A Wide-Angle Finite-Element Beam Propagation Method

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***Abstract*— A wide-angle finite-element beam propagation method based on the Padé approximation is developed. Considerable improvement in accuracy over the paraxial approximation is achieved with virtually no additional computation. In the present algorithm, the quadratic element, transparent boundary condition, adaptive reference index, and adaptive grid are effectively UTILIZED.**

I. Introduction

The BEAM propagation method (BPM) is at present widely used for the study of light propagation in longitudinally varying waveguides, and there are a great number of versions of the BPM [1]; fast Fourier transfom BPM (FFT-BPM), finite difference BPM (FD-BPM), and finite-element BPM (FE-BPM) [2]–[6]. Conventional BPM's based on the Fresnel or paraxial approximation cannot treat wide-angle propagation. Recently, wide-angle algorithms based on the Padé approximation have been reported for the FD-BPM [7], [8] and the FFT-BPM [9], but not for the FE-BPM. In the FFT-BPM a weakly guiding structure is assumed, and in the FD-BPM it is, in general, difficult to introduce an adaptive discretization method which is well suited for the FE-BPM [5]. In this letter, a newly developed wide-angle FE-BPM using a Padé approximant operator is presented.

II. Basic Equation

We consider a planar (two-dimensional) optical waveguide as shown in Fig. 1 where n is the refractive index, y and z are the transverse and propagation directions, respectively, and there is no variation in the x direction. With these assumptions we get the following:

$$\frac{\partial}{\partial y}\left(p\frac{\partial \Phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(p\frac{\partial \Phi}{\partial z}\right)+k_0^2 q\Phi=0 \quad (1)$$

where $\Phi = E_x$ with E_x being the x component of electric field, $p = 1$, and $q = n^2$ for the TE modes, and $\Phi = H_x$ with H_x being the x component of magnetic field, $p = 1/n^2$, and $q = 1$ for the TM modes. Substituting a solution of the form

$$\Phi(y,z)=\phi(y,z)\exp(-jk_0 n_0 z) \quad (2)$$

into (1), we obtain the following equation for the slowing varying complex amplitude ϕ:

$$p\frac{\partial^2 \phi}{\partial z^2}-2jk_0 n_0 p\frac{\partial \phi}{\partial z}+\frac{\partial}{\partial y}\left(p\frac{\partial \phi}{\partial y}\right)+k_0^2(q-n_0^2 p)\phi=0 \quad (3)$$

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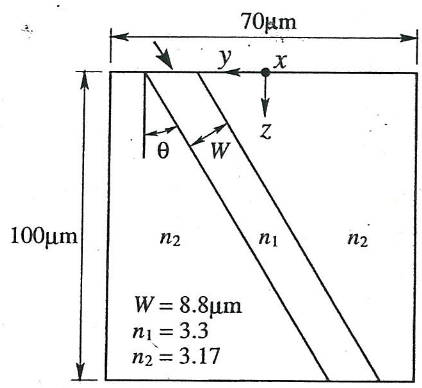

Fig. 1. Planar optical waveguide.

where n_0 is the reference refractive index, and the term $\partial p/\partial z$ is neglected for the TM modes.

III. Finite-Element Discretization

Dividing the waveguide cross section (computational window) into the quadratic (second-order) line elements [10], applying the standard FEM to (3), and using the transparent boundary condition [4], [6], we obtain

$$[M]\frac{d^2\{\phi\}}{dz^2}-2jk_0 n_0[M]\frac{d\{\phi\}}{dz}+([K]-k_0^2 n_0^2[M])\{\phi\}+[K]_\Gamma\{\phi\}=\{0\} \quad (4)$$

where $\{\phi\}$ is the global electric or magnetic field vector, $\{0\}$ is a null vector, $[K]$ and $[M]$ are the conventional finite-element matrices, and the matrix $[K]_\Gamma$ is related to the computational window edge [4], [6].

We may formally rewrite (4) in the form

$$-2jk_0 n_0[M]\frac{d\{\phi\}}{dz}=-\frac{([\tilde{K}]-k_0^2 n_0^2[M])\{\phi\}}{1-\dfrac{1}{2jk_0 n_0}\dfrac{d}{dz}} \quad (5)$$

with

$$[\tilde{K}]=[K]+[K]_\Gamma.$$

Utilizing the Padé recurrence relation [7]–[9] and replacing the z derivative in the denominator of (5) by

$$\frac{d}{dz}\simeq\frac{1}{2jk_0 n_0}[M]^{-1}([\tilde{K}]-k_0^2 n_0^2[M]) \quad (6)$$

the following Padé equation is obtained:

$$-2jk_0 n_0[\tilde{M}]\frac{d\{\phi\}}{dz}+([\tilde{K}]-k_0^2 n_0^2[M])\{\phi\}=\{0\} \quad (7)$$

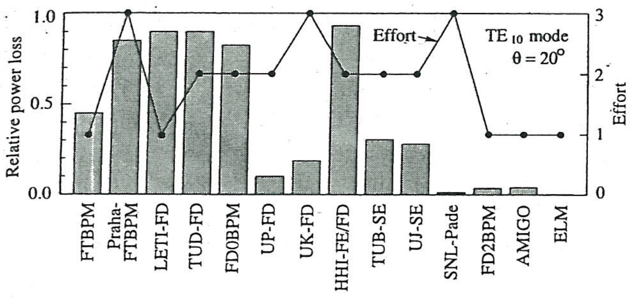

Fig. 2. Relative power loss and effort for the TE_{10} mode and $\theta = 20°$.

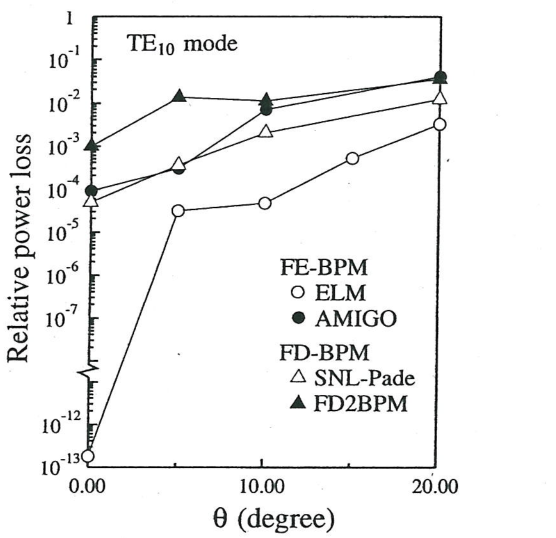

Fig. 3. Relative power loss as a function of tilt angle for the TE_{10} mode.

TABLE I
BRIEF DESCRIPTION OF THE BPM PROGRAMS

Program	Equation	Discretization	Grid	TE_0 ($\theta = 0°$)		TE_{10} ($\theta = 20°$)	
				M	N	M	N
FTBPM	Helmholtz	FFT	uniform	256	100	256	1000
Praha-FTBPM	Helmholtz	FFT	uniform	512	100	4096	2500
LETI-FD	Fresnel	FDM	uniform	1024	200	1024	200
TUD-FD	Fresnel	FDM	non-uniform	1100	1000	1100	1000
FD0BPM	Fresnel	FDM	uniform	1701	100	1701	1000
UP-FD	Fresnel	FDM	uniform	401	250	1201	2000
UK-FD	Fresnel	FDM	uniform	1401	1000	1401	1000
TUB-SE	Fresnel	FDM	uniform	1401	100	1401	1000
UJ-SE	Fresnel	FDM	uniform	1024	1000	1024	1000
HHI-FE/FD	Fresnel	FEM	uniform	1024	500	1024	2000
SNL-Pade	Padé	FDM	uniform	1501	1000	6001	3421
FD2BPM	Padé	FDM	uniform	512	100	2048	1000
AMIGO	Fresnel	FEM	adaptive	42	21	1311	1429
ELM	Padé	FEM	adaptive	61	20	1311	800

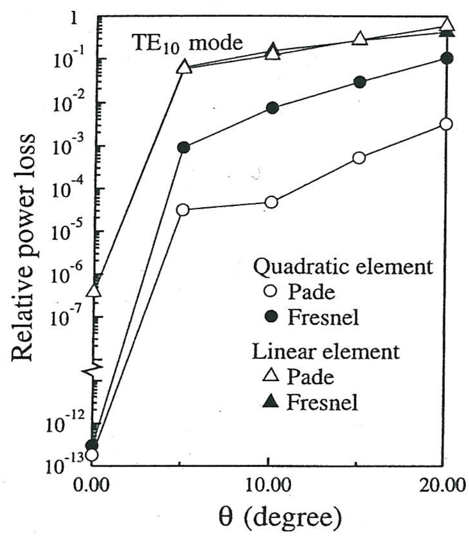

Fig. 4. Comparison between the Padé and the Fresnel approximant FE-BPM.

with

$$[\tilde{M}] = [M] + \frac{1}{4k_0^2 n_0^2}([\tilde{K}] - k_0^2 n_0^2 [M]). \tag{8}$$

The Fresnel or paraxial equation is easily obtained from (7) by replacing the matrix $[\tilde{M}]$ by $[M]$.

Applying the Crank–Nicholson algorithm for the propagation direction z to (7) yields

$$[A]_i \{\phi\}_{i+1} = [B]_i \{\phi\}_i \tag{9}$$

with

$$[A]_i = -2jk_0 n_{0,i}[M]_i + 0.5\Delta z([\tilde{K}]_i - k_0^2 n_{0,i}^2 [M]_i) \tag{10}$$

$$[B]_i = -2jk_0 n_{0,i}[M]_i - 0.5\Delta z([\tilde{K}]_i - k_0^2 n_{0,i}^2 [M]_i) \tag{11}$$

where Δz is the propagation step size, and the subscripts i and $i+1$ denote the quantities related to the ith and $(i+1)$th propagation steps, respectively. Here, in order to improve numerical accuracy, the adaptive reference index [5], [6] and the adaptive grid [5] are introduced.

IV. BENCHMARK TEST

We consider the TE mode propagating in a tilted straight optical waveguide shown in Fig. 1 where the waveguide supports 11 guided modes at the wavelength of 1.55 μm. The benchmark problem consists of a sequence of simulations; fundamental mode with a tilt angle of 0° and 10th-order mode with tilt angles between 0° and 20° [1].

Table I gives an overview of various types of BPM programs [1] where M is the number of discretization points along the cross-section and N the number of propagation steps. Our wide-angle FE-BPM is incorporated into an optical CAD system, ELM (expert lightwave modeling system). The overall relative power loss, i.e., the power transfer to any other guided mode, for the fundamental mode propagating in the untilted waveguide ($\theta = 0°$) is less than -30 dB for all programs [1]. Fig. 2 shows the power loss and the "effort" for the 10th-order mode propagating in the most tilted waveguide ($\theta = 20°$) where the efforts are divided into three classes to measure the efficiency of each program [1]. We can see that only, four programs, SNL-Padé, FD2BPM, AMIGO, and ELM, are able to propagate the eigenmode with an acceptable power loss less than -15 dB, and that ELM has the highest accuracy (power loss less than -20 dB) and the lowest effort (see Table I). Fig. 3 shows the evolution of power loss with increasing tilt angle for the above four programs. In ELM, $N = 20$, 200, 400, 600, and 800 for the $\theta = 0°$, 5°, 10°, 15°, 20° cases, and $M = 1311$ for all cases.

We can see again that ELM has the highest accuracy irrespective of tilted angles. A comparison between the Padé and the Fresnel (paraxial) approximant FE-BPM is shown in Fig. 4 where the values of M and N are the same as those of ELM in Fig. 3. The Padé approximant approach with quadratic

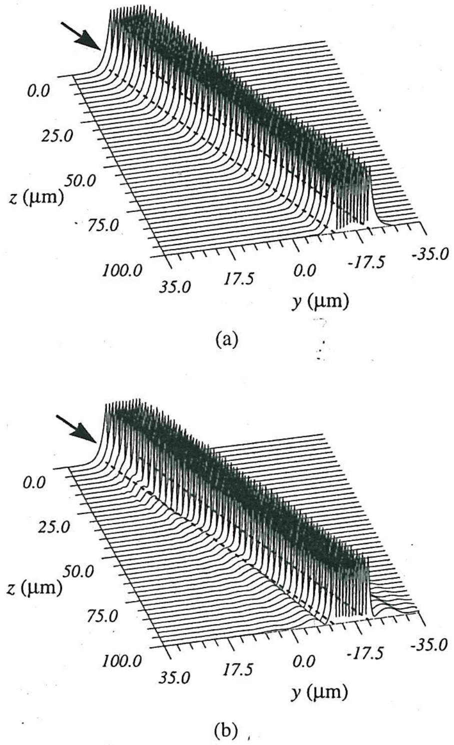

Fig. 5. Field pattern for the TE_{10} mode and $\theta = 20°$. (a) Padé approximation. (b) Fresnel approximation.

elements incorporated into ELM is very effective in reducing numerical errors. When using conventional linear (first-order) elements [10], no changes in the Padé and Fresnel results are observed. Fig. 5 shows the electric field patterns of the 10th-order mode for a tilt angle of 20° where quadratic elements are used in both the Padé and Fresnel approximant approaches. The nonphysical mode behavior being the source of the power loss to radiation modes is observed in the Fresnel results.

V. Conclusion

A wide-angle FE-BPM with quadratic elements based on the Padé approximation was developed. Numerical results were compared with those of other BPM algorithms. It demonstrates considerable improvement in accuracy over the paraxial FE-BPM with virtually no additional computation.

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