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## Abstract of Doctoral Dissertation

Degree requested    Doctor of Science    Applicant's name    Xiaotian Si

### Title of Doctoral Dissertation

Number-Phase Fluctuations in Isolated Superconductors  
(孤立超伝導体の粒子数\*位相揺らぎ)

One of the most controversial issues in the Bardeen-Cooper-Schrieffer (BCS) theory, which is remarkably successful in describing weak-coupling superconductors, may be the superposition over the number of condensed particles in their variational ground-state wave function.

This is apparently incompatible with particle-number conservation, which manifestly holds in any closed system, as noted by Schrieffer from the beginning and emphasized by Peierls and Leggett.

On the other hand, the superposition was used by Anderson in the context of Bose-Einstein condensation to discuss emergence of a well-defined macroscopic phase, called spontaneously broken gauge symmetry, as the key ingredient for superfluidity and the Josephson effect.

Thus, particle-number fluctuations seem indispensable for bringing macroscopic coherence to the system, which were originally traced by Anderson to the exchange of particles between subsystems.

However, question may be raised regarding this identification because there are definitely no fluctuations in the total particle number in any closed system.

$$\Delta N = 0 \Rightarrow \Delta \Phi \rightarrow \infty?$$

If we consider that the fluctuations of particle  $\Delta N$  number is absolutely zero, does that lead to an infinite fluctuation in the superconducting phase? Are the fluctuations real or a mere artifact in the mathematical treatment of superconductivity? If the former is the case, where do they originate? How can we define a macroscopic wave function with a well-defined phase in isolated superconductors? We aim to answer these questions by improving the BCS wave function with a fixed particle number.

Weak-coupling superconductors have been described theoretically within the mean-field framework. The corresponding ground-state with  $N$  fermions has been identified as the anti-symmetrized product of  $N/2$  Cooper-pairs with no superposition,

which is given by

$$|\Phi_N^{\text{BCS}}\rangle = A_{N/2}^{-1/2} (\hat{\pi}_{\text{cp}}^\dagger)^{N/2} |0\rangle,$$

and may thereby have no well-defined phase.

We improve the Bardeen-Cooper-Schrieffer wave function with a fixed particle number so as to incorporate many-body correlations beyond the mean-field treatment, which is given by

$$|\Phi_N^{\text{Corr.}}\rangle = B_N^{-1/2} \exp(\hat{\pi}_4^\dagger) |\Phi_N^{\text{BCS}}\rangle.$$

It is shown in our study that the correlations lower the ground-state energy far more than Cooper-pair condensation in the weak-coupling region.

This fact implies that, formally speaking, Cooper-pair condensation should be studied only after the correlations effects have been incorporated.

Moreover, they naturally bring a superposition over the number of condensed particles. Thus, Cooper-pair condensation is special among the various bound-state formations of quantum mechanics in that number fluctuations are necessarily present in the condensate through the dynamical exchange of particles with the non-condensate reservoir.

Finally, we propose  $\Delta N_{\text{Cooper-pairs}} \Delta \varphi \geq 1$  as the uncertainty relation relevant to the number-phase fluctuations in superconductors and superfluids, where the number of condensed particles  $N_{\text{Cooper-pairs}}$  is used instead of the total particle number  $N_{\text{total}}$ . The formula implies that a macroscopic phase  $\varphi$  can be established even in number-fixed superconductors and superfluid since  $\Delta N_{\text{Cooper-pairs}} \gg 1$ .

Thus, the superposition is a real physical entity that exists in any isolated superconductor or superfluid. Note in this context that the superposition and coherence have so far been discussed mostly in terms of condensed particles alone.

This thesis is organized as follows;

We will discuss the improved ground-wave function with many-body correlations. Section 2 presents formulation. Section 3 gives numerical results. Section 4 presents concluding remarks. We also present details that deriving the formalism from Appendix.