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Transient Behavior of a Stress-Strain Curve within Cottrell-Stokes Law†

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By employing stress-strain constitutive relationships within Gilman-Johnston and Alexander-Haasen models, transient behavior of calculated stress-strain curves is examined in the light of Cottrell-Stokes law. It is pointed out that the incorporation of the interaction force between mobile and immobile dislocations are indispensable to satisfy Cottrell-Stokes law.

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I. Introduction

Although the experimental and theoretical studies on dislocation behavior in plastic deformation have been recently advanced, the theory of *collective behavior of dislocations* have not been well established yet. In order to describe the *collective behavior of dislocations*, the evolution of the mobile dislocation densities, N_m, and the variation of average velocity, $\bar{v}$, of moving dislocations should be properly formulated. The body of former studies on the *collective behavior of dislocations* exclusively focused on yield point and work-hardening behavior under a constant external condition such as deformation temperature and cross-head speed. Few works have been performed on transient behavior caused by a sudden change of deformation condition. In the present study, as a key to the analysis of the transient behavior, Cottrell-Stokes law[1] is evoked.

We describe the stress-strain curve at temperatures T_1 and T_2, by $\sigma(\varepsilon, T_1)$ and $\sigma(\varepsilon, T_2)$, respectively, and assume that deformation temperature changes from T_1 to T_2 ($T_1 > T_2$) at a strain ε^*. When only mobility of dislocations, which is manifested by the velocity $\bar{v}$, is controlled by temperature and multiplication behavior is independent of that, a discontinuous jump of the stress, $\sigma(\varepsilon^*, T_2) - \sigma(\varepsilon^*, T_1)$, is realized and the stress-strain curve of T_2 is followed in the subsequent deformation. Whereas in the opposite case, that is, only the multiplication behavior has temperature dependency, no jump of the stress is expected and a resulting stress-strain curve follows continuously from $\sigma(\varepsilon, T_1)$ with the inclination, $d\sigma(\varepsilon, T_2)/d\varepsilon$. In the actual case, however, both mobility and multiplication processes have temperature dependences and a certain amount of the discontinuous change of the stress $\Delta\sigma$, which is less than $|\sigma(\varepsilon, T_2) - \sigma(\varepsilon, T_1)|$, is anticipated. Most importantly for this discontinuity of the stress $\Delta\sigma$, Cottrell and Stokes observed that $\Delta\sigma/\sigma$ is independent of the strain. This is the Cottrell-Stokes law and $\Delta\sigma/\sigma$ is termed Cottrell-Stokes ratio.

In this paper, we discuss transient behavior of theoretically calculated stress-strain curves in view of Cottrell-Stokes law. In particular, we focus on two representative traditional theories: Gilman-Johnston's[3] and Alexander-Haasen's models[4].

II. A Brief Summary of Theories

1. Constitutive equation of stress-displacement

The constitutive equation to describe the stress-displacement relation is given in the following form[2],

$$\frac{d\tau_{app}}{dy} = \frac{K \cdot \beta}{C} \cdot \left(1 - \frac{l_0 \cdot \beta \cdot \dot{\varepsilon}}{S_c}\right), \tag{1}$$

where τ_{app} is the applied shear stress, y is the displacement of the cross-head and S_c is a cross-head speed. K, β, C and l_0, are, respectively, the rigidity of a machine-sample system, Schmid factor, the initial cross section area of the sample and the initial gauge length of the sample. The strain rate, $\dot{\varepsilon}$, is defined as

$$\dot{\varepsilon} = N_m \cdot \bar{v} \cdot b, \tag{2}$$

where b is the magnitude of Burgers vector. Substitution of eq. (2) into eq. (1) yields the differential equation

$$\frac{d\tau_{app}}{dy} = \frac{K \cdot \beta}{C} \cdot \left(1 - \frac{l_0 \cdot \beta \cdot N_m \cdot \bar{v} \cdot b}{S_c}\right), \tag{3}$$

which correlates microscopic dislocation behavior with macroscopically observed stress-strain relations. In order

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to draw a stress-strain curve, one needs to give an expression for the average dislocation velocity, $\bar{v}$, and mobile dislocation density, N_m. In the present study, we adopted two representative formulations described in the next section.

2. Gilman-Johnston model and Alexander-Haasen model

From experimental observations, Gilman and Johnston[3] provided the velocity, $\bar{v}$, and the density, N_m, in the following forms,

$$\bar{v}=B_0\cdot\tau_{\mathrm{eff}}^m\cdot\exp\left(-\frac{Q}{k_B\cdot T}\right)=K(T)\cdot\tau_{\mathrm{eff}}^m, \tag{4}$$

and

$$N_m=C_1+C_2\cdot y^{1/2}, \tag{5}$$

where both B_0, m, C_1 and C_2 are constants to be determined experimently. Q, k_B and T are, respectively, an activation energy, Boltzmann constant and the absolute temperature. τ_{eff} is the effective stress defined in the next section. Based on these equations they reproduced yield point behavior quite satisfactorily.

Alexander and Haasen[4] assumed that the increase in the total length of mobile dislocations is proportional to the area swept by moving dislocations. Thus

$$dN_m=\delta\cdot N_m\cdot\bar{v}\cdot dt, \tag{6}$$

where t is the time and δ is a proportionality coefficient which is assumed to be an increasing function of τ_{eff} expressed as

$$\delta=B\cdot\tau_{\mathrm{eff}}, \tag{7}$$

where B is a material parameter specifying the multiplication rate. Based on eqs. (6) and (7), a differential equation describing the evolution (devolution) of mobile dislocation densities is derived as

$$\frac{dN_m}{dy}=\frac{1}{S_c}\cdot B\cdot\tau_{\mathrm{eff}}\cdot N_m\cdot\bar{v}. \tag{8}$$

III. Calculation Results and Discussion

In both models in the previous section, τ_{eff} in eqs. (4) and (7) is the effective stress defined as

$$\tau_{\mathrm{eff}}=\tau_{\mathrm{app}}-\tau_{\mathrm{in}}, \tag{9}$$

where τ_{in} is the internal stress. In the present study, two kinds of expression for the internal stress are assumed. One is

$$\tau_{\mathrm{in}}=\tau_m=\frac{1}{2}A\cdot N_m^{1/2} \tag{10}$$

and the other is

$$\tau_{\mathrm{in}}=\tau_m+\tau_i=\frac{1}{2}A\cdot N_m^{1/2}+C(T)\cdot\varepsilon^{1/2}, \tag{11}$$

where

$$A=\frac{\mu b}{B_s}. \tag{12}$$

μ is the stiffness of the material and B_s characterizes the interaction between mobile dislocations. In the former case, the internal stress originates from interaction among mobile dislocations, τ_m, while for the latter case, an additional contribution being proportional to $\varepsilon^{1/2}$ simulates the interaction between mobile and immobile dislocations, τ_i, and $C(T)$ is the function which increases with the decrease of temperature. Parameters and constants employed in the present study are tabulated in **Table 1**. In this calculation, it is assumed that the temperature is reduced forom 298 to 248 K.

When eq. (10) is adopted with Gilman-Johnston model, the temperature dependency is conveyed only in the average dislocation velocity ($\bar{v}$) in eq. (4). In fact, the sudden change of the temperature from T_1 to T_2 induces the jump of the stress and the resultant stress-strain curve coincides exactly with the one for T_2 as shown for three different strains in **Fig. 1**.

Table 1 The employed parameters and constants in this calculation. These values are typical for the aluminum-magnesium dilute solid solution.

Parameters and constants	Values
C	7.5×10^{-6} (m^2)
l_0	2.0×10^{-2} (m)
b	2.8635×10^{-10} (m)
μ	2.8×10^{10} (Pa)
S_c	1.2×10^{-4} (m/s)
Q	4.73×10^{-20} (J)
k_B	1.38×10^{-23} (J/K)
K	5.0×10^{3} (Pa·m)
B	5.0×10^{-5}
C_1	2.0×10^{4} (m^{-2})
C_2	2.5×10^{8} (m^{-2})
β	0.4
m	1.0
B_s	3.3
$C(T)$	5.0×10^4(T_1=298 K) 8.0×10^4(T_2=248 K)

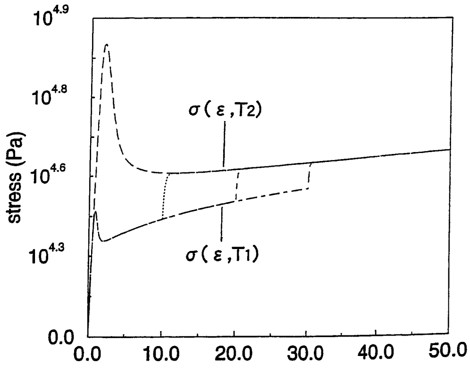

Fig. 1 Transient behavior of stress-strain curve associated with temperature change from T_1 to T_2. In this calculation, temperature dependency is assumed only for the mobility ($\bar{v}$) of mobile dislocations.

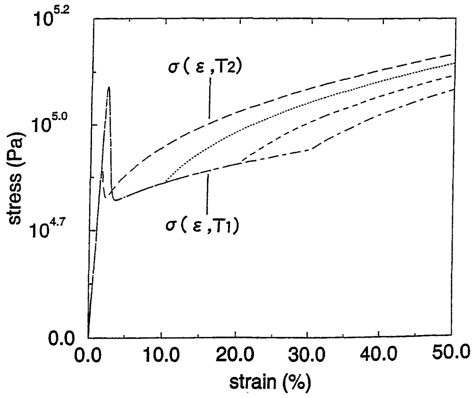

Fig. 2 Transient behavior of stress-strain curve associated with temperature change from T_1 to T_2. Temperature dependency is assumed only for multiplication process (N_m).

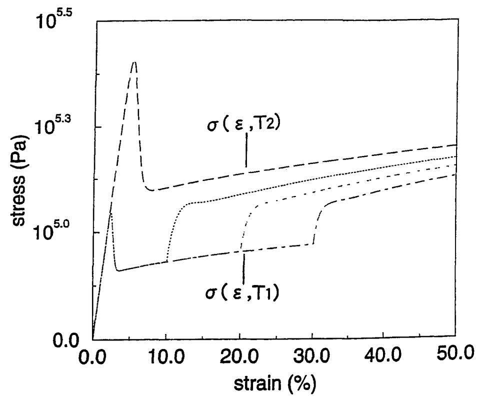

Fig. 3 Transient behavior of stress-strain curve associated with temperature change from T_1 to T_2. Temperature dependencies are assumed both for $\bar{v}$ and N_m.

In order to examine the temperature dependence of the multiplication process, Alexander-Haasen model is adopted with the internal stress given by eq. (10). For the velocity $\bar{v}$ in eq. (8), eq. (4) is employed with a constant value assigned to $K(T)$, which enables one to derive the sole effect of the temperature dependency of N_m. As shown in **Fig. 2**, the stress-strain curve shows no discontinuous jump of the stress at any strains. Furthermore the inclination $d\sigma(\varepsilon, T_2)/d\varepsilon$, at the onset is observed to be nearly the same as that of $\sigma(\varepsilon, T_2)$ for each strain.

The effect of the temperature on both mobility and multiplication behavior are studied within Alexander-Haasen model. In this case, the temperature dependency of the velocity is explicitly incorporated through $K(T)$ in eq. (4). With the internal stress expression eq. (10), the transient behavior is calculated as shown in **Fig. 3**. Certainly, a discontinuous stress change, $\Delta\sigma$, is observed. But the magnitude of $\Delta\sigma$ is less than $\sigma(\varepsilon, T_2)-\sigma(\varepsilon, T_1)$. This is regarded as an intermediate situation between two extreme cases shown in Figs. 1 and 2. Unfortunately, however, the Cottrell-Stokes ratio are not kept to be a constant value as shown in **Fig. 4**.

Finally, Alexander-Haasen model is employed with the internal stress given by eq. (11). The transient behaviors are demonstrated in **Fig. 5**. In this case, the Cottrell-Stokes ratio stays a constant value as can be seen in **Fig. 6** when appropriate values of $C(T)$ tabulated in Table 1 are employed.

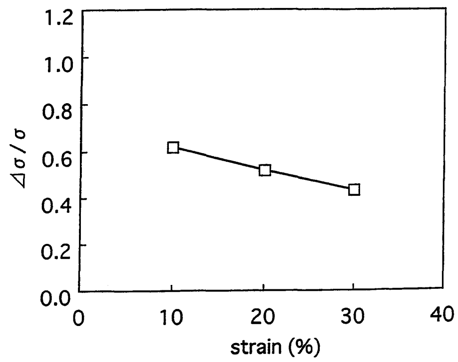

Fig. 4 Cottrell-Stokes ratio corresponding to Fig. 3.

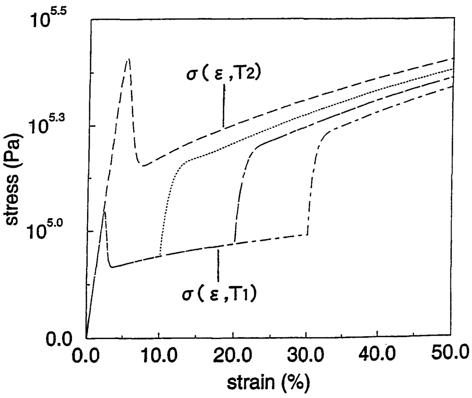

Fig. 5 Transient behavior of stress-strain curve associated with temperature change from T_1 to T_2. Temperature dependencies are assumed both $\bar{v}$ and N_m. Also, immobile-mobile dislocation interaction force is taken into consideration.

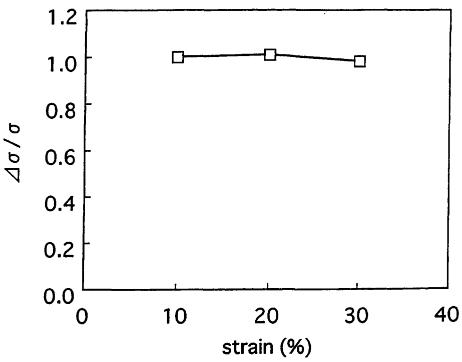

Fig. 6 Cottrell-Stokes ratio corresponding to Fig. 5.

IV. Conclusions

In the present study, we examined the transient behavior of stress-strain curve associated with the sudden change of temperature. Within the Gilman-Johnston and

Alexander-Haasen models, the temprature dependencies of dislocation mobility ($\bar{v}$) and multiplication process (N_m) are well studied both in a separate and in a synthesized manner. It is pointed out that the interaction between mobile and immobile dislocations play essential role to observe Cottrell-Stokes law. It is believed that the transient behavior due to the change of strain rate can be studied within the same framework.

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