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<td>Koshiba, M.; Tsuji, Y.</td>
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A Wide-Angle Finite-Element Beam Propagation Method

Masanori Koshiba, Senior Member, IEEE and Yasuhide Tsuji

Abstract—A wide-angle finite-element beam propagation method based on the Padé approximation is developed. Considerable improvement in accuracy over the paraxial approximation is achieved with virtually no additional computation. In the present algorithm, the quadratic element, transparent boundary condition, adaptive reference index, and adaptive grid are effectively utilized.

I. INTRODUCTION

THE BEAM propagation method (BPM) is at present widely used for the study of light propagation in longitudinally varying waveguides, and there are a great number of versions of the BPM [1]; fast Fourier transform BPM (FFT-BPM), finite difference BPM (FD-BPM), and finite-element BPM (FE-BPM) [2]–[6]. Conventional BPM's based on the Fresnel or paraxial approximation cannot treat wide-angle propagation. Recently, wide-angle algorithms based on the Padé approximation have been reported for the FD-BPM [7], [8] and the FFT-BPM [9], but not for the FE-BPM. In the FFT-BPM a weakly guiding structure is assumed, and in the FD-BPM it is, in general, difficult to introduce an adaptive discretization method which is well suited for the FE-BPM [5]. In this letter, a newly developed wide-angle FE-BPM using a Padé approximant operator is presented.

II. BASIC EQUATION

We consider a planar (two-dimensional) optical waveguide as shown in Fig. 1 where \( n \) is the refractive index, \( y \) and \( z \) are the transverse and propagation directions, respectively, and there is no variation in the \( x \) direction. With these assumptions we get the following:

\[
\frac{\partial}{\partial y} \left( p \frac{\partial \Phi}{\partial y} \right) + \frac{\partial}{\partial z} \left( p \frac{\partial \Phi}{\partial z} \right) + k_0^2 q \Phi = 0
\]

where \( \Phi = E_x \) with \( E_x \) being the \( x \) component of electric field, \( p = 1 \), and \( q = n^2 \) for the TE modes, and \( \Phi = H_z \) with \( H_z \) being the \( x \) component of magnetic field, \( p = 1/n^2 \), and \( q = 1 \) for the TM modes. Substituting a solution of the form

\[
\Phi(y, z) = \phi(y, z) \exp(-j k_0 n_0 z)
\]

into (1), we obtain the following for the slowing varying complex amplitude \( \phi \):

\[
p \frac{\partial^2 \phi}{\partial z^2} - 2j k_0 n_0 p \frac{\partial \phi}{\partial z} + \frac{\partial}{\partial y} \left( p \frac{\partial \phi}{\partial y} \right) + k_0^2 (q - n_0^2 p) \phi = 0
\]

where \( n_0 \) is the reference refractive index, and the term \( \partial p/\partial z \) is neglected for the TM modes.

III. FINITE-ELEMENT DISCRETIZATION

Dividing the waveguide cross section (computational window) into the quadratic (second-order) line elements [10], applying the standard FEM to (3), and using the transparent boundary condition [4], [6], we obtain

\[
[M] \frac{d^2 \phi}{dz^2} - 2j k_0 n_0 [M] \frac{d \phi}{dz} + ([K] - k_0^2 n_0^2 [M]) \phi + [K]_r \phi = \{0\}
\]

where \( \{\phi\} \) is the global electric or magnetic field vector, \( \{0\} \) is a null vector, \( [K] \) and \( [M] \) are the conventional finite-element matrices, and the matrix \( [K]_r \) is related to the computational window edge [4], [6].

We may formally rewrite (4) in the form

\[
-2j k_0 n_0 [M] \frac{d \phi}{dz} = \frac{([K] - k_0^2 n_0^2 [M]) \phi}{1 - \frac{1}{2j k_0 n_0} \frac{d}{dz}}
\]

with

\[
[K] = [K] + [K]_r.
\]

Utilizing the Padé recurrence relation [7]–[9] and replacing the \( z \) derivative in the denominator of (5) by

\[
\frac{d}{dz} \simeq \frac{1}{2j k_0 n_0} [M]^{-1} ([K] - k_0^2 n_0^2 [M])
\]

the following Padé equation is obtained:

\[
-2j k_0 n_0 [M] \frac{d \phi}{dz} + ([K] - k_0^2 n_0^2 [M]) \{\phi\} = \{0\}
\]

Fig. 1. Planar optical waveguide.

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the adaptive grid [5] are introduced. The benchmark problem consists of a sequence of simulations; fundamental mode with a tilt angle of 0° and 10th-order mode with tilt angles between 0° and 20° [1].

The Fresnel or paraxial equation is easily obtained from (7) by replacing the matrix $[M]$ by $[M]$. Applying the Crank–Nicholson algorithm for the propagation direction $z$ to (7) yields

$$[A]_i \{\phi\}_{i+1} = [B]_i \{\phi\}_i$$

with

$$[A]_i = -2jk_0n_0,i[M]_i + 0.5\Delta z(\hat{K})_i - k_0^2n_0,i[M]_i$$  \hspace{0.5cm} (9)

$$[B]_i = -2jk_0n_0,i[M]_i - 0.5\Delta z(\hat{K})_i - k_0^2n_0,i[M]_i$$  \hspace{0.5cm} (10)

where $\Delta z$ is the propagation step size, and the subscripts $i$ and $i+1$ denote the quantities related to the $i$th and $(i+1)$th propagation steps, respectively. Here, in order to improve numerical accuracy, the adaptive reference index [5], [6] and the adaptive grid [5] are introduced.

## IV. BENCHMARK TEST

We consider the TE mode propagating in a tilted straight optical waveguide shown in Fig. 1 where the waveguide supports 11 guided modes at the wavelength of 1.55 $\mu$m. The benchmark problem consists of a sequence of simulations; fundamental mode with a tilt angle of 0° and 10th-order mode with tilt angles between 0° and 20° [1].

Table I gives an overview of various types of BPM programs [1] where $M$ is the number of discretization points along the cross-section and $N$ the number of propagation steps. Our wide-angle FE-BPM is incorporated into an optical CAD system, ELM (expert lightwave modeling system). The overall relative power loss, i.e., the power transfer to any other guided mode, for the fundamental mode propagating in the tilted waveguide ($\theta = 0^\circ$) is less than −30 dB for all programs [1]. Fig. 2 shows the power loss and the “effort” for the 10th-order mode propagating in the most tilted waveguide ($\theta = 20^\circ$) where the efforts are divided into three classes to measure the efficiency of each program [1]. We can see that only, four programs, SNL-Padé, FD2BPM, AMIGO, and ELM, are able to propagate the eigenmode with an acceptable power loss less than −15 dB, and that ELM has the highest accuracy (power loss less than −20 dB) and the lowest effort (see Table I). Fig. 3 shows the evolution of power loss with increasing tilt angle for the above four programs. In ELM, $N = 20, 200, 400, 600, and 800$ for the $\theta = 0^\circ, 5^\circ, 10^\circ, 15^\circ, 20^\circ$ cases, and $M = 1311$ for all cases.

We can see again that ELM has the highest accuracy irrespective of tilted angles. A comparison between the Padé and the Fresnel (paraxial) approximant FE-BPM is shown in Fig. 4 where the values of $M$ and $N$ are the same as those of ELM in Fig. 3. The Padé approximant approach with quadratic

#### TABLE I

<table>
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<tr>
<th>Program</th>
<th>Equation</th>
<th>Discretization</th>
<th>Grid</th>
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<td>FTBPM</td>
<td>Rayleigh</td>
<td>uniform</td>
<td>512</td>
</tr>
<tr>
<td>Probabilistic FTBPM</td>
<td>Rayleigh</td>
<td>uniform</td>
<td>512</td>
</tr>
<tr>
<td>LITT-FD</td>
<td>Fresnel</td>
<td>non-uniform</td>
<td>1100</td>
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<tr>
<td>TIO-FD</td>
<td>Fresnel</td>
<td>uniform</td>
<td>1024</td>
</tr>
<tr>
<td>FD2BPM</td>
<td>Fresnel</td>
<td>uniform</td>
<td>1024</td>
</tr>
<tr>
<td>SNL-Pade</td>
<td>Padé</td>
<td>uniform</td>
<td>1501</td>
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<tr>
<td>FD2BPM</td>
<td>Padé</td>
<td>uniform</td>
<td>512</td>
</tr>
<tr>
<td>AMIGO</td>
<td>Padé</td>
<td>adaptive</td>
<td>42</td>
</tr>
<tr>
<td>ELM</td>
<td>Padé</td>
<td>adaptive</td>
<td>41</td>
</tr>
</tbody>
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Fig. 2. Relative power loss and effort for the TE10 mode and $\theta = 20^\circ$.

Fig. 3. Relative power loss as a function of tilt angle for the TE10 mode.

Fig. 4. Comparison between the Padé and the Fresnel approximant FE-BPM.
elements incorporated into ELM is very effective in reducing numerical errors. When using conventional linear (first-order) elements [10], no changes in the Padé and Fresnel results are observed. Fig. 5 shows the electric field patterns of the 10th-order mode for a tilt angle of 20° where quadratic elements are used in both the Padé and Fresnel approximant approaches.

The nonphysical mode behavior being the source of the power loss to radiation modes is observed in the Fresnel results.

V. CONCLUSION

A wide-angle FE-BPM with quadratic elements based on the Padé approximation was developed. Numerical results were compared with those of other BPM algorithms. It demonstrates considerable improvement in accuracy over the paraxial FE-BPM with virtually no additional computation.

REFERENCES