Abstract of Doctoral Dissertation

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Title of Doctoral Dissertation

On the Falk invariant of an arrangement

(超平面配置の Falk 不変量)

Abstract:

A hyperplane H is an affine subspace with codimension 1 in $\mathbb{C}^l$, and an arrangement $\mathcal{A}$ is a finite set of hyperplanes. Let $M = M(\mathcal{A}) = \mathbb{C}^l \setminus \bigcup_{H\in\mathcal{A}} H$ be the complement of the arrangement of $\mathcal{A}$. There are many interesting topological objects related to the complement M. One of them is the fundamental group $G = \pi_1(M)$. And from the fundamental group, we can get its lower central series:

$$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq \cdots,$$

where $G_i = [G_{i-1}, G_1]$, for $i \geq 2$.

We can get some important information from the series. To study such problems, Falk introduced a multiplicative invariant ϕ_3 that is the rank of the abelian group G_3 / G_4 and posed as an open question to give a combinatorial interpretation of the rank for some arrangements. We call it **Falk invariant**.

In my thesis, I mainly introduced my work with Michele Torielli during my doctoral course on calculating the Falk invariant for some arrangements.

At the first two chapters, I gave some definitions and theorems about hyperplane arrangements and graph theory. These are what we need in the following chapters. To make the definitions clear, I added some necessary examples.

In the third chapter, we described a combinatorial formula for the Falk invariant of a signed graphic arrangement that do not have a B_2 as subarrangement., where B_2 is the graph containing two vertices, two loops on the vertices, and two edges where one is positive and the other one is negative. Finally, we computed one example using the formula we got.

In the fourth chapter, we gave a formula for the Falk invariant ϕ_3 of the arrangements that are canonical linear gain representations of gain graphs that do not have a subgraph isomorphic to B_2, or loops adjacent to a θ-graph with only three edges and with at most triple parallel edges. And we gave the matroidal interpretation.

In the fifth chapter, firstly we computed the Falk invariant for an additive gain graphic arrangement which is an additive graph via its complete lift representation in which there are no loops and there are at most double parallel edges, then we applied the formula to calculate the cone of Shi, Linial and semiorder arrangements.