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# Topology Optimization of Metamaterial Using Gaussian-Basis Functions

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**Abstract.** This paper presents topology optimization of electromagnetic metamaterial using the on/off method based on the Normalized Gaussian network (NGnet). In this work, the conductor shape printed on a dielectric slab of fixed dimensions is optimized to have a negative macroscopic permeability at a prescribed frequency band for realization of metamaterial. It is shown that the proposed method successfully provides the metamaterial that works at any desired operating frequency.

**Keywords:** FDTD, Genetic Algorithm, Metamaterial, NGnet, Topology Optimization

## 1. Introduction

Electromagnetic metamaterials are artificially fabricated structures that exhibit unique properties not found readily in nature. Because the electromagnetic (EM) metamaterials have many potentials in engineering applications, they have attracted considerable attention. Most EM metamaterials are realized as unit structure of metallic inclusion on a dielectric substrate repeated periodically. Negative permittivity can be realized by the lattice wire structure [1]. To realize negative permeability, a structure consisting of split ring resonators (SRR) has been developed [2]. There have been proposed many variations of SRR to satisfy diverse applications and requirements such as [3]. These variations include changes in the ring radius and width as well as the gap between two rings, and also replacement of the circular ring with the rectangular ring. All these trial and error methods have, however, serious difficulties in design of EM metamaterial that works at a desired frequency.

To overcome the above-mentioned difficulty, a design method based on the topology optimization for EM metamaterial has been proposed [4]. This method can provide the EM metamaterial for any desired operating frequency. This method, however, results in “gray” permeability which ranges from the permeability of vacuum to the maximum value because it is based on the density method in which the permeability is treated as a continuous function. Moreover, this method cannot perform global search because of the use of sensitivity method for optimization.

In this paper, we propose a topology optimization method as an alternative for universal design method of EM metamaterials. The proposed method employs the on/off method based on the Normalized Gaussian network (NGnet) that has been shown effective for the topology

design of various EM apparatus including rotating machines [5] and lens antennas [6]. In contrast to the density method, the on/off method does not generate "gray" permeability. Moreover, the proposed method that employs the genetic algorithm as an optimization algorithm can perform global search.

This paper is organized as follows: the NGnet-based topology optimization method for EM metamaterial is formulated in the second section. The third section discusses the numerical results on design of EM metamaterial for different operating frequencies, the impact of the choice of the basis function on the results and the choice of the optimal solution based on the robustness. The fourth section provides the concluding remarks.

## 2. Topology Optimization Method

### 2.1 Extraction of effective permeability

We consider the periodic structure composed of unit cells which are illuminated by a plane electromagnetic wave shown in the Figure 1. The unit cell, $d$=2.5 mm, $l$=0.25 mm, $\varepsilon$=4.4, is composed including a planar metallic layer printed on a dielectric slab. Since the unit cell is inhomogeneous, the effective permeability is introduced by field computations through homogenization. A widely used method has been proposed in [7, 8], which is under the assumption that the effective homogeneous medium has the same scattering characteristics as the inhomogeneous medium. The impedance $z$ and the refractive index $n$ are expressed in terms of the scattering parameters $S_{11}, S_{21}$ computed from FDTD simulation as follows:

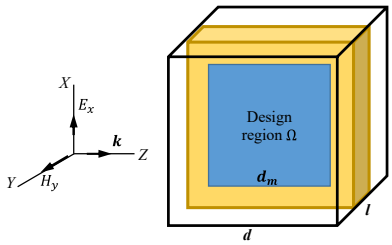



Figure 1: Unit cell of metamaterial for optimization problem

$$z = \sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}} \tag{1a}$$

$$e^{inkd} = \frac{S_{21}}{1 - S_{11}\dfrac{z-1}{z+1}} \tag{1b}$$

where $i, k$ and $d$ denote the imaginary unit, the wave number of air and thickness of the unit cell. The effective permeability can be obtained from

$$\mu = nz \tag{2}$$

### 2.2 NGnet-based topology optimization method

In FDTD simulation, the design region is subdivided into several cells. The material attribute, (dielectric, metal) of each FDTD cell is determined from the shape function defined by

$$y(\boldsymbol{x}) = \sum_{i=1}^{N} w_i\, b_i(\boldsymbol{x}) \tag{3}$$

where $\boldsymbol{x}$ and $w_i$ denote the position vector, weighting coefficients respectively and $b_i(\boldsymbol{x})$ is

the normalized Gaussian function expressed by Gaussian function $G_i(\boldsymbol{x})$ given by

$$b_i(\boldsymbol{x}) = G_i(\boldsymbol{x}) / \sum_{j=1}^{N} G_j(\boldsymbol{x}) \tag{4}$$

The material attribute of a cell is set to metal when $y(\boldsymbol{x}) \geq 0$, otherwise dielectric. Process of shape determination using NGnet on/off method is schematically shown in Figure 2.

As for optimization, $w_i$ is continually updated by the micro genetic algorithm ($\mu$GA) [9] to solve the optimization problem defined by

$$\mu''(\omega_0) \longrightarrow \min. \tag{5}$$

where $(*)''$ denotes the imaginary part, and $\omega_0$ is the target frequency. We choose this objective function to avoid complexity in dealing with the real part of permeability as objective function, which has been verified in [4].

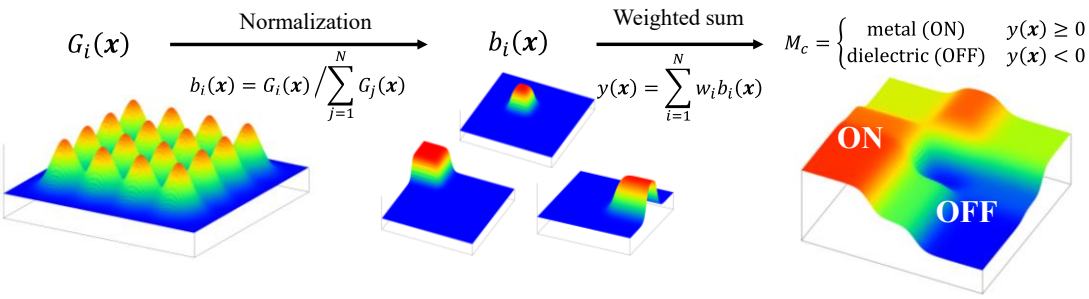



Figure 2: Process of shape determination using NGnet on/off method

## 3. Optimization results

### 3.1 Frequency Characteristics

The design region $\Omega$ for the topology optimization, shown in Figure 1, is a square whose edge length is $d_m$=2 mm. The unit cells are arranged periodically in $X$ and $Y$ directions. The Mur-type absorbing condition is imposed on the surfaces perpendicular to $Z$ direction. The Gaussian disposition of design region $\Omega$ is shown in the Figure 3. The circles of radius with the standard deviation $\sigma$, which are the contours of the Gaussian functions, are set in contact with each other.

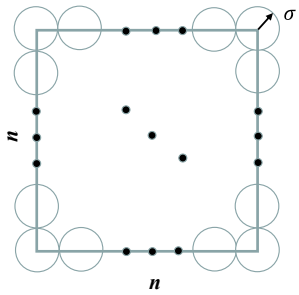



Figure 3: Arrange of Gaussian basis functions in design region

Firstly, validity of the proposed method is tested. The number of Gaussian functions $N$ is set to 11×11 where the standard deviation $\sigma$ is 0.1mm and the target frequency is set to 15 GHz. Figure 4 depicts the convergence process of objective function $\mu''(\omega_0)$ during the optimization. The effective permeability of the optimized shape is plotted against frequency in Figure 5. According to the Figure 5, the real part of the permeability takes negative values at 15-16.3 GHz, and the minimum of the imaginary part of permeability is -4.193.

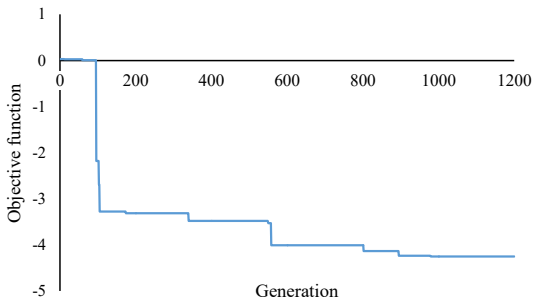



Figure 4: Convergence process of Genetic Algorithm

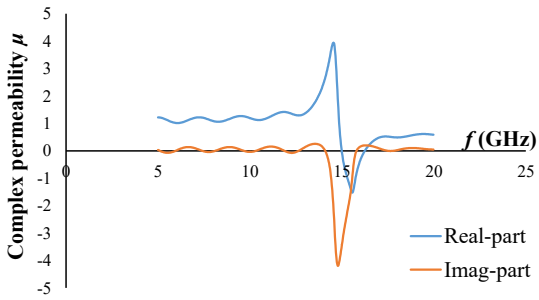



Figure 5: Effective permeability of the optimized shape

To test the universality of the proposed method, the target frequency is also set to 13 GHz and 17 GHz. The frequency characteristics of the effective permeability at these frequencies are plotted in Figure 6. As we can see from these results, the real part of the permeability takes negative values in the band near the target frequency.

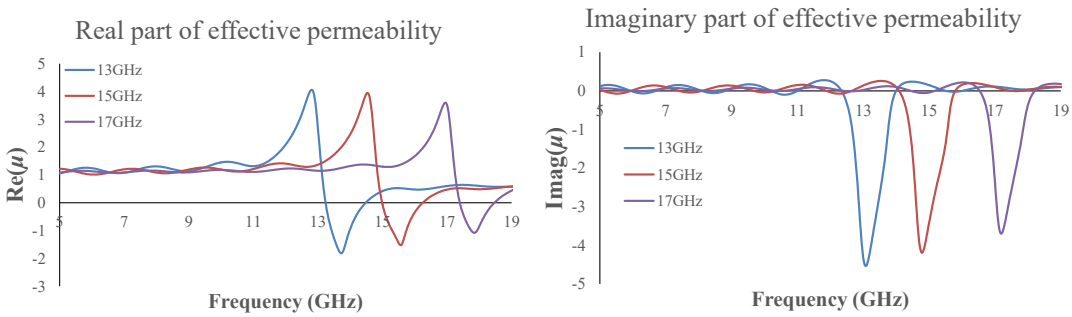
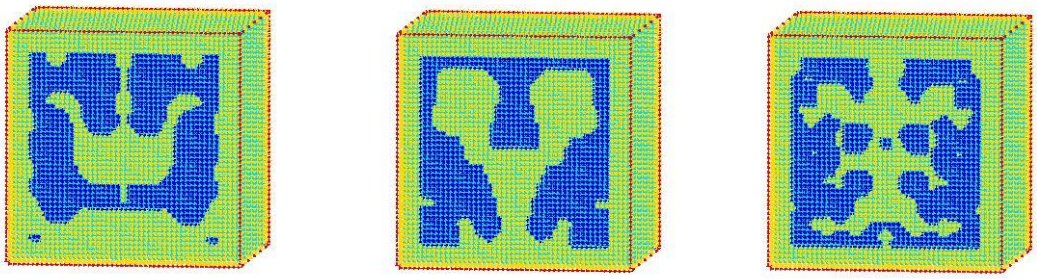



Figure 6: Frequency characteristics of effective permeability at different target frequencies

### 3.2 Impact of Gaussian Basis Functions

The number and standard deviation of Gaussian functions would have influences on optimization results and frequency performance. In this subsection, these effects are discussed where at the target frequency is fixed to 15 GHz.

We consider the influence of the number of Gaussians $N$ on the optimal solutions. We change $N$ from 11×11 to 6×6 and 17×17, respectively. The contour lines of $\sigma$ for the Gaussian functions are kept in contact with each other as shown in Figure 3. The optimized shapes are shown in Figure 7, and the corresponding frequency characteristics are shown in Figure 8. As we can see, the three optimization results are completely different metallic shapes although there are no significant differences in those frequency responses. These results strongly suggest that there are no unique solutions to the optimization problem.



(a) 6×6 (b) 11×11 (c) 17×17

Figure 7: Optimized shapes for different number $N$ of Gaussians

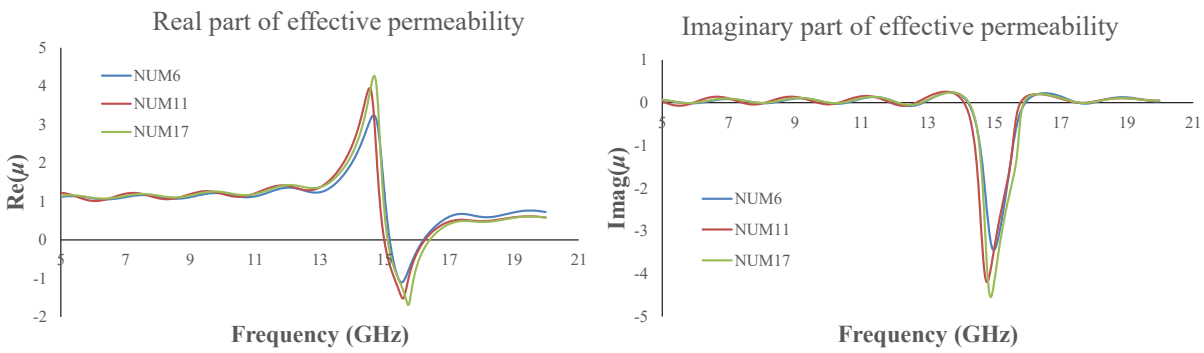



Figure 8: Frequency characteristics of effective permeability for different number $N$ of Gaussians

We next consider the influence of arrangement of the Gaussians. The number of Gaussian functions is kept to be 11×11, whereas the Gaussian functions are partial overlapped by multiplying the factors 1.15, 1.25, 1.35 and 1.45 to the standard deviation $\sigma$. The overlapped disposition is shown in Figure 9.

The optimized shapes are shown in Figure 10. We again observe different shapes coming from the lack of the uniqueness of solution. Moreover, the corresponding frequency characteristics are shown in Figure 11. It can be seen from these results that the frequency performance of the overlapped Gaussian arrangement is relatively better than that of non-overlapped arrangement. It is found that either $1.15\sigma$ or $1.25\sigma$ gives the best performance.

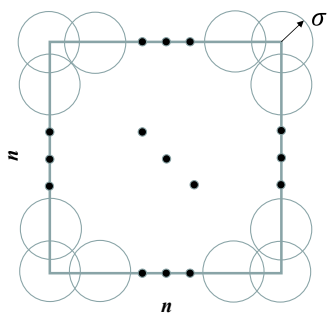
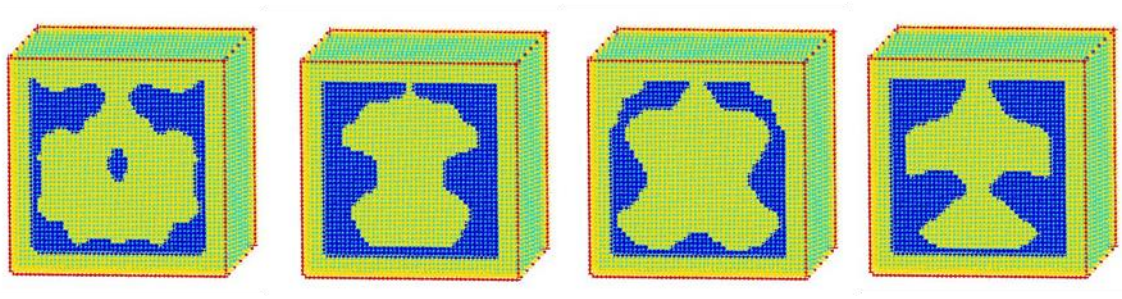



Figure 9: Partial overlapped disposition



(a) 1.15$\sigma$ (b) 1.25$\sigma$ (c) 1.35$\sigma$ (d) 1.45$\sigma$

Figure 10: Optimized shapes for different standard deviations

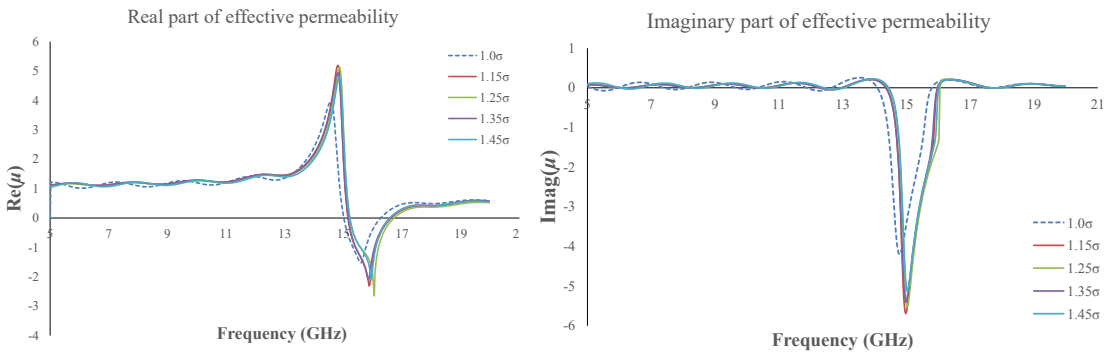



Figure 11: Frequency characteristics of effective permeability for different standard deviations

### 3.3 Classification of Solutions and Consideration of Robustness

As suggested by results mentioned in 3.1 and 3.2, the optimized shapes possess a great diversity. In this subsection, we make classification of the solutions obtained by the proposed method. Then we propose a method to choose an optimal solution from them with reference to the robustness against the change in the permittivity of the substrate.

We obtain the diverse solutions by performing the optimization repeatedly where the Gaussian functions with deviation of 1.15$\sigma$ are arranged at 11×11 grid points, and the target frequency is set to 15 GHz. The relative permeability would not well converge to sufficiently small values depending on the trial of GA. For this reason, the selection criterion

$$\mu''(\omega_0) < -4.0 \tag{6}$$

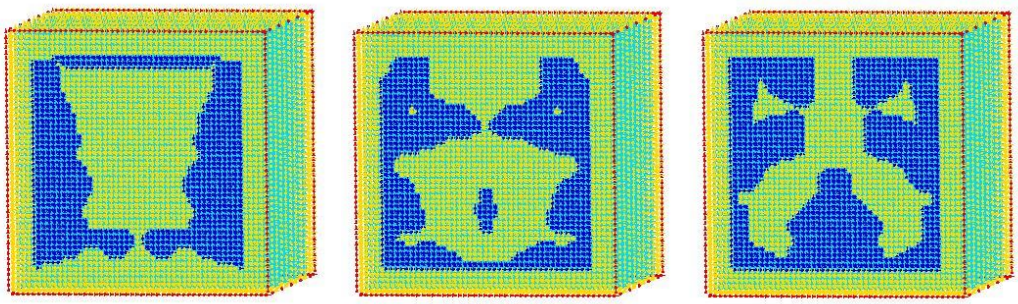
is introduced. We repeated the optimization process 30 times and obtained 26 eligible results. Among these selected shapes, they are classified into two categories, 15 C type and 8 Y type, while other 3 shapes have no common geometric features. If there are metallic islands in the middle of the C ring, we refer to this type as C+ type. As a result, there are 7 C+ type shapes among 15 C type shapes. The typical structures of C, C+ and Y type are shown in Figure 12.

In reality, permittivity of the substrate would deviate from the specified value due to manufacturing error and time degradation. For this reason, the EM metamaterial that is robust against change in the permittivity would be suitable. We propose here a method to choose a candidate for realization from the diverse optimal solutions on the basis of the robustness. In the original model in Figure 1, permittivity of the dielectric substrate is set to $\varepsilon$=4.4. We consider relative

error of permittivity in a range of ±5% ($\Delta\varepsilon$=-0.2~0.2) to test the robustness of the typical shapes. The objective function $F$ is plotted against permittivity ε in Figure 13, where the C type seems most robust. To quantify the robustness, we introduce the index defined by

$$E = \sqrt{\left(\frac{\Delta F}{\Delta\varepsilon_-}\right)^2 + \left(\frac{\Delta F}{\Delta\varepsilon_+}\right)^2} \tag{7}$$

The values of $E$ are summarized in the Table 1. It is concluded from Table 1 that the C type that has the strongest robustness is most suitable for realization.



(a) C type (b) C+ type Y type

Figure 12: Typical classification shapes

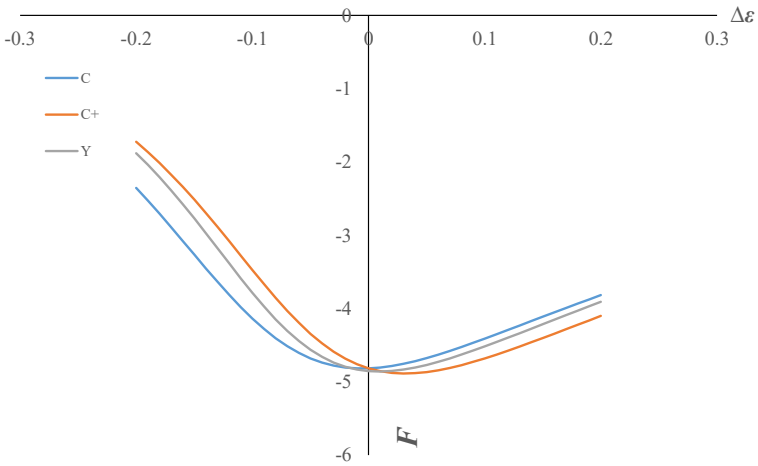



Figure 13: Objective Function distribution against permittivity increment

Table 1: Robustness index $E$ for three different types

| | C | C+ | Y |
|---|---|---|---|
| $E$ | 13.28 | 15.86 | 15.59 |

## 4. Conclusion

In this work, topology optimization of EM metamaterial with negative permeability using Normalized Gaussian network has been proposed. The proposed method has been shown to provide optimal solutions for different target frequencies. It has been shown that the Gaussian basis functions should have small overlapping. Moreover, it has been numerically suggested that there is no uniqueness in the optimization problem. There are three different types in the optimal shapes. A method to choose an optimal solution based on the robustness has been proposed. The C type has been shown suitable for realization because of the strongest robustness.

In the future work, we will manufacture the EM metamaterial based on the optimization result to experimentally measure its performance.

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