $\mathbf{R}(t) = \mathbf{R}$ $\forall\ t \in N$. An open set in this product topology is expressed as $\mathbf{U} = \sum_{t=1}^{\infty}\mathbf{U}_t$ where each $\mathbf{U}_t$ is open in $\mathbf{R}(t)$ and $\mathbf{U}_{\mathrm{t}}$ is equal to $\mathbf{R}(t)$ for all but finite number of t. This product topology is also metrizable and the convergence of a sequence $\{\mathbf{z}^n\}_{n=1}^{\infty}$ to $\mathbf{z}$ in $\mathbf{s}^N$ is characterized by $\mathbf{z}_t^n \to \mathbf{z}_t$ as $n \to \infty$ for $t = 1, 2, \cdot\cdot$. For $\mathbf{z} \in \mathbf{s}$, define $\mathbf{z}(t)$ and $\widehat{\mathbf{z}}(t)$ be the head of $\mathbf{z}$ up to $t, (\mathbf{z}_1, \cdot\cdot, \mathbf{z}_t, 0, \cdot\cdot)$, and the tail of $\mathbf{z}$ after $t, (0, \cdot\cdot, \mathbf{z}_{t+1}, \cdot\cdot)$. Then, $\mathbf{z}(t) \to \mathbf{z}(\widehat{\mathbf{z}}(t) \to \mathbf{0})$ holds in the coordinatewaise convergence topology as $t \to \infty$. From this fact, this product topology on $\mathbf{s}^N$ is also called as the coordinatewise convergence topology. l_∞ is a subset of $\mathbf{s}$, defined as $\{\mathbf{z} \in \mathbf{s} : \sup_{1\le t<\infty}|\mathbf{z}_t| = ||\mathbf{z}_t||_\infty < \infty\}$, where $||\mathbf{z}_t||_\infty = \sup_{1\le t<\infty}|\mathbf{z}_t|$ is called a supnorm. l_1 is also a subset of $\mathbf{s}$, defined as $\{\mathbf{z} \in \mathbf{s} : \sum_{t=1}^{\infty}|\mathbf{z}_t| = ||\mathbf{z}||_1 < \infty\}$, where $||\mathbf{z}||_1 = \sum_{t=1}^{\infty}|\mathbf{z}_t|$ is called a l_1-norm. Both of l_∞ and l_1 are Banach spaces under the associated norms, respectively. Although l_∞ is the norm dual of l_1, the norm dual of l_∞ contains l_1 as its proper subset and is called the space of purely finitely additive measures on natural numbers denoted *ba*. $l_1^{+(+)}$,$(l_\infty^{+(+)})$ is defined as $\{\mathbf{z} \in l_\infty^N : \mathbf{z} > (\gg)\ 0\}$. Then the $||\cdot||_\infty$-closure of $(l_\infty^N)^{++}$ is equal to (l_∞^+). In the case of l_∞, $\mathbf{z}(t) = (\mathbf{z}_1, \cdot\cdot, \mathbf{z}_t, 0, \cdot\cdot)$, the head of $\mathbf{z}$ after $t(\widehat{\mathbf{z}}(t) = (0, \cdot\cdot, \mathbf{z}_{t+1}, \cdot\cdot)$, the tail of $\mathbf{z}$ up to t, then, $\mathbf{z}(t) \to \mathbf{z}(\widehat{\mathbf{z}}(t) \to \mathbf{0})$ holds $\mathbf{z} \in l_\infty$ in the weak * $\sigma(l_\infty, l_1)$-topology, and hence, coordinatewaise convergence topology, as $t \to \infty$.

Note that when $\mathbf{R}^N = \mathbf{R}^{(N)}(t)\ \forall\ t \in N$ is used its coordinate, instead of $\mathbf{R}(t) = \mathbf{R}$, for the Cartesian product $\Pi_{t=1}^{\infty}\mathbf{R}^{(N)}(t)$, $\mathbf{s}^{(N)}$ is used for this Cartesian product $\Pi_{t=1}^{\infty}\mathbf{R}^{(N)}(t)$ endowed with the product topology where summation norm $|z_{\mathrm{t}}| = \sum_{i=1}^{N}|z_{\mathrm{it}}|$ is used for each $\mathbf{R}^{(N)}(t) = \mathbf{R}^N$. l_∞^N and $l_1^{(N)}$ are defined similarly, and the results hold for l_∞ and l_1 also holds as well. Then the $||\cdot||_\infty$-closure of $(l_\infty^N)^{++}$ is equal to $(l_\infty^N)^+$. Note that $\mathbf{s}^N$ is the Cartesian product $\Pi_{t=1}^{\infty}\mathbf{R}^N(t)$ endowed with the product topology where summation norm $|z_{\mathrm{t}}| = \sum_{i=1}^{N}|z_{\mathrm{it}}|$ is used for each $\mathbf{R}^N(t) = \mathbf{R}^N$. The following notation of inequality signd are used in this space $\mathbf{s}^N$. $\mathbf{x} \ge \mathbf{y} \Leftrightarrow \mathbf{x}_{it} \ge \mathbf{y}_{it}$ for $i = 1, \cdot\cdot N$, $t = 1, 2, \cdot\cdot\cdot$. $\mathbf{x} > \mathbf{y} \Leftrightarrow \mathbf{x} \ge \mathbf{y}$ and for some t with $\mathbf{x}_{it} > \mathbf{y}_{it}$ for $i = 1, \cdot\cdot, N$. $\mathbf{x} \gg \mathbf{y} \Leftrightarrow \forall\ i = 1, \cdot\cdot, N$, $t = 1, 2, \cdot\cdot$. Also $(\mathbf{s}^N)^{+(+)}$ is defined by $(\mathbf{s}^N)^{+(+)} = \{\mathbf{y} \in \mathbf{s}^N : \mathbf{y} \ge (>)\ \mathbf{0}\}$. Since $(\mathbf{s}^N)^+$ is a closed subset, thus $(\mathbf{s}^N, >)$ defines an ordered topological vector space. Note that the closure of $(\mathbf{s}^N)^{++}$ is equal to $(\mathbf{s}^N)^+$. Moreover, $\mathbf{e}_t \in \mathbf{s}^N$ is defined such as $\mathbf{e}_{is} = 1$ for $i = 1, \cdot\cdot N$ if $s = t$ and $\mathbf{e}_{is} = 0$ otherwise and the unit vector $\mathbf{e}$ is defined by $\mathbf{e} = \sum_{t=1}^{\infty}\mathbf{e}_t$. l_1^N is a subset of $\mathbf{s}^N$ and defined as $\{\mathbf{z} \in \mathbf{s}^N : \sum_{t=1}^{\infty}|\mathbf{z}_t| = ||\mathbf{z}||_1 < \infty\}$. This is a Banach space under the l_1 norm $||\cdot||_1$.

The topology employed mainly in l_∞ is the weak $*$ topology $\sigma(l_\infty, l_1)$ which is the weakest (Hausdorff) locally convex topological vector space on l_∞ to make l_1 its topological dual space, *i.e*, the set of continuous (with respect to this topology) linear functional defined on l_∞ and is called the *weak* topology on l_∞. Similarly, there is also the strongest topology to make l_1 as its topological dual space and is called the *Mackey* $\tau(l_\infty, l_1)$ topology on l_∞. In fact, these topologies can be defined similarly if there is a dual pairing $(\mathbf{X}, \mathbf{X}')$ of a pair of vector spaces $\mathbf{X}$ and $\mathbf{X}'$ with a (non-singular) bilinear form $(\mathbf{x}, \mathbf{x}') \to \mathbf{R}$ on $\mathbf{X} \times \mathbf{X}'$. Then $\sigma(\tau)(\mathbf{X}, \mathbf{X}')$ is the weakest(strongest) (Hausdorff) locally convex topological vector space which has $\mathbf{X}'$ as its topological dual space. Since $(\mathbf{X}', \mathbf{X})$ can be treated as another dual pairing, indeed, $\sigma(\tau)(\mathbf{X}', \mathbf{X})$ can be defined similarly as well. The weak topology $\sigma(\mathbf{X}', \mathbf{X})$ is usually called the weak * topology. Then a net $\{\mathbf{x}_\alpha\}_{\alpha\in A}$ in $\mathbf{X}$ converges to an element $\mathbf{x} \in \mathbf{X}$ with respect to $\sigma(\mathbf{X}, \mathbf{X}')$ as $\alpha \uparrow$ if and only if $|\mathbf{x}' \cdot (\mathbf{x}_\alpha - \mathbf{x})|$ converges to 0 as $\alpha \uparrow$ for any $\mathbf{x}' \in \mathbf{X}'$. Also a net $\{\mathbf{x}_\alpha\}_{\alpha\in A}$ in $\mathbf{X}$ converges to an element $\mathbf{x} \in \mathbf{X}$ with respect to $\tau(\mathbf{X}, \mathbf{X}')$ as $\alpha \uparrow$ if and only if $\sup\{|\mathbf{x}' \cdot (\mathbf{x}_\alpha - \mathbf{x})|\mathbf{x}' \in \mathbf{A}\}$ converges 0 on each convex, balanced, and weak* $\sigma(\mathbf{X}', \mathbf{X})$ compact subset $\mathbf{A}$ of $\mathbf{X}'$ as $\alpha \uparrow$. Note that a set $\mathbf{A}$ is balanced if $\mathbf{x} \in \mathbf{A}$ and $|\lambda| \le 1$ imply $\lambda\mathbf{x} \in \mathbf{A}$. Note also that when $\mathbf{x}_\alpha \to \mathbf{x}(\alpha \uparrow)$ with respect to $\tau(\mathbf{X}, \mathbf{X}')$, $\mathbf{x}_\alpha \to \mathbf{x}(\alpha \uparrow)$ with respect to $\sigma(\mathbf{X}, \mathbf{X}')$. Since l_∞ is the topological norm dual space of l_1, *i.e*, the topological dual space with respect to the norm topology on l_1, and hence (l_∞, l_1) is a dual pairing, the weak $\sigma(l_\infty, l_1)$ topology is indeed the weak* topology(Dunford–Schwartz(1958, VI 8.5,

p. 189)). Aliplantis–Brown–Burkinsshaw(1989, Chapter 2) is succinct introduction to this material. For details on this material, see Shaefer(1966) and Robertson–Robertson(1973). An important property of the Mackey $\tau(l_\infty, l_1)$ topology is following: If $\mathbf{x}^n \to \mathbf{0}$ holds in $\tau(l_\infty, l_1)$ as $n \to \infty$, then $\mathbf{y}^n \to \mathbf{0}$ holds in $\tau(l_\infty, l_1)$ as $n \to \infty$ as well for any $\{\mathbf{y}^n\}_{n=1}^{\infty}$ with $|\mathbf{y}^n| \leq |\mathbf{x}^n|$ for $n \geq 1$(Bewley(1972, p. 535) and Back(1988, Footnote 5, 91)). In particular, $\widehat{\mathbf{x}}(t) = (0, \cdots, x_t, x_{t+1}, \cdots) \to \mathbf{0}$ in $\tau(l_\infty, l_1)$, hence, $\sigma(l_\infty, l_1)$ as $n \to \infty$ for $\mathbf{x} \in l_\infty$.

In the case of general $\mathbf{L}_\infty$ case, weak* $\sigma(\mathbf{L}_\infty,\mathbf{L}_1)$-open lower sections of preference in (T–6) are characterized as follows. Since μ is assumed σ-finite, from the definition of σ-finite measure, there is an increasing sequence of measurable sets $\{\mathbf{M}_n\}_{n=1}^{\infty}$ satisfying $\cup_{n\geq 1}\mathbf{M}_n = \mathbf{\Omega}$ and $\mu(\mathbf{M}_n) < \infty$ for $n \geq 1$. Then, a decreasing sequence of measurable subset $\{\mathbf{E}_n\}_{n=1}^{\infty}$ with $\cap_{n\geq 1}\mathbf{E}_{\mathrm{n}} = \emptyset$. Define $\mathbf{E}_n = (\mathbf{M}_n)^c$ for $n \geq 1$. Then $\mathbf{x} \cdot \chi_{E_n} \to \mathbf{0}$ holds in $\tau(\mathbf{L}_\infty, \mathbf{L}_1)$ as $n \to 0$ for $\mathbf{x} \in \mathbf{L}_\infty$. (Bewley(1972, Appendix 1 (24) p. 534). Since the weak* $\sigma(\mathbf{L}_\infty, \mathbf{L}_1)$ topology is coarser than Mackey $\tau(\mathbf{L}_\infty, \mathbf{L}_1)$, if a sequence converges in the Mackey $\tau(\mathbf{L}_\infty, \mathbf{L}_1)$ topology, then it also converges in the weak* $\sigma(\mathbf{L}_\infty, \mathbf{L}_1)$ topology. Thus, $\chi_{E_n} \to \mathbf{0}$ holds in the weak* $\sigma(\mathbf{L}_\infty,\mathbf{L}_1)$ topology, and hence, $c \cdot \chi_{E_n} \to \mathbf{0}$ also holds in the weak* $\sigma(\mathbf{L}_\infty,\mathbf{L}_1)$ topology for $c > 0$. Thus, given $c > 0$, $\mathbf{z} \in \mathbf{P}_{ik}(\mathbf{x})$ (*i.e.*, $\mathbf{x} \in \mathbf{P}_{ik}^{-1}(\mathbf{z})$) implies $\mathbf{z} \in \mathbf{P}_{ik}(\mathbf{x} + c \cdot \chi_{E_n})$ (*i.e.*, $\mathbf{x} + c \cdot \chi_{E_n} \in \mathbf{P}_{ik}^{-1}(\mathbf{z})$) for sufficiently large n. Thus an additional gain accruing from a rise of the consumption by a constant amount c over $\mathbf{E}_n$ has only a slight impact to the preference for n sufficiently large. Nite that $\mathbf{x} + c \cdot \chi_{E_n} \in \mathbf{C}_{ik}^T$ must be assumed implicitely for n sifficiently large to make the argument sense. This holds when $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ is complehensive upward so that $[\mathbf{C}_{ik}^T + \mathbf{L}_\infty^+] \subset \mathbf{C}_{ik}^T$ holds.

Appendix B: Proofs

Proof of Theorem 1) Consider a class of $\mathcal{F}$ of all finite dimensional subspaces F of l_∞ containing $\{\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}$ and ω_{ik} for (i,k) $\in$ I, and $\mathbf{e}^{\mathrm{k}}$ for k $\in$ J$\}$. For each F, define a new world free trade economy obtained by restricting economic data of the original world economy on finite dimensional subspace F. Then new production set $\mathbf{Y}^{\mathrm{T}} \cap F$ is closed and $(\widehat{\mathbf{Y}^{\mathrm{T}} \cap F}) = \widehat{\mathbf{Y}^{\mathrm{T}}} \cap F$ is compact under (T–2). When (T–2)$'$ is used instead of (T–2), $(\mathbf{Y}^{\mathrm{T}} \cap \mathbf{F}) \cap [-(\mathbf{Y}^{\mathrm{T}} \cap F)] = \{\mathbf{0}\}$ holds besides compactness of $(\widehat{\mathbf{Y}^{\mathrm{T}} \cap F})$. From the existence of quasi-equilibrium with domestic income transfers in world free trade economies with a finite number of commodities, $\mathcal{E}^F$ has a quasi-equilibrium with domestic income transfers $(\mathbf{x}_{\mathrm{ik}}^F, \mathbf{y}_{\mathrm{k}}^F, \mathbf{p}^F)_{(\mathrm{i,k})\in\mathrm{I}}$ with $\mathbf{p}^F \neq \mathbf{0}$ under world free trade in both cases with (T–2) and with (T–2)$'$.[40] Since preferences of consumers in the k-th country satisfy periodwise monotonicity, $\mathbf{p}^F \cdot \mathbf{z}^{\mathrm{k}} \geq 0$ holds for $\mathbf{z}^{\mathrm{k}} \in ((l_\infty^{\mathrm{k}})^+ \cap F)$. Since this holds for each k $\in$ J and N is equal to $\sum_{\mathrm{k}\in\mathrm{J}} N_{\mathrm{k}}^{\mathrm{T}}$, $\mathbf{p}^F \cdot \mathbf{z} \geq 0$ holds for $\mathbf{z} \in (l_\infty^+ \cap F)$. $\mathbf{p}^F$ is positive on F. Also since $\mathbf{Y}$ is a convex cone and $\mathbf{0} \in \mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}$ holds for k $\in$ J, $\mathbf{p}^F \cdot \mathbf{y}_{\mathrm{k}}^F = 0$ holds. F contains $\mathbf{e}$, a $||\cdot||_\infty$-interior point of l_∞^+ from the definition. Then the positive continuous linear functional $\mathbf{p}^F$ on F has a positive and $||\cdot||_\infty$-continuous extension π^F over l_∞.[41] π^F is a non-zero element of ba since $\pi^F|_F = \mathbf{p}^F \neq \mathbf{0}$ holds. We can use a price normalization expressed by $||\pi^F|| = 1$. Then positivity of π^F implies $\pi^F \cdot \mathbf{e} = 1$.

Since $(\mathbf{x}_{\mathrm{ik}}^F, \mathbf{y}_{\mathrm{k}}^F)_{(\mathrm{i,k})\in\mathrm{I}}$ is a feasible allocation in $\mathcal{E}^F$, it is also feasible in the dynamic world free trade economy. For any $F \in \mathcal{F}$, $\mathbf{x}_{\mathrm{ik}}^F \in \widehat{\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}}$ and $\mathbf{y}^F \in \widehat{\mathbf{Y}^{\mathrm{T}}}$ hold for (i,k) $\in$ I, where $\mathbf{y}^F$ is $\sum_{\mathrm{k}\in\mathrm{J}} \mathbf{y}_{\mathrm{k}}^F$. $\widehat{\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}}$ and $\widehat{\mathbf{Y}_{\mathrm{k}}^{\mathrm{T}}}$ are weak* $\sigma(l_\infty, l_1)$-compact from, in particular, (T–8)((T–8)$'$). $\{\mathbf{x}_{\mathrm{ik}}^F\}_{F^{\mathrm{D}}\in\mathcal{F}^{\mathrm{D}}}$ and $\{\mathbf{y}^F\}_{F^{\mathrm{D}}\in\mathcal{F}^{\mathrm{D}}}$ are nets in $\widehat{\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}}$ and $\widehat{\mathbf{Y}^{\mathrm{T}}}$ directed by set inclusion over $\mathcal{F}$. $\{\pi^F\}_{F\in\mathcal{F}} \subset \mathbf{B}^* = \{\pi \in ba : ||\pi|| \leq 1\}$ is also a net directed by set inclusion over $\mathcal{F}$. $\mathbf{B}^*$ is weak* $\sigma(ba, l_\infty)$-compact.[42] Thus each of the nets has a converging subnet. By passing subnet if necessary, we may assume $\mathbf{x}_{\mathrm{ik}}^F \to \mathbf{x}_{\mathrm{ik}}^D \in \widehat{\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}}$, $\mathbf{y}^F \to \mathbf{y}^D \in \widehat{\mathbf{Y}^{\mathrm{T}}}$, and $\pi^F \to \pi^D \in \mathbf{B}^*$ as $F \uparrow$ over $\mathcal{F}$.

The balance condition, $\sum_{(\mathrm{i,k})\in\mathrm{I}} \mathbf{x}_{\mathrm{ik}}^F = \mathbf{y}^F + \omega$, in each subeconomy $\mathcal{E}^F$ and weak* $\sigma(l_\infty, l_1)$-convergence of $\mathbf{x}_{\mathrm{ik}}^F$ to $\mathbf{x}_{\mathrm{ik}}^D$ and $\mathbf{y}^F$ to $\mathbf{y}^D$ as $F \uparrow$ gives rise to $\sum_{(\mathrm{i,k})\in\mathrm{I}} \mathbf{x}_{\mathrm{ik}}^D = \mathbf{y}^D + \omega$ in the limit. The

[40] See Kubota(1997a, Proposition 3 – 1).
[41] Krein-Rutman's theorem, Shaefer(1966, Corollary 2, p. 192). See also Jones(1987, Lemma 1, p. 96).
[42] By Alaoglu's theorem.

allocation $(\mathbf{x}_{\rm ik}^D, \mathbf{y}_{\rm k}^D)_{\rm (i,k)\in I}$ is also feasible under world free trade, where $\mathbf{y}_{\rm k}^D \in \mathbf{Y}_{\rm k}^D$ is selected so that $\mathbf{y}^D$ is $\sum_{\rm k\in J} \mathbf{y}_{\rm k}^D$. Also the weak* $\sigma(ba, l_\infty)$-convergence of π^F to π^D implies $||\pi^F|| = \pi^F \cdot \mathbf{e} = 1 \to \pi^D \cdot \mathbf{e} = ||\pi^D|| = 1$ holds in the limit. π^D is non-zero. Again from the weak* $\sigma(ba, l_\infty)$-convergence of π^F to π^D, positivity of π^F implies $\pi^F \cdot \mathbf{z} \geq 0 \to \pi^D \cdot \mathbf{z} \geq 0$ holds for $\mathbf{z} \in l_\infty^+$. π^D is also positive.

Let $\mathbf{x}_{\rm ik} \in \mathbf{P}_{\rm ik}(\mathbf{x}_{\rm ik}^D)$ and $\mathbf{y}_{\rm k} \in \mathbf{Y}_{\rm k}^D$ for (i,k) $\in$ I and k $\in$ J. $\mathbf{P}_{\rm ik}^{-1}(\mathbf{x}_{\rm ik})$ is weak* $\sigma(l_\infty, l_1)$-open in $\mathbf{C}_{\rm ik}^D$ from (T–5). From weak* $\sigma(l_\infty, l_1)$-convergence of $\{\mathbf{x}_{\rm ik}^F\}_{F\in\mathcal{F}}$ to $\mathbf{x}_{\rm ik}^D$, there is some $F_0 \in \mathcal{F}$ such that $\mathbf{x}_{\rm ik}^{\rm F} \in \mathbf{P}_{\rm ik}^{-1}(\mathbf{x}_{\rm ik})$ holds for any $F \in \mathcal{F}$ with $F \supset F_0$. Define a subspace F_1 satisfying $F_1 \supset [F_0 \cup \{\mathbf{x}_{\rm ik}, \mathbf{y}_{\rm k} : \text{(i,k)} \in \text{I and k} \in \text{J}\}]$. Since for any $F(\supset F_1) \in \mathcal{F}$, $(\mathbf{x}_{\rm ik}^F, \mathbf{y}_{\rm k}^F, \mathbf{p}^F)_{\rm (i,k)\in I}$ is a quasi-equilibrium with domestic income transfers under world free trade in $\mathcal{E}^F$ and π^F is an extension of $\mathbf{p}^F$,

$$\begin{aligned}\pi^F \cdot \mathbf{x}_{\rm ik} \geq \tau_{\rm ik}(\pi^F) &= \pi^F \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^F(\omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \\ &\geq \pi^F \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^F(\mathbf{y}_{\rm k} + \omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \end{aligned} \tag{1}$$

holds. From the weak* $\sigma(ba, l_\infty)$-convergence of $\{\pi^F\}_{F\in\mathcal{F}}$ to π^D, (1) yields

$$\begin{aligned}\pi^D \cdot \mathbf{x}_{\rm ik} \geq \tau_{\rm ik}(\pi^D) &= \pi^D \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^D \cdot (\omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \\ &\geq \pi^D \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^D \cdot (\mathbf{y}_{\rm k} + \omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \end{aligned} \tag{2}$$

in the limit. (2) holds for any $\mathbf{x}_{\rm ik} \in \mathbf{P}_{\rm ik}(\mathbf{x}_{\rm ik}^D)$. Also from (2), $\pi^D \cdot \mathbf{y}_{\rm k} \leq 0$ holds. Periodwise monotonicity of preference implies $\mathbf{x}_{\rm ik}^D + \delta\mathbf{e}_1 \in \mathbf{P}_{\rm ik}(\mathbf{x}_{\rm ik}^D)$ for $\delta > 0$. (2) then implies $\pi^D \cdot (\mathbf{x}_{\rm ik}^D + \delta\mathbf{e}_1) \geq \tau_{\rm ik}(\pi^D)$ for $\delta > 0$. Letting $\delta \to 0$ gives rise to

$$\begin{aligned}\pi^D \cdot \mathbf{x}_{\rm ik}^D \geq \tau_{\rm ik}(\pi^D) &= \pi^D \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^D \cdot (\omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \\ &\geq \pi^D \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^D \cdot (\omega_{\rm k} + \mathbf{y}_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}). \end{aligned} \tag{3}$$

Summing over (3) over I with $\mathbf{y}_{\rm k} = \mathbf{y}_{\rm k}^D$ for k $\in$ J gives rise to

$$\begin{aligned}\pi^D \cdot \sum_{\rm (i,k)\in I} \mathbf{x}_{\rm ik}^D \geq \sum_{\rm (i,k)\in I} \tau_{\rm ik}(\pi^D) &= \sum_{\rm k\in J}\sum_{\rm i\in I_k} \tau_{\rm ik}(\pi^D) = \pi^D \cdot \omega \\ &\geq \pi^D \cdot \sum_{\rm (i,k)\in I} \mathbf{x}_{\rm ik}^{\rm A} + \pi^D \cdot \sum_{\rm k\in J}(\mathbf{y}_{\rm k}^D + \omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \\ &= \pi^D \cdot \sum_{\rm k\in J}(\mathbf{y}_{\rm k}^D + \omega_{\rm k}) = \pi^D \cdot \sum_{\rm (i,k)\in I} \mathbf{x}_{\rm ik}^D. \end{aligned} \tag{4}$$

(4) implies $\pi^D \cdot \sum_{\rm k\in J} \mathbf{y}_{\rm k}^D = 0$ and $\pi^D \cdot \mathbf{y}_{\rm k}^D = 0$ holds for k $\in$ J. For k $\in$ J, profit maximization of $\mathbf{y}_{\rm k}^D$ holds with π^D. Also (4) implies that (3) holds with equality for (i,k) $\in$ I. Thus

$$\pi^D \cdot \mathbf{x}_{\rm ik}^D = \tau_{\rm ik}(\pi^D) = \pi^D \cdot \mathbf{x}_{\rm ik}^{\rm A} + \theta_{\rm ik}\pi^D \cdot (\omega_{\rm k} - \sum_{\rm h\in I_k} \mathbf{x}_{\rm hk}^{\rm A}) \tag{5}$$

follows for (i,k) $\in$ I. Then, (2) and (5) imply that $\mathbf{x}_{\rm ik}^D$ minimizes expenditure over $\mathbf{P}_{\rm ik}(\mathbf{x}_{\rm ik}^D)$ with π^D for (i,k) $\in$ I. $(\mathbf{x}_{\rm ik}^D, \mathbf{y}_{\rm k}^D, \pi^D)_{\rm (i,k)\in I}$ therefore a quasi-equilibrium with domestic income transfers under world free trade.

Proof of Theorem 2) Let $(\mathbf{x}_{\rm ik}^D, \mathbf{y}_{\rm k}^D, \pi^D)_{\rm (i,k)\in I}$ be a quasi-equilibrium with domestic income transfers under world free trade with $\pi^D \in ba^+\backslash\{\mathbf{0}\}$ found in theorem 1 above. Note that from (5), $\pi^D \cdot \mathbf{x}_{\rm ik}^D = \tau_{\rm ik}(\pi^D)$ holds. Let π_c^D be the l_1-part of π^D in the Yosida–Hewitt decomposition. We prove below that π_c^D is non-zero and also a quasi-equilibrium price with domestic income transfers under world free trade.

We show first that expenditure minimization holds at $\mathbf{x}_{\mathrm{ik}}^D$ with π_c under the exclusion assumption (T–5) for consumption set. Let $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$. As is mentioned in the last section, (T–5) gives rise to $\{\mathbf{x}_{\mathrm{ik}}^n\}_{n=1}^{\infty} \subset \mathbf{C}_{\mathrm{ik}}^D$ such that $\mathbf{x}_{\mathrm{ik}}^n \to \mathbf{x}_{\mathrm{ik}}$ in the weak* $\sigma(l_\infty, l_1)$ topology and $\pi_p^D \cdot \mathbf{x}_{\mathrm{ik}}^n \to 0$ as $n \to \infty$, where π_p^D is the purely finitely additive part of π^D in the Yosida–Hewitt decomposition theorem. From its weak* $\sigma(l_\infty, l_1)$-lower semi-continuity, $\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ is weak* $\sigma(l_\infty, l_1)$-open in $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$. Since $\mathbf{x}_{\mathrm{ik}}^n \to \mathbf{x}_{\mathrm{ik}}$ holds in $\mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ with respect to the weak* $\sigma(l_\infty, l_1)$ as $n \to \infty$, there is N satisfying $\mathbf{x}_{\mathrm{ik}}^n \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ for $n > N$. Also $\mathbf{x}_{\mathrm{ik}}^D$ minimizes expenditure over $\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ with π^D. Thus,

$$\pi^D \cdot \mathbf{x}_{\mathrm{ik}}^n = \pi_c^D \cdot \mathbf{x}_{\mathrm{ik}}^n + \pi_p^D \cdot \mathbf{x}_{\mathrm{ik}}^n \geq \tau_{\mathrm{ik}}(\pi^D) = \pi^D \cdot \mathbf{x}_{\mathrm{ik}}^D \tag{6}$$

holds for $n > N$. Since $\mathbf{x}_{\mathrm{ik}}^n \to \mathbf{x}_{\mathrm{ik}}$ holds in the weak* $\sigma(l_\infty, l_1)$ topology and $\pi_p^D \cdot \mathbf{x}_{\mathrm{ik}}^n \to 0$ holds as $n \to \infty$, (6) gives rise to

$$\pi_c^D \cdot \mathbf{x}_{\mathrm{ik}} \geq \tau_{\mathrm{ik}}(\pi^D) = \pi^D \cdot \mathbf{x}_{\mathrm{ik}}^D \tag{7}$$

in the limit. From periodwise monotonicity of preference, $\mathbf{x}_{\mathrm{ik}}^D + \delta \mathbf{e}_1 \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ holds for $\delta > 0$, and hence $\pi_c^D \cdot (\mathbf{x}_{\mathrm{ik}}^D + \delta \mathbf{e}_1) = \pi_c^D \cdot \mathbf{x}_{\mathrm{ik}}^D + \delta \pi_c^D \cdot \mathbf{e}_1 \geq \tau_{\mathrm{ik}}(\pi^D)$ holds for $\delta > 0$. Thus letting $\delta \to 0$ yields

$$\pi_c^D \cdot \mathbf{x}_{\mathrm{ik}}^D \geq \tau_{\mathrm{ik}}(\pi^D) = \pi^D \cdot \mathbf{x}_{\mathrm{ik}}^D (\geq \pi_c^D \cdot \mathbf{x}_{\mathrm{ik}}^D) \tag{8}$$

since $\pi_c^D \cdot \mathbf{e}_1$ is a well-defined finite number. Since $\mathbf{x}_{\mathrm{ik}} + \theta_{\mathrm{ik}}(\sum_{\mathrm{h} \in \mathrm{I_k}} \omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}}) \notin l_\infty^+$ may happen to some (i,k) $\in$ I, we can not get $\tau_{\mathrm{ik}}(\pi^D) = \tau_{\mathrm{ik}}(\pi_c^D)$. This is in fact a reason for employing an alternative exclusion assumption (T–5) to consumption set instead of exclusion assumption. We show below that $\tau_{\mathrm{ik}}(\pi_c^D) = \tau_{\mathrm{ik}}(\pi^D)$ holds for (i,k) $\in$ I. For this purpose, we need the mixture condition (T–3) to production set instead of the exclusion assumption. Once $\tau_{\mathrm{ik}}(\pi_c^D) = \tau_{\mathrm{ik}}(\pi^D)$ holds, then $\mathbf{x}_{\mathrm{ik}}^D$ minimizes expenditure over $\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ with π_c^D.

Let $\mathbf{y}^D$ be $\mathbf{y}'$ in (T–3). Then as in mentioned in the last section, for $\mathbf{y} \in \mathbf{Y}^{\mathrm{T}}$, there is $\{\mathbf{y}^n\}_{n=1}^{\infty} \subset \mathbf{Y}^{\mathrm{T}}$ such that $\mathbf{y}^n \to \mathbf{y}$ in the weak* $\sigma(l_\infty, l_1)$ and $\pi^D \cdot \mathbf{y}^n \to \pi^D \cdot \mathbf{y}^D$ as $n \to \infty$. Then from profit maximization of $\mathbf{y}$ with π^D, $\pi^D \cdot \mathbf{y}^D = \pi_c^D \cdot \mathbf{y}^D + \pi_p^D \cdot \mathbf{y}^D \geq \pi^D \cdot \mathbf{y}^n = \pi_c^D \cdot \mathbf{y}^n + \pi_p^D \cdot \mathbf{y}^n$ for $n \geq 1$ implies $\pi_c^D \cdot \mathbf{y}^D + \pi_p^D \cdot \mathbf{y}^D \geq \pi_c^D \cdot \mathbf{y} + \pi_p^D \cdot \mathbf{y}^D$ in the limit. Thus

$$\pi_c^D \cdot \mathbf{y}^D \geq \pi_c^D \cdot \mathbf{y} \tag{9}$$

holds and $\mathbf{y}^D$ also maximizes profit even with π_c^D. In particular, $\pi_c^D \cdot \mathbf{y}^D \geq 0$ and $\pi_c^D \cdot \mathbf{y}^D \geq \pi_c^D \cdot \mathbf{y}_{\mathrm{k}}^D$ hold. Similarly let $\mathbf{y}^D$ be $\mathbf{y}$ in (T–3). Then for $\mathbf{y}' \in \mathbf{Y}$, there is $\{\mathbf{y}'^n\}_{n=1}^{\infty} \subset \mathbf{Y}^{\mathrm{T}}$ satisfying $\mathbf{y}'^n \to \mathbf{y}^D$ in the weak* $\sigma(l_\infty, l_1)$ and $\pi_p^D \cdot \mathbf{y}'^n \to \pi_p^D \cdot \mathbf{y}'$ as $n \to \infty$. Since $\pi^D \cdot \mathbf{y}^D = \pi_c^D \cdot \mathbf{y}^D + \pi_p^D \cdot \mathbf{y}^D \geq \pi \cdot \mathbf{y}'^n = \pi_c^D \cdot \mathbf{y}'^n + \pi_p^D \cdot \mathbf{y}'^n$ holds for n from profit maximization of $\mathbf{y}^D$, $\pi_c^D \cdot \mathbf{y}^D + \pi_p^D \cdot \mathbf{y}^D \geq \pi_c^D \cdot \mathbf{y}^D + \pi_p^D \cdot \mathbf{y}'$ follows in the limit. Thus

$$\pi_p^D \cdot \mathbf{y}^D \geq \pi_p^D \cdot \mathbf{y}' \tag{10}$$

holds for $\mathbf{y}' \in \mathbf{Y}^D$. In particular, $\pi_p^D \cdot \mathbf{y}^D \geq 0$ and $\pi_p^D \cdot \mathbf{y}^D \geq \pi_p^D \cdot \mathbf{y}_{\mathrm{k}}^D$ hold. Since (T–1) implies $\pi^D \cdot \mathbf{y}^D = \pi_c^D \cdot \mathbf{y}^D + \pi_p^D \cdot \mathbf{y}^D = 0$ and $\pi_c^D \cdot \mathbf{y}^D$, $\pi_p^D \cdot \mathbf{y}^D \geq 0$ hold, $\pi_c^D \cdot \mathbf{y}^D = \pi_p^D \cdot \mathbf{y}^D = 0$ follows. Then $\pi_c^D \cdot \mathbf{y}_{\mathrm{k}}^D = \pi_p^D \cdot \mathbf{y}_{\mathrm{k}}^D = 0$ holds for k $\in$ J. Also then $\pi_c^D \cdot \mathbf{y}_{\mathrm{k}}^D \geq \pi_c^D \cdot \mathbf{y}_{\mathrm{k}}$ and $\pi_p^D \cdot \mathbf{y}_{\mathrm{k}}^D \geq \pi_p^D \cdot \mathbf{y}_{\mathrm{k}}$ hold for $\mathbf{y}_{\mathrm{k}} \in \mathbf{Y}_{\mathrm{k}}^D$. In particular,

$$\pi_c^D \cdot \mathbf{y}_{\mathrm{k}}^D = 0 \geq \pi_c^D \cdot (\sum_{\mathrm{h} \in \mathrm{I_k}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}} - \omega_{\mathrm{hk}}) \text{ and } \pi_p^D \cdot \mathbf{y}_{\mathrm{k}}^D = 0 \geq \pi_p^D \cdot (\sum_{\mathrm{h} \in \mathrm{I_k}} \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}} - \omega_{\mathrm{hk}}) \tag{11}$$

follow.

Since $(\mathbf{x}_{\mathrm{ik}}^D, \mathbf{y}_{\mathrm{k}}^D)_{(\mathrm{i,k}) \in \mathrm{I}}$ is a feasible allocation in the world economy, from (8),

$$\begin{aligned} \pi^D \cdot \omega = \pi^D \cdot (\mathbf{y}^D + \omega) &= \pi^D \cdot \sum_{(\mathrm{i,k}) \in \mathrm{I}} \mathbf{x}_{\mathrm{ik}}^D (= \sum_{(\mathrm{i,k}) \in \mathrm{I}} \tau_{\mathrm{ik}}(\pi^D)) \\ &= \pi_c^D \cdot \sum_{(\mathrm{i,k}) \in \mathrm{I}} \mathbf{x}_{\mathrm{ik}}^D = \pi_c^D \cdot (\mathbf{y}^D + \omega) = \pi_c^D \cdot \omega (= \sum_{(\mathrm{i,k}) \in \mathrm{I}} \tau_{\mathrm{ik}}(\pi_c^D)) \end{aligned} \tag{12}$$

holds. From (12), $\pi_p^D \cdot \omega = 0$ and hence $\pi_p^D \cdot \omega_{\mathrm{k}} = 0$ holds for k $\in$ J. Since $\tau_{\mathrm{ik}}(\pi_p^D) = \pi_p^D \cdot \mathbf{x}_{\mathrm{ik}}^{\mathrm{A}} + \theta_{\mathrm{ik}}\pi_p^D \cdot \sum_{\mathrm{h}\in \mathrm{I_k}}(\omega_{\mathrm{hk}} - \mathbf{x}_{\mathrm{hk}}^{\mathrm{A}}) \geq 0$ holds from (11), and $\sum_{\mathrm{i}\in \mathrm{I_k}} \tau_{\mathrm{ik}}(\pi_p^D) = \pi_p^D \cdot \omega_{\mathrm{k}} = 0$ holds from the definition $\tau_{\mathrm{ik}}(\cdot)$,

$$\tau_{\mathrm{ik}}(\pi_p^D) = 0 \tag{13}$$

follows for i $\in$ $\mathrm{I_k}$. Thus $\tau_{\mathrm{ik}}(\pi^D) = \tau_{\mathrm{ik}}(\pi_c^D)$ holds for i $\in$ $\mathrm{I_k}$. This implies $\mathbf{x}_{\mathrm{ik}}^D$ minimizes expenditure $\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ with π_c^D from (7) and $\pi_c^D \cdot \mathbf{x}_{\mathrm{ik}}^D = \tau_{\mathrm{ik}}(\pi_c^D)$ holds.

We need to establish $\pi_c^D \neq \mathbf{0}$. (T–9) gives rise to there are $\overline{\mathbf{x}}_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}}^D$, $\overline{\mathbf{y}}_{\mathrm{k}} \in \mathbf{Y}_{\mathrm{k}}^D$, and $r > 0$ such that $\sum_{(\mathrm{i,k})\in \mathrm{I}} \overline{\mathbf{x}}_{\mathrm{ik}} \leq \sum_{\mathrm{k}\in \mathrm{J}}(\overline{\mathbf{y}}_{\mathrm{k}} + \omega_{\mathrm{k}}) - r\mathbf{e}$ holds. The positivity of π^D, then, yields $\pi^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}} \overline{\mathbf{x}}_{\mathrm{ik}} < \pi^D \cdot \sum_{\mathrm{k}\in \mathrm{J}}(\overline{\mathbf{y}}_{\mathrm{k}} + \omega_{\mathrm{k}}) \leq \pi^D \cdot \sum_{\mathrm{k}\in \mathrm{J}} \omega_{\mathrm{k}} = \sum_{(\mathrm{i,k})\in \mathrm{I}} \tau_{\mathrm{ik}}(\pi^D)$. There is thus some (i′,k′) $\in$ I with $\pi^D \cdot \overline{\mathbf{x}}_{\mathrm{i'k'}} < \tau_{\mathrm{i'k'}}(\pi^D)$. Then

$$\pi_c^D \cdot \overline{\mathbf{x}}_{\mathrm{i'k'}} \leq \pi^D \cdot \overline{\mathbf{x}}_{\mathrm{i'k'}} < \tau_{\mathrm{i'k'}}(\pi^D) = \tau_{\mathrm{i'k'}}(\pi_c^D) \tag{14}$$

holds. This implies $\pi_c^D \neq \mathbf{0}$ and so $\pi_c^D > \mathbf{0}$. $(\mathbf{x}_{\mathrm{ik}}^D, \mathbf{y}_{\mathrm{k}}, \pi_c^D)_{(\mathrm{i,k})\in \mathrm{I}}$ therefore is a quasi-equilibrium with domestic income transfers with prices in l_1 under world free trade.

Proof of Theorem 3) Let $(\mathbf{x}_{\mathrm{ik}}^D, \mathbf{y}_{\mathrm{k}}^D, \mathbf{p}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ be a quasi-equilibrium with domestic income transfers in the dynamic world free trade economy, where $\mathbf{p}^D$ is in l_1. The existence of such a quasi-equilibrium follows from theorem 2. Note that $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}}^D = \tau_{\mathrm{ik}}(\mathbf{p}^D)$ holds for (i,k) $\in$ I. We need to establish that every consumer in the world has a cheaper point than $\tau_{\mathrm{ik}}(\mathbf{p}^D)$ under the world adequacy assumption (T–9) and the modified irreducibility assumption (T–10). From (T–9), there are $\overline{\mathbf{x}}_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ for (i,k) $\in$ I , $\overline{\mathbf{y}} \in \mathbf{Y}^{\mathrm{T}}$, and $r > 0$ such that $\sum_{(\mathrm{i,k})\in \mathrm{I}} \overline{\mathbf{x}}_{\mathrm{ik}} < \overline{\mathbf{y}} + \omega - r\mathbf{e}$ holds. Then $\mathbf{p}^{\mathrm{D}} \in l_1^+ \backslash \{0\}$ implies $\mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}} \overline{\mathbf{x}}_{\mathrm{ik}} < \mathbf{p}^D \cdot (\overline{\mathbf{y}} + \omega) \leq \mathbf{p}^D \cdot \omega = \sum_{(\mathrm{i,k})\in \mathrm{I}} \tau_{\mathrm{ik}}(\mathbf{p}^D)$. Thus some of consumers in the world economy have cheaper points when each of their income level is $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Let I^1 be the set of the consumers who have cheaper points than $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Then I^1 is non-empty. Then $\mathbf{x}_{\mathrm{ik}}^D$ of every consumer in I^1 satisfies the demand condition with his income equal to $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Thus for (i,k) $\in$ I^1, $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)$ implies $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D)$. Then $\mathbf{x}_{\mathrm{ik}} \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)]$ also implies $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D)$ for them. Let I^2 be the complement of I^1 in I. For (i,k) $\in$ I^2, $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}}^{\mathrm{T}}$ implies $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}} \geq \tau_{\mathrm{ik}}(\mathbf{p}^D)$. Suppose that I^2 is non-empty. Then, for (h,j) $\in$ I^2, $\mathbf{p}^D \cdot \mathbf{x}_{\mathrm{hj}}^{\mathrm{A}} \geq \mathbf{p}^D \cdot \mathbf{x}_{\mathrm{hj}}^D = \tau_{\mathrm{hj}}(\mathbf{p}^D) = \mathbf{p}^D \cdot \mathbf{x}_{\mathrm{hj}}^D + \theta_{\mathrm{hj}}\mathbf{p}^D \cdot \sum_{\mathrm{h}\in \mathrm{I_k}}(\omega_{\mathrm{hj}} - \mathbf{x}_{\mathrm{hj}}^{\mathrm{A}}) \geq \mathbf{p}^D \cdot \mathbf{x}_{\mathrm{hj}}^{\mathrm{A}}$ implies

$$\tau_{\mathrm{hj}}(\mathbf{p}^D) = \mathbf{p}^D \cdot \mathbf{x}_{\mathrm{hj}}^D = \mathbf{p}^D \cdot \mathbf{x}_{\mathrm{hj}}^{\mathrm{A}}. \tag{15}$$

Since $(\mathbf{x}_{\mathrm{ik}}^D, \mathbf{y}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ is a feasible allocation in the world economy, the modified irreducibility assumption (T–1) implies that there are $(\widetilde{\mathbf{x}}_{\mathrm{ik}}^D)_{(\mathrm{i,k})\in \mathrm{I}^1}$, $\widetilde{\mathbf{y}}^D \in \mathbf{Y}^{\mathrm{T}}$, and $\widetilde{\mathbf{z}}^D \in \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{C}_{\mathrm{hj}}^{\mathrm{T}}$ satisfying

$$\begin{gathered}(\mathbf{x}_{\mathrm{ik}}^D + \widetilde{\mathbf{x}}_{\mathrm{ik}}^D) \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^D)] \text{ for (i,k)} \in \mathrm{I}^1 \text{ and} \\ \sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}}^D = \widetilde{\mathbf{y}}^D - \mathbf{y}^D + \alpha \sum_{(\mathrm{h,j})\in \mathrm{I}^2} (\mathbf{x}_{\mathrm{hj}}^{\mathrm{A}} - \widetilde{\mathbf{z}}_{\mathrm{hj}}) \text{ for } \alpha > 0.\end{gathered}$$

For (i,k) $\in$ I^1, $\mathbf{p}^D \cdot (\widetilde{\mathbf{x}}_{\mathrm{ik}}^D + \mathbf{x}_{\mathrm{ik}}^D) > \tau_{\mathrm{ik}}(\mathbf{p}^D) = \mathbf{p}^D \cdot \mathbf{x}_{\mathrm{ik}}^D$ or $\mathbf{p}^D \cdot \widetilde{\mathbf{x}}_{\mathrm{ik}}^D > 0$ holds. Thus

$$\mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}}^D > 0 \tag{16}$$

holds. Then

$$\begin{aligned}\mathbf{p}^D \cdot (\widetilde{\mathbf{y}}^D - \mathbf{y}^D) &= \mathbf{p}^D \cdot [\sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}}^D + \alpha \sum_{(\mathrm{h,j})\in \mathrm{I}^2} (\widetilde{\mathbf{z}}_{\mathrm{hj}} - \mathbf{x}_{\mathrm{hj}}^{\mathrm{A}})] \\ &= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}}^D + \alpha \mathbf{p}^D \cdot \sum_{(\mathrm{h,j})\in \mathrm{I}^2} (\mathbf{z}_{\mathrm{hj}} - \mathbf{x}_{\mathrm{hj}}^{\mathrm{A}})] \\ &= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}}^D + \alpha[\mathbf{p}^D \cdot \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{z}_{\mathrm{hj}} - \tau_{\mathrm{hj}}(\mathbf{p}^D)] \\ &\geq \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}}^D > 0\end{aligned} \tag{17}$$

follows from (15) and (16). On the other hand,

$$\mathbf{p}^D \cdot (\widetilde{\mathbf{y}}^D - \mathbf{y}^D) \leq 0 \tag{18}$$

follows from the profit maximization of $\mathbf{y}^D$ at $\mathbf{p}^D$. (18) is, however, a contradiction to (17). Thus I^2 must be empty and hence every consumer in the world economy has a cheaper point than $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Then lemma 1 implies that $\mathbf{x}^D_{\mathrm{ik}}$ becomes a demand point with $\mathbf{p}^D$ for every consumer in the world economy. $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D_{\mathrm{k}}, \mathbf{p}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ therefore is a competitive equilibrium with domestic income transfers under world free trade. Since $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ is affordable to each consumer in the world economy and $\mathbf{x}^D_{\mathrm{ik}}$ is a demand point to him, he is as well off with $\mathbf{x}^D_{\mathrm{ik}}$ than with $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$. Thus, in this dynamic world free trade economy over a discrete-time infinite horizon, all consumers in the worlds economy is as well off under world free trade with grandmont-McFadden's domestic income transfers than under autarky. This world free trade is therefore actually preferable to autarky.

Proof of Theorem 4) Let $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D_{\mathrm{k}}, \mathbf{p}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ be a quasi-equilibrium with domestic income transfers in the dynamic world free trade economy, where $\mathbf{p}^D$ is in l_1. The existence of such a quasi-equilibrium follows from theorem 2. Note that $\mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{ik}} = \tau_{\mathrm{ik}}(\mathbf{p}^D)$ holds for (i,k) $\in$ I. We need to establish that every consumer in the world has a cheaper point than $\tau_{\mathrm{ik}}(\mathbf{p}^D)$ under the world adequacy assumption (T–9) and the strong irreducibility assumption (T–11). From (T–9), there are $\overline{\mathbf{x}}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ for (i,k) $\in$ I , $\overline{\mathbf{y}} \in \mathbf{Y}^{\mathrm{T}}$, and $r > 0$ such that $\sum_{(\mathrm{i,k})\in \mathrm{I}} \overline{\mathbf{x}}_{\mathrm{ik}} < \overline{\mathbf{y}} + \omega - r\mathbf{e}$ holds. Then $\mathbf{p}^{\mathrm{D}} \in l_1^+ \backslash \{0\}$ implies $\mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}} \overline{\mathbf{x}}_{\mathrm{ik}} < \mathbf{p}^D \cdot (\overline{\mathbf{y}} + \omega) \leq \mathbf{p}^D \cdot \omega = \sum_{(\mathrm{i,k})\in \mathrm{I}} \tau_{\mathrm{ik}}(\mathbf{p}^D)$. Thus some of consumers in the world economy have cheaper points when each of their income level is $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Let I^1 be the set of such consumers. I^1 is non-empty. By lemma 1, $\mathbf{x}^D_{\mathrm{ik}}$ of every consumer in I^1 satisfies the demand condition with his income equal to $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Thus for (i,k) $\in \mathrm{I}^1$, $\mathbf{x}'_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}^D_{\mathrm{ik}})$ implies $\mathbf{p}^D \cdot \mathbf{x}'_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D)$. Then $\widetilde{\mathbf{x}}_{\mathrm{ik}} \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}^D_{\mathrm{ik}})]$ also implies $\mathbf{p}^D \cdot \widetilde{\mathbf{x}}_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D)$ for them. Let I^2 be the complement of I^1. For (h,j) $\in \mathrm{I}^2$, $\mathbf{x}_{\mathrm{hj}} \in \mathbf{C}^{\mathrm{T}}_{\mathrm{hj}}$ implies $\mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{hj}} = \tau_{\mathrm{hj}}(\mathbf{p}^D)$. Suppose that I^2 is non-empty. Then, for (h,j) $\in \mathrm{I}^2$,

$$\tau_{\mathrm{hj}}(\mathbf{p}^D) = \mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{hj}} = \mathbf{p}^D \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{hj}}. \tag{19}$$

Since $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ is a feasible allocation in the world economy, the strong irreducibility assumption (T–11) implies that there are $(\widetilde{\mathbf{x}}^D_{\mathrm{ik}})_{(\mathrm{i,k})\in \mathrm{I}^1} \in \sum_{(\mathrm{i,k})\in \mathrm{I}^1} \mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$, $\widetilde{\mathbf{y}}^D \in \mathbf{Y}^{\mathrm{T}}$, and $\widetilde{\mathbf{z}}^D \in \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{C}^{\mathrm{T}}_{\mathrm{hj}}$ satisfying

$$\widetilde{\mathbf{x}}^D_{\mathrm{ik}} \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}^D_{\mathrm{ik}})] \text{ for (i,k)} \in \mathrm{I}^1 \text{ and}$$
$$\sum_{(\mathrm{i,k})\in \mathrm{I}^1} \widetilde{\mathbf{x}}^D_{\mathrm{ik}} = \widetilde{\mathbf{y}}^D - \alpha \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \widetilde{\mathbf{z}}_{\mathrm{hj}} \text{ for } \alpha > 0.$$

For (i,k) $\in \mathrm{I}^1$, $\mathbf{p}^D \cdot \widetilde{\mathbf{x}}^D_{\mathrm{ik}} > \tau_{\mathrm{ik}}(\mathbf{p}^D) = \mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{ik}}$ or $\mathbf{p}^D \cdot (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) > 0$ holds. Thus

$$\mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) > 0 \tag{20}$$

holds. Then

$$\begin{aligned}
\mathbf{p}^D \cdot (\widetilde{\mathbf{y}}^D - \mathbf{y}^D) &= \mathbf{p}^D \cdot \Big(\sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) + \sum_{(\mathrm{h,j})\in \mathrm{I}^2} (\widetilde{\mathbf{z}}_{\mathrm{hj}} - \mathbf{x}^D_{\mathrm{hj}})\Big) \\
&= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) + \mathbf{p}^D \cdot \sum_{(\mathrm{h,j})\in \mathrm{I}^2} (\widetilde{\mathbf{z}}_{\mathrm{hj}} - \mathbf{x}^D_{\mathrm{hj}}) \\
&= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) + \Big(\sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{p}^D \cdot \widetilde{\mathbf{z}}_{\mathrm{hj}} - \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{p}^D \cdot \mathbf{x}^D_{\mathrm{hj}}\Big) \\
&= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) + \Big(\sum_{(\mathrm{h,j})\in \mathrm{I}^2} \mathbf{p}^D \cdot \widetilde{\mathbf{z}}_{\mathrm{hj}} - \sum_{(\mathrm{h,j})\in \mathrm{I}^2} \tau_{\mathrm{hj}}(\mathbf{p}^D)\Big) \\
&= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) + \sum_{(\mathrm{h,j})\in \mathrm{I}^2} (\mathbf{p}^D \cdot \widetilde{\mathbf{z}}_{\mathrm{hj}} - \tau_{\mathrm{hj}}(\mathbf{p}^D)) \\
&= \mathbf{p}^D \cdot \sum_{(\mathrm{i,k})\in \mathrm{I}^1} (\widetilde{\mathbf{x}}^D_{\mathrm{ik}} - \mathbf{x}^D_{\mathrm{ik}}) > 0 \qquad (21)
\end{aligned}$$

follows from (19) and (20). On the other hand,

$$\mathbf{p}^D \cdot (\widetilde{\mathbf{y}}^D - \mathbf{y}^D) \leq 0 \qquad (22)$$

follows from the profit maximization of $\mathbf{y}^D$ at $\mathbf{p}^D$. (22) is, however, a contradiction to (21). Thus I^2 must be empty and hence every consumer in the world economy has a cheaper point than $\tau_{\mathrm{ik}}(\mathbf{p}^D)$. Then lemma 1 implies that $\mathbf{x}^D_{\mathrm{ik}}$ becomes a demand point with $\mathbf{p}^D$ for every consumer in the world economy. $(\mathbf{x}^D_{\mathrm{ik}}, \mathbf{y}^D_{\mathrm{k}}, \mathbf{p}^D)_{(\mathrm{i,k})\in \mathrm{I}}$ therefore is a competitive equilibrium with domestic income transfers under world free trade. Since $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$ is affordable to each consumer in the world economy and $\mathbf{x}^D_{\mathrm{ik}}$ is a demand point to him, he is as well off with $\mathbf{x}^D_{\mathrm{ik}}$ than with $\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}$. Thus, in this dynamic world free trade economy over a discrete-time infinite horizon, all consumers in the worlds economy is as well off under world free trade with grandmont-McFadden's domestic income transfers than under autarky. This world free trade is therefore actually preferable to autarky.

Appendix C:Autarky Economy

Since Grandmont–McFadden's result on gains from trade assumes the existence of competitive equilibrium under autarky in each country in a dynamic world economy over a discrete-time infinite horizon, this appendix lists the conditions which yield such a competitive equilibrium under autarky in each country.[43] We assume that there are K number of countries in the world economy and $\mathrm{I_k}$ number of infinitely-lived consumers in each k-th country for $\mathrm{k} \in \mathrm{J} = \{1, \cdot\cdot, \mathrm{K}\}$. We also assume that N is the number of goods available for the dynamic world economy in each period. For simplicity, we assume that N is constant over time. N number of goods is available under world free trade. But some of them may be unavailable to k-th country under the state of autarky. Let $N^{\mathrm{A}}_{\mathrm{k}}$ be the set of goods available to k-th country under the state of autarky. For simplicity, we assume $N^{\mathrm{A}}_{\mathrm{k}}$ constant over time. The elements of coordinates in $N \backslash N^{\mathrm{A}}_{\mathrm{k}}$ are all zero in actual consumptions and productions of in k-th country under autarky. Consumption sets and production set available under autarky are thus smaller than those under world free trade. Then, for those coordinates in $N \backslash N^{\mathrm{A}}_{\mathrm{k}}$, the requirement of the aggregate adequacy assumption used in the case of general competitive economies over a discrete-time infinite horizon does not hold. In this situation, we need to restrict the underlying economic data of k-th country defined originally in l_∞ to its subset l'_∞, where l'_∞ is defined as $\mathbf{z} \in l'_\infty$ implies $\mathbf{z}_{\mathrm{i}} = \mathbf{0}$ for $\mathrm{i} \in N \backslash N^{\mathrm{A}}_{\mathrm{k}}$.

Let ω_{ik} and $\mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}(\subset (l'_\infty)^+)$ be an initial endowment and an consumption set of the i-th consumer in the k-th country available under autarky. Let $\mathbf{P}_{\mathrm{ik}}$ be preference of the i-th consumer in the k-th

[43] For the details on this appendix, in particular, the proofs, see Kubota(1998).

country restricted on $\mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$.[44] Also let $\mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$ is a production set available to this country under state of autarky.[45] A vector $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ is called an allocation under autarky if $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ for all $\mathrm{i} \in \mathrm{I}_{\mathrm{k}}$ and $\mathbf{y}_{\mathrm{k}} \in \mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$ hold. It is also called feasible if $\sum_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}} \mathbf{x}_{\mathrm{ik}} = \mathbf{y}_{\mathrm{k}} + \omega_{\mathrm{k}}$, where ω_{k} is the aggregate initial endowment of this country, $\sum_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}} \omega_{\mathrm{ik}}$. A *competitive equilibrium under autarky* is a vector $(\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}, \mathbf{y}^{\mathrm{A}}_{\mathrm{k}}, \mathbf{p}^{\mathrm{A}}_{\mathrm{k}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ with $\mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \in ba'\backslash\{\mathbf{0}\}$ satisfying

(1) $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ is a feasible allocation under autarky.

(2) $\mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \mathbf{x}^{\mathrm{A}}_{\mathrm{ik}} \le \mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \omega_{\mathrm{ik}}$ and $\mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \mathbf{x}_{\mathrm{ik}} > \mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \omega_{\mathrm{ik}}$ holds for $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}})$.[46]

(3) $\mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \mathbf{y}^{\mathrm{A}}_{\mathrm{k}} = 0$ and $\mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \mathbf{y}_{\mathrm{k}} \le 0$ holds for $\mathbf{y}_{\mathrm{k}} \in \mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$.

We use the following assumptions for the existence of competitive equilibrium under autarky of the k-th country.

(A–1) $\mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$ is a non-empty convex cone with vertex at $\mathbf{0}$.

(A–2) $\mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$ is closed with respect to the weak* $\sigma(l_\infty, l_1)$-topology.

(A–3) For $\mathbf{y}_{\mathrm{k}} \in \mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$, there are T and M such that for $t > T$ there is $\overline{\mathbf{y}^t}_{\mathrm{k}} \in \{\mathbf{z} \in \mathbf{c}'_0(\equiv \mathbf{c}_0 \cap l_\infty{}') : ||\mathbf{z}||_\infty \le M\}$ satisfying $\widetilde{\mathbf{y}^t_{\mathrm{k}}} = \overline{\mathbf{y}}_{\mathrm{k}}(t) + \widehat{\overline{\mathbf{y}^t}}(t) = (\mathbf{y}_{\mathrm{k}1}, \cdot\cdot, \mathbf{y}_{\mathrm{k}2}, \overline{\mathbf{y}^t}_{\mathrm{k}t+1}, \cdot\,\cdot) \in \mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$.

(A–4) $\mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ is a non-empty subset of $(l_\infty{}')^+$ and closed in the weak* $\sigma(l_\infty, l_1)$-topology. $\omega_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ holds.

(A–5) For $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$, there is T such that for $t > T$, there is $\overline{\mathbf{x}^t}_{\mathrm{ik}} \in \{\mathbf{z} \in (l'_\infty)^+ : \mathbf{z} \le \omega_{\mathrm{ik}}\}$ satisfying $\widetilde{\mathbf{x}^t}_{\mathrm{ik}} = \mathbf{x}_{\mathrm{ik}}(t) + \widehat{\overline{\mathbf{x}^t}}_{\mathrm{ik}}(t) = (\mathbf{x}_{\mathrm{ik}_1}, \cdot\,\cdot, \mathbf{x}_{\mathrm{ik}_t}, \overline{\mathbf{x}^t}_{\mathrm{ik}t+1}, \cdot\,\cdot) \in \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$.

(A–6) For $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$, $\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$ and $\mathbf{P}^{-1}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$ are open in $\mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ with respect to the weak* $\sigma(l_\infty, l_1)$-topology, and $\mathbf{x}_{\mathrm{ik}} \notin co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})]$ holds.[47]

(A–7) $\mathbf{x}_{\mathrm{ik}} \in \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ and $\mathbf{x}'_{\mathrm{ik}}(\in (l'_\infty)^+) > \mathbf{x}_{\mathrm{ik}}$ imply $\mathbf{x}'_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$.

(A–8) $(\mathbf{Y}^{\mathrm{A}}_{\mathrm{k}} + \omega_{\mathrm{k}}) \cap (l'_\infty)^+$ is $||\cdot||_\infty$-bounded.

(A–9) $(\mathbf{Y}^{\mathrm{A}}_{\mathrm{k}} + \omega_{\mathrm{k}} - \mathbf{C}^{\mathrm{A}}_{\mathrm{k}}) \cap \{\mathbf{z} \in \mathbf{s}'^+ : \mathbf{z} \ge r \cdot \mathbf{e}^{\mathrm{A}}_{\mathrm{k}}$ for some $r > 0\} \ne \emptyset$, where $\mathbf{C}^{\mathrm{A}}_{\mathrm{k}} = \sum_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}} \mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ and $\mathbf{e}^{\mathrm{A}}_{\mathrm{k}}$ is the restriction of $\mathbf{e}$ over l'_∞.

(A–10) Whenever $\{\mathrm{I}^1_{\mathrm{k}}, \mathrm{I}^2_{\mathrm{k}}\}$ is a non-trivial partition of I_{k} and $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ is a feasible allocation under autarky, then there exist $(\widetilde{\mathbf{x}}_{\mathrm{ik}})_{\mathrm{i}\in\mathrm{I}^1_{\mathrm{k}}}$, $\widetilde{\mathbf{y}}_{\mathrm{k}} \in \mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$ and $\widetilde{\mathbf{z}}_{\mathrm{k}} \in \sum_{\mathrm{h}\in\mathrm{I}^2_{\mathrm{k}}} \mathbf{C}^{\mathrm{A}}_{\mathrm{hk}}$ satisfying

$$(\mathbf{x}_{\mathrm{ik}} + \widetilde{\mathbf{x}}_{\mathrm{ik}}) \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})] \text{ for } \mathrm{i} \in \mathrm{I}^1_{\mathrm{k}} \text{ and}$$
$$\sum_{\mathrm{i}\in\mathrm{I}^1_{\mathrm{k}}} \widetilde{\mathbf{x}}_{\mathrm{ik}} = \widetilde{\mathbf{y}}_{\mathrm{k}} - \mathbf{y}_{\mathrm{k}} + \sum_{\mathrm{h}\in\mathrm{I}^2_{\mathrm{k}}} \alpha_{\mathrm{hk}}(\omega_{\mathrm{hk}} - \widetilde{\mathbf{z}}_{\mathrm{hk}}) \text{ for some } \alpha_{\mathrm{hk}} > 0.$$

Under these assumptions, we have the following theorem on the existence of competitive equilibrium under autarky with prices in $l'_1\backslash\{\mathbf{0}\}$.

Theorem A : *Under* (A–1), $\cdots$, (A–9), *and* (A–10), *the k-th country has a competitive equilibrium under autarky with prices in* $l'_1\backslash\{\mathbf{0}\}$.

[44] If $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}}$ is his consumption set available under world free trade, then $\mathbf{C}^{\mathrm{A}}_{\mathrm{ik}}$ is defined as $\mathbf{C}^{\mathrm{T}}_{\mathrm{ik}} \cap l_\infty{}'$.

[45] If $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}}$ is a production set available to this country under world free trade, then $\mathbf{Y}^{\mathrm{A}}_{\mathrm{k}}$ is defined by $\mathbf{Y}^{\mathrm{T}}_{\mathrm{k}} \cap l_\infty{}'$.

[46] When only second part of (2) is replaced with $\mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \mathbf{x}_{\mathrm{ik}} \ge \mathbf{p}^{\mathrm{A}}_{\mathrm{k}} \cdot \omega_{\mathrm{ik}}$ for $\mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}})$ and the others are same, then $(\mathbf{x}^{\mathrm{A}}_{\mathrm{ik}}, \mathbf{y}^{\mathrm{A}}_{\mathrm{k}}, \mathbf{p}^{\mathrm{A}}_{\mathrm{k}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ is called a quasi-equilibrium under autarky.

[47] Note $\mathbf{P}^{-1}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$ is defined as $\{\mathbf{x}'_{\mathrm{ik}} \in \mathbf{C}_{\mathrm{ik}} : \mathbf{x}_{\mathrm{ik}} \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}'_{\mathrm{ik}})\}$. Also $\mathbf{R}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})$, the weakly preferred set to $\mathbf{x}_{\mathrm{ik}}$, is defined as $[\mathbf{P}^{-1}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})]^c$.

This competitive equilibrium price is in l_1^+ because of periodwise monotonicity of preferences (A–7). Note also an allocation $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ under autarky is called (weakly) Pareto optimal when there is no feasible allocation under autarky which is (strongly) Pareto improving to $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$. The associated theorem implies that the allocation of a competitive equilibrium under autarky is weakly Pareto optimal under autarky.[48] Moreover, when preferences satisfy weak desirability of present consumption, *i.e.*, $\mathbf{x}_{\mathrm{ik}} \in \mathbf{R}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}})$ and $\delta > 0$ imply $\mathbf{x}_{\mathrm{ik}} + \delta\mathbf{e}_1 \in \mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}})$, then allocation $(\mathbf{x}_{\mathrm{ik}}^{\mathrm{A}}, \mathbf{y}_{\mathrm{k}}^{\mathrm{A}})_{\mathrm{i}\in\mathrm{I}}$ of a competitive equilibrium under autarky is Pareto optimal under autarky.

A similar result as as the above also holds when (A-10) is replaced with the following strong irreducibility assumption (A-11).

(A–11): Whenever $\{\mathrm{I}_{\mathrm{k}}^1, \mathrm{I}_{\mathrm{k}}^2\}$ is a non-trivial partition of I_{k} and $(\mathbf{x}_{\mathrm{ik}}, \mathbf{y}_{\mathrm{k}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}}$ is a feasible allocation under autarky, then there exist $(\widetilde{\mathbf{x}}_{\mathrm{ik}})_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}^1} \in \sum_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}^1} \mathbf{C}_{\mathrm{ik}}^{\mathrm{A}}$, $\widetilde{\mathbf{y}}_{\mathrm{k}} \in \mathbf{Y}_{\mathrm{k}}^{\mathrm{A}}$ and $\widetilde{\mathbf{z}}_{\mathrm{k}} \in \sum_{\mathrm{h}\in\mathrm{I}_{\mathrm{k}}^2} \mathbf{C}_{\mathrm{hk}}^{\mathrm{A}}$ satisfying

$$\widetilde{\mathbf{x}}_{\mathrm{ik}} \in co[\mathbf{P}_{\mathrm{ik}}(\mathbf{x}_{\mathrm{ik}})] \text{ for } \mathrm{i} \in \mathrm{I}_{\mathrm{k}}^1 \text{ and}$$
$$\sum_{\mathrm{i}\in\mathrm{I}_{\mathrm{k}}^1} \widetilde{\mathbf{x}}_{\mathrm{ik}} = \widetilde{\mathbf{y}}_{\mathrm{k}} - \sum_{\mathrm{h}\in\mathrm{I}_{\mathrm{k}}^2} \widetilde{\mathbf{z}}_{\mathrm{hk}}.$$

Note that we assume that agents of k-th country do not take a regime switch from autarky to free trade in favor of them and they take this as an additional constrain to their behavior. We assume that each of two regime starts from the first period and continue once it is adopted. Then, we compare entire two consumption streams in the arguments on the dynamic gains from trade. One of these consumption streams is obtained once autarky policy is chosen from the first period and the other is obtained once free trade policy is chosen from the first period. The agents are assumed to behave under the constraint that the coordinates of $N\backslash N_{\mathrm{k}}^{\mathrm{A}}$ are zero. Then, their decision become irrelevant to the prices of the unavailable goods associated with $N\backslash N_{\mathrm{k}}^{\mathrm{A}}$. By deleting these unavailable goods and rearranging the indices for available goods, we could transform this autarky economy defined in l'_∞ into an economy in $l_\infty^{N_{\mathrm{k}}^{\mathrm{A}}}$. We can identify the autarky economy in l'_∞ and the associated economy in $l_\infty^{N_{\mathrm{k}}^{\mathrm{A}}}$. The above assumptions also hold in the economy transformed into $l_\infty^{N_{\mathrm{k}}^{\mathrm{A}}}$ as well. Thus the transformed economy in $l_\infty^{N_{\mathrm{k}}^{\mathrm{A}}}$ satisfies the hypotheses of theorem A. The economy in $l_\infty^{N_{\mathrm{k}}^{\mathrm{A}}}$ has a competitive equilibrium with price $\mathbf{p}_{\mathrm{k}}^{\mathrm{A}}$ in $(l_1^{N_{\mathrm{k}}^{\mathrm{A}}})^+\backslash\{\mathbf{0}\}$.

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