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Child costs on marriage market:
Game theoretical study

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1. Introduction

Recently, declining birthrate, late marriage and unmarried are progressing in Japan. By Declining birthrate society measures white paper (2019), in Japan, total fertility rate has remained below 1.5 from 1994 and is continuously lower than the population replacement level. In Japan, in 1985-2015, the unmarried rate of 30-34 years old increased from 10.4% to 34.6% for women and for man, from 28.2% to 47.1%, and "lifetime unmarried rate" showing the proportion of those who have never married until the age of 50 has increased from 4.3% to 14.1% for women and from 3.9% to 23.4% for men.

Income, child-raising costs, opportunity cost for other activities can be considered as factors of declining birthrate and non-marriage. In this paper, I focus on the child-raising costs. Browning, Chiappori and Weiss (2014) concludes that couples who drew a good match quality on meeting are more willing to marry and to invest in children because they expect the marriage to be more stable. They assumed that two-stage model and a couple owe the child-raising costs equally only in the first period, if a couple has the child. In this paper, we assume that child-raising costs occur also in the second period. If they divorced, we assume that the child care expenses in the second stage are asymmetric. This cost asymmetricity effects on the decisions of whether to marry or not, whether to make a child or not.

2. Model

Consider a society in which there is an equal number of men and women. All individuals are ex ante identical and live for two periods. Each person consumes his or her income Y if he or she lives alone. If they married, a couple share consumption and each consumes $2Y$. In addition, they receive non-economic emotional values θ from marriage. If a man and a woman remain single, they will receive zero emotional values from marriage. This emotional values θ is randomly distributed, and different couples draw different values of θ at the time of marriage. At the beginning of each period, they match in the marriage market. On matching, emotional values θ is revealed and matched man and woman decide whether to marry or not. During each period, there is a shock ε to the emotional values that is revealed at the end of the period. Given a cohort, we assume that marriage binds for at least one period. Having observed the shock at the end of the first period, the couple decide whether to divorce or not. The random values θ and ε are assumed to be independent across partners. We denote the distributions of θ and ε by $G(\theta)$ and $F(\varepsilon)$ with densities $g(\theta)$ and $f(\varepsilon)$ respectively. We assume that the distributions of θ has some negative mean and the distributions of ε has zero mean and they are symmetrical around their mean. At the end of the first period divorce may occur and at the beginning of the second period remarriage may occur. Remarriage is possible only with a single person who never married before or have divorced. The probability of each person match with a single individual at the beginning of the second period equals the proportion in the population of single person who never married before or have divorced.

When a man and a woman form a family by marriage in the first period, they can have a child. We assume that there is no out-of-wedlock birth and a couple can produce only one child in the first period for simplicity. When giving birth to a child in the first period, we assume that the cost of giving birth to a child is c_1>0 equally for male and female. In the second period, the cost of raising a child assumes to be $2c_2$, c_2>0. If the first marriage continues in the second period, parents equally owe the child-rearing costs. When a couple divorce, we assume that the mother has the custody right and the mother owes $2c_2$-c_d and the father owes c_d, where c_d >0 to raise their natural child in the succeeding period. The utility of a child depends on whether or not she lives with the natural parents. When the child lives with both natural parents, her utility equals to q_m, and if she does not live with only her mother or in a stepfamily, it equals to q_d, where q_m > q_d >0. The biological parents treat the utility of the child as a public good and additively benefit from the natural child in the second period.

3. Individual choices

3.1 The last stage: the remarriage decision

We first analyze the marriage, fertility and divorce decisions of individuals who take the conditions in the marriage market as given. We start from the last available choice which is marriage at the second period and see backwards. Two unattached people who meet at the beginning of the second period will marry if and only if their drawn θ satisfies

$2Y + \theta \geq Y,$

where the left side denotes the payoff when they get married and the right side denotes when they remain single. This inequality is rewritten as follows:

$\theta \geq -Y.$ (1)

This simple marriage rule holds because each partner gains Y+θ from the marriage, and if one of the partners has a child, the benefits from the child are the same whether the child lives with only her mother or in a stepfamily.

We denote the probability of remarriage conditioned on a matching in the second period by

$\gamma = 1 - G(-Y)$ (2)

and the expected emotional values conditioned on marriage in the second period by

$\beta = E(\theta/\theta \geq -Y).$ (3)

The probability of meeting an unattached person as remarriage candidate at the beginning of the second period is denoted by u. The probability that unattached individual will meet a fitting single person whom he or she will choose to marry is p=$u\gamma$. The expected utility of an unattached male is therefore

$V_{2,n}^m = p(2Y + \beta) + (1 - p)Y + n(q_d - c_d)$ (4)

and the expected utility of an unattached female is

$V_{2,n}^f = p(2Y + \beta) + (1 - p)Y + n(q_d - 2c_2 + c_d)$ (5)

where n=1 if the unattached individual has a natural child and n=0 otherwise.

3.2 The intermediate stage: the divorce decision

A married man will decide to divorce if and only of the θ drawn at the beginning of the first period and the ε drawn at the end of the first period are such that

$2Y + \theta + \varepsilon + nq_m < V_{2,n}^{m}$ (6)

and a married woman

$2Y + \theta + \varepsilon + nq_m < V_{2,n}^{f}$. (7)

These can be rewritten as $\theta+\varepsilon<h_n^m$, $\theta+\varepsilon<h_n^f$, where

$h_n^m \equiv -Y + p(Y+\beta) - n(q_m - q_d + c_d)$ (8)

and

$h_n^f \equiv -Y + p(Y+\beta) - n(q_m - q_d + 2c_2 - c_d)$ (9)

are the expected net gain from divorce.

The probability of divorce for a married man with initial emotional values θ is given by $F(h_n^m - \theta)$ and for a married woman, $F(h_n^f - \theta)$. Divorce probability may differ between male and female unless $c_d = c_2$. Since marriage can be established under the agreement of both sexes, the marriage shall not last if one of the partners wants to divorce. Divorce probability depends on θ, n and $p=u\gamma$. It rises with the number of unattached person as remarriage candidate u which rises remarriage probability p. However, the effect of having a child n on the probability of divorce may differ between mother and father. When the child support cost from the father c_d is large enough, for the expected net gain from divorce of the couple with the child, the father's becomes smaller and the mother's becomes larger, and vice versa. Therefore, divorce is more likely to occur if the difference in the child support cost from the father c_d when divorced and c_2 when not divorced is sufficiently large.

The influence of remarriage prospects on the decision to divorce makes a link between the aggregate divorce rate and the individual divorce decision. If many persons decide to divorce, then the numbers of remarriage candidates u is high, which increases remarriage probability p and the net gain from divorce h_n^m, h_n^f and thus divorce probability.

3.3 The first stage: the marriage and fertility decision

At the beginning of the first period, all individuals are single and have a matching with opposite sex and observe their drawn emotional value θ. Then, they decide whether to marry and whether to have a child. The expected lifetime utility on marriage conditioned on n are given by

$W_{1,n}^m = 2Y + \theta - nc_1 + \int_{h_n^m-\theta}^{\infty}(2Y + n\,q_m + \theta + \varepsilon)f(\varepsilon)d\varepsilon + F(h_n^m - \theta)V_{2,n}^m$, (10)

$W_{1,n}^f = 2Y + \theta - nc_1 + \int_{h_n^f-\theta}^{\infty}(2Y + n\,q_m + \theta + \varepsilon)f(\varepsilon)d\varepsilon + F(h_n^f - \theta)V_{2,n}^f$. (11)

Differentiating $W_{1,n}^i$, $i=m, f$ with respects to θ yields

$\frac{\partial W_{1,n}^i}{\partial\theta} = 2 - F(h_n^i - \theta)$. (12)

Therefore, expected lifetime utilities on marriage are increasing in the emotional values θ. The values of marrying without a child and the values of with a child are continuous, increasing and convex functions of θ.

The decision whether to have a child has a stopping rule because $\partial W_{1,n}^i/\partial\theta>0$, $i=m, f$, $n=0$, 1 for all θ. There exists a unique value of θ, θ_c^i, $i=m$, f that is determined by the condition $W_{1,0}^i = W_{1,1}^i$ holds. If a couple decides marriage and derives $\theta > \theta_c^i$, then they have a child. When spouse i and j ($i, j=m, f, i\neq j$) draw some θ where $\theta_c^j > \theta > \theta_c^i$, spouse i does not hope to have a child. Then such a couple decides not to have a child.

Expected lifetime utilities of those who do not marry in the first period are given by

$W_1^m = Y + V_{2,0}^m$ (13)

for man and

$W_1^f = Y + V_{2,0}^f$ (14)

for woman and $W_1^m = W_1^f$. Thus, marriage in the first period will be realized if and only if
$\max[W_{1,0}^m, W_{1,1}^m] \geq W_1^m$ (15)
and
$\max\left[W_{1,0}^f, W_{1,1}^f\right] \geq W_1^f$. (16)
Thus, we have θ_m^m which is determined by the condition that (15) holds as and equality and θ_m^f by (16). Since the maximum is an increasing function of θ, θ_m^m is unique and θ_m^f is also unique as well. We can drive
$\frac{\partial W_{1,1}^i}{\partial \theta} > \frac{\partial W_{1,0}^i}{\partial \theta}, i = f, m$ (17)
by $h_1^i < h_0^i$ since $q_m - q_d + c_d > 0$ and from (8) (9) (12),
$\frac{\partial W_{1,0}^f}{\partial \theta} = \frac{\partial W_{1,0}^m}{\partial \theta}$,
$\frac{\partial W_{1,1}^f}{\partial \theta} \geq \frac{\partial W_{1,1}^m}{\partial \theta}$, if $c_2 \leq c_d$,
$\frac{\partial W_{1,1}^f}{\partial \theta} < \frac{\partial W_{1,1}^m}{\partial \theta}$, if $c_2 > c_d$.

We can find θ^0 such that θ^0 satisfies $W_{1,0}^i = W_1^i, i = 1, 2$. If $\theta_c^i < \theta^0$, then $\theta_c^i < \theta_m^i$ and $\theta_m^i < \theta^0$. If $\theta_c^i \geq \theta^0$, then $\theta_c^i \geq \theta_m^i$ and $\theta_m^i = \theta^0$. We can consider following cases.

case1: $\theta_c^f < \theta_m^f$ and $\theta_c^m < \theta_m^m$
If $\theta_m^f \leq \theta_m^m$, then they do not marry and do not have a child if $\theta < \theta_m^m$, and they get marry and have a child if $\theta \geq \theta_m^m$. If $\theta_m^f > \theta_m^m$, they do not marry and do not have a child if $\theta < \theta_m^f$, and they get marry and have a child if $\theta \geq \theta_m^f$.

case2: $\theta_c^f < \theta_m^f$ and $\theta_c^m = \theta_m^m$
$\theta_m^f \leq \theta_m^m$ holds. They do not marry and do not have a child if $\theta < \theta_m^m$, and they get marry and have a child if $\theta \geq \theta_m^m$.

case3: $\theta_c^f < \theta_m^f$ and $\theta_c^m > \theta_m^m$
$\theta_m^f = \theta_m^m = \theta^0$ holds. They do not marry and do not have a child if $\theta < \theta_m^m$, and they get marry but do not have a child if $\theta_m^m \leq \theta < \theta_c^m$, and they get marry and have a child if $\theta_c^m < \theta$.

case4: $\theta_c^f = \theta_m^f$ and $\theta_c^m \leq \theta_m^m$
$\theta_m^f \geq \theta_m^m$ holds. If $\theta < \theta_m^f$, and they get marry and have a child if $\theta \geq \theta_m^f$.

case5: $\theta_c^f = \theta_m^f$ and $\theta_c^m > \theta_m^m$
$\theta_m^f = \theta_m^m = \theta^0$ holds. If $\theta < \theta_m^f$, they do not marry and do not have a child. If $\theta_m^f \leq \theta < \theta_c^m$, they get marry but do not have a child. If $\theta_c^m < \theta$, they get marry and have a child.

case6: $\theta_c^f > \theta_m^f$ and $\theta_c^m \leq \theta_m^m$
$\theta_m^f \geq \theta_m^m$ holds. If $\theta < \theta_m^f$, they do not marry and do not have a child. If $\theta_m^f \leq \theta < \theta_c^f$, they get marry but do not have a child. If $\theta_c^f < \theta$, they get marry and have a child.

case7: $\theta_c^f > \theta_m^f$ and $\theta_c^m \leq \theta_m^m$
$\theta_m^f \geq \theta_m^m$ holds. If $\theta_c^f < \theta_c^m$, they do not marry and do not have a child if $\theta < \theta_m^f$, and they get marry but do not have a child if $\theta_m^f \leq \theta < \theta_c^m$, and they get marry and have a child if $\theta_c^m < \theta$. If $\theta_c^f \geq \theta_c^m$, they do not marry and do not have a child if $\theta < \theta_m^f$, and they get marry but do not have a child if $\theta_m^f \leq \theta < \theta_c^f$, and they get marry and have a child if $\theta_c^f < \theta$.

4. Future works

By incorporating the asymmetry of child-rearing costs for the second term into the model, we showed that marriage selection and choice with children are in a more complicated situation. Moreover, we can derive $\partial W_{1,n}^i/\partial p > \partial W_1^i/\partial p > 0,\ i = 1, 2$. This implies that the remarriage probability p has the effect of increasing the lifetime expected utility and effects more strongly on married people in the first period. Since $d\theta_m^i/dp > d\theta_c^i/dp, i = 1, 2$, the remarriage probability effects more strongly on marriage decision than fertility. As an extension of this analysis, it is possible to consider an asymmetry in childcare cost in the first period and/or labor income that takes a relationship of trade-off with child-rearing time.

Reference

Browning, M., Chiappori, P.-A. and Weiss, Y. (2014) “Economics of the family,” Cambridge University Press.
Cabinet Office, Government of Japan (2019) “A 2018 Declining birthrate society measures white paper.”