Fast backprojection-based reconstruction of spectral-spatial EPR images from projections with
the constant sweep of a magnetic field

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Abstract

In this paper, we introduce a procedure for the reconstruction of spectral-spatial EPR images using projections acquired with the constant sweep of a magnetic field. The application of a constant field-sweep and a predetermined data sampling rate simplifies the requirements for EPR imaging instrumentation and facilitates the backprojection-based reconstruction of spectral-spatial images. The proposed approach was applied to the reconstruction of a four-dimensional numerical phantom and to actual spectral-spatial EPR measurements. Image reconstruction using projections with a constant field-sweep was three times faster than the conventional approach with the application of a pseudo-angle and a scan range that depends on the applied field gradient. Spectral-spatial EPR imaging with a constant field-sweep for data acquisition only slightly reduces the signal-to-noise ratio or functional resolution of the resultant images and can be applied together with any common backprojection-based reconstruction algorithm.

Highlights:

• The constant sweep of a magnetic field is proposed for spectral-spatial EPR imaging.
• This approach simplifies the rapid acquisition of a large number of EPR projections.
• Image reconstruction is three times faster with constant-sweep projections.
• The proposed imaging approach insignificantly reduces the quality of the final images.

Keywords: 4D EPR imaging, spectral-spatial EPR, image reconstruction, backprojection.
1. Introduction

Spectral-spatial EPR imaging is an imaging modality that enables the spatially resolved measurement of EPR spectra [1-3]. It has been shown to be particularly useful for functional imaging, where the EPR spectrum of a paramagnetic compound, a spin probe, changes depending on its microenvironment. The additional spectral dimension can be processed to give images of the functional properties of the object being investigated. Some important in vivo applications of functional EPR imaging include the visualization of tumor oxygenation [4-8], spatially-resolved measurements of tissue redox status [9-11], visualization of myocardial oxygenation and ischemia-induced acidosis [12-14], and brain oximetry [15-17].

Four-dimensional (4D, 3 spatial and 1 spectral dimension) EPR imaging is required to obtain fully spatially resolved spectral data. This can be achieved by collecting EPR projections using magnetic field gradients of different strengths at multiple spatial orientations. Typically, a very large number of EPR projections (starting from several thousand and more) are required to acquire an EPR image with fairly good spatial and spectral resolution. In this context, the time required for data acquisition becomes a matter of critical importance. Pulsed EPR techniques appear to be attractive for the rapid acquisition of a large number of 4D EPR projections. However, the application of pulsed EPR methods is mainly limited to the use of spin probes with long relaxation times, e.g., trityl radicals [5, 7] or perdeuterated nitroxide radicals [18]. An alternative EPR modality known as the rapid-scan technique is focused on direct detection of the EPR absorption profile without the application of Zeeman field modulation or lock-in amplifiers [19-21]. Although this technique looks very promising for the rapid acquisition of a large amount of experimental data, there has been only one report on the application of the rapid scan technique for in vivo EPR imaging [22].

Continuous-wave (CW) EPR remains the most commonly used modality for EPR measurements. Recent developments in fast field scanning CW-EPR have made it possible to acquire single EPR projections in as little as 10 – 50 ms [23-25]. In these EPR instruments, field-sweeping Helmholtz coils are driven by an analog sawtooth-like signal, and data acquisition is synchronized with the magnetic field scan. Application of the fast field-scanning technique would make it possible to acquire a 4D spectral-spatial EPR image in a matter of several minutes. However, in the conventional approach for spectral-spatial EPR imaging, the projections are collected with a variable field scan range that depends on the applied field gradient [1-4, 12]. To reduce the time required for image acquisition several previous works have suggested using a reduced sweep width that depends on the applied field gradient and the geometry of the imaged object [26, 27].
The implementation of a variable field sweep width might be associated with certain technical
difficulties, including the need for the use of sophisticated field driving circuits and proper
synchronization of the magnetic field sweep with data acquisition. Moreover, if projections are
scanned with a fast sweep of the magnetic field, the time-constant of the Helmholtz coil may affect
the linearity of field scanning. Therefore, a feedback system or proper calibration of the driver
current/voltage output depending on the scan speed could be required.

Thus, it may be beneficial to collect the EPR data using a constant field-sweep and to implement
all the necessary corrections and adjustments into the reconstruction procedure. In this paper, we
present a simple yet efficient approach for calculating the projections of a spectral-spatial EPR
image and backprojecting the experimental data collected with the constant sweep of a magnetic
field. The application of a constant-sweep imaging approach simplifies image reconstruction and
considerably reduces the time required for computations. The proposed approach can be used with
any type of backprojection-based reconstruction technique such as filtered backprojection (FBP),
algebraic reconstruction technique (ART), maximum likelihood expectation maximization (MLEM),
and others.

2. Theory

In spectral-spatial EPR imaging, the spatial orientation of the magnetic field gradient is
traditionally given in spherical coordinates with polar and azimuthal angles, \((\alpha, \beta)\), and the
additional spectral coordinate is described by introducing a pseudo-angle, \(\varphi\):

\[
\tan \varphi = \frac{L \| \mathbf{G} \|}{\Delta B},
\]

where \(L\) is the field of view, \(\mathbf{G} = (G_x, G_y, G_z)\) is the magnetic field gradient and \(\Delta B\) is the spectral
window. In these terms, the projection \(P_{\alpha, \beta, \varphi}(r)\) of a four-dimensional EPR image, \(f(B, x)\), can be
calculated as the integral along the direction specified by angles \(\alpha, \beta\) and \(\varphi\):

\[
P_{\alpha, \beta, \varphi}(r) = \int_{-\infty}^{\infty} \int_{\mathbb{R}^3} f(B, x) \delta((x \cdot n) \sin \varphi + B \cos \varphi - r) dB \, dx.
\]

Here we used the vector notation \(\mathbf{x} \equiv (x, y, z)\) for the 3D spatial coordinate and
\(n = (\cos \alpha \sin \beta, \sin \alpha \sin \beta, \cos \beta)\) for the geometric normal; \(r\) is the radial distance in 4D space
and \(B\) is the magnetic field. The meaningful length of an EPR projection is determined by the
spectral window and by the difference in local magnetic field between the most remote parts of the
sample due to the presence of a field gradient:
\[
\Delta B + \sqrt{3} L \|G\| = \frac{\Delta B}{\cos \varphi} \left( \cos \varphi + \sqrt{3} \sin \varphi \right).
\] (3)

Thus, the magnetic field sweep required for the projection \(P_{\alpha,\beta,\varphi}(r)\) in Eq. 2 can be determined by

\[
\text{field sweep} = \frac{2 \Delta B}{\cos \varphi},
\] (4)

where coefficient \(2 = \max(\cos \varphi + \sqrt{3} \sin \varphi)\) represents the main diagonal of a four-dimensional object. Note that the field sweep determined by Eq. 4 is always bigger than the meaningful length of a projection (Eq. 3) and reaches it only when \(\varphi = 60^\circ\).

In a discrete case, the reconstruction of a four-dimensional image with a matrix size of \(m^4\) voxels would require a set of projections with \(2m\) data points acquired with a sampling rate of

\[
\text{step} = \frac{\Delta B}{\cos \varphi} \cdot \frac{1}{m}.
\] (5)

The position corresponding to the image voxel with coordinates \((i_B, i_x, i_y, i_z)\) on the projection \(P_{\alpha,\beta,\varphi}(r)\) can be calculated as:

\[
r = \left[(i_x, i_y, i_z) \cdot \mathbf{n}\right] \sin \varphi + i_B \cos \varphi.
\] (6)

An illustration of how a projection can be computed using the pseudo-angle is presented in Fig. 1A. For the sake of simplicity, a two-dimensional case (1D spatial + spectral) is demonstrated.

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**Fig. 1.** (A) Two-dimensional sketch of the concept of spectral-spatial EPR imaging using a pseudo-angle, \(\varphi\). The projection is calculated by summarizing the data along the direction specified by the angle \(\varphi\). (B) Computation of projection with a constant sweep of the magnetic field. The image matrix is sheared along the spectral axis according to the applied field gradient. Projection is calculated by summarizing the data along the direction perpendicular to the axis.
Another approach for the computations may exploit the direct definition of an EPR projection

\[ P_G(B) = \int_{\mathbb{R}^2} f(B + G \cdot x, x) \, dx. \]  

To cover the desired field of view of the image, the magnetic field sweep for the projections should satisfy the following equation:

\[ \text{field sweep} = \Delta B + \sqrt{3} L G_{max}, \]  

where \( G_{max} \) is the maximum field gradient used for the measurements. With a constant sweep width, it is possible to set the projection data sampling rate to be the same as in the image to be reconstructed:

\[ \text{step} = \frac{\Delta B}{m}. \]  

In this case, the position of a voxel \((i_x, i_y, i_z, i_B)\) on the projection \(P_G(B)\) can be calculated as:

\[ B = [(i_x, i_y, i_z) \cdot G] + i_B. \]  

Thus, a constant-sweep projection can be calculated by shearing the image matrix according to the applied field gradient followed by the integration of data along the direction perpendicular to the spectral axis. A two-dimensional illustration for the computation of a projection using this approach is presented in Fig. 1B.

To calculate the projection of a spectral-spatial image or to reconstruct an EPR image from experimental data using a backprojection procedure, all voxels of the image should be placed in or taken from the proper position on the projection. In general, the voxel coordinates determined by Eq. 6 and Eq. 10 are not integers, and thus interpolation is needed to calculate the projection or to draw a specific value from the discrete data of the experimental projections. In this work, we used linear interpolation for the calculations. Linear interpolation was chosen because it can be implemented in just a few operations to provide a fast and satisfactory result. In addition, elements of the image were accessed as in a single-dimensional array because straight-through indexing is much faster than having to address the data of a multidimensional array. The complete source code file for the MATLAB executable function used for the reconstruction of 4D EPR images from projections with a constant field-sweep is provided in the Supplemental Materials (available online).

3. Methods

3.1. Numerical 4D EPRI phantom
All computations were performed in the MATLAB development environment (MathWorks, Inc., Natick, MA, USA) running on an Apple iMac (Mid 2010) computer. A four-dimensional numerical phantom with $64^3$ points for 3D-spatial coordinates and 128 points for the spectral coordinate was generated. The simulated field-of-view and spectral window were 25 mm and 0.5 mT, respectively. The sample consisted of four cylindrical objects with a single EPR line in each. A Voigt function (convolution of Gaussian and Lorentzian functions) was used to simulate the EPR signals. For all of the objects, the Gaussian linewidth was set to 13 µT and the Lorentzian linewidths were 20, 22, 28 and 30 µT. The integral intensities of the spectral lines for the four objects were 1.0, 1.5, 1.0 and 1.5 arbitrary units, respectively.

Two sets of EPR projections were generated: one with the field sweep adjusted to the gradient and another with a constant field sweep. For each case, a total of 3375 (15^3) projections with 512 data points were simulated. An incrementally ramped field gradient with 15 steps with a maximum value of 0.2 mT/cm for each of the X-, Y- and Z-directions was used for the simulation. The field sweep for the variable sweep projections was determined using Eq. 4. Projections with a constant field-sweep were calculated for 2 mT (Eq. 8). To investigate the effect of instrumental noise on reconstructed images, artificial white Gaussian noise was added to the simulated projections. The signal-to-noise ratio for the data was determined as

$$SNR = \frac{S_{pp}}{2\sqrt{2} SD_{noise}},$$

where $S_{pp}$ is the peak-to-peak intensity of the EPR signal and $SD_{noise}$ is the standard deviation of noise.

3.2. CW-EPRI phantom measurements

A phantom was prepared for spectral-spatial EPR imaging. The sample consisted of four flame-sealed glass tubes (6 mm internal diameter and 25 mm length) filled with oxygen-free (nitrogen-bubbled) and air-equilibrated solutions of trans-3,4-dicarboxy-2,2,5,5-tetra(2H3)methylpyrrolidin-(3,4-2H2)-(1-15N)-1-oxyl ($^{2}$H, $^{15}$N-DCP) in phosphate-buffered saline. The concentration of the radical in the tubes was 1.0 mM and 1.5 mM, respectively. The $^{2}$H, $^{15}$N-DCP radical was synthesized and kindly provided by Dr. Igor Kirilyuk (Novosibirsk Institute of Organic Chemistry, Russia).

EPR measurements were performed using a laboratory-built CW-EPR imager/spectrometer operating at 750 MHz equipped with a multi-coil parallel-gap resonator (22 mm in diameter and 30 mm in length). The details of the EPR instrumentation have been reported previously [23, 24, 28].
The following EPR settings were used for image acquisition: duration of field scanning, 100 ms; time constant of lock-in amplifier, 0.1 ms; number of data points per scan, 512; magnetic field sweep, 2 mT; modulation amplitude, 20 µT; modulation frequency, 90 kHz; and incident RF power, 2.2 mW. The center field was set to the low field component of the EPR spectrum. EPR projections were acquired under a magnetic field gradient of $15 \times 15 \times 15$ of equal steps with a maximum value of 0.2 mT/cm for each of the X-, Y- and Z-directions (total of 3375 projections). The total image acquisition time was 7.5 min.

### 3.3. Image reconstruction

Four-dimensional spectral-spatial EPR images were reconstructed using the algebraic reconstruction technique (ART). Reconstruction was performed in the MATLAB development environment with the ART function implemented in C-language for speed. The source code for the function is provided online in the Supplemental Materials. The matrix size for image reconstruction was $64^3$ points for 3D-spatial coordinates and 128 points for the spectral coordinate with a field of view of 2.5 cm and a spectral window of 0.5 mT. When reconstruction was performed by a pseudo-angle approach, the projections were downsampled to 256 points and scaled by dividing by the cosine of the corresponding pseudo-angle, $\cos \varphi$.

After each ART iteration, spectral data of the images were fitted with the convolution of Gaussian and Lorentzian functions (Voigt profile) with a 5 % threshold for peak-to-peak intensity. Fitting was performed by the Levenberg-Marquardt algorithm with the Voigt function calculated using a rational approximation of the complex error function [29]. The known position of the EPR signal and a constant Gaussian linewidth of 13 µT were used to fit the spectral data. After fitting, image spatial data were smoothed using a 3D Gaussian filter with a standard deviation of 1 voxel width. A total of 12 ART iterations were performed for the reconstruction and provided consistent results and good convergence with the initial projections.

### 4. Results and Discussion

#### 4.1. Comparison of the reconstruction approaches

In image reconstruction based on a backprojection procedure, each voxel must be processed through all of the experimental projections at least once. For a four-dimensional image and a projection count of several thousand, the number of operations required for reconstruction could become extremely large. For example, FBP reconstruction of a 4D image with a matrix size of $64^3 \times$
128 voxels from a set of $15^3$ projections should include a for-loop which would be repeated $1.1 \times 10^{11}$ times. The application of iterative reconstruction techniques, such as ART or MLEM, which are less sensitive to experimental noise and typically provide more solid solutions than FBP, would increase this number in proportion to the total number of iterations. While computers are continually getting faster and can perform parallel computations, reducing the number of numerical operations within the reconstruction loop should still be considered good practice.

The two approaches presented in Figs. 1A and 1B illustrate how a projection can be calculated using the pseudo-angle and by shearing the image matrix. In both cases, the reverse procedure, i.e. backprojection, can be accomplished in a similar manner by applying the data from the projections to the image. Application of the pseudo-angle approach implies that a proper position on the projections will be calculated for each voxel of the spectral-spatial EPR image. However, if the projections were acquired with a constant-sweep magnetic field and the spectral coordinate of reconstructed EPR image has the same data sampling rate, it becomes unnecessary to compute the field index position during reconstruction, and the projection data can be accessed consecutively point by point. This considerably reduces the number of floating-point operations and pointer arithmetics required for tossing the data between the image and its projections. Moreover, due to the identical data sampling rate, the coefficients for linear data interpolation remain constant within a single vector of spectral data and can be calculated only once outside the field-sweeping for-loop of the reconstruction algorithm (see the source code file in the Supplemental Materials). Therefore, the application of a constant field sweep and a predefined data sampling rate can facilitate reconstruction of spectral-spatial EPR images by a backprojection procedure.

To compare the time required for reconstruction by a traditional technique using a pseudo-angle and by the proposed constant-sweep approach, a 4D numerical phantom and two sets of projections were generated (see Methods). Image reconstruction was performed with an ART algorithm, which was chosen because it converges relatively quickly and typically provides a good solution. Figure 2 shows the EPR image that was reconstructed from the projections with a constant-sweep magnetic field. A very similar image was obtained when reconstruction was performed through the use of a pseudo-angle from the projections with a gradient-adjusted field sweep (data not shown). The computation time for a single ART cycle was 6 minutes for the constant-sweep approach and 19 minutes with a pseudo-angle. Therefore, the application of a constant field-sweep and a predefined data sampling rate could increase the speed of reconstruction of spectral-spatial EPR images by more than three-fold. When EPR projections are
collected using either a standard or optimized sweep-width, it is also possible to achieve faster image reconstruction by adding zeros to the projections and interpolating them with a constant data-sampling rate.

**Fig. 2.** Spectral-spatial EPR image reconstructed from the numerically generated projections with a constant sweep of the magnetic field. (A) 3D surface-rendered image of EPR signal intensity calculated with a 30 % threshold. (B) Map of the integral intensity and (C) map of the Lorentzian linewidth for the central cross-section. Image matrix size is $64^3 \times 128$ points (3D spatial $\times$ spectral dimension).

### 4.2. Spectral-spatial EPR phantom measurements

To demonstrate the utility of the proposed reconstruction approach for actual EPR measurements, we prepared a phantom with isotopically substituted nitroxyl radical $^{2}$H,$^{15}$N-DCP. The spatial orientation of the sample differed from that of the numerical phantom. EPR projections were acquired under the constant sweep of a magnetic field of 2 mT. The signal-to-noise ratio (SNR) of the experimental data as calculated for the zero-gradient projection was 56. The image in Fig. 3 was obtained after reconstruction. As can be seen from Fig. 3, the shape of the reconstructed object properly corresponds to the original sample and the observed Lorentzian linewidths of oxygen-free and air-saturated radical solutions can be clearly distinguished. However, the integral intensity of the tube located at the upper central part of the image appears to be less than might be expected for a 1 mM radical solution. This observation can most likely be explained by the lower intensity of the $B_1$-field nearby the gap of the EPR resonator as compared to the areas in the vicinity.
of the current loops. The measured spectral intensity also decreases along the sample with increasing distance from the RF feed point of the EPR resonator (data not shown).

Fig. 3. Spectral-spatial EPR image of a $^2$H, $^{15}$N-DCP phantom sample. The sample consisted of four tubes with oxygen-free and air-saturated solutions of the radical at concentrations of 1.0 mM and 1.5 mM. (A) 3D surface-rendered image of EPR signal intensity calculated with a 30 % threshold. (B) Map of the integral intensity and (C) map of the Lorentzian linewidth for the central cross-section. Field of view of the image is 25 mm; spectral window, 0.5 mT; image matrix size, $64^3 \times 128$ points (3D spatial $\times$ spectral dimension).

The Lorentzian linewidths for all of the four tubes in the phantom accurately reproduced the values measured spectroscopically (Table 1). As might be expected, the EPR spectral linewidths of 1.5 mM radical solutions are slightly larger than those of 1 mM solutions due to concentration-induced line-broadening. The histogram of the Lorentzian linewidth data calculated with a 30 % intensity threshold is presented in Fig. 4. While the histogram peaks for 1.0 and 1.5 mM radical solutions strongly overlap each other, oxygen-free and air-saturated solutions are fully separated.
<table>
<thead>
<tr>
<th>$^{2}\text{H},^{15}\text{N}-\text{DCP}$</th>
<th>N$_2$-bubbled</th>
<th>Air-saturated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 mM</td>
<td>$19.8 \pm 0.9$ (19.3)</td>
<td>$27.2 \pm 1.1$ (28.5)</td>
</tr>
<tr>
<td>1.5 mM</td>
<td>$20.7 \pm 0.8$ (21.3)</td>
<td>$27.9 \pm 0.8$ (29.4)</td>
</tr>
</tbody>
</table>

Table 1. Calculated values of the Lorentzian linewidth for tubes from the phantom sample presented in Fig. 3. The data are means ± SD µT; values in parentheses were measured spectroscopically.

Fig. 4. Histogram of Lorentzian linewidth data for the spectral-spatial EPR phantom. The data for 1.0 and 1.5 mM radical are presented in green and blue, respectively.

4.3. Optimization of spectral-spatial EPR images

Generally, reconstruction of an image from a limited number of measured projections is an ill-posed problem with multiple possible solutions. The most direct way to resolve the ambiguity and improve both the spatial and spectral resolution of the image is to increase the number of EPR projections. Therefore, EPR instrumentation that is capable of rapid data acquisition and a time-efficient reconstruction algorithm are desirable for 4D EPR imaging. However, several approaches have recently been proposed for the reconstruction of EPR images acquired with a limited number of measured projections [30-33]. These methods exploit certain additional assumptions during reconstruction such as maximum entropy, minimum total variance, piece-wise constancy or spatial sparsity of the sample. Although the application of such criteria during the reconstruction process may considerably improve the quality of the final images, in real experiments, researchers do not possess any information about the nature of the spin probe distribution in the object being examined. Therefore, any EPR image solution that satisfies experimental projections and does not contradict the spectral properties of the applied spin probe should be taken into consideration.

To improve the quality of reconstructed images, we fitted the spectral data of images with the Voigt function after each ART iteration. Fitting was performed with the known EPR peak field position and Gaussian linewidth, which is typical in real EPR oximetry experiments. Fitting forces the algorithm to converge to a pure "EPR image" with nothing but EPR spectra on the spectral coordinate and significantly improves the spatial resolution of the image and the accuracy of
functional measurements. A similar result can be achieved by the application of a parametric approach for spectral-spatial EPR imaging that uses non-linear regression for image reconstruction [34].

4.4. Influence of experimental noise on EPR images

When EPR projections are acquired with a constant-sweep magnetic field, only a part of the experimental data is used for image reconstruction because the meaningful length of EPR projections (Eq. 3) is always smaller than the scan range determined by Eq. 8, with an exception for the maximum gradient. The projection acquisition scheme used in our experiments with a scan width of 2 mT and a maximum field gradient of 0.2 mT/cm for each spatial direction causes 30.5% of the experimental data to be collected in vain. The percent of unused data will be even higher if a larger field-gradient and scan-width are used. This suggests that the application of a constant field-sweep for spectral-spatial EPR imaging might be less effective than the conventional imaging approach and might have some unfavorable effects on the resultant images, e.g., greater susceptibility to experimental noise.

To compare the two approaches for spectral-spatial EPR imaging, we performed reconstruction of the numerical phantom shown in Fig. 2. from projections with artificially added white noise. The SNR for both sets of projections was 58. A total of 12 ART iterations were used for reconstruction and no additional data fitting or filtering was applied. After reconstruction, the SNR for the spectral data of the images was calculated. The SNR for the most intense object on the image was 8.4 ± 1.0 when reconstruction was performed with projections with a variable field sweep and 6.1 ± 0.7 with a constant-sweep magnetic field. However, it should be noted that reconstruction by the pseudop-angle approach was performed using projections that were downsampled to 256 points, while all 512 data points were used for the constant-sweep reconstruction (see Image reconstruction in the Methods). This downsampling averages white noise and decreases its intensity by a factor of $\sqrt{2}$. Thus, to make a proper comparison, constant-sweep projections were passed through a low-pass filter with a bandwidth equal to the corresponding downsampled projection:

$$\text{bandwidth} = 512 \cos \varphi.$$  \hspace{1cm} (12).

Specifically, a low-pass filter with a Fourier domain bandwidth of 256 points was applied to the high-gradient projection (phi = 60°, bandwidth = 256 points for 2 mT), and all 512 points were used for the zero-gradient projection (phi = 0°, bandwidth = 512 points for 2 mT). This bandwidth exactly corresponds to the sampling scheme of the projections with a gradient-dependent field sweep of
256 data points for $1/\cos(\phi)$ mT. When the filtered projections were used for constant-sweep reconstruction, the SNR of the resultant image was $8.1 \pm 1.0$, which is very close to the value obtained with variable-sweep projections and a pseudo-angle reconstruction approach.

4.5. Comparison of functional resolution

To compare the functional resolution of the methods, we fitted spectral data of the images reconstructed from projections with added white noise with a Voigt profile. The fitting results for each object on the images with a 30% intensity threshold are summarized in Fig. 5. For both imaging approaches, the mean values of the Lorentzian linewidth properly reproduce the initial parameters for simulation. The mean values of the integral intensity were smaller than what was expected for the samples with an EPR signal of 1.5 arbitrary units, because the same threshold was used for low- and high-intensity objects. The standard deviation of the measured parameters from the mean values was similar for both reconstruction methods.

![Fig. 5. The calculated values for the integral intensity and Lorentzian linewidth for numerical phantoms with added white noise. (A) Conventional approach for image reconstruction. (B) Reconstruction using constant-sweep projections. The data are shown as means ± SD.](image)

Thus, both imaging approaches yield similar SNR and functional resolution, despite the fact that in the constant-sweep method the data from projections with a low field-gradient are collected in a less effective manner. Specifically, 30.5% of the constant-sweep data are not needed for image reconstruction. If we assume that instrumental noise has a white Gaussian nature, an ideal EPR experiment with an optimized scan-width and equal intensity of noise in every projection could
decrease the standard deviation of noise on the images by 16.6 %. However, if the comparison is to
the traditional imaging approach, this value would be smaller because high- and low-gradient
projections have different noise intensities. This is because the total intensity of the EPR signal on
the projections of an image is determined only by the sample and does not depend on the gradient
orientation or magnitude. However, when EPR projections are acquired with a scan range adjusted
to the field gradient, as is required in the conventional imaging approach (Eq. 4), the plain sum of
the EPR signal for the discrete data points decreases in proportion to the applied field sweep (note
that the field integral remains constant). For this reason, measured projections must be scaled
before a backprojection-based reconstruction, i.e., they must be divided by the cosine of the
corresponding pseudo-angle, \( \cos \varphi \). After scaling, the absolute intensity of noise on high-gradient
projections becomes greater than that on projections with a low field-gradient, and this amplified
noise is transferred to the images during subsequent reconstruction. For this reason, in the
traditional approach for spectral-spatial EPR imaging, major image distortions originate from the
noise on high-gradient projections and projections with a low field-gradient have less of an impact
on the SNR of the final images. The sweep-width of projections from sets with variable and
constant field-sweep become closer at higher field-gradients, and this explains why the application
of constant-sweep method insignificantly reduces the quality of the final images. In real EPR
imaging experiments, this can be compensated by higher stability, linearity and reproducibility of
the magnetic field sweep during fast projection scanning.

In summary, we have introduced a simple procedure for the reconstruction of spectral-spatial
EPR images from projections acquired with a constant field-sweep. This approach seeks to achieve
the rapid swapping of data between the image and its projections and can be applied with any
common backprojection-based reconstruction techniques, such as FBP, ART, MLEM or others (note
that proper adjustment of filter bandwidth in the Fourier domain might be needed for
reconstruction by FBP). Spectral-spatial EPR images can be reconstructed 3-times faster using
constant-sweep projections compared to the traditional approach using a pseudo-angle and
projections with a scan range that depends on the applied field gradient. Constant sweep of the
magnetic field allows for the rapid scanning of large number of EPR projections and, when used
with the proposed reconstruction technique, could be advantageous for the acquisition of high-
resolution 4D spectral-spatial EPR images.

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