FLEXURAL BEHAVIOR OF REINFORCED CONCRETE BEAMS REPAIRED WITH ULTRA-HIGH PERFORMANCE FIBER REINFORCED CONCRETE (UHPFRC)

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Abstract

Over the past few decades, premature deterioration of reinforced concrete structures exposed to severe environmental actions and mechanical loading has become a serious problem. Previous studies have shown that the use of ultra-high performance fiber reinforced concrete (UHPFRC) improves the structural response and extends the durability of concrete structures. In this study, the flexural behavior of reinforced concrete beams retrofitted with UHPFRC is investigated and experimental results are compared with 3-D finite element analysis. The experiments were performed on reinforced concrete beams repaired in tension and compression zone, with UHPFRC of varying thicknesses. The flexural strength of repaired beams was investigated by four-point bending test and compared with that of reference beam without repair. Experimental and analytical results indicate that the ultimate flexural strength of RC beams repaired with UHPFRC in tension and compression zone is increased, with the increase of UHPFRC thickness. Thereafter, a parametric study was carried out by using MSC/Marc simulation to investigate the influence of tensile properties of UHPFRC and yield strength of tension steel on the flexural capacity of repaired beams. The investigation shows that the UHPFRC improves stiffness and delay the formation of localized cracks, thus, improving the resistance and durability of repaired beams.

Keywords: fiber reinforced concrete; finite element analysis; flexural strength; failure mode

1. Introduction

In the recent years, sustainable infrastructure is essential for economic growth and prosperity. Sustainable infrastructures are those requiring the minimum intervention during their lifetime. Reinforced concrete structures show poor performance in terms of structural behavior and durability under severe environmental conditions and high mechanical loading [1]. Therefore, the rehabilitation of deteriorated concrete structures is a major problem from the sustainability point of view. The development of new, cost effective repair method and design is essential to extend the life of reinforced concrete structures. Many researchers have used the fiber cementitious materials as one of the promising and cost effective repair method application [2, 3]. The use of these materials result in improvement of tensile and fatigue performance. Several kinds of fiber cementitious materials such as fiber reinforced concrete (FRC), high-performance concrete (HPC) and ultra-high performance fiber reinforced concrete (UHPFRC) have been used to meet the requirement of sustainable infrastructures [4].

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Increasing requirements of load bearing capacity, durability, and safety concern of concrete structures also demand the UHPFRC for repair and maintenance [5].

In general, UHPFRC can be described as a composite material comprised of relatively large proportion of steel fibers, low water-binder ratio and high micro silica content, thus, making the composite with superior characteristics such as self-compacting, very high strength, high modulus of elasticity and extremely low permeability that prevents the ingress of detrimental substances such as water and chloride ions [6-8]. Typical strengths of UHPFRC are of 150 to 200 MPa and 7 to 11 MPa in compression and tension respectively [9, 10]. Moreover, the tensile behavior of this material consists of four domains and high fracture energy. In first domain, stress is increased linearly without any crack formation. In second domain, the stress is again increased with the formation of micro-cracks until tensile strength is reached and distribution of small crack is widened (strain hardening zone). As the tensile strength is reached, localized macro-cracks form and propagate in the third domain. Finally, no more stress is transferred through these localized macro-cracks and final fracture takes place [11]. Because of these properties, UHPFRC has increased resistance against environmental degradation of concrete and high mechanical loading. Thus, UHPFRC is a promising material to significantly improve structural resistance and durability of deteriorated concrete structures.

The present study demonstrates the flexural behavior of reinforced concrete beams retrofitted with UHPFRC and the influence of material parameters on the structural response. For this, finite element analysis is carried out by using a nonlinear FEM software i.e. MSC/Marc to compare the structural behavior with the experimental results. The experimental and analytical results disclose that the ultimate strength and stiffness of reinforced concrete beams, which were repaired in the tension zone, are increased with the increase of UHPFRC thickness. Thereafter, a parametric study is conducted to investigate the influence of tensile properties of UHPFRC on the RC beams repaired with UHPFRC in tension zone, because the cracking strength strongly influences the flexural behavior. For RC beams repaired with UHPFRC in the compression zone, the influence of yield strength is investigated on the structural behavior of composite beam. The investigation demonstrates that the structural performance of UHPFRC can be utilized in rehabilitation and strengthening of existing concrete structures because high compressive and tensile strength helps in lowering the deflections under service conditions.

2. Experimental methodology

2.1 Overview

Four-point bending tests were carried out on beams at different cross-sectional repair positions with the incorporation of UHPFRC under optimized laboratory conditions. Katrin Habel proposed the three basic configurations to exploit the advantageous material properties of UHPFRC in rehabilitation and strengthening of existing concrete structures such as (P) for protection purposes, (PR) that can be designed either only for protection or additionally for an increase in resistance, and (R) for protection and increase of resistance [11]. Therefore, in the present study, two configurations namely: (P) and (PR) were employed for repairing of reinforced concrete (RC) beams. Table 1 lists the experimental cases of cross-sectional repair of upper and lower positions with UHPFRC. The repaired thickness was varied 20 mm, 40 mm, and 60 mm. The repair
thickness of 20 and 40 mm corresponds to the waterproofing of structural element i.e. (P) configuration, while for (PR) configuration, relatively thicker layer of repair (60 mm) was used for strengthening of beams.

2.2 Specimen description

Figure 1(a) shows the proposed geometry of reference beam (B-0) for an experiment without the incorporation of UHPFRC. The length and span of these beams are 3000 mm and 2800 mm respectively. The shear to span ratio of the beams is 2.8 and D10 stirrups are provided at 200 mm intervals. Figure 1(b) presents the cross section of seven different specimens, with the width and height of 250 mm and 400 mm respectively. The repair thickness position for upper and lower repair and specimen designation is illustrated. Two D16 bars are provided both in tension and compression zone in longitudinal direction of the beam. The beam is designed as flexural failure type with the tolerance of 3.1.

2.3 Materials

Ordinary Portland cement, coarse aggregates having particle size ≤ 20 mm as well as fine aggregates of 5 mm were used to prepare the ordinary concrete. The compressive strength and Young’s modulus of the concrete at the start of the experiment were 29.7 MPa and 24.3 GPa. Prior to experiment, RC beams were left for a period of 186 days. The UHPFRC is composed of steel fibers, steel wool, and water reducing agent. The length and diameter of steel fiber was 13 mm and 0.16 mm respectively. The compressive strength and Young’s modulus of UHPFRC after 42 days were 156.3 MPa and 34.6 GPa respectively, while the mechanical properties of UHPFRC and concrete are listed in Table 2. Figure 2 displays the tensile behavior of UHPFRC obtained from the uniaxial tension test on dumbbell specimen with 13 mm thickness. The test was conducted under displacement control at a rate of 0.05 mm/min up to the maximum load, thereafter; displacement rate of 0.5 mm/min was maintained up to the failure of specimen. The tensile stress increases linearly until cracks starts forming within the UHPFRC. The average strain at maximum tensile stress was 4800 µ. Table 3 summarizes the geometric and mechanical properties of longitudinal rebar and shear stirrups obtained by tension test of bare bar. The rebars used in the experiment were D16 SD345 and D10 SD345 with 16 mm and 10 mm diameter respectively.

2.4 Specimen preparation method

The RC beams with ordinary concrete were casted in the steel mould and retarder was sprayed on the concrete surface during casting in order to delay the setting of concrete. Concrete surface was washed out after 24 hours by high-pressure water-jet to expose the aggregate as shown in Figure 3(a). This method of repair has been used to remove damaged concrete surface and UHPFRC was applied in October 2014, on steel girder bridge with 160 mm thick reinforced concrete deck located in Sapporo, Japan [12]. After 144 days, a layer of UHPFRC was casted over the RC beam as shown in Figure 3(b). The UHPFRC of lower repaired beams were also casted after inverting the beam such that lower part of the beam became the top part during casting. For one week, UHPFRC surface was kept under wet condition by spray curing. After curing, the specimens were kept in the laboratory until the time of testing.
2.5 Test setup and loading method

The flexural test was conducted at the age of 186 days. The repaired beam specimen was simply supported on 2800 mm span by round bars and two concentrated line loads were applied at third points through H-shape girder and round bars as shown in Figure 4. The distance between the two concentrated loads was 800 mm. The test was monotonically conducted under load control condition until the failure of specimens or the limitation of jack stroke. The deflection and mechanical strains were recorded at each load step. At each stage of loading, the beam was carefully inspected in order to detect the first crack and record the load corresponding to the first crack.

3. Flexural analysis

The flexural capacity and the load-deflection behavior of the reference and retrofitted beams are determined by flexural model, which is the extension of the commonly used bending design model for reinforced concrete [11]. The moment resistance of composite UHPFRC-concrete element can be calculated based on the distribution of stresses caused by bending. The strain distribution along the height of section is determined based on following assumptions.

1. Plane sections remain plane. In other words, the distribution of strain is linear along the height of beam (Bernoulli hypothesis).
2. The bond between the UHPFRC layer and concrete is perfect and there is no sliding at the interface (monolithic behavior).

The moment resistance of reference and the beams retrofitted with UHPFRC layer under bending are calculated with the model given in Figure 5. Firstly, the strain top concrete fiber and neutral axis depth are assumed. Then, the strain distribution over the cross section is defined to calculate the compressive and tensile stresses in the cross section by using the constitutive laws. The forces are then calculated by multiplying the stresses with the area the cross section and the neutral axis is adjusted until the equilibrium of normal forces is established. When equilibrium is achieved, the bending moment is determined by taking the sum of the moments caused by all forces about a single point. The process is repeated with different strains in the top fiber of concrete to determine the cracking, yielding and maximum moment of the section [13]. Based on calculated moments, the corresponding loads are calculated for the beam under four-point bending.

The deflection calculations are carried out based on the ACI 318 provision for reinforced concrete member. ACI 318 recommends the effective moment of inertia, $I_{eff}$ for deflection calculation. The effective moment of inertia, $I_{eff}$ is calculated using Branson equation and used to compute the deflection after cracking [14, 15].

\[
I_{eff} = \left( \frac{M_{cr}}{dM_a} \right)^3 \times I_{gross} + \left[ 1 - \left( \frac{M_{cr}}{dM_a} \right)^3 \right] \times I_{crack} \tag{1}
\]
Where $M_c$, $M_d$ are the cracking moment and maximum moment in the beam, $I_{ef}$ is the effective moment of inertia, $I_{gros}$ is the gross section moment of inertia, and $I_{crack}$ is the transformed cracked moment of inertia. The simplified load-deflection curves obtained from flexural analysis are compared with the experimental and FEM load-deflection curves in Figure 11.

4. Numerical analysis

4.1. Analytical Model

Finite element analysis is performed by using a nonlinear FEM software i.e. MSC/Marc. The quarter models are adopted, because load conditions are symmetric as shown in Figure 6(a). As for the element, 8-nodes 3D solid elements of type 7 are used for the plate, support, concrete, and UHPFRC. The assumed strain procedure is flagged through the GEOMETRY option. The assumed strain formulation improves the bending behavior. This increases the stiffness assembly costs per element, but it improves the accuracy [16]. The steel reinforcement is idealized using truss elements of type 9 with the node points defined such that each rebar element is sharing common nodes with the concrete solid elements. This approach is called discrete idealization of rebar with concrete. The perfect bond is assumed between reinforcing bar, concrete and UHPFRC. This is justified because the steel fiber leads to increased bond strength, especially in the post-yielding regime. The use of steel fibers also increases the energy and ductility of bond failure [17]. Moreover, the bond slips of deformed bar does not affect much the behavior of reinforced beam before yielding and stress remains below the yielding everywhere besides the vicinity of the crack plane. The reinforcement layout for quarter symmetric beam and rebar area is given in Figure 6(b). The nonlinear material properties of steel are input using von Mises yield criteria. The cross-sectional area is input through GEOMETRY option. Default area is equal to 1.0 [16]. The Buyukozturk model is used for modeling of concrete failure and is defined by the following equations [18].

\[
f = \beta \sqrt{3J_1} + \gamma J_1^2 + 3J_2 - \overline{\sigma}^2
\]

\[
J_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}
\]

\[
J_2 = \frac{1}{2} \sum_{i<j} \sigma_{ij} S_{ij}
\]

\[
S_{ij} = \begin{bmatrix}
\sigma_{11} - \sigma_{33} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} - \sigma_{33} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_{11}
\end{bmatrix}
\]

\[
\sigma_{it} = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33})
\]

Where $\overline{\sigma}$ is the equivalent stress, which is equal to one-third of uniaxial compressive strength and stress invariants are represented by $J_1$ and $J_2$. $\beta$ and $\gamma$ are material constants having values of 0 and 0.2 respectively. The cracking model is defined through the cracking option in MSC/MARC software. The UHPFRC material models are defined as hypo-elastic material by user-subroutine. Sarkar and Haris reported that the bond strength of UHPFRC is greater than normal concrete if proper surface roughness is provided under slant shear test [19]. The bond between UHPFRC and concrete is assumed perfect because the water jetting method of repair is used to expose the aggregate of existing concrete surface. Therefore, the roughness of the
concrete surface is improved and pouring UHPFRC on the rough surface has good bond strength. The model is divided into total 2400 elements for concrete and the meshing in UHPFRC layer is finer than in normal concrete.

The boundary conditions are necessary to get the unique solution of the analytical model. To obtain the same behavior of experimental beam, the analytical must be the constraint at points of symmetry, the supports, and loading point. Since the quarter of the beam is used for the model, the planes of symmetry are required at the internal faces. At the plane of symmetry, the displacement perpendicular to the plane is held zero. Rollers are used to show the symmetry condition at the internal face. The support is modeled as a roller by giving constraint to a single line of nodes on the plate. The load is a force, P/4, applied by load increment of displacement control with 4000 increments. The boundary conditions for quarter symmetric beam are shown in Figure 6(c).

4.2. Material properties and constitutive models

4.2.1. Reinforcing bar

Figure 7 shows the constitutive law of reinforcing bar. In this analysis, linear elasticity is used for \( 0 \leq \sigma_s \leq f_y \), and a linear hardening elastoplasticity model is applied until a specified ultimate strain is reached in tension and compression. The yield strength, \( f_y \), and ultimate strength, \( f_{uw} \), of reinforcing bar SD345-D16 are provided with 386 MPa and 546 MPa respectively. The ultimate strain \( \varepsilon_u \) used in this study is 240000 \( \mu \). As for SD345-D10 reinforcement, the yield strength, \( f_y \), and ultimate strain, \( \varepsilon_u \), are 376 MPa and 280000 \( \mu \) respectively. The Poisson’s ratio and elastic modulus, \( E_c \), of 0.3 and 200 GPa for both steels are used in this study.

4.2.2. Concrete

In this study, the concrete is assumed homogenous and initially isotropic. The uniaxial compressive stress-strain relationship for the concrete model as shown in Figure 8(a) is obtained by using the multi-linear equations for concrete by MacGregor 1992 [20].

\[
\begin{align*}
f_c &= \varepsilon E_c & 0 \leq \varepsilon \leq \varepsilon_c \\
f_c &= \frac{\varepsilon E_c}{1 + \left( \frac{\varepsilon}{\varepsilon_c} \right)^2} & \varepsilon_c \leq \varepsilon \leq \varepsilon_u \\
f_c &= f'_c & \varepsilon_u \leq \varepsilon \leq \varepsilon_{cu} \\
\varepsilon_u &= \frac{f'_c}{E_c}
\end{align*}
\]

The uniaxial compressive strength of concrete, \( f'_c \), and initial modulus of elasticity of concrete, \( E_c \), are input as 29.7 MPa and 24.3 GPa respectively. Under uniaxial compression, the concrete strain, \( \varepsilon_u \), corresponding to the peak stress, \( f'_c \), is usually 0.002. The crushing strain, \( \varepsilon_{cu} \), suggested by ACI committee 318 and used in the analysis is 0.003. The Poisson’s ratio, \( \nu_c \), under uniaxial compressive stress is within the range of 0.15-0.22, with representative value of 0.19 or 0.2. However, in this study, the Poisson’s ratio of concrete is assumed to be 0.2.
The uniaxial tensile strength, \( f_t \) is difficult to measure. For this study, the value is calculated by using following equation.

\[
f_t = 0.23(f'_t)^{1/3}
\]  

(10)

In tension zone, the linear elasticity is used before cracking, i.e. until the tensile strength of concrete, \( f_t \), is reached. When cracking of concrete takes place, a discrete model is used to represent the macro crack behavior. It is known that cracked concrete of RC member can still carry the tensile stress in the direction normal to the crack, which is known as tension stiffening [21]. In this study, a linear softening elastoplasticity model is adopted to model the tension-stiffening phenomenon as shown in Figure 8(b). The softening modulus, \( E_s \) is selected as 2.43 GPa and corresponding strain, \( \varepsilon^* \), at which tension stiffening stress reduces to zero is 0.001.

During the post-cracking stage, the cracked concrete can still transfer the shear forces through aggregate interlocking or shear friction, which is termed as shear retention. The shear retention is modeled by introducing the shear retention factor, which is an input parameter to reduce the shear modulus after cracking. Numerous analytical results have demonstrated that shear retention factor has value between 0 and 1 [22]. The shear retention factor is selected as 0.4 in this analysis.

### 4.2.3. UHPFRC

Figure 9 shows the uniaxial compressive stress-strain relationship for the UHPFRC model, which is modeled as tri-linear as, proposed in AFGC 2013 [23]. The linear-elastic stress rise represents well the test results. The plastic plateau is small or even nonexistent, since the observed stress decrease is strong, and compressive behavior is rather brittle. The 42 days uniaxial compressive strength, \( f_{Uc} \) and initial modulus of elasticity, \( E_{Uc} \) of UHPFRC are 156.3 MPa and 34.6 GPa respectively. The Poisson’s ratio, \( \nu_{Uc} \) for UHPFRC was determined to be 0.22 to 0.24 [23]. However, in this study, the Poisson’s ratio of UHPFRC is assumed 0.22.

The tensile behavior of UHPFRC is modeled in two parts i.e. softening and hardening. A bilinear relation is idealized using the average tensile properties obtained by tension test for strain smaller than \( \varepsilon_{Ut,max} \) as shown in Figure 10(a). The cracking strength \( f_{Ut,1} \) and ultimate tensile strength, \( f_{Ut,max} \), are provided with 7.4 MPa and 10.1 MPa, respectively, obtained by experiment. The average maximum tensile strain, \( \varepsilon_{Ut,max} \), in the strain-hardening domain is 4800 \( \mu \). Beyond hardening at, \( f_{Ut,max} \), the formation of factitious crack with an opening, \( W_{Ut} \), is assumed following a bilinear softening law as shown in Figure 10(b). The crack opening, \( W_{Ut} \), is transformed into the strain by using reference length, \( LR \) [23].

\[
\varepsilon = \varepsilon_{ut} + \frac{W_{Ut}}{LR}
\]  

(11)

Where \( LR \) is the reference length and \( W_{Ut} \) is the factitious crack width. The reference length, \( LR \) is a modeling parameter that is dependent on material properties and geometry of the RC beams. The reference value, \( LR \) is estimated as two third of the total section height by AFGC 2013 and used in the analysis as 266.67 mm. The maximum crack width, \( W_{Ut,2} \), is approximately half of the fiber length, which is 6.5 mm for the fiber of
length 13 mm. The stress and crack width corresponding to point C are assumed to be 3.03 MPa and 2 mm, respectively as shown in Figure 10(b).

5. Parametric study

For the parametric study, the tensile properties of UHPFRC are varied to investigate their influence on structural response. There are two approaches to determine the UHPFRC tensile behavior. In approach-I, the test results of the uniaxial tensile test of UHPFRC are used as input parameters for analytical model and the simulated results are compared with the results of the beam tests. In approach-II, the results of the beam tests are simulated by using the assumed UHPFRC tensile properties. The tensile properties of UHPFRC obtained by the tensile test are overestimated because notch concentrates the failure process into a given section. However, the tensile properties are also reduced due to fiber segregation in UHPFRC, when used as repair material [25]. Therefore, the cracking strength, \( f_{Ut,1} \), and ultimate tensile strength, \( f_{Ut,max} \), are reduced by 10%, 20% and 30% of the reference value in this study.

The yield strength of rebar is reduced due to localized yielding at the crack vicinity and bond effect in the experiment. For smeared idealization, this effect can be taken into account by the following equation based on parametric study of Salem and Maekawa, and the equation gives the average yield stress in the analytical model [26].

\[
f_{yo} = f_t - \frac{f_t}{2\rho}
\]  

Where \( f_{yo} \) is the reduced yield strength, \( f_t \) is the tensile strength of concrete and \( \rho \) is reinforcement ratio in RC zone. The yield strength of tension rebar is reduced to 325 MPa from 386 MPa using the above equation to investigate the structural response of the RC beams. Moreover, the yield strength of tension rebar calculated by flexural model using the experimental yielding load gives the same strength of 325 MPa.

6. Results and discussions

6.1. Load deflection curves

In Figure 11, the relationship between load and deflection at the center of the span is shown along with ACI deflection, yielding load, and the load at crack initiation. The analytical model captures well the nonlinear load deflection response of the beams up to failure. Three stages (a) linear elastic-uncracked, (b) elastic-cracked, (c) ultimate stage can be distinguished in the curves. In general, the load-deflection curves for the beams from the analytical analysis are not in close agreement with the experimental results. In linear elastic-uncracked stage, the analytical load-deflection curves are slightly stiffer than the experimental curves. After linear elastic-uncracked stage, the stiffness of the finite element model is again higher than of the experimental beams. As for this, many effects cause higher stiffness in the analytical model. First, dry shrinkage, heat evolution during the hydration process and handling of RC beams cause the micro-cracks in the concrete of RC beam specimens, while the analytical models do not include such micro-cracks. Because of these effects, the stiffness of
experimental beam specimens reduces due to presence of micro-cracks. Moreover, the perfect bond between the concrete and steel reinforcement is assumed in the finite element analysis, but the assumption would not be true for the experimental RC beam specimens. As bond slip occurs, the composite action between concrete and steel reinforcing bar is lost. Therefore, the overall stiffness of the RC beam specimens is expected to be lower than that of finite element models.

6.1.1. Reference specimen

Figure 11(g) shows the load deflection curve for the reference specimen for experimental and analytical results. The results obtained from finite element analysis are not in close agreement with the experimental data. The experimental and analytical bending failure load of reference specimen (B-0) was 118.9 kN and 121.23 kN respectively. The cracking initiates at 30 kN and continues to progress until crushing of concrete during the experiment. However, the analytical model predicts the cracking load of 43.28 kN and the linear response until the yielding load of tensile reinforcement is reached. The yielding load predicted in analytical model was 106.67 kN. This load is consistent with the predicted tension rebar yielding at a load of 108.5 kN in flexural model. However, the yielding load of tension rebar observed during the experiment was 88.30 kN. The FEM curve is higher than the experimental because of this difference in yielding load. As for this, it is thought that the yield strength of rebar is reduced in RC beam specimens during the experiment, because of localized yielding at cracks.

6.1.2. Top repair series

Figures 11(a) to 11(c) show the load-deflection curves for the top repair beams for experimental and analytical results. The maximum load capacity increases with the increase of UHPFRC thickness in experimental and analytical result. In top repair beams, the BU-20 specimen shows the same bending behavior as that of reference beam (B-0) in experimental and analytical results. After yielding of rebar, the experimental and FEM results indicate that the load increases with a constant gradient up to 142.2 kN and 142.88 kN in load deflection relationship respectively. The crushing of UHPFRC repaired beam was observed at 142.2 kN which is 1.2 times higher than that of B-0. In BU-40 and BU-60 specimens, the load-displacement relationship and crack condition were the same as that of BU-20. The experimental maximum load of BU-40 was 1.25 times higher compared to B-0. In BU-60, the experiment was terminated earlier because of destruction signs and the maximum load was 137.0 kN. The analytical maximum load of BU-60 was 147.78 kN. The experimental results are not in good agreement with the analytical results after yielding because of reduction of yielding load due to localized yielding during experiment as shown in Figures 11(a) to 11(c). However, the analytical and ACI deflection results are in acceptable agreement.

6.1.3. Bottom repair series

Figures 11(d) to 11(f) show the load-deflection curves for the bottom repair beams for experimental and analytical results. In general, the flexural capacity of the beams with bottom repair increases with increase of UHPFRC thickness. This is attributed to the high strength and strain hardening of UHPFRC. In BL-20 specimen, no increase in flexural capacity was observed compared to the reference specimen in the experiment, while the
analytical and flexural model show increase in capacity. This is because the localized macro cracks lead to the deterioration of bond between existing concrete and UHPFRC during the experiment. The crack opening takes place with the increase in load and the crushing of upper edge concrete at the load of 118.9 kN was observed in the experiment. However, the analytical curve is higher than the experimental curve with maximum load of 141.31 kN. The analytical curve agrees well with the simplified load-deflection curve obtained from flexural model. The flexural model predicts the yielding of tensile reinforcement to start at approximately 131.30 kN, which is consistent with the numerical analysis. In BL-40 and BL-60 specimens, the tensile resistance of UHPFRC leads to increase in ultimate load to 1.22 and 1.31 times reference beam capacity in the experiment, while more increase in capacities are seen in analytical and flexural model results because the tensile properties of UHPFRC are obtained from the tensile test. The tensile properties are overestimated during the test of UHPFRC specimen, while the reduction in tensile strength takes place for thicker UHPFRC repair layer because of segregation of fibers. However, an increasing thickness is advantageous, since it leads to smaller deformations for a given load and formation of localized macro-cracks at higher loads.

6.2. Correlation between analytical model with the experiment

The cracking load and maximum load for all the tested specimens are correlated with the results obtained from flexural model presented in this study. The input properties of material model are the same as that of obtained from the material test. Table 4 lists the cracking load and maximum load calculated by flexural model and finite element analysis of the tested specimen along with the observed experimental loads. The maximum and cracking load obtained by calculation using the flexural model and finite element analysis show good correlation that confirms the applicability of FEM model. For the reference beam, the predicted maximum load during the flexural and finite element analysis is close to the observed experiment load with the coefficient of variation (COV) of 3.8%. For top and bottom repair beams, the maximum coefficient of variation between the predicted and observed loads are 4.6% and 14.81% respectively. This shows that the top repair experimental maximum load is closer to the predicted maximum load of finite element analysis than that of the bottom repair series. However, the comparison between the predicted cracking loads from flexural model and finite element analysis shows acceptable agreement with the experimental cracking load for bottom repair series with the maximum COV of 5.79%. The cracking load increases with the increase of UHPFRC thickness because thicker UHPFRC layer leads to an increase of the height of compressive stress block, thus, delay of localized macro-cracks formation results in improving the protection function under service conditions. The analytical cracking load of the reference beam is 1.32 times higher than that of calculated by flexural model. This is because of the reason that the finite element model is relatively homogeneous as compared to the experimental RC beam specimens having micro-cracks. The analytical cracking load of the top repair beams is not in acceptable agreement with the calculated cracking load using flexural model. Moreover, the analytical cracking load is not increased significantly with the increase in UHPFRC thickness because there is no extra tensile force to balance by the compressive force produced by UHPFRC in the compression zone. The comparison reveals that the predicted cracking loads using the flexural and analytical analysis are in acceptable agreement with the experimental for BU-40 and BU-60 with the COV of 5.28% and 9.68% respectively.
6.3. Strain distributions and failure modes

Figure 12 shows the predicted strain distribution from analytical and flexural model along with the experimental at mid span of the beams. The experimental strain is measured using strain gauges attached to the compression and tension rebar at mid span and strain distribution is calculated for the height of whole section using linear curve. In case of the flexural model analysis, neutral axis depth and the strain in top compression fiber are assumed for calculation the strain at location of rebars and extreme tension fiber based on the Bernoulli assumption i.e. the plan section remain plane and the strain distribution is linear. In general, the strain predicted using analytical model shows nonlinear behavior along the depth of the beam. The strain distributions obtained from finite element analysis and flexural model are in acceptable agreement. The experimental strain distribution is obtained by the rebar strain measurement during the experiments.

Figure 12(g) show the strain distributions for the reference beam (B-0). The strain distribution is nonlinear along the depth of the reference beam. The finite element analysis overestimates the strain compared to flexural model and experimental strains. The strain in top fiber reached the crushing strain value and neutral axis lies at a distance of 1/11th of the beam depth from top fiber both in analytical and flexural model. Hence, the reference beam fails by crushing of concrete at failure load. The crushing of concrete was observed during the experiment at failure as the experimental strain distribution reveals that strain in top fiber of the beam exceeds the crushing strain.

Figure 12(a) to 12(c) shows the strain distributions for top repair series. The top repair series show the same behavior of strain distributions as that of reference beam (B-0). However, the neutral axis is shifted up in the UHPFRC layer approximately at 1/20th of the beam depth from top fiber. The strain in bottom fiber increased with increase of UHPFRC thickness. It is clear from the analytical and flexural strain distribution that the maximum compressive strain in top fiber of the BU-20 and BU-60 beams is reached the crushing strain value. Hence, the BU-20 and BU-60 specimens fail by crushing of UHPFRC in the analysis. For BU-20 specimen, the crushing of UHPFRC was observed during the experiment and strain distribution show acceptable agreement with analytical and flexural model strains. For BU-40 specimen, the experimental strain distribution show that the UHPFRC strain remained below the crushing, whereas the tensile rebar shows fracture during the experiment as shown in Figure 12(b). The failure mode of BU-60 specimen was not observed during the experiment because the experiment was terminated before the final failure of the beam. The strains in UHPFRC and rebar measured during experiment are smaller than the flexural model and analytical analysis.

Figure 12(d) to 12(f) show the strain distribution for the bottom repair beams along a section at 100 mm away from the mid span of the beams at failure load. All the bottom repair beams show almost the same strain distribution as that of the reference beam (B-0). The compressive strain in top fiber for all the beams exceeds the crushing strain of the concrete and neutral axis lies at the same depth as that of reference beam (B-0). The strain at level of tensile reinforcement is also less than the ultimate strain i.e. 24% of the rebar. Therefore, the fracture of rebar is not observed in the analysis. It is clear from the strain distribution that all the beams repaired in tension zones fail by the crushing of concrete. However, the experimental strain distribution shows that the tensile rebar fractures because the ultimate strain was reached in case of BL-40 and BL-60 specimens during the experiment.

The failure modes of the specimens obtained from the FEM model and experiment are listed in Table 4. The failure modes observed during the experiments are of three types (1) CC: concrete crushing, (2) UC:
UHPFRC crushing, and (3) R: rebar fracture. The comparison shows that the failure modes of experimental and finite element analysis are different because of fracture of rebar due to formation of localized macro-cracks during the experiment.

### 6.4. Crack patterns

Figure 13 shows the comparison of analytical and experimental cracks. The comparison reveals that experimental crack patterns have few macro-cracks observed visually while the analytical crack patterns have many cracks at regular spacing. The spacing of cracks depends on the meshing size. In the analysis, stresses and strains are calculated at integration points of the concrete solid elements. A cracking sign is represented by a line perpendicular to the direction of principal stress or principal cracking strain vector. This cracking line represents the orientation of cracks in analytical results. The crack appears when a principal tensile stress exceeds the tensile strength of concrete. A side face of a quarter beam model is used to demonstrate the crack patterns. The program records a cracking strain vector at each applied load step. In general, there are three types of cracks: (a) flexural cracks, (b) compressive cracks and (c) diagonal tensile cracks that appear in different regions of the beam.

Figure 13(a) to 13(d) show the experimental and analytical crack patterns for the reference and top repair beams. Before the loading, the experimental beams have very small cracks because of fresh concrete shrinkage, segregation, and handling of specimen. At 40 kN, the flexural cracks start to appear at midspan. These cracks are vertical straight lines occurring at the integration points of concrete solids. When applied load increases, vertical flexural cracks spread horizontally from midspan to the support and are shown at yield deflection $1\delta_y$. The neutral axis moves to the top surface with the progress of cracking. At a higher load i.e. $5\delta_y$ and $10\delta_y$, diagonal tensile cracks appear in shear span. Finally, at the end of loading, few compressive cracks appear under the loads and concrete crushing observed in experimental reference beam. The experimental cracks are difficult to observe because the meshing size limit the cracking spacing and crack location is predetermined in analytical model.

Figure 13(e) to 13(g) show the experimental and analytical crack patterns for the bottom repair beams. The experimental bottom repair beams also show some horizontal crack near the interface of concrete before the application of loading. For BL-20, the flexural cracks are shown at 80 kN and start to appear in ordinary concrete in constant moment region. The same cracking pattern is observed in BL-40 and BL-60 at 100 kN. The cracking load increases because of increasing thickness of UHPFRC, so the crack formation is delayed under service conditions. The cracks start to propagate horizontally and vertically as the loading increases from $1\delta_y$ to $5\delta_y$ and induce additional diagonal and flexural cracks. The cracks start to propagate with increasing loading in a direction perpendicular to the UHPFRC layer. In experimental beams, the cracks in UHPFRC layer produced after the formation of cracks in ordinary concrete. The analytical cracks showed good agreement with experiment and the same pattern is observed.
6.5. Parametric study results

6.5.1. Influence of the yield strength of the tension rebar

Figure 14(a) to 14(d) show the influence of yield strength of the tension rebar on the load-deflection curves for the reference and top repair beams for experimental and analytical results. In general, the reduction of the yield strength of the tension rebars result in lowering the analytical curves to the experimental curves. The comparison discloses that the yield load of the beams is reduced approximately by 10% compared to original analysis. The ultimate load is reduced significantly in the finite element analysis because the lower yield strength result in reducing the stress block height. After cracking, the stiffness of the analytical curve is still higher than the experimental curve because of different mechanism such as shrinkage and micro-cracking are not considered in the finite element model. After yielding of rebar, the deformation of the beams increases significantly in analytical model, because the strain in steel suddenly jumps to hardening after cracking for a given moment compared to original model.

6.5.2. Influence of the cracking strength of the UHPFRC

Figure 15(a) to 15(c) show the influence of cracking strength $f_{Ut,1}$ and ultimate tensile strength $f_{U,t, max}$ on the load-deflection curves for the bottom repair beams for experimental and analytical results. The FEM line represents the analysis of the beam with original parameters. Whereas, the $f_{t,1}$-line shows the effect of reducing yield strength of tension rebar by adopting the tensile test values. The comparison reveals that the flexural capacity is reduced approximately by 14% by the reduction of yield strength of tension rebar. In general, the reduction of cracking strength and ultimate tensile strength leads to the reduction of ultimate and yielding load, and analytical load-deflection curves approach the experimental curves. After first cracking, the stiffness of the finite element models is reduced because the depth of Whitney stress block reduces. Localized macro-cracks form at lower load compared to the original analysis. This comparison shows that the cracking strength $f_{U,cr,1}$ and ultimate tensile strength $f_{U,t,1}$ are approximately 30% lower than the mean experimental values. As for this, the tensile properties of UHPFRC depend on the thickness of UHPFRC layer because thicker UHPFRC layer lowers the tensile strength. This may be explained by fiber segregation in UHPFRC layer.

7. Conclusions

In this study, the existing concrete structure is intended to repair with UHPFRC as cross-sectional restorative material. The bending behavior of beam is investigated for either tension or compression cross-section repaired with UHPFRC having different thickness. The results show that the use of UHPFRC in structural elements made of cementitious materials leads to higher stiffness and to an increased bending capacity compared to the previously repaired thickness. Moreover, the significantly delayed crack formation prevents the formation of localized macro-cracks in the UHPFRC layer under service conditions, thus guaranteeing the protection function. The fracture modes change significantly, lowering the deflection of the beams. The parametric study reveals that the tensile properties of UHPFRC obtained by tensile test cannot be applied to structural element analysis and further research is essential to deduce the tensile properties of UHPFRC. In
future based on the results of this study and additional experiments, the structural performance of UHPFRC as a strengthening material can be utilized.

8. Acknowledgments

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References


Table 1: Experimental cases

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Repair location</th>
<th>Repair Thickness (mm)</th>
<th>Configuration</th>
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<tr>
<td>B-0</td>
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<td>BU-20</td>
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<td>P</td>
</tr>
<tr>
<td>BU-40</td>
<td></td>
<td>40</td>
<td>P</td>
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<tr>
<td>BU-60</td>
<td></td>
<td>60</td>
<td>PR</td>
</tr>
<tr>
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<td>P</td>
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<td></td>
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Table 2: Properties of concrete and UHPFRC

<table>
<thead>
<tr>
<th>Material</th>
<th>Age (days)</th>
<th>Compressive strength (MPa)</th>
<th>Young’s modulus (GPa)</th>
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<td>29.7</td>
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<td>UHPFRC</td>
<td>42</td>
<td>156.3</td>
<td>34.6</td>
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Table 3: Geometric and mechanical properties of rebars

<table>
<thead>
<tr>
<th>Rebar</th>
<th>Diameter (mm)</th>
<th>Cross-sectional area (mm$^2$)</th>
<th>Yield strength (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Elongation (%)</th>
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<tr>
<td>Longitudinal (D16, SD345)</td>
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<td>Shear stirrup (D10, SD345)</td>
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<td>71.33</td>
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Table 4: Comparison between experimental and analytical results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Maximum Load (kN)</th>
<th>Cracking Load (kN)</th>
<th>Failure modes</th>
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<tr>
<td></td>
<td>EXP</td>
<td>ANA</td>
<td>EXP</td>
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<td>BL-0</td>
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<td>BU-20</td>
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<td>BL-60</td>
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<td>205.15</td>
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</table>

COV: Coefficient of variation, CC: concrete crushing, UC: UHPFRC crushing, R: rebar fracture
Figure 1(a): Reference beam (B-0) geometry

Figure 1(b): Beam cross sections with different repair thickness

Figure 2: Stress-strain curve of uniaxial tensile test for UHPFRC
Figure 3: (a) Concrete surface after water jetting, (b) casting of UHPFRC

Figure 4: Flexural test setup

Figure 5: Description of flexural model
Figure 6: (a) Full RC model, (b) Reinforcement layout, (c) Boundary conditions

Figure 7: Stress-strain model for reinforcing bar

Figure 8: (a) Uniaxial compressive stress-strain curve for concrete, (b) Tension stiffening model
Figure 9: Uniaxial compressive stress-strain curve for UHPFRC

Figure 10: Tensile stress-strain curve for UHPFRC, (a) hardening, (b) softening
Figure 11: Load-deflection behaviors of experiment and FEM analysis for (a) BU-20, (b) BU-40, (c) BU-60, (d) BL-20, (e) BL-40, (f) BL-60, (g) B-0 with ACI deflection, yielding load and crack initiation
Figure 12: Strain distribution at failure load for, (a) BU-20, (b) BU-40, (c) BU-60, (d) BL-20, (e) BL-40, (f) BL-60 and (g) B-0
Figure 13: Crack patterns of experiment and FEM analysis for (a) B-0, (b) BU-20, (c) BU-40, (d) BU-60, (e) BL-20, (f) BL-40, (g) BL-60
Figure 14: Influence of yield strength of tension rebar on structural response for (a) BU-20, (b) BU-40, (c) BU-60, (d) B-0

Figure 15: Influence of cracking strength of UHPFRC on structural response for (a) BL-20, (b) BL-40, (c) BL-60