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Trade and Welfare in General Equilibrium : A Discrete-time Infinite Horizon Case.

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Abstract

This paper extends the results on trade and welfare obtained in Ohyama(1972) in the case of a traditional world economy with a finite number of goods to the one of a world economy over a discrete-time infinite horizon with l_∞ , the space of all bounded sequences, as the underlying commodity space. The case with l_∞ is a typical special case of economies with infinite number of goods. In this paper, it is shown that the main results ottained in Ohyama(1972) still hold in the world economy over a discrete-time infinite horizon by following the method used in Ohyama(1972). It turns out that Ohyama(1972)'s method is, indeed, very general in a sense that it also applies to more general cases including economies with infinitely many goods.

1 Introduction

Ohyama(1972) considers several propositions on trade and welfare in a simple traditional world trade model with a finite number of commodities from the viewpoint of general equilibrium. It establishes a general result on trade and welfare based on Hicks-type welfare criterion and it is applied to many situations including world free trade and world trade with tariffs and so on. One of the important concepts in Ohyama(1972) is a self financing tariff scheme and one of the main results on trade and welfare is that world trade with self-financing tariffs is still better than autarky. There are also important results on terms of trade improvement and price divergences, and on customs unions.

All of the results in Ohyama(1972) are obtained in a traditional world economy with a finite number of commodities. Since an infinite horizon model is standard even in international trade theory recently, it is quite important to establish the general result on trade and welfare in such an infinite horizon world economy as Ohyama(1972) did for a traditional

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finite horizon world economy. Thus, the aim of this paper is to generalize these results to a dynamic world economy over an discrete time infinite horizon.[1]

In a traditional world economy with a finite number of commodities, the interpretation of a price of a good is quite simple. The value of a commodity bundle is expressed as an inner product of a price vector and the commodity bundle in a finite dimensional vector space so that a value of a good is the corresponding coordinate of the price vector.[2] In a world economy with an infinite number of commodities, although the value of a commodity bundle is expressed as an inner product of a price vector and the commodity bundle in an infinite dimensional vector space, a value of a good may be zero even the corresponding price vector is non-zero.[3] Thus the interpretation of a price of good in this case is quite different to the finite dimensional case even where the value of a commodity bundle is expressed as an inner product of a price vector and a commodity bundle. When the price vector is chosen from l_1, a value of a good is the corresponding coordinate of the price vector as in the finite dimensional case.[4]

In this paper, it is shown that, as long as the price vector is chosen from l_1, the basic results of Ohyama(1972) obtained in a simple traditional world trade economy with a finite number of goods are extended to the one in a dynamic world economy over a discrete-time infinite horizon with l_∞ as the commodity space.[5]

This paper is organized as followed. Next section constructs a basic competitive model for the following arguments of this paper. Then section 3 establishes the main result of trade and welfare based on Hick type welfare criterion and considers the situation with net tariff revenue. Section 4 discusses the terms of trade improvement and price divergence. Section 5 considers the welfare aspect of customs unions and focuses on tariff-compensated customs union. The final section picks up several issues relating the topics treated in this paper. The Appendix is for mathematical notes and notations used in this paper.

2 Model

Consider a country in a dynamic world trade economy over a discrete-time infinite horizon with l_∞ as the commodity space. At each period, there is a given and constant number of

[1] For this purpose, this paper uses l_∞, the space of all bounded sequences of l-dimensional vectors, as the underlying commodity space and ba, the space of all bounded and finitely additive measures on l_∞ as the general price space, where the innner product $p \cdot x$ of $p \in ba$ and $x \in l_\infty$ is well-defined. Note that l_1, the set of summeble sequences, is a proper subset of ba. See Appentix.

[2] The innner product $p \cdot x$ of $p = (p_1, \cdots, p_l)$ and $x = (x_1, \cdots, x_l)$ is just equal to $\sum_{i=1}^{l} p_i x_i$ so that the price of good $i = p_i$ is followed by $p_i = p \cdot e^i$ for $e^i = (0, \cdots, 1, \cdots, 0)$ $i = 1, \cdots, l$.

[3] In the case of l_∞, some of price vectors in ba such as the Banach limit, has this property. In general, Yoshida-Hewitt Theorem says that a vector in ba is decomposed uniquely to summable l_1 part and pfa, purely finitely additive part, where the latter has this property. For $p = (p^t)_{t=0}^{\infty} \in l_1$, the inner product $p \cdot x = \sum_{t=0}^{\infty} p^t \cdot x^t$ for $x = (x^t)_{t=0}^{\infty} \in l_\infty$, so that the price of good i at time t becomes p_i^t for $i = 1, \cdots, l$, $t = 0, 1, \cdots,$.

[4] To find an equilibrium price in l_1, it is important to assume that the production set is a convex cone and satisfies so-called exclusion condition, which implies that the proction process is always possible to be shut down at any time. See Bewley(1972), Prescott-Lucas(1972), and Storky-Lucas(1989) for details. Also for the existence of such world free trade equilibrium with Grandmond-McFadden type domestic income transfers, see Kubota(2018).

[5] Although the general price space is ba, we restrict the prices belonging to l_1^+ to make the price of a good meaningful. Note that some of the following definition, propositions, theorems, and corollary still hold even with prices in ba.

l goods. Let Y denote the aggregate production set of the country, so that $Y = \sum_{f=1}^{F} Y^f$, where Y^f is the production set of the firm f.[6] The consumption set for each consumer in this country is assumed to be l_∞^+, the non-negative orthant of l_∞, i.e., the set of bounded sequences of l-dimensional vectors. We also assume that consumer h has a complete pre-ordered preference R^h on l_∞^+.[7] Let x and y be a vector of l_∞^+ and a vector of Y denoted the aggregate consumption bundle and the aggregate production of the country. When x^h is a consumption vector of consumer h and y^f is a production vector of firm f, $x = \sum_{h=1}^{H} x^h$ and $y = \sum_{f=1}^{F} y^f$. A negative(positive) coordinate of y denotes input(output). Let z be the aggregate excess demand vector or net import vector, i.e., $z = x - y - \omega$, where $\omega(\in l_\infty^+) = \sum_{h=1}^{H} \omega^h$ is the aggregate initial endowment vector of the country and $\omega^h(\in l_\infty^+)$ is the initial endowment vector for consumer h.

Let $q = (q^t)_{t=1}^{\infty}(\in l_1^+)$ be a price vector prevailing in the world market. Let $p = (p^t)_{t=1}^{\infty}(\in l_1^+)$, $p_r = (p_r^t)_{t=1}^{\infty}(\in l_1^+)$, and $p_c = (p_c^t)_{t=1}^{\infty}(\in l_1^+)$ be a general domestic price vector, domestic production price vector, and domestic consumption price vector, respectively, prevailing in the domestic market of the country. Let t_i^t, r_i^t, and c_i^t be the ad valorem rate of tariff to the import of good i, the ad valorem rate of subsidy on the production of good i, and the ad valorem rate of tax to the consumption of good i, at time $t = 0, 1, \cdots$. Then, $p_i^t = (1 + t_i^t)q_i^t$, $p_{ri}^t = (1 + r_i^t)p_i^t$, and $p_{ci}^t = (1 + c_i^t)p_i^t$ for $i = 1, , \cdots, l$, $t = 0, 1, \cdots$. Note that when the good i at time t is imported(exported), i.e., $z_i^t > (<) 0$, $t_i^t > 0$ means an import tariff(export subsidy) and $t_i^t < 0$ means an import subsidy(export tariff). When T^t, R^t, and C^t are the diagonal matrices of the ad valorem rates of t_i^t, r_i^t, and c_i^t, respectively, $t = 0, 1, \cdots$, then, $p^t = q^t(I + T^t)$, $p_r^t = p^t(I + R^t)$, and $p_c^t = p^t(I + C^t)$, respectively, $t = 0, 1, \cdots$. Note that once T^t, R^t, and C^t are given by the government of the country, respectively, $t = 0, 1, \cdots$, a corresponding domestic production price vector $p_r = (p_r^t)_{t=1}^{\infty}(\in l_1^+)$ and a corresponding domestic consumption price vector $p_c = (p_c^t)_{t=1}^{\infty}(\in l_1^+)$ are known easily from a general domestic price vector $p = (p^t)_{t=1}^{\infty}(\in l_1^+)$. Also when $\mathbf{T}, \mathbf{R}$, and $\mathbf{C}$ are the infinite dimensional diagonal block matrices whose diagonal blocks are given by T^t, R^t, and C^t, $t = 0, 1, \cdots$, then, $p = q(\mathbf{I} + \mathbf{T})$, $p_r = p(\mathbf{I} + \mathbf{R})$,and $p_c = p(\mathbf{I} + \mathbf{C})$, respectively.[8] Of course, if there are no domestic taxes or subsidies on production and consumption, p is equal to the price vector that the producers and consumers face. If, further, there are no tariffs and subsidies on international trade, p is also equal to q. The consumers and the producers in the country are assumed to behave competitively as taking the prices as given.

Definition (2-1) *A competitive trade equilibrium with income transfers for the country* is a vector $(x^h, y^f, p)_{i=1,f=1}^{H,F} = (\widetilde{x}, \widetilde{y}, p)$ with $p \in l_1^+ \backslash \{0\}$ satisfying

(*i*) $x'^h \in P^h(x^h)$ implies $p_c \cdot x'^h (= \sum_{t=1}^{\infty} p_c^t \cdot x'^{th}) > p_c \cdot x^h (= \sum_{t=1}^{\infty} p_c^t \cdot x^{th}), h = 1, \cdots, H$.[9]

[6] Note that to make sure that the equilibrium price is from l_1, Y^f need to be a convex cone with the vertex as the origin as in Bewley(1972). This condition is cstandard in traditional world trade models such as Ricardian model and Heckscher-Ohlin model.

[7] Strict preference from this R^h is denoted by P^h.

[8] See Apendix for these concepts.

[9] Here we assume an implicit but feasible lump-sum income transfers over the consumers in the country

(ii) $y'^f \in Y^f$ implies $p_r \cdot y'^f (= \sum_{t=1}^{\infty} p_r^t \cdot y'^{tf}) \leq p_r \cdot y^f (= \sum_{t=1}^{\infty} p_r^t \cdot y^{tf}), f = 1, \cdots, F.$[10]

(iii) $q \cdot (x - y - \omega) = 0$ or $q \cdot z = 0$.

Definition (2-2) *A competitive equilibrium with income transfers under autarky for the country* is a vector $(x^h, y^f, p)_{i=1,f=1}^{H,F} = (\widetilde{x}, \widetilde{y}, p)$ with $p \in l_1^+ \backslash \{0\}$, when (iii) in the above is replaced with following $(iii)'$:

$(iii)'$ $(x - y - \omega) \leqq 0, p \cdot (x - y - \omega) = 0$, or $z \leqq 0, p \cdot z = 0$.

Note that (iii) is the trade balance condition as the budget constraint for the entire country. On the other hand, $(iii)'$ is the standard recourse constraint for the county with the free good condition.[11] Also all government revenue is assumed to be redistributed over the consumers in some lump-sum way.

3 A General Theorem on Welfare Comparison

The aim of this section is to generalize the general result on trade and welfare obtained in Ohyama(1972) in traditional world trade economy with a finite number of goods to the one in a dynamic world trade economy over a discrete-time infinite horizon.

Consider two different situation S' and S''.[12] Let $(\widetilde{x'}, \widetilde{y'}, p')$ and $(\widetilde{x''}, \widetilde{y''}, p'')$ be an competitive equilibrium with income transfers under situation S' and one under situation S''. We use Hicks type welfare criterion as our welfare criterion and defined as following:

Definition (3-1) $\widetilde{x''}$ is *non-inferior to* $\widetilde{x'}$, denoted by $\widetilde{x''} R \widetilde{x'}$, if $p_c'' \cdot x''(= \sum_{h=1}^{H} p_c'' \cdot x''^h) \geqq$

$p_c'' \cdot x'(= \sum_{h=1}^{H} p_c'' \cdot x'^h)$ holds.

Then the following result holds.

Proposition (3-1) *Suppose that the consumers satisfy the local non-satiation condition.*[13] *If* $\widetilde{x''}$ *is non-inferior to* $\widetilde{x'}$, $\widetilde{x''}$ *is better to* $\widetilde{x'}$ *in the sense of Hicks, i.e.,* $\widetilde{x''}$ *can not be Pareto dominated by* $\widetilde{x'}$.

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in s sense that the sum of all lump-sum income transfers and subsidies is zero so that the total income is equal to the total expenditure as expressed in (iii).

[10] When Y^f is a convex cone with the vertex as the origin so sa to make sure that the equilibrium price is from l_1, its profis is 0 and hence $p_r \cdot y^f (= \sum_{t=1}^{\infty} p_r^t \cdot y^{tf}) = 0$ holds.

[11] When we compare two trade situations, the consumption set of the consumers is assumed to be unchanged and equal to l_∞^+. When we compare one trade situation and one under autarky, the consumption set of the consumers may be expanded from the autarky situation to trade situation since some goods may be available under world trade. To make the argument simple, it may be assumed that even in this situation, the consumption sets are still same as and equal to l_∞^+.

[12] Followings are some example of S' and S'' : S' is the situation under autarky and S'' is the one under free trade, or S' is the situation under free trade and S'' is the one under trade with tariffs, and so forth.

[13] The local non-satiationn condition for a consumer h is following : $\exists (x'^{hn})_{n=1}^{\infty}$ such that $x'^{hn}\ P^h x'^h$ for $n = 1, 2, \cdots$ and $x'^{hn} \to x'^h (n \to \infty)$ uniformly, i.e., $||x'^{hn} \to x'^h||_\infty \to 0 (n \to \infty)$. When each good at aech time is desirable, this local-non-satiation condition holds.

Proof Suppose that $\widetilde{x''}$ is Pareto dominated by $\widetilde{x'}$ so that $x'^h R^h x''^h$ for $h = 1, \cdots, H$ and $x'^k P^k x''^k$ for some k. (i) implies $p''_c \cdot x'^k > p''_c \cdot x''^k$ for this k. From the local non-satiation condition, $\exists (x'^{hn})_{n=1}^{\infty}$ such that $x'^{hn} \to x'^h (n \to \infty)$ uniformly and $x'^{hn}\ P^h x'^h$ for $n = 1, 2, \cdots$ so that (i) and the transitivity of preferences imply $p''_c \cdot x'^{hn} > p''_c \cdot x''^h$ for $n = 1, 2, \cdots$. Since any element of ba is continuous with respect to the uniform convergence on l_∞ and l_1 is a subset of ba, $p''_c \cdot x'^{hn} \to p''_c \cdot x'^h (n \to \infty)$ and hence $p''_c \cdot x'^h \geqq p''_c \cdot x''^h$ for $h = 1, \cdots, H$. Then summing these inequality ends up with $\sum_{h=1}^{H} p''_c \cdot x'^h > \sum_{h=1}^{H} p''_c \cdot x''^h$, which is a contradiction to $\sum_{h=1}^{H} p''_c \cdot x''^h \geqq \sum_{h=1}^{H} p''_c \cdot x'^h$ followed from the non-inferiority of $\widetilde{x''}$ over $\widetilde{x'}$. *Q.E.D.*

Suppose that the situation S'' is the one under world trade. As in Ohyama(1972), $p''_c \cdot (x'' - x') = p''_c \cdot \sum_{h=1}^{H} (x''^h - x'^h)$ is decomposed into several meaningful parts. From the definitions of $p''^t = q''^t(I + T''^t), p''^t_r = p''^t(I + R''^t)$, and $p''^t_c = p''^t(I + C^t)$, $t = 0, 1, \cdots$, it follows that $p''^t_r = q''^t(I + T''^t)(I +' R^t)$,and $p''^t_c = q''^t(I + T''^t)(I + C''^t)$, $t = 0, 1, \cdots$. Then, for $t = 0, 1, \cdots$,

$$\begin{aligned} p''^t_c \cdot (x''^t - x'^t) &= q''^t(I + T''^t)(I + C''^t) \cdot [(z''^t + y''^t + \omega''^t) - (z'^t + y'^t + \omega'^t)] \\ &= q''^t \cdot (z''^t - z'^t) + q''^t \cdot T''^t \cdot (z''^t - z'^t) + q''^t(I + T''^t)C''^t \cdot (x''^t - x'^t) \\ &+ q''^t(I + T''^t)[(y''^t - y'^t) + (\omega''^t - \omega'^t)]. \end{aligned}$$

Since $q''^t(I + T''^t) = p''^t = p''^t_r - p''^t R''^t$, for $t = 0, 1, \cdots$,

$$\begin{aligned} p''^t_c \cdot (x''^t - x'^t) &= q''^t(I + T''^t)(I + C''^t) \cdot [(z''^t + y''^t + \omega''^t) - (z'^t + y'^t + \omega'^t)] \\ &= q''^t \cdot (z''^t - z'^t) + q''^t \cdot T''^t \cdot (z''^t - z'^t) + p''^t C''^t \cdot (x''^t - x'^t) + p''^t R''^t \cdot (y'^t - y''^t) \\ &+ p''^t_r \cdot (y''^t - y'^t) + p''^t \cdot (\omega''^t - \omega'^t)], \end{aligned}$$

follows for $t = 0, 1, \cdots$ .Thus, summing over t gives rise to

$$\begin{aligned} \sum_{t=1}^{\infty} p''^t_c \cdot (x''^t - x'^t) &= p''_c \cdot (x'' - x') \\ &= \sum_{t=1}^{\infty} q''^t(I + T''^t)(1 + C''^t) \cdot [(z''^t + y''^t + \omega''^t) - (z'^t + y'^t + \omega'^t)] \\ &= \sum_{t=1}^{\infty} q''^t \cdot (z''^t - z'^t) + \sum_{t=1}^{\infty} q''^t \cdot T''^t \cdot (z''^t - z'^t) + \sum_{t=1}^{\infty} p''^t C''^t \cdot (x''^t - x'^t) \\ &+ \sum_{t=1}^{\infty} p''^t R''^t \cdot (y'^t - y''^t) + \sum_{t=1}^{\infty} p''^t_r \cdot (y''^t - y'^t) + \sum_{t=1}^{\infty} p''^t \cdot (\omega''^t - \omega'^t)], \end{aligned}$$

Thus,

$$\begin{aligned} p''_c \cdot (x'' - x') &= q''(\mathbf{I} + \mathbf{T}'')(1 + \mathbf{C}'') \cdot [(z'' + y'' + \omega'') - (z' + y' + \omega')] \\ &= q'' \cdot (z'' - z') + q'' \cdot \mathbf{T}'' \cdot (z'' - z') + p''\mathbf{C}'' \cdot (x'' - x') \\ &+ p''\mathbf{R}'' \cdot (y' - y'') + p''_r \cdot (y'' - y') + p'' \cdot (\omega'' - \omega')] \end{aligned}$$

follows. From this expression, the general theorem on trade and welfare obtained in Ohyama(1972) in traditional world trade economy with a finite number of goods is extended to the one in a dynamic world trade economy over an infinite horizon.

Theorem (3-1) *Suppose that the following condition holds:* $q'' \cdot (z'' - z') + q''T'' \cdot (z'' - z') + p''C'' \cdot (x'' - x') + p''R'' \cdot (y' - y'') + p''_r \cdot (y'' - y') + p'' \cdot (\omega'' - \omega')] \geqq 0$. *Then* $\widetilde{x''} R \widetilde{x'}$, *i.e.,* $\widetilde{x''}$ *is non-inferior to* $\widetilde{x'}$.

Proof Straightforward from the definition of welfare criterion and the above equation. *Q.E.D.*

When the first term is positive, i.e., $q'' \cdot z''(= 0) > q'' \cdot z'$ holds, the revealed preference argument implies that the gains are obtained from situation S' to S''. Also the the first term also captures the terms of trade change effect since $q'' \cdot z'' = q' \cdot z' = 0$ implies $q'' \cdot (z'' - z') = q'' \cdot (-z') = (q'' - q') \cdot (-z')$. The second, third, and the fourth terms express the gains in the government's net revenue arising from tariffs, one from taxes and subsidies from consumption and one from taxes and subsidies from production, when $q'' = q', \mathbf{T}'' = \mathbf{T}', \mathbf{C}'' = \mathbf{C}', \mathbf{R}'' = \mathbf{R}'$ hold. The last two terms are the gain in producers' profits and the one in the income from initial endowment changes, respectively, when $p'' = p'$ and $p''_r = p'_r$ hold.

Consider that there is no changes in the initial endowments and the production set between situations S' and S''. Then, the profit maximization condition implies $p''_r \cdot (y'' - y') + p'' \cdot (\omega'' - \omega') = p''_r \cdot (y'' - y') \geqq 0$. Thus if $q'' \cdot (z'' - z') + q''\mathbf{T}'' \cdot (z'' - z') + p''\mathbf{C}'' \cdot (x'' - x') + p''\mathbf{R}'' \cdot (y' - y'') \geqq 0$ holds, the condition in the theorem holds. Thus the following result is obtained.

Corollary (3-1) *Suppose that there is no taxes and subsidies on consumptions and productions and no changes in the initial endowments and in the production set between situations* S' *and* S''. *When the condition,* $q'' \cdot (z'' - z') + q''\mathbf{T}'' \cdot (z'' - z') \geqq 0$, *holds,* $\widetilde{x''} R \widetilde{x'}$, *i.e.,* $\widetilde{x''}$ *is non-inferior to* $\widetilde{x'}$.

Define free trade as a situation with $\mathbf{T} = \mathbf{C} = \mathbf{R} = 0$ and trade restricted by tariffs as a trade situation with $sign(t_i^t) = sign(z_i^t)$ for $i = 1, \cdots, l, t = 0, 1, \cdots$. Then Corollary (3-1) gives rise to the following basic results on trade and welfare.

Theorem (3-2) *Suppose that there is no changes in the initial endowments and the production sets. Then,*

(1) *free trade is not inferior to autarky.*

(2) *Trade restricted by tariffs is not inferior to autarky.*

Proof Since free trade means $\mathbf{T}''(= \mathbf{C}'' = \mathbf{R}'') = 0$, the condition in the corollary becomes $q'' \cdot (z'' - z') = -q'' \cdot z' \geqq 0$. Then (1) follows from $q'' \geqq 0$ and $z' \leqq 0$ under autarky. As to (2), note that the condition in the corollary in this case becomes $q''\mathbf{T}'' \cdot z'' - p'' \cdot z' \geqq 0$. Since trade restricted by tariffs implies $t_i^t z_i^t \geqq 0$, for $i = 1, \cdots, l, t = 0, 1, \cdots$, and hence $T''^t z^t \geqq 0$ for $t = 0, 1, \cdots$, or $\mathbf{T}''z''$ holds, $q'' \geqq 0$ implies $q''\mathbf{T}'' \cdot z'' \geqq 0$. Since $p'' \geqq 0$ and $z' \leqq 0$ under autarky imply $p'' \cdot z' \leqq 0$ holds. Thus, the desired condition holds which implies (2). *Q.E.D.*

Note that any restricted trade with tariffs does not include any trade subsidies. When some trade subsidies are allowed, $t_i^t z_i^t \geqq 0$ may not holds for time $i = 1, \cdots, l$ and for some $t = 0, 1, \cdots$. Thus, $T''^t z^t \geqq 0$ may not hold for some $t = 0, 1, \cdots$, and hence $\mathbf{T}''z'' \geqq 0$ may not holds. Then even with $q'' \geqq 0$, $q''\mathbf{T}'' \cdot z'' \geqq 0$ may not hold. When some trade subsidies are also allowed besides tariffs, the following is used.[14]

[14] Trade subsidies are treated as negative tariffs.

Definition (3-1) A scheme of trade tariffs is called *self-financing* when $q\mathbf{T} \cdot z \geqq 0$ holds.

Under self-financing trade tariffs, (2) of Theorem (3-2) is extended as follows. The original result is obtained by Ohyama(1972) in a traditional world trade economy with a finite number of goods. The following result is its extension to the one in a dynamic world trade economy over a discrete-time infinite horizon.

Theorem (3-3) *Suppose that there is no taxes and subsidies on consumptions and productions and no changes in the initial endowments and the production sets, and that the autarky price is non-negative. Then, trade with self-financing tariffs is not inferior to autarky.*

Proof Note that in this case the condition in Corollary (3-1) becomes $q''\mathbf{T}'' \cdot z'' - p'' \cdot z' \geqq 0$. Since trade with self-financing tariffs implies $q''\mathbf{T}'' \cdot z'' \geqq 0$ and $p'' \geqq 0$ and $z' \leqq 0$ under autarky imply $p'' \cdot z' \leqq 0$, the desired condition holds which implies the result. *Q.E.D.*

4 The Terms of Trade Improvement and Price Divergence

Let q' and q'' be two external price vectors under two different trade situations, S' and S''. Suppose that there are no tariffs and no domestic taxes and subsidies in these both situations.

Definition (4-1) q'' is said to be *a terms of trade improvement relative to* q' if $(q''-q') \cdot z'(= q'' \cdot z') < 0$.

The following is immediate from Corollary (3-1).

Theorem (4-1) *Suppose that there is no changes in the initial endowments and free trade prevails under the new trade situation. Then, a terms of trade improvement is not inferior to the country.*

Proof $\mathbf{T}'' = \mathbf{C}'' = \mathbf{R}'' = 0$ and $\omega'' = \omega'$ implies that the condition for welfare improvement in Corollary (3-1) becomes $q'' \cdot (z'' - z') = -q'' \cdot z' \geqq 0$, which follows from the terms of trade improvement. *Q.E.D.*

Next consider the price divergence of two external prices under two trade situations from the autarky price. To make commodity-wise price comparison meaningful, the prices are restricted to satisfy its sum equal to 1, i.e., $\sum_{t=1}^{\infty} p^t = \sum_{t=1}^{\infty}\sum_{i=1}^{l} p_i^t = 1$. Notice that some of non-zero prices from *ba* have zero value at each good of each period and it is quite uninteresting to use these prices for commodity-wise price comparison. Thus it is important to find l_1-prices for commodity-wise price comparison. This is one of aspects different from classical trade model with a finite number of goods.

Let p^0 be a price under autarky. Define $\delta_i'^t$ and $\delta_i''^t$ as $q_i'^t = (1+\delta_i'^t)p_i^{0t}, q_i''^t = (1+\delta_i''^t)p_i^{0t}$ for $i = 1, \cdots, l, t = 0, 1, \cdots$. Similarly, let δ'^t and δ''^t for $t = 0, 1, \cdots$, be two $(l \times l)$ diagonal matrices whose i- the diagonal elements are $\delta_i'^t$ and $\delta_i''^t$ $i = 1, \cdots, l$. Then $q'^t = (I + \delta'^t)p^{0t}, q''^t = (I + \delta''^t)p^{0t}$ for $t = 0, 1, \cdots$. Moreover, let δ' and δ'' be two infinite dimensional diagonal matrices whose t- the diagonal block matrices are δ'^t and δ''^t, $t = 0, 1, \cdots$. Then $q' = (\mathbf{I} + \delta')p^0, q'' = (\mathbf{I} + \delta'')p^0$.

Definition (4-2) q'' is said to *diverge more than* q' *from* p^0 if $\delta_i''^t \geqq \delta_i'^t$ for $\delta_i'^t > 0$ and $\delta_i''^t \leqq \delta_i'^t$ for $\delta_i'^t < 0$ with strict inequality for at least one i and t either for $\delta_i''^t \geqq \delta_i'^t$ or $\delta_i''^t \leqq \delta_i'^t$.

This means that if a commodity is imported(exported) after opening of trade, its price is lower(higher) than the one under autarky. Note that the concept of price divergence is a binary relation of the price vectors which is defined relative to the autarky price vector p^0 , and is denoted by $q'' \succeq q'$.

Let introduce the assumption based on the law of comparative advantage:

Assumption (A) $\delta_i'^t < (>)0$ for $z_i'^t > (>)0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$.[15]

This means that if a good is imported(exported) after opening to world trade, its price after trade is lower(higher) than the one under autarky.

Theorem (4-2) *Suppose that Assumption (A) holds and that there is no change in the endowments of the economy. Suppose also that free trade prevails under* S''*as well as under* S'*. Then if* $q'' \succeq q'$ *holds,* S'' *is not inferior to* S'*.*

Proof If the condition $q'' \cdot (z'' - z') + q''\mathbf{T}'' \cdot (z'' - z') \geqq 0$ holds, Corollary (3-1) implies $\widetilde{x''} R \widetilde{x'}$. Since free trade prevails under S'', $\mathbf{T}'' = 0$. Thus, it is enough to establish $q'' \cdot (z'' - z') = -q'' \cdot z' \geqq 0$ for the result from balanced trade condition, $q' \cdot z' = 0$. Since $q'' \cdot z' = q'' \cdot z' - q' \cdot z' = [(\mathbf{I} + \delta'')p^0 \cdot z' - [(\mathbf{I} + \delta')p^0 \cdot z'] = (\delta'' - \delta')p^0 \cdot z'$, and

$q'' \succeq q'$ and Assumption (A) implies $\delta_i''^t \leqq (\geqq)\delta_i'^t < (>)0$ for $z_i'^t > (<)0$ with strict inequality for at least one i and t, $[\delta'' - \delta']p^0 \cdot z' = \sum_{t=1}^{\infty}\sum_{i=1}^{l}(\delta_i''^t - \delta_i'^t)p_i^{0t} \cdot z_i'^t = q'' \cdot z' < 0$ and hence $-q'' \cdot z' > 0$ hold. *Q.E.D.*

Consider three situation, S'', S', as well as $\overline{S}$. Note that $\overline{S}$ need not to be the autarky situation. Denote $\overline{q} = (I + \overline{\delta})p^0$ as for q'' and q'.

Lemma (4-1) *The price diversion relation* $\succeq$ *is transitive,i.e.,* $q'' \succeq q'$, $q' \succeq \overline{q} \Rightarrow q'' \succeq \overline{q}$.

Proof From $q' \succeq \overline{q}$, $\overline{\delta}_i^t > 0 \Rightarrow \delta_i'^t \geqq \overline{\delta}_i^t > 0$ and $\overline{\delta}_i^t < 0 \Rightarrow \delta_i'^t \leqq \overline{\delta}_i^t < 0$. Similarly, from $q'' \succeq q', \delta_i'^t > 0 \Rightarrow \delta_i''^t \geqq \delta_i'^t > 0$ and $\delta_i'^t < 0 \Rightarrow \delta_i''^t \leqq \delta_i'^t < 0$. Thus, $\overline{\delta}_i^t > 0 \Rightarrow \delta_i''^t \geqq \delta_i'^t \geqq \overline{\delta}_i^t > 0$ and $\overline{\delta}_i^t < 0 \Rightarrow \delta_i''^t \leqq \delta_i'^t \leqq \overline{\delta}_i^t < 0$ hold and hence $q'' \succeq \overline{q}$ follows. *Q.E.D.*

Suppose that similar condition as of Assumption (A) holds at $\overline{S}$. $\overline{\delta_i^t} < (>)0$ for $\overline{z_i^t} > (<)0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$. Since Theorem (4-2) and this condition imply that $q' \succeq \overline{q} \Rightarrow \widetilde{x'} R \widetilde{\overline{x}}$. Similarly, Theorem (4-2) and Assumption (A) imply that $q'' \succeq q' \Rightarrow \widetilde{x''} R \widetilde{x'}$. Then $q'' \succeq q'$ implies $q'' \succeq \overline{q}$ from Lemma (4-1) so that $\widetilde{x''} R \widetilde{\overline{x}}$ holds from Theorem (4-2) and the condition. Thus as long as changes are occurred though price divergence $\succeq$, R is transitive under the condition on the law of comparative advantage.

Define the set $Q(\overline{q}, p^0) = \{q : q \succeq \overline{q}\} \cup \{\overline{q}\}$. This is the set of world price vectors which diverge more from $\overline{q}$. Note that since $\succeq$ is transitive from Lemma (4-1), $q'' \succeq q'$ and $q' \in Q(\overline{q}, p^0)$ imply $q'' \succeq \overline{q}$ and hence $q'' \in Q(\overline{q}, p^0)$. Also $q'' \succeq \overline{q}$ implies $\widetilde{x''} R \widetilde{\overline{x}}$, and hence $q'' \in Q(\overline{q}, p^0)$ implies $\widetilde{x''} R \widetilde{\overline{x}}$.

[15] Since there are an infinite number of goods, it may be difficult to hold Assumption (A). When $\delta' z' = \sum_{t=0}^{\infty} \delta'^t z'^t < 0$ holds, it may be said that the theory of comparative advantage holds on average according to Deardorff(1980).

Now consider the relation between price divergence and terms of trade improvement. Suppose that $q' \cdot \overline{z} < 0$ and $q'' \cdot \overline{z} < 0$. This means that q' and q'' represent terms of trade improvement relative to $\overline{S}$. Suppose further that $q'' \cdot \overline{z} < q' \cdot \overline{z}$ holds. This situation is defined as follows.

Definition (4-3) The degree of terms of trade improvement is called *greater at S'' than at S'* if $q'' \cdot \overline{z} < q' \cdot \overline{z} < 0$ holds.

It is interesting to find out when greater price divergence implies greater terms of trade improvement. Let introduce the assumption that the law of comparative advantage holds over $Q(\overline{q}, p^0)$, i.e., Assumption (A) holds for any $q' \in Q(\overline{q}, p^0)$. This is a generalization of Assumption (A) and (B).

Assumption (B) $\delta_i'^t < (>) 0$ for $z_i'^t > (<) 0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$, where $z_i'^t$ is the component of net import vector z associated with $q' \in Q(\overline{q}, p^0)$.

Then $\overline{q} \in Q(\overline{q}, p^0)$ implies the above condition. Also since $q' \in Q(\overline{q}, p^0)$ implies $q' \succeq \overline{q}$ and hence $\overline{\delta}_i^t > 0 \Rightarrow \delta_i'^t (\geqq \overline{\delta}_i^t) > 0$ and $\overline{\delta}_i^t < 0 \Rightarrow \delta_i'^t (\leqq \overline{\delta}_i^t) < 0$ hold, $z_i'^t \leqq (\geqq) 0$ for $\overline{z_i^t} \leqq (\geqq) 0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$ hold from Assumption (B). Thus, trade pattern is unchanged from S' from $\overline{S}$. Then the following result holds.

Theorem (4-3) *Under Assumption (B), $q'', q' \in Q(\overline{q}, p^0)$ and $q'' \succeq q'$ imply $q'' \cdot \overline{z} < q' \cdot \overline{z} < 0$.*

Proof Since Assumption (B) implies $\delta_i'^t < (>) 0$ for $\overline{z_i^t} > (<) 0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$, as mentioned above, $q' \cdot \overline{z} = q' \cdot \overline{z} - \overline{q} \cdot \overline{z} = [(\mathbf{I} + \delta')p^0 \cdot \overline{z} - (\mathbf{I} + \overline{\delta})p^0 \cdot \overline{z}] = (\delta' - \overline{\delta})p^0 \cdot \overline{z} = \sum_{t=1}^{\infty} \sum_{i=1}^{l} (\delta_i'^t - \overline{\delta}_i^t) p_i^{0t} \cdot \overline{z}_i'^t$ implies $q' \cdot \overline{z} = (\delta' - \overline{\delta})p^0 \cdot \overline{z} < 0$. Since $q'' \cdot \overline{z} = q'' \cdot \overline{z} - \overline{q} \cdot \overline{z} = [(\mathbf{I} + \delta'')p^0 \cdot \overline{z} - (\mathbf{I} + \overline{\delta})p^0 \cdot \overline{z}] = (\delta'' - \overline{\delta})p^0 \cdot \overline{z}$, $q'' \cdot \overline{z} - q' \cdot \overline{z} = (\delta'' - \delta')p^0 \cdot \overline{z} = \sum_{t=1}^{\infty} \sum_{i=1}^{l} (\delta_i''^t - \delta_i'^t) p_i^{0t} \cdot \overline{z}_i'^t$. Since $\delta_i'^t < (>) 0$ for $\overline{z_i^t} > (<) 0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$ and $q'' \succeq q'$ imply $\delta_i''^t \leqq (\geqq) \delta_i'^t < (>) 0$ for $\overline{z_i^t} > (<) 0$ for $i = 1, \cdots, l, t = 0, 1, \cdots$, so that $q'' \cdot \overline{z} - q' \cdot \overline{z} = (\delta'' - \delta')p^0 \cdot \overline{z} = \sum_{t=1}^{\infty} \sum_{i=1}^{l} (\delta_i''^t - \delta_i'^t) p_i^{0t} \cdot \overline{z}_i'^t < 0$ or $q'' \cdot \overline{z} < q' \cdot \overline{z}$. Thus, $q'' \cdot \overline{z} < q' \cdot \overline{z} < 0$ holds. *Q.E.D.*

Note that the converse does not necessarily hold. It may happen that terms of trade improvement occur even without greater price divergence. Thus, it is interesting to find out when the converse holds. Consider a subset $\overline{Q}$ of $Q(\overline{q}, p^0)$ where $q'', q' \in \overline{Q}$ implies either $q'' \succeq q'$ or $q' \succeq q''$. Note that there exist many of such sets. The converse holds on $\overline{Q}$.

Proposition (4-1) *Suppose that there is no change in the endowment of the country. Suppose also that free trade prevails in both S'' and S'. Then, under Assumption (C), if $q'' \cdot \overline{z} < q' \cdot \overline{z} < 0$ holds for $q'', q' \in \overline{Q}$, $q'' \succeq q'$* holds.

Proof Since $q'', q' \in \overline{Q}$ implies either $q'' \succeq q'$ or $q' \succeq q''$, $\overline{Q} \subset Q(\overline{q}, p^0)$ implies $q'' \succeq q' \succeq \overline{q}$ or $q' \succeq q'' \succeq \overline{q}$ hold. Since $q' \succeq q'' \succeq \overline{q}$ and Assumption (B) implies $q' \cdot \overline{z} < q'' \cdot \overline{z} < 0$ from Theorem (4-3), which is a contradiction to $q'' \cdot \overline{z} < q' \cdot \overline{z} < 0$. Thus, $q'' \succeq q' \succeq \overline{q}$ holds. *Q.E.D.*

Thus, greater terms of trade improvement occurs only from greater price divergence for $q'', q' \in \overline{Q}$, and hence both are equivalent over $\overline{Q}$. Note that although $q' \cdot \overline{z} < 0$ and $q'' \cdot \overline{z} < 0$ imply that both of S'' and S' are not inferior to $\overline{S}$ from Theorem (4-1), it is unclear which is not inferior to the other between S'' and S'. But when greater terms of trade improvement occurs over $\overline{Q}$, it is a desirable change in welfare. Thus it becomes clear which is not inferior to the other between S'' and S' when greater terms of trade improvement occurs over $\overline{Q}$

Theorem (4-4) *Suppose that there is no change in the endowment of the country. Suppose also that free trade prevails in both S'' and S'. Then, under Assumption (B), if $q'' \cdot \overline{z} < q' \cdot \overline{z} < 0$ holds for $q'', q' \in \overline{Q}$, $\widetilde{x''} R \widetilde{x'} (R\widetilde{\overline{x}})$.*

Proof Since Proposition (4-1) implies $q'' \succeq q' \succeq \overline{q}$ holds for $q'', q' \in \overline{Q}$, Theorem (4-2) implies $\widetilde{x''} R \widetilde{x'} (R\widetilde{\overline{x}})$. *Q.E.D.*

Thus, Theorem (4-4) imply that when our attention is restricted on $\overline{Q}$, greater terms of trade improvement gives rise to welfare gain since it is given by greater price divergence from Proposition (4-1).

5 The Customs Unions Issue

This section considers customs unions issue. One of important results in Ohyama(1972) is relating to this issue. The customs unions abolish all tariffs among the member countries, and it sets up common external tariff to the non-member countries. The basic argument is to treat customs union as one country as treated before so that income redistributions in customs union are performed even over members countries.

Suppose that there are N countries in the world and $K(< N)$ countries formed a customs union. Let S' and S'' represent the pre-union and the post-union situations. Suppose that there is no domestic taxes and subsidies before and after the union. Then applying Corollary (3-1) gives rise to the following result.

Proposition (5-1) *If the condition $(q' - q'') \sum_{k=1}^{K} z'_k + q'' T'' (\sum_{k=1}^{K} z''_k - \sum_{k=1}^{K} z'_k) \geqq 0$ is satisfied, the post-union situation is not inferior to the pre-union situation for the customs union as a whole.*

Proof Since the member countries of the customs union is considered as one country owing to the income distributions coordinations among member countries in the post union situation, Corollary (3-1) implies this result once ω' and ω'' are replaced with $\sum_{k=1}^{K} \omega'_k$ and $\sum_{k=1}^{K} \omega''_k$, respectively, where ω'_k and ω''_k are the k-th country's initial endowments under the pre-union and the post-union. *Q.E.D.*

Next consider a special type of the customs union which is called a tariff-compensating customs union in Ohyama(1972).

Definition (5-1) A customs union is called *tariff-compensating* if its common external tariffs is set to keep same the volume and composition of the net trade with the rest of the world before forming the union.

This means that $\sum_{k=1}^{K} z''_k = \sum_{k=1}^{K} z'_k$ holds at the post-union situation. Then the following result is obtained from Proposition (5-1).

Theorem (5-1) *A tariff-compensating customs union is not inferior to the pre-union situation for the customs union as a whole.*

Proof Since

$$
\begin{aligned}
&(q' - q'')\sum_{k=1}^{K} z'_k + q''\mathbf{T}''(\sum_{k=1}^{K} z''_k - \sum_{k=1}^{K} z'_k) \\
&= (q' - q'')\sum_{k=1}^{K} z'_k + q''\mathbf{T}''(\sum_{k=1}^{K} z'_k - \sum_{k=1}^{K} z'_k) \\
&= (q' - q'')\sum_{k=1}^{K} z'_k = q'\sum_{k=1}^{K} z'_k - q''\sum_{k=1}^{K} z'_k \\
&= q'\sum_{k=1}^{K} z'_k - q''\sum_{k=1}^{K} z''_k = 0
\end{aligned}
$$

holds, the condition in the above Proposition (5-1) holds with equality.$Q.E.D.$

Note first that since the non-member countries can have the same consumption bundles of the pre-union even after the post-union, a tariff-compensating customs union does not hurt the non-member countries. Therefore, it is also beneficial for the world as a whole.

Kemp-Wan(1976) and Griolis(1981) also established the similar result. They use q' as q'', and $p'' - q'$ as $q''T''$, where p'' is the common free trade price vector in the union. Thus, in Kemp-Wan(1976) and Griolis(1981), the condition necessary to be shown holds with equality as in the above proposition.[16] One difference between Kemp-Wan(1976) and Griolis(1981) is the way to find p''. To create a tariff-compensating customs union with its union free trade price vector p'', Kemp-Wan(1976) uses 2nd fundamental theorem of welfare economics with using as its aggregate initial endowment adding $(-\sum_{k=1}^{K} z'_k)$ to $\sum_{k=1}^{K} \omega_k$. Then they set the difference of $p'' - q'$ as $q''T''$. On the other hand, Griolis(1981) uses Grandmond-McFadden(1972)'s existence theorem of gain from free trade to get the same result. Another difference between Kemp-Wan(1976) and Griolis(1981) is that the former need income distribution over member countries but the latter need only income distribution policy within each country.

6 Conclusion

Although Ohyama(1972) argued trade and welfare in a traditional Arrow-Debreu framework with a finite number of goods, the method used there is also applicable to a Bewley framework with an infinite number of goods. In this sense, Ohyama(1972)'s method is indeed very general. Since an infinite horizon model is standard even in international trade theory as well as international finance recently, it is quite important to establish the general result on trade and welfare in such an infinite horizon world economy as Ohyama(1972) did for a traditional finite horizon world economy. The main aim of this paper is at this point. Since an economy with the discrete-time infinite horizon case is a special case of economies with infinite dimensional commodity spaces, trade and welfare argument done in this paper following Ohyama(1972) will be applicable to more general economies with infinite dimensional commodity spaces.[17]

Also it is important to remind that the concept of price system is a little different in an infinite horizon economy from the one in a traditional finite horizon economy. In order

[16] Since the 1st term is $q' - q'' = q' - q' = 0$ and the 2nd term is $\sum_{k=1}^{K} z''_k - \sum_{k=1}^{K} z'_k = \sum_{k=1}^{K} z'_k - \sum_{k=1}^{K} z'_k = 0$, the above equation holds with equality.

[17] See for example, Aliprantis-Brown-Birkinshaw(1989), MasColell-Zame(1991) and Becker-Boyd(1997) for the analysis of economies with infinite dimansional commoditiy spaces.

to make price comparison meaningful in an infinite horizon economy, it is important to get summable price.[18] For this purpose, the conditions such as discounting future and stoppable production process at anytime are important, but these conditions did not appear in a traditional economy with a finitely many goods.

In Ohyama(1972), there is also a result on trade and welfare though economic growth. Even an economic growth occurs in a traditional finite horizon world trade economy, it is possible to treat it in R^l. However, when economic growth occurs at a positive growth rate in an infinite horizon world trade economy, consumption paths and production paths become unbounded so that they are not in l_∞.[19] Since this paper uses l_∞ as the commodity space, any argument on economic growth are not picked up. Since long-run growth is also an important topics in an infinite horizon world trade economy, it is desirable to extend the results obtained in this paper to the ones in an infinite horizon growing world trade economy.

Appendix : Mathematical Notes

Let s^l be the Cartesian product $\Pi_{t=1}^{\infty} R(t)$ with $R(t) = R^l \ \forall t \in N$.[20] l_∞, a subset of s^l, is defined as $\{z \in s^l : \sup_{1\leq t<\infty} ||z_t|| = ||z||_\infty < \infty\}$, where $||z||_\infty = \sup_{1\leq t<\infty} ||z_t||$ is called a supnorm. l_1 is also a subset of s, defined as $\{z \in s^l : \sum_{t=1}^{\infty} ||z_t|| = ||z||_1 < \infty\}$, where $||z||_1 = \sum_{t=1}^{\infty} ||z_t||$ is called a l_1-norm. l_1 is considered as a general price space including present value expression of price sequences such as one using an constant interest rate. For $z\in l_\infty$, $z(t) = (z_1, \cdot\cdot, z_t, 0, \cdot\cdot)$ is the head of z after $t \in N$ and $\widehat{z}(t) = (0, \cdot\cdot, z_{t+1}, \cdot\cdot)$, the tail of z after $t \in N$. $e = (e^l, \cdot\cdot, e^l, \cdot\cdot) \in l_\infty$ is the element of l_∞ whose coordinates are all $e^l = (1, \cdot\cdot, 1) \in \mathbf{R}^l$. Both of l_∞ and l_1 are Banach spaces, i.e., complete normed spaces, under the associated norms, respectively.

When B is a Banach space, the norm dual space of B, B^*, is defined as the set of norm-continuous linear functional on B, and expressed as $p \cdot x$ for $x \in B$and $p \in B^*$.[21] Note that B^* is also a Banach space. Note also that l_∞ is the norm dual of l_1, while the norm dual of l_∞ contains l_1 as its proper subset and is ba, the space of bounded finitely additive measures on natural numbers. For $p = (p_t)_{t=0}^{\infty} \in l_1$, $p \cdot x = \displaystyle\sum_{t=0}^{\infty} p_t \cdot x_t$ for $x = (x_t)_{t=0}^{\infty} \in l_\infty$ and $ba\backslash l_1$ is pfa, the space of purely finitely additive measures on natural numbers. Yoshida-Hewitt theorem says that ba is uniquely decomposed into l_1 and pfa. An example of pfa is the famous Banach limit.[22] An important property of an element π of pfa is that $\pi \cdot e \neq 0$ holds but $\pi \cdot e(t) = 0 \ \forall t \in N$. In a sense, π has non-zero value only in a far future and sometimes called a bubble. When the l_1-part of non-zero p of ba is zero, p is in pfa and hence it

[18] Kubota(2018) generalizes the existence theorem of gains from free trade obtained in Grandmond-McFadden(1972) in a traditional finite horizon case to the one in an infinite horizon case. Note that it is easy to show that the results obtained in this paper also hold in a world economy over a continuous-time infinite horizon where $p \cdot x = \int p(t)x(t)dt$ is used instead of $p \cdot x = \sum_{t=0}^{\infty} p_t \cdot x_t$.

[19] When price p is in l_1 and a sequence x is unbounded, $p \cdot x$ may not converge and becomes $\pm\infty$.

[20] s^l is usually endowed with the product topology. An open set in this product topology is expressed as $\mathbf{U} = \sum_{t=1}^{\infty} \mathbf{U}_t$ where each $\mathbf{U}_t$ is open in $\mathbf{R}(t)$ and $\mathbf{U}_t$ is equal to $\mathbf{R}(t)$ for all but finite number of t. This product topology is also metrizable and the convergence of a sequence $\{\mathbf{z}^n\}_{n=1}^{\infty}$ to $\mathbf{z}$ in $\mathbf{s}^l$ is characterized by $\mathbf{z}_t^n \to \mathbf{z}_t$ as $n \to \infty$ for $t = 1, 2, \cdot\cdot$. From this fact, this product topology on $\mathbf{s}^l$ is also called as the coordinatewise convergence topology. For $\mathbf{z} \in \mathbf{s}$, define $\mathbf{z}(t)$ and $\widehat{\mathbf{z}}(t)$ be the head of $\mathbf{z}$ up to t, $(\mathbf{z}_1, \cdot\cdot, \mathbf{z}_t, 0, \cdot\cdot)$, and the tail of $\mathbf{z}$ after t, $(0, \cdot\cdot, \mathbf{z}_{t+1}, \cdot\cdot)$. Then, $\mathbf{z}(t) \to \mathbf{z}(\widehat{\mathbf{z}}(t) \to \mathbf{0})$ holds in the coordinatewaise convergence topology as $t \to \infty$.

[21] Somethimes it is also expressed as (p, x) or $< p, x >$ for $x \in B$ and $p \in B^*$.

[22] See Dunford-Schwaltz(1958) and Rao-Rao(1983) for these facts. For economics applications, see, for example Radner(1967), Bewley(1972), Prescott-Lucas(1972), Majumdar(1974), Giles-Leroy(1992), and Lucas-Stokey(1989).

is quite difficult to make commodity-wise price comparison meaningful. In order to make commodity-wise price comparison meaningful, it is very important to make the l_1-part of an equilibrium price non-zero.[23] For this purpose, as shown in Bewley(1972), it is important to put myopic preferences characterized as discounting the future for consumers and the exclusion condition to production sets.[24]

Suppose that $(A_t)_{t=0}^{\infty}$ is a series of $(l\times l)$ diagonal matrices with a property $\sup_{0\leq t<\infty}||A_t|| < \infty$ where $||A_t|| = \max|a_{ij}^t|_{i,j=1}^l$ for $t = 0, 1, 2, \ldots$. Suppose that A is an infinite dimensional diagonal block matrix express as $A = \begin{bmatrix} A_0 & \cdots & \cdots & \cdots & \cdots \\ \vdots & A_1 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & A_t & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$. Then, $Ax = \sum_{t=0}^{\infty} A_t x_t$ and $|Ax| < \infty$ for $x \in l_\infty$. Similarly, $pA = \sum_{t=0}^{\infty} p_t A_t$ and $|pA| < \infty$ for $p \in l_1$. Moreover, $pA \cdot x = p \cdot Ax = \sum_{t=0}^{\infty} p_t A_t x_t$ and $|pA \cdot x| < \infty$ for $x \in l_\infty$ and for $p \in l_1$.

References

[1] Aliprantis, C. D., D. J. Brown, and O., Burkinshaw.(1989):*Existence and Optimality of Competitive Equilibria.* (Springer-Verlag, New York)

[2] Back, K.(1988):"Structure of consumption sets and existence of equilibria in infinite dimensional spaces." *Journal of Mathematical Economics* 22, p. 61 - 71.

[3] Baldwin, R.(1992):"Measurable Dynamic Gains from Trade." *Journal of Political Economy* 100, pp. 162 - 74.

[4] Becker, R. A. and J. H. Boyd III.(1997):*Capital Theory, Equilibrium Analysis, and Recursive Utility.* (Blackwell, Oxford)

[5] Bewley, T.(1972):"Existence of equilibria in economies with infinitely many commodities." *Journal of Economic Theory* 4, pp. 514 - 540.

[6] Bhaskara Rao, K. P. S., and M. Bhaskara Rao.(1983):*Theory of Charges.* (Academic Press, London)

[23] The topology on l_∞ employed for this purpose is the weak $*$ topology $\sigma(l_\infty, l_1)$, the weakest linear topology which makes l_1 as its the space of continuous linear functionals with respect to this topology. As this topology, $\mathbf{z}(t) \to \mathbf{z}(\widehat{\mathbf{z}}(t) \to \mathbf{0})$ holds $\mathbf{z} \in l_\infty$ in the weak $*$ $\sigma(l_\infty, l_1)$-topology as $t \to \infty$. Note that $\widehat{\mathbf{e}}(t) \to \mathbf{0}$ in the weak $*$ $\sigma(l_\infty, l_1)$-topology as $t \to \infty$ since $|p \cdot \widehat{\mathbf{e}}(t)| = |\sum_{\tau=t}^{\infty}\sum_{i=1}^{l} p_{\tau_i}| \leqq \sum_{\tau=t}^{\infty}\sum_{i=1}^{l} |p_{\tau_i}| \to 0$ as $t \to \infty$, $\forall p \in l_1$.

[24] Suppose a prefence is continuous with respect to the weak $*$ $\sigma(l_\infty, l_1)$-topology. Suppoose further that a prefence is monotonic and hence $\mathbf{x} + \mathbf{e}(1)$ is prefered over $\mathbf{x}$. Then since $\widehat{\mathbf{e}}(t) \to \mathbf{0}$ in the weak $*$ $\sigma(l_\infty, l_1)$-topology as $t \to \infty$, $\mathbf{x} + \mathbf{e}(1)$ is still prefered over $\mathbf{x}+ \widehat{\mathbf{e}}(t)$ for sufficinetly latge t and $\mathbf{x} + \mathbf{e}(1) - \widehat{\mathbf{e}}(t)$ is still prefered over $\mathbf{x}$. The former implies that current rise in consumption is better than a rise in consumption in the far future. The latter implies that current rise in consumption and constant fall in consumption in the far future is still better than $\mathbf{x}$. This prefence discounts the future and is called myopic. For details, see, for example, Brown-Lewis(1981), Epstein(1987), Back(19888), and Boyd-McKenzie(1992).

[7] Boyd, J. H. III, and L. W. McKenzie.(1993):"The existence of competitive equilibrium over infinite horizon with production and general consumption sets. "*International Economic Review* 34, pp. 1 - 20.

[8] Brown, D. J., and L. W. Lewis.(l981):"Myopic economic agents." *Econometrica* 49, pp. 359 - 68.

[9] Corden, W. M.(1984):"The normative theory of international trade." Chapter 2 of *Handbook of International Economics* Vol.1 ed. by R. W. Jones and P. B. Kenen, p. 63 – 130. (North-Holland, Amsterdam)

[10] Deardorff, A. V.(1980):"The general validity of the law of comparative advantage." *Journal of Political Economy* 88, pp. 942 – 957.

[11] Debreu, G.(1959):*Theory of Value.* (Wiley, New York)

[12] Dixit, A.(1981):"The export of capital theory." *Journal of International Economics* 11, pp. 279 - 294.

[13] ———,and V, Norman.(1980):*The Theory of International Trade.* (Cambridge University Press, Cambridge)

[14] Dunford, N., and J. Schwartz.(1958):*Linear Operators Part 1:General Theory.* (Interscience, New York)

[15] Epstein, L.(1987):"Impatience." in *New Palgrave Dictionary* Vol. 2. pp. 720 - 723.

[16] Giles, C., and S. F. Leroy.(1992):"Bubbles and charges." *International Economic Review* 33, pp. 323 - 339.

[17] Grandmont, J. M. and D, McFadden.(1972):"A technical note on classical gains from trade." *Journal of International Economics* 2, pp. 109 - 125.

[18] Kemp, M, C.(1962):"The gain from international trade." *Economic Journal* 72, pp. 802 - 819.

[19] ————.(1976):*Three Topics in the Theory of International Trade.* (North-Holland, Amsterdam)

[20] ————.(1995):*The Gains from Trade:essays in International Trade Theory.* (Routledge, NewYork)

[21] ————, and H. Wan.(1972):"The gains from free trade." *International Economic Review* 13, pp. 509 - 522.

[22] ————————.(1976):"An elementary proposition concerning the formation of customs unions." *Journal of International Economics* 6, p. 95 - 97.

[23] ————————.(1993):*The Welfare Economics of International Trade.* (Harwood Academic Publishers, Switzerland)

[24] Kubota, H.(2018):"On dynamic gains from free trade : a discrete-time infinite horizon case." Discussion Paper Series A No.330 (Faculty of Economics, Hokkaido University, Sapporo, Japan)

[25] Krueger, A.O. and H. Sonnenschein(1967):"The terms of trade, the gains from trade and price divergence." *International Economics Review* 8, pp. 121 - 127.

[26] Majumdar, M.(1974):"Some general theorems on efficiency prices with infinite dimensional commodity space." *Journal of Economic Theory* 5, pp. 1 - 13.

[27] ——————.(1975):"Some remarks on optimal growth with intertemporally dependent preferences." *Review of Economic Studies* 42, pp. 147 - 153.

[28] Mas-Colell, A., and W. R. Zame.(1989):"Equilibrium theory in infinite dimensional space." Chapter 34 in *Handbook of Mathematical Economics* Vol. 4. ed. by W. Hildenbrand and H. Sonnenschein.

[29] McKenzie, L. W.(2002):*Classical General Equilibrium Theory* (MIT Press, Canbridge, MA)

[30] Ohyama, M.(1972):"Trade and welfare in general equilibrium." *Keio Economic Studies* 9, pp. 37 - 73. Also in Ohyama(2016, Chap. 4).

[31] ————.(2016):*Macroeconomics, Trade, and Social Welfare.* (Springer, Japan)

[32] Prescott, E. C., and R. E. Lucas.(1972):"Price systems in infinite dimensional space." *International Economic Review* 13, pp. 416 - 422.

[33] Radner, R.(1967):"Efficiency Prices for Infinite Horizon Production Programs." *Review of Economic Studies* 34, pp. 187 - 193.

[34] Samuelson, P. A.(1939):"The gain from international trade." *Canadian Journal of Economics and Political Science* 5, pp. 195 - 205.

[35] ——————.(1950):"Evaluation of real national income." *Oxford Economic Paper N.S.*1, pp. 1 - 29.

[36] ——————.(1956):"Social Indifference curves." *Quarterly Journal of Economics* 70, pp. 1 - 22.

[37] ——————.(1962):"The gain from international trade once again." *Economic Journal* 72, pp. 820 - 29.

[38] Smith, M. A. M.(1979):"Intertemporal gain from trade" *Journal of International Economics* 8, pp. 239 - 248.

[39] ——————-.(1984):"Capital theory and trade theory." Chapter 6 in *Handbook of International Economics Vol.* 1, ed. by R. W. Jones and P. B. Kenen. (North-Holland, Amsterdam)

[40] Srinivasan, T. N. and J. Bhagwati.(1980):"Trade and welfare in a steady state." Chapter 9 in *Flexible Exchange Rates and the Balance of Payments: Essays in Memory of Egon Sohmen*, ed. by J. Chipman and C. Kindleberger. (North-Holland, Amsterdam)

[41] Stokey, N. L. and R. E. Lucas(1989):*Recursive Methods in Economic Dynamics.* (Harvard University Press, Cambridge, MA)

[42] Takayama, A(1972):*International Trade:An Approach to the Theory.* (Holt, Rinehart and Winston, New York)

[43] Woodland, A(1982):*International Trade and Resource Allocations* (North Holland, New Nork)

[44] Yosida, K., and E. Hewitt.(1952):"Finitely additive measures." *Transactions of the American Mathematical Society* 72, pp. 46 – 66.