Group delay spread analysis of coupled-multicore fibers: A comparison between weak and tight bending conditions

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Group delay spread of coupled three-core fiber is investigated based on coupled-wave theory. The differences between supermode and discrete core mode models are thoroughly investigated to reveal applicability of both models for specific fiber bending condition. A macrobending with random twisting is taken into account for random modal mixing in the fiber. It is found that for weakly bent condition, both supermode and discrete core mode models are applicable. On the other hand, for strongly bent condition, the discrete core mode model should be used to account for increased differential modal group delay for the fiber without twisting and short correlation length, which were experimentally observed recently. Results presented in this paper indicate the discrete core mode model is superior to the supermode model for the analysis of coupled-multicore fibers for various bent condition. Also, for estimating GDS of coupled-multicore fiber, it is critically important to take into account the fiber bending condition.

Key words: Coupled-multicore fiber, space-division multiplexing, group delay spread.
I. Introduction

Mode-division-multiplexing (MDM) transmission technique has attracted considerable attention as a promising technology for increasing network capacity. Few-mode fibers (FMFs) used for MDM transmission have also been intensively studied and various designs of FMF have been proposed [1]. Among them, coupled-multicore fibers (CMCFs) are intensively studied recently, since the measured group delay spread (GDS) is significantly lower than that predicted by differential modal group delay (DMGD) [2] of the eigenmodes of the fiber. The reduced GDS leads to simpler configuration of MIMO receiver, and hence, longer system reach [3].

The phenomenon is caused by random modal mixing in fibers and theoretical methods estimating GDS has been developed [4-6]. The modal coupling is induced by various fiber perturbations, such as, core-diameter and core-distance deformations [5], random macrobending [7], and macrobending with uniform or random twisting [7-9]. Especially, recent experimental studies [8,9] show macrobending with twisting can be a dominant mechanism for random modal coupling. The effect of macrobending with random twisting on GDS was theoretically analysed in [7] for coupled 3-core fiber (3CF) [2] based on coupled-wave theory (CWT). In [7], since the supermodes of straight 3CF was used for the basis of CWT, it is difficult to use the theory for strongly bent condition. For example, in [8], although increased DMGD was observed for short length (up to 60 m) of strongly bent 3CF without twisting, the phenomenon cannot be explained with the supermode model in [7]. Furthermore, in earlier theoretical studies [5-7], the correlation length was assumed to be ∼ 10 m, which is the typical polarization mode dispersion correlation length of single mode fibers. However, experimental results for short length fiber in [8] indicate the length is much shorter than 10 m for strongly bent condition. As discussed in [10], the installed fiber can be divided into two regions: bend challenged spun and main transmission line. In the bend challenged spun, the length of the fiber is much shorter than 1 km and bending condition is tight (bending radius, $R$, $\ll$ 1 m). In the main transmission line, the length is long (1 ∼ 1000 km) and bending condition is gentle ($R >> 0.1$ m) [10]. To extract the ultimate performance of CMCF in various installed condition, general theoretical models for estimating GDS of CMCF is necessary.

In this paper, GDS of 3CF [2] is investigated based on CWT with supermode or discrete core mode (DCM) basis. The differences between supermode and DCM models are thoroughly
investigated to reveal applicability of both models for specific fiber bending condition. A macrobending with random twisting is taken into account for random modal mixing in the fiber. It is found that for weakly bent condition, both supermode and discrete core mode models are applicable. On the other hand, for strongly bent condition, the discrete core mode model should be used to account for increased differential modal group delay for the fiber without twisting and short correlation length, which were experimentally observed recently. Results presented in this paper indicate the discrete core mode model is superior to the supermode model for the analysis of CMCFs for various bent condition. Also, considering fiber bending condition is critically important for estimating GDS. Finally, the effect of effective index fluctuations on GDS is investigated. If the magnitude of core-diameter fluctuation is that of reported state-of-the-art fiber [11], the effect of fluctuation on GDS is negligible for the fiber with twisting.

II. Theory

A. Fiber structure and coupled-wave theory

Figure 1 shows the cross-sectional structure of the fiber considered here. Three identical cores are arranged in an equilateral triangle with the side length of $\Lambda$. The core radius is $a = 6.2 \, \mu m$ and the refractive index difference, $\Delta = 0.185\%$ [2] (normalized frequency, $V = 2.22$). The wavelength is $1.55 \, \mu m$. Figure 1 also shows the electric field distributions of three supermodes of this fiber for $\Lambda = 29 \, \mu m$ [2]. First mode has almost uniform distributions for three cores. Second and third modes are degenerate and have zero intensity at the center of the fiber. For this fiber, calculated DMGD between first and second (third) modes without random modal coupling is 212 ps/km. We only consider one polarization state since the modes are degenerate in terms of polarization. We assume that the fiber is uniformly bent to $x$ direction with the constant bending radius of $R$. The fiber with the length of $L$ is divided into $M$ segments with the segment length of $\Delta L$. Each segment has randomly generated twist angle $\Delta \theta_i$ relative to the previous segment. $\Delta \theta_i$ has Gaussian distribution with the mean value of 0 rad and the standard deviation (STD) of $\sigma_0$ rad. In each segment, the structure is assumed to be uniform.
To estimate GDS, we use CWT, in which the field amplitude and phase are fully taken into account, which is essential for treating the change of modal group delay due to perturbations [12]. Field coupling equations for \( N \) guided mode system in one segment are given by

\[
\frac{d a_m}{dz} = -j \beta_m a_m - j \sum_{m \neq n}^N \kappa_{mn} a_n
\]

where \( a_m \) and \( \beta_m \) are the field amplitude and propagation constant of \( m \)th guided mode. \( \kappa_{mn} \) is the coupling coefficient between \( m \)th and \( n \)th guided modes and given in section 2-B and 2-C for supermode and DCM models.

**B. Supermode model**

If the bending condition is gentle and the guided mode in 3CF can be considered as that of straight 3CF, we can use unbent supermodes as the basis of CWT. In this case, the coupling coefficient \( \kappa_{mn} \) is given by

\[
\kappa_{mn} = \omega \varepsilon_0 \oint \oint \left( n_{\text{bend}}^2(x', y', R) - n_{\text{st}}^2(x', y') \right) E_m^* \cdot E_n dxdy
\]

\[
\oint \oint \vec{i}_z \cdot (E_m^* \times H_m + E_m \times H_m^* ) dxdy
\]

Here, \( x' = x \cos \theta - y \sin \theta \) and \( y' = x \sin \theta + y \cos \theta \) are the rotated coordinate with respect to the original ones with the rotation angle \( \theta \). \( \omega \) is the angular frequency, \( \varepsilon_0 \) is the free-space permittivity, \( n_{\text{bend}} \) and \( n_{\text{st}} \) are the refractive index distributions for bending and straight waveguides. \( E_m \) and \( H_m \) are transverse electric and magnetic field distributions of \( m \)th mode. Refractive index distribution under the bending radius \( R \) is given by

\[
n_{\text{bend}}^2(x', y', R) = n_{\text{st}}^2(x', y') \left( 1 + \frac{2x'}{R} \right).
\]

Finite-element method [13] is used for the calculation of the coupling coefficient. In this model, \( \kappa_{mn} = \kappa_{nm} \). The index distribution differences between bent and straight waveguides make the coupling coefficient between different supermodes nonzero, leading to modal coupling. It should be noted that “bent supermode” cannot be used for the basis of CWT if the segment is uniform since these bent modes are orthogonal each other and the coupling coefficient given by (2) is zero (index difference term in the numerator is zero).
C. Discrete core mode model

If the bending condition is tight, the guided modes of 3CF tend to localize in each core since refractive index distribution is no longer symmetric as in equation (3). From (3), it is obvious that right half of the fiber has larger refractive index and the left half of the fiber has lower refractive index if the origin is taken at the fiber center. And the difference with original refractive index, \( n_{st} \), is larger for large \( |x| \) and small \( R \). Therefore, for tight bending condition, the refractive index distribution of the fiber is highly asymmetric and the coupling between cores are suppressed, leading to localized mode for each core, rather than forming supermode.

Figure 2 shows three “bent” guided modes of each core of 3CF with \( R = 100 \) mm. Here, each guided mode is calculated for only one core (assuming other cores are cladding) and core outline in the Figure is plotted just for reference. In this case, we should take DCM as the basis of CWT and the DCM is \( LP_{01} \) mode of each core as shown in Fig. 2. In this model, the coupling coefficient between two cores is given by

\[
\kappa_{mn} = \frac{\omega \varepsilon_0}{4} \int \int \left( \frac{\delta n^2}{2} \mathbf{E}_m^{\ast} \cdot \mathbf{E}_n + \frac{i m}{2} \int \mathbf{E}_m^{\ast} \times \mathbf{H}_m + \mathbf{E}_n \times \mathbf{H}_n^{\ast} \right) dxdy. \tag{4}
\]

Here, \( m \) and \( n \) are core number (\( m, n = 1,2,3 \)). \( \delta n^2 \) is the refractive index distribution difference between original fiber cross section and fiber with only core \( n \). More concretely, \( \delta n^2 = n_{st}^2 - n_n^2 \), where \( n_{st} \) is the refractive index distribution of three-core fiber (there are three cores) and \( n_n \) is the refractive index distribution of the fiber, only having core \( n \). This is the usual definition of the coupling constant [14].

The coupling coefficient between two cores for straight fiber calculated by (4) is 11.4 m\(^{-1}\). The effective index of core \( m (= 1,2,3) \) at the \( i \)th step, \( n_{eff,m,i} \) is given by [15]

\[
n_{eff,m,i} = n_{eff,m} \left( 1 + \frac{D \cos \phi_{m,i}}{R} \right), \tag{5}
\]

where \( n_{eff,m} \) is the effective index of \( m \)th core without bending, \( D \) is the core-center position from the fiber center, and \( \phi_{m,i} \) is the angle of the \( m \)th core. These parameters are denoted in Fig. 3. When two cores are aligned on the same \( x \) position, two cores are phase matched and the strong coupling occurs. Since the position of each core is randomly changed with propagation distance as explained in 2-A, random coupling between cores takes place. For the coupling coefficient,
because the coupling due to phase matching is dominant in this model rather than the magnitude of coupling coefficient, we use the coupling coefficient for straight fiber for simplicity.

**D. Group delay spread**

The solution of (1) can be written as

\[ \mathbf{a}(L) = \mathbf{T}(\omega) \mathbf{a}(0) \]  

(6)

where \( a \) is a column vector, \([a_1 \ a_2 \ \cdots \ a_N]^T\), and \( \mathbf{T} \) is a total transmission matrix. By using \( \mathbf{T} \), so-called group delay operator (GDO) [16] can be defined as

\[ \text{GDO}(\omega) = j \mathbf{T}(\omega)^{-1} \frac{d\mathbf{T}(\omega)}{d\omega}. \]  

(7)

The GDS (\( \sigma_{gd}^2 \)) is given by [4-7]

\[ \sigma_{gd}^2 = \frac{1}{N} \sum_{i=1}^{N} \tau_i^2 \]  

(8)

where \( \tau_i \) is the \( i \)th eigenvalue of GDO and is normalized as \( \sum \tau_i = 0 \). The angled brackets denote the ensemble average. It was recently revealed in [6] that this quantity corresponds to the width of intensity impulse response of the recent MDM fiber [2] in the strong coupling regime.

**III. Results**

**A. Difference between supermode and DCM models**

Figure 4 (a) shows GDS obtained by supermode and DCM models in [ps/m] as a function of fiber curvature (1/R) for 1-km 3CF without twisting (\( \sigma_0 = 0 \) rad). Horizontal dashed line is GDS calculated by DMGD. GDS obtained by DCM model agree with that obtained by supermode model up to \( 1/R \approx 1 \) m\(^{-1}\) and after that GDS is increased for DCM model and almost constant and lower than DMGD for supermode model. This is reasonable result because in the supermode model, guided modes of straight 3CF are used as the basis of CWT, GDS is always smaller than DMGD. Since measured results in [8] shows clear GDS increase for large value of 1/R, DCM model should be used for tight bending condition. For weak bending condition (up to \( 1/R \approx 1 \) m\(^{-1}\)
1) both model gives almost the same results. The computational time for both models is similar since the matrix size is the same. Figure 4 (b) shows GDS obtained by supermode and DCM models in [ps/m] as a function of fiber curvature (1/R) for 1-km 3CF with random twisting (σ₀ = 2 rad). Here, ΔL is taken as 10 m, which is the typical polarization mode dispersion correlation length of single mode fibers. Results are averaged over 100 fiber realizations. For all curvature, GDS is significantly decreased compared with 3CF without twisting (Fig4(a)). The results obtained by both models are consistent up to 1/R ≈ 1 m⁻¹ and after that GDS is increased for DCM model and almost constant for supermode model. Therefore, the upper limit of the curvature, at which GDSs obtained by both models are similar, is not sensitive to the twisting. Since DCM model can be used for both weak and tight bending condition, DCM model is superior to the supermode model for the analysis of GDS of CMCF with macrobend and random twisting model. It should be noted that there is a possibility that the supermode model is easier to use than DCM model for different physical fiber condition, such as random (weak) bending model presented in [7].

Next, the relationship between GDS and the correlation length is investigated. Here, we want to show the correlation length for tight bending condition is much shorter than that for weak bending condition for obtaining the same value of GDS. To compare these two conditions, we have to determine the reference value of GDS, and the experimental results in [2] are chosen for this purpose. Measured GDS in [2] is given by

\[
GDS = \kappa \sqrt{L}
\]  \hspace{1cm} (9)

where \( \kappa^2 = 275 \text{ ps}^2/\text{km} \), whose value is fitted by measured GDS [2,5].

Figure 5 shows GDS of 3CF as a function of transmission distance calculated for R = 5000 mm calculated by supermode model (weak bending condition). ΔL is taken as 10 m, which is the typical polarization mode dispersion correlation length of single mode fibers. Upper dashed line shows GDS calculated by DMGD. Dots are given by (9). For σ₀ = 2 rad, calculated GDS is well fitted to the reference value obtained by (9).

For comparing weak and tight bending conditions, since the state of twisting seems to be not so different for different values of R, we assume that σ₀ is equal for both conditions (σ₀ = 2 rad) and see the difference of the correlation length.
Figure 6 shows GDS of 3CF as a function of transmission distance for $R = 100$ mm calculated by DCM model (tight bending condition) and $\sigma_0 = 2$ rad, for various values of $\Delta L$. Upper dashed line shows GDS calculated by DMGD of bent core mode and dots are given by (9). GDS is reduced for smaller $\Delta L$. For $R = 100$ mm, reference values (obtained by (9)) are well fitted to the line given by $\Delta L = 0.5$ m, which is 1/20 of weak bending condition ($\Delta L = 10$ m for $R = 5000$ mm). The correlation length for tight bending condition to obtain the same GDS for gentle bending is much shorter than that for gentle bending. The result is consistent to measured results in [8], where strong modal coupling is observed for tightly bent short piece of 3CF ($\sim 60$ m). Note that, we do not discuss that which condition is absolutely correct compared with the measured results in [2] (in [2], there are no description on the experimental bending condition). The results in [2] is just used for reference value and we discuss the relative difference in the correlation length between two conditions.

Figure 7 shows GDS of 100-km 3CF calculated by DCM model as a function of $1/R$ for 3CF with random twisting ($\sigma_0 = 2$ rad) for various values of $\Delta L$. For smaller values of $\Delta L$, GDS is significantly reduced because there are a lot of random sections in the fiber. From the results of Figs. 5, 6, and 7, following important design considerations on strongly CMCF can be derived. Currently, the lab-experiment for measuring GDS of CMCF is usually done for tight bending condition (the fiber is spooled to a bobbin, whose $R \approx 100$ mm, or $1/R \approx 10$ m$^{-1}$). In that case, the fitted correlation length from the measured results is much shorter than 1 m as shown in Fig. 7. If one estimates the GDS of the fiber based on this lab-fitted correlation length, GDS for 100-km fiber is 860, 70, and 140 ps for $1/R = 0.01$, 0.1, and 10 m$^{-1}$ as in Fig. 7. However, as stated in Introduction, the majority of the installed fiber is in weak bending condition and for weak bending condition, there is a possibility that the correlation length seems to be $\sim 10$ m, which is the typical polarization mode dispersion correlation length of single mode fibers [5,6]. If we use $\Delta L = 10$ m, estimated GDSs are 3750 and 405 ps for $1/R = 0.01$ and 0.1 m$^{-1}$ as in Fig. 7 and they are very different from those obtained by using tight bending correlation length. Therefore, when estimating GDS of CMCF, special care should be taken: fitting parameters obtained in lab-experiment may not be valid for estimating GDS of the fiber in the field.

B. Effects of core diameter fluctuations
In [9], it was experimentally observed that there is an optimum core pitch ($\Lambda$) for reducing GDS of coupled two-core fiber. Theoretically speaking, if $\Lambda$ is large, the difference of effective index between different supermodes is reduced, leading to strong modal coupling in fibers. However, due to the fabrication imperfection, especially, core diameter difference, $\Delta a$, modes are decoupled for too large $\Lambda$ and not coupled, leading to the existence of optimum core pitch. The assumed $\Delta a$ between two cores in [9] is about 2%, which is relatively large compared with that of well-fabricated fiber. For example, it was reported that $\Delta a$ is about 0.03% for state-of-the-art highly nonlinear fiber [11]. Although there is always fabrication imperfection in real fiber, it is interesting to see how $\Delta a$ of state-of-the-art fiber affects to GDS, in other words, ultimate performance of CMCF.

We assume that core-diameter $a$ is fluctuated in longitudinal direction. 0.03% fluctuation for $a = 6.2$ $\mu$m core is about 0.002 $\mu$m. The effective index change for the fluctuation is about $4 \times 10^{-5} \%$. To take into account it, random $n_{\text{eff}}$ fluctuation, $\delta n_{\text{eff},m,i}$, is given for each segment by Gaussian distribution with the mean value of 0 and the variance of $\sigma_{n_{\text{eff}}} = 4 \times 10^{-7} n_{\text{eff},m}$, and $8 \times 10^{-7} n_{\text{eff},m}$. Therefore, $n_{\text{eff},m,i}$ is given by

$$n_{\text{eff},m,i} = \left(n_{\text{eff},m} + \delta n_{\text{eff},m,i} \left(1 + \frac{D \cos \phi_{m,i}}{R}\right)\right). \quad (9)$$

Note that, $m$ is the mode number for supermode model and core number for DCM model. Figure 8 shows GDS as a function of transmission distance for $R = 5000$ mm and $\Delta L = 10$ m. Without twisting ($\sigma_\theta = 0$), slight fluctuation in the effective index greatly affects GDS for long distance transmission. On the other hand, for $\sigma_\theta = 2$ rad, the effect of index fluctuation is almost negligible since the modal coupling due to twisting is dominant. Figure 9 shows the same one as Fig. 8 but for $R = 100$ mm and $\Delta L = 0.5$ m. For the tight bending condition, index fluctuation for the state-of-the-art fiber is almost negligible regardless of twisting.

**IV. Conclusion**

We have investigated GDS of 3CF for weak and tight bending condition by CWT. While for weak bending condition, both supermode and DCM models can be used, for tight bending condition, DCM model is preferable due to the isolated nature of guided modes in 3CF. DCM
model successfully accounts for GDS increase in tight bending condition without twisting effect and short correlation length in tight bending condition, which were experimentally observed recently in [8]. These results indicate that for estimating GDS of coupled-multicore fiber, it is critically important to consider fiber bending condition. The effect of effective index fluctuation of state-of-the-art fiber is also investigated, and it is negligible for the fiber with twisting.

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**References**


Fig. 1 Fiber cross section and supermodes of 3CF.

Fig. 2 Bent guided modes of each core of 3CF for $R = 100$ mm.

Fig. 3 A schematic of bent fiber and parameters used for modeling.
Fig. 4 GDS of 1-km 3CF as a function of fiber curvature for (a) $\sigma_0 = 0$ and (b) $\sigma_0 = 2$ rad.
Fig. 5 GDS as a function of transmission distance for weak bending condition calculated by supermode model \((R = 5000 \text{ mm})\).

Fig. 6 GDS as a function of transmission distance for tight bending condition calculated by DCM model \((R = 100 \text{ mm})\).
Fig. 7 GDS of 100-km 3CF as a function of fiber curvature for $\sigma_0 = 2$ rad.

Fig. 8 GDS as a function of transmission distance for weak bending condition ($R = 5000$ mm and $\Delta L = 10$ m) with and without index fluctuations.
Fig. 9 GDS as a function of transmission distance for tight bending condition ($R = 100$ mm and $\Delta L = 0.5$ m) with and without index fluctuations.