Evaluation of interfacial strength between fiber and matrix based on cohesive zone modeling

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Abstract

This paper presents a measurement technique of interfacial strength considering non-rigid bonding on a fiber/matrix interface modeled as a cohesive surface. By focusing on the stress concentration near a fiber crack obtained from a single-fiber fragmentation test, the stress contours in matrix observed by photoelasticity can be related to the interfacial strength by defining a characteristic length. An equation expressing the relationship between the characteristic length on the stress contour and the interfacial strength was derived, and validated using finite element analysis. The primary advantage of proposed measurement technique is that only a single fiber crack, which usually occurs within elastic deformation of matrix, is required for the evaluation of interfacial strength, whereas saturated fiber fragmentation is necessary in the conventional method. Herein, a sample application was demonstrated using a single carbon fiber and epoxy specimen, and an average interfacial strength of 23.8 MPa was successfully obtained.
Keywords
B. Interface/interphase, B. Fragmentation, B. Fiber/matrix bond, C. Cohesive interface modelling

1. Introduction
The interface between fiber and matrix plays an important role in the overall load-bearing performance of the composite structure. In particular, the bonding quality of the interface determines the stress transfer from the matrix to the fiber and vice versa [1, 2]. The bonding quality of the interface has been assessed through the development of interface models. An early model representing stress transfer at an interface was introduced by Cox and Kelly-Tyson [3, 4]. Kim et al. then modified this model by introducing the bonding quality parameter, i.e., the interfacial strength (\( t_o \)) was regarded as the stress required to initiate an interfacial crack [5]. Moreover, the bonding quality has also been determined using energy-based approaches [6–8].

The similarity between the abovementioned models is that the interface is assumed to be a two-dimensional surface with a rigid bonding condition. However, recent studies have shown that the interface is, in fact, a three-dimensional thin layer (also called an interphase) having mechanical properties that are different from those of both the fiber and the matrix [9–11]. Several studies have indicated that non-rigid bonding is formed at the interface regardless of the strength of the bonding condition [12–14]. Therefore, earlier models may have evaluated the bonding quality inaccurately [15, 16].

Recently, a surface-based cohesive model that defines the interface as a non-rigid bond has been attracting attention [17–19]. It provides an improved interpretation of the
real interface condition by introducing the traction ($t$) - separation ($\delta$) curve. However, characterization of the maximum traction, which is equivalent to $t_o$, from the $t$ - $\delta$ curve has not been well established because it is difficult to experimentally measure the $t$ - $\delta$ curve along the interface [20]. Among familiar experimental methods of push-out testing [21], micro bond testing [22], and single fiber fragmentation testing (SFFT) [23], SFFT is appropriate to consider the interface as a non-rigid bond because it replicates actual stress transfer in real fiber/matrix composites. It also has an advantage of the easier preparation of specimens [24, 25]. Our group previously proposed a method based on SFFT for evaluating $t_o$ without requiring the $t$ - $\delta$ curve observation [26]. It utilizes contours of principal stress difference ($\Delta \sigma$) in a matrix by defining a characteristic length ($L_t$), which can be obtained directly via photoelastic analysis [27-29]. Moreover, our method requires only a single fiber crack in the specimen whereas conventional SFFT requires complete fiber cracks generated until saturation. Thus, the influence of plastic deformation on the matrix can be significantly reduced.

This paper introduces a theoretical analysis of $\Delta \sigma$ contours to establish a novel technique of evaluating $t_o$ at the debonding interface. First, an equation showing the relationship between the $\Delta \sigma$ contour and $t_o$ is derived from the stress distribution of the matrix near the fiber crack during SFFT. FEA is then conducted to examine the derived equation. The application of $L_t$ for the measurement of $t_o$ is finally demonstrated using a carbon fiber/epoxy sample. A photoelastic image representing the $\Delta \sigma$ concentration in the matrix is analyzed by an image processing technique [30], to determine $t_o$ from the $L_t$ value using the derived equation.

2. Theory

2.1. Surface-based cohesive model for interface
Strain ($\varepsilon_a$) was imposed on a SFFT specimen consisting of a single fiber surrounded by a matrix to initiate fiber fragmentation, as shown in Figs. 1a and 1b. Shear traction ($t_s$) and separation ($\delta_s$) developed in the interface soon after a fiber crack was generated, and then increased in response to the applied $\varepsilon_a$. When the relationship between $t_s$ and $\delta_s$ is assumed to be simply expressed by the $t_s-\delta_s$ curve as shown in Fig. 2, three interfacial properties can be defined: interfacial stiffness ($K_o$), $t_o$, and the interfacial fracture toughness ($G_c$). $K_o$ is a parameter that relates $t_s$ and $\delta_s$ in a bonding condition, where

$$t_s = K_o \delta_s.$$  

(1)

The real interface condition can be more suitably represented by the definition of $K_o$ than the assumption of rigid bonding, because it implies that the interface can undergo separation even though $t_s$ does not exceed $t_o$. Here, $t_o$ is defined as the $t_s$ required to initiate the debonding process, represented by degradation of $K_o$. Further, $G_c$, which can be obtained from the total area below the $t_s-\delta_s$ curve, is defined as the energy release rate required to cause debonded interface.

The position on the interface where $t_s$ reaches a maximum can be found by examining the stress distribution in the matrix. Although it is difficult to observe the $t_s$ distribution along the interface directly, $t_o$ can be obtained by observing the $\Delta \sigma$ contour near the interface. It simply corresponds to the point of maximum $\Delta \sigma$ because $\Delta \sigma$ is a representative of principal shearing stress. Therefore, $\Delta \sigma$ can be an indicator of the maximum $t_s$, which is equal to $t_o$.

In order to confirm the relationship between $t_s$ and the $\Delta \sigma$ contour, FEA using Abaqus 6.14 was conducted for an axisymmetric model, as shown in Fig. 3. It represents a region near a fiber crack, indicated by the dashed line in Fig. 1b. The single
fiber and matrix were assumed to be linearly elastic materials, because the first fiber crack usually appears in the elastic range of the matrix. The mechanical properties of the fibers and matrices used in the FEA are shown in Table 1. The model geometry and the number of elements are shown in Table 2. Three sample cases with the different interfacial properties shown in Table 3 were examined. The range of \( t_o \) was determined based on previous study [9]. The values of \( G_c \) and \( K_o \) were suitably selected so that FEA calculation converges under \( \varepsilon_a \) of less than 3%, which is comparable to experimental results. Note that stress state of matrix for SFFT can be represented by axisymmetric model because of low volume fraction of fiber \((a << b)\) even though the actual cross section of SFFT specimen is not axisymmetric [26, 31].

Fig. 4a shows the \( t_s \) and \( d\sigma /dz \) distribution along the interface obtained from a simulation result for a carbon fiber-hard epoxy composite with case-1 interfacial properties in Table 3. The \( t_s \) becomes maximum value of 30 MPa, which is identical to \( t_o \) defined in case-1, at the boundary \((z_o)\) between the bonded and debonding regions. It almost linearly decreases in the bonded region, resulting in almost constant \( d\sigma /dz \) near the fiber crack. The \( \Delta\sigma \) contours in the matrix shown in Fig. 4b clearly show that \( t_o \) causes a highly concentrated \( \Delta\sigma \) near the fiber crack. The black points on each contour in Fig. 4b indicate the positions located farthest from the interface, represented by the maximum radius \((r_{max})\). Subtracting the fiber radius \((a)\), it can be used as an indicator of \( t_o \) since our previous work found a linear relationship between them [26]. Therefore, it was defined as a characteristic length, \( L_t \).

\[
L_t = r_{max} - a
\]  

(2)  

Among the contours in Fig. 4b, those with a \( \Delta\sigma \) of 50 MPa or higher are ideal to find the maximum \( t_s \), because the location of \( L_t \) corresponds well to the \( z_o \). The \( L_t \)-based
approach is not effective when the Δσ contours are far from the interface because the
maximum radius on contours gradually shift to the left due to the edge effects of the
calculation area.

2.2. Equation relating $L_t$ and $t_o$

The equation relating $L_t$ and $t_o$ is derived by the Δσ equation from stress state in the
bonding region near the interface ($z \equiv z_0$) where the deformations of matrix is linearly
elastic. It is expressed by the axial ($\sigma_z$), radial ($\sigma_r$), and shear stress on the rz axis ($\tau_{rz}$).

$$\frac{\Delta \sigma}{2} = \sqrt{\left(\frac{1}{2}(\sigma_z - \sigma_r)\right)^2 + \tau_{rz}^2} \quad (3)$$

In an axisymmetric model of the bonding region, the above stresses are related with
hoop stress ($\sigma_\theta$) and axial displacement ($u_z$) by equilibrium and the stress-strain relations
for perfectly elastic and isotropic matrices.

$$\frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} + \frac{\partial \tau_{rz}}{\partial r} = 0 \quad (4)$$

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{1}{r}(\sigma_r - \sigma_\theta) = 0 \quad (5)$$

$$\frac{\partial u_z}{\partial z} = \frac{1}{E} \left[ \sigma_z - v(\sigma_r + \sigma_\theta) \right] \quad (6)$$

$$\frac{\partial u_z}{\partial r} = \frac{2(1 + v)}{E} \tau_{rz} \quad (7)$$

where $E$ and $v$ are the elastic modulus and Poisson ratio, respectively. It should be noted
that the radial displacement ($u_r$) was assumed to be zero near the interface because $a$ is
very small compared with the matrix radius ($b$). The boundary conditions of the matrix
model shown in Fig. 3 are expressed as

$$\tau_{rz}(a, z) = t_z; \tau_{rz}(b, z) = 0, \quad (8)$$

$$\sigma_r(a, z) = -q ; \sigma_r(b, z) = 0, \quad (9)$$
\[ u_z(b, z) = \varepsilon_a z, \] (10)

where \( q \) is the stress due to the Poisson ratio difference of the fiber and matrix. This can be calculated from the following equation [32].

\[
q = \frac{\varepsilon_a (v_m - v_f)}{\left(\frac{1 - v_f - 2v_f^2}{E_f} + \frac{1 + v_m}{E_m}\right)},
\] (11)

where \( f \) and \( m \) refer to the fiber and matrix, respectively.

Intensive studies have been conducted by Zhao et al. and Nair et al. regarding the decay function of \( \tau_{rz} \) along \( r \) axis [31, 33]. These studies have shown that, for a very low fiber-volume fraction model \((a \ll b)\) such as in the case of an SFFT specimen, the simplest \( \tau_{rz} \) that satisfies Eq. 4 and the boundary condition of Eq. 8 became the following equation.

\[
\tau_{rz} = \frac{a}{r} t_s
\] (12)

Here, the \( t_s \) is assumed to be a linear function of \( z \) based on the FEA result shown in Fig. 4a. Considering a constant value of \( dt_s/dz \) near the fiber crack, the radial and hoop stresses can be obtained from Eq. 5 with the boundary conditions of Eq. 9 under the assumption of uniform axial stress along \( z \) direction and low fiber-volume fraction \((a \ll b)\).

\[
\sigma_r = -\frac{a^2}{r^2} q
\] (13)

\[
\sigma_\theta = \frac{a^2}{r^2} q + a \frac{dt_s}{dz}
\] (14)

The \( \sigma_z \) is derived by obtaining \( u_z \) from Eqs. 6 and 7, and then substituting the stresses of Eqs. 12, 13, and 14.

\[
\sigma_z = E_m \varepsilon_a + a \frac{dt_s}{dz} \left( v_m + 2(1 + v_m) \ln \frac{r}{b} \right)
\] (15)
It is reasonable that the $\sigma_r$, $\sigma_\theta$, and $\sigma_z$, are only functions of $r$ (uniform along $z$ direction) because they actually have little effect to the overall stress state compared with shear stress. Substituting Eqs. 12, 13, and 15 into Eq. 3, $t_s$ can be expressed as a function of $\Delta \sigma$ and the distance from the interface to the $\Delta \sigma$ contour ($r$).

$$t_s = \frac{r}{2a} \sqrt{\Delta \sigma^2 - \left(E_m \varepsilon_a + a \frac{dt_s}{dz} \left(v_m + 2(1 + v_m) \ln \frac{r}{b} + \frac{a^2}{r^2} q \right)\right)^2}$$ (16)

Focusing on the maximum radius on a $\Delta \sigma$ contour near the interface, where $r = L_t + a$ and $t_s = t_o$, $t_o$ can be expressed in terms of $L_t$.

$$t_o = \frac{(L_t + a)}{2a} \sqrt{\Delta \sigma^2 - \left(E_m \varepsilon_a + a \frac{dt_s}{dz} \left(v_m + 2(1 + v_m) \ln \frac{L_t + a}{b} + \frac{a^2}{(L_t + a)^2} q \right)\right)^2}$$ (17)

Further, the constant $dt_s/dz$ parameter must be obtained to apply Eq. 17 for evaluating $t_o$, but it is difficult to theoretically derive the value. Therefore, FEM results were utilized to estimate $dt_s/dz$ parameter. As an example of $t_s$ and $dt_s/dz$ distributions along the interface shown in Fig. 4a, $dt_s/dz = -249$ MPa/mm at a point of $t_o = 30$ MPa was obtained by approaching from the bonding region. The $L_t$ values corresponding to $\Delta \sigma$ contours between 40 and 65 MPa were then measured and substituted into Eq. 17 in order to obtain $t_o$. The same procedure was repeated for other parameters in Table 3.

Results were calculated for both the carbon fiber/hard epoxy and glass fiber/soft epoxy models, and are plotted in Figs. 5 and 6, respectively. The dashed lines indicate the actual $t_o$ input in the FEA, whereas the data-points indicate the estimated $t_o$ using Eq. 17 with $L_t$ and $dt_s/dz$ values. Measurements excluding $dt_s/dz$ effect ($dt_s/dz = 0$) are also plotted here, because it has been neglected in a large number of theoretical analyses due to simplification [5–10]. These figures clearly show that more accurate $t_o$ values are obtained when $dt_s/dz$ is included in the equation. Specifically, $t_o$ values are
overestimated when $dt_s/dz$ is neglected. These results show that $t_o$ can be accurately
evaluated by measuring $L_t$. Moreover, these findings indicate that the assumptions
applied in the derivation of Eq. 17 are reasonable and yield good agreement with the
FEA results.

2.3. Non-dimensional analysis for practical use

Although Eq. 17 produces good results regarding evaluation of $t_o$, it is not
practically useful if FEA must be conducted to obtain $dt_s/dz$ parameter. Therefore, Eq.
17 was rearranged to a non-dimensional form to identically evaluate $dt_s/dz$ term. Both
sides of Eq. 17 were divided by $\Delta \sigma$ in order to obtain a general form that is independent
of material properties.

$$t' = \frac{L'_t}{a} + \frac{1}{\Delta \sigma}$$ (18)

where $t' = t_o/\Delta \sigma$ and $L' = L_t/a$.

The relationship between $t'$ and $L'$ is plotted in Eq. 18 for both the carbon fiber/hard
epoxy and glass fiber/soft epoxy models. It was found that the relationship is identical
regardless of materials and can be approximated by a simple linear equation, which was
implied in our previous work [26].

$$t_o = \left(0.21 \frac{L_t}{a} + 0.45\right) \Delta \sigma$$ (19)

Through a linear approximation of Eq. 18, expressed by Eq. 19, the $dt_s/dz$ value is no
longer required for the $t_o$ estimation. By selecting the appropriate $\Delta \sigma$ contour, which
should be as close as possible to the interface, and by measuring $L_t$ from the SFFT
experiment, the $t_o$ can be evaluated directly. Thus, Eq. 19 contributes significantly
towards the effective and efficient measurement of $t_o$. Moreover, it can be applied to any
relationship of $t_o - \delta$, curve on the interface since our proposed method only focuses on the location of $z_o$.

### 3. Experiment

#### 3.1. Experimental procedure

SFFT was conducted to estimate $t_o$ by proposed method of $L_t$ measurement. Single carbon fiber (HTS30 3K, TOHO Tenax) and epoxy resin (KONISHI Chemical Co., Ltd.) were prepared to create an SFFT specimen with 2-mm thickness ($h$). The experimental setup consists of a polychromatic light and a microscope with a digital camera attached. A micro-tensile testing machine was located under the microscope so that the specimen was placed at the center of two polarizers. The retarders were also used to create circularly polarized light for the elimination of isoclinic and isochromatic interaction noise. A detailed schematic of the apparatus is shown in Fig. 8.

Stress contours can be observed because epoxy has two refraction indexes, as it is a birefringence material. The presence of two refraction indexes generates relative retardation expressed in the fringe order ($N$). Thus, $N$ indicates $\Delta \sigma$, which is connected to the stress-optic coefficient ($f_\sigma$).

$$\Delta \sigma = \frac{f_\sigma N}{h} \quad (20)$$

First, $f_\sigma$ of the pure-epoxy specimen, meaning no carbon fiber embedded, was obtained from a bending test [34]. $\Delta \sigma$ distribution under bending load and an image of continuous colored band corresponding to $N$ can be simultaneously recorded. $f_\sigma$ was then calculated by using Eq. 20. Next, $\varepsilon_a$ was applied to a specimen of single fiber-embedded epoxy to capture the $\Delta \sigma$ contours near the fiber crack. On a certain $\varepsilon_a$, a fiber crack appeared and caused a $\Delta \sigma$ concentration near the interface. An image of colors
corresponding to $\Delta \sigma$ contours was then captured by the camera through the microscope, and then the colors were extracted and converted to hue-saturation-value (HSV) system values. The conversion of colors to these values eliminates errors in color comparison, so that accurate results can be assured. Finally, $L_t$ was measured from the $\Delta \sigma$ contours near the interface, and applied to Eq. 19 to obtain $t_o$.

3.2. Results and Discussion

Fig. 9a shows captured color image from bending test correspond to $N$ with bending load of 6.2 N. The black color band in the specimen indicates no stress. Focusing on tensile stress distribution in the upper side of specimen, the $\Delta \sigma - N$ curve of the epoxy, shown in Fig. 9b, indicated that the $f_{\sigma}$ of the epoxy specimen was 7.9 MPa.mm. The $E_m$ of 0.67 GPa was also obtained from a tensile test.

On the SFFT, a fiber crack appeared at the $\varepsilon_a$ of a 1.4%, which is still within the elastic range. The color distribution near the fiber crack visualized through an image processing was shown in Fig. 10a. The colors of every pixel related to $N$ values of 2.56, 2.52, and 2.50 were extracted, and then plotted as contours in the $r\theta$ axis, as shown in Fig. 10b. Three contours were selected for the measurement of $t_o$. Through application of Eq. 20, these contours were found to be $\Delta \sigma$ values of 10.1, 9.9, and 9.8 MPa, respectively. These $\Delta \sigma$ values must be corrected to compensate for the axisymmetric effect because the carbon fiber has a circular shape that leads to axisymmetry in the projection. The corrected $\Delta \sigma$ ($\sigma_c$) can be calculated from [27, 29, 35] as

$$\sigma_c = \frac{h(\Delta \sigma - E_m \varepsilon_a)(b - a)}{2\left\{bm - \frac{1}{2}\left(mb + (a + L_t)^2 \ln\left(\frac{m+b}{a+b}\right)\right)\right\}} + E_m \varepsilon_a, \quad (21)$$

where $m$ is obtained from...
\[ m = [b^2 - (L_t - a)^2]^{0.5}. \]  

\( \sigma_c \) values of 32.6, 29.3, and 26.9 MPa were obtained from the calculations for the \( \Delta \sigma \) values of 10.1, 9.9, and 9.8 MPa, respectively. The \( L_t, a, \) and \( \sigma_c \) were measured and substituted into Eq. 19. As a result, the \( t_o \) values of 23.4, 29.3, and 31.5 MPa were finally obtained. The same procedure was repeated for 26 other stress contours from three specimens, and resulted in an average value of 23.8 MPa.

A conventional SFFT that considers the interface as being a rigid bond was also conducted for the same specimens. Fig. 11 shows fragmentation process of fiber on initial and saturated conditions. The images were captured without installing retarders on photoelastic tools in order to observe location of fiber cracks. The analysis of conventional SFFT follows Ref. 36 which clearly explain the procedure. The averaged \( t_o \) value from the conventional SFFT was 33.7 MPa. The detail comparison of \( t_o \) evaluation between our analysis and conventional SFFT analysis is shown in Fig. 12. It is confirmed that \( t_o \) is overestimated unless non-rigid bonding is considered.

Furthermore, our proposed procedure to evaluate \( t_o \) is easier and more straightforward compared to the conventional SFFT, because it requires only measurement of \( L_t \) based on the stress response of the matrix to the interface.

**Conclusion**

A method to evaluate the interfacial strength (\( t_o \)) between fiber and matrix has been developed based on the cohesive damage model. The characteristic length (\( L_t \)) indicating \( t_o \) was introduced and measured from a \( \Delta \sigma \) contour in epoxy matrix. Hence, a theoretical analysis was conducted to obtain the relationship between \( t_o \) and \( L_t \). From a non-dimensional analysis, it was found that the normalized \( t_o (t' \) and \( L_t (L' \) have a
linear relationship independently determined from material properties. A sample application to carbon fiber-epoxy composite was demonstrated to evaluate the proposed technique. A photoelastic analysis in conjunction with an SFFT experiment was conducted to capture the stress contours, clearly visualized through image processing techniques. The calculated result yielded an average \( t_0 \) value of 23.8 MPa, which is almost 30% lower than one obtained from conventional SFFT analysis. The overestimation of conventional method implies the importance of debonding process of the interface.

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**References**


Figure Legends

Fig. 1. Fiber fragmentation process; (a) before and (b) after.

Fig. 2. Shear traction ($t_s$)-separation ($\delta_s$) curve.

Fig. 3. Axisymmetric model of fiber/matrix composite near fiber crack.

Fig. 4. FEA result for carbon fiber-hard epoxy composite with case-1 interfacial properties: (a) $t_s$ and $dt_s/dz$ curves along simulated interface; (b) $\Delta\sigma$ contours in matrix.

The black dots indicate the maximum radius ($r_{max}$) with respect to the $r$-axis.

Fig. 5. Measurement of $t_o$ for carbon fiber-hard epoxy composite. The black and white marks indicate measurements with and without considering $dt_s/dz$ respectively.

Fig. 6. Measurement of $t_o$ for glass fiber-soft epoxy composite. The black and white marks indicate measurements with and without considering $dt_s/dz$ respectively.

Fig. 7. Linear relationship approximation between $t'$ and $L'$.

Fig. 8. Schematic of apparatus used for photoelastic analysis.

Fig. 9. (a) colored band observed under bending load and (b) epoxy stress-fringe order curve.

Fig. 10. (a) color captured near fiber crack using photoelastic technique and (b) plotted $\Delta\sigma$ contours from experiment.

Fig. 11. Fiber cracks appearance on SFFT (a) initial and (b) saturated conditions.

Fig. 12. Comparison of $t_o$ evaluation between conventional SFFT and our experimental analysis.
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Table 1. Mechanical properties of fibers and epoxy used in simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Elastic modulus ($E$)</th>
<th>Poisson ratio ($\nu$)</th>
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<tr>
<td>Carbon fiber(^{a})</td>
<td>240 GPa</td>
<td>0.2</td>
</tr>
<tr>
<td>Glass fiber(^{b})</td>
<td>80 GPa</td>
<td>0.22</td>
</tr>
<tr>
<td>Hard epoxy(^{c})</td>
<td>2 GPa</td>
<td>0.4</td>
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<tr>
<td>Soft epoxy(^{c})</td>
<td>1 GPa</td>
<td>0.4</td>
</tr>
</tbody>
</table>

\(^{a}\)Toho Tenax’s Datasheet, \(^{b}\)ASM handbook vol 21: Composites, \(^{c}\)assumption

Table 2. Model parameters used in simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Fiber radius ($a$)</td>
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</tr>
<tr>
<td>Matrix radius ($b$)</td>
<td>70 µm</td>
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<tr>
<td>Model length ($L$)</td>
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<tr>
<td>Fiber element</td>
<td>10500 els. (7×1500)</td>
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<tr>
<td>Matrix element</td>
<td>82500 els. (55×1500)</td>
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Table 3. Interfacial properties of sample cases examined via simulation.

<table>
<thead>
<tr>
<th>Interfacial properties</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
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<tr>
<td>Interfacial stiffness ($K_o$)</td>
<td>$2 \times 10^4$ MPa/mm</td>
<td>$2 \times 10^4$ MPa/mm</td>
<td>$2 \times 10^4$ MPa/mm</td>
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<tr>
<td>Interfacial strength ($t_o$)</td>
<td>30 MPa</td>
<td>20 MPa</td>
<td>40 MPa</td>
</tr>
<tr>
<td>Interfacial fracture toughness ($G_c$)</td>
<td>0.04 mJ/mm$^2$</td>
<td>0.03 mJ/mm$^2$</td>
<td>0.05 mJ/mm$^2$</td>
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