The Dynamics of Group Knowledge and Belief

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Group knowledge

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I know that if you got at least 70 points you pass the exam, you know that you got 85 points. Together we know that you pass the exam.

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The police know my speed was too high $S_{\text{Police}}\text{toofast}$

I know that if you got at least 70 points you pass the exam, you know that you got 85 points. Together we know that you pass the exam. $D_{\{\text{you,me}\}}\text{pass}$

Group knowledge, dynamics and ability

$\langle\text{this talk}\rangle C_{\text{all of you}}$:

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$\langle\text{this talk}\rangle C_{\text{all of you}}$:

- Group knowledge is all about *dynamics*

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$\langle \text{this talk} \rangle C_{\text{all of you}}$:

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- Group *ability* is fundamental in reasoning about group knowledge

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Group knowledge, dynamics and ability

$\langle \text{this talk} \rangle C_{\text{all of you}}$:

- Group knowledge is all about *dynamics*
- Group *ability* is fundamental in reasoning about group knowledge
- Until recently common knowledge has received most attention in the dynamic epistemic logic literature
  - I will focus a little more on distributed knowledge

Plan

- Background: multi-agent epistemic/doxastic logic
- Group knowledge
- Group belief
  - Generalised
- Adding dynamics
- Group ability and group knowledge
  - General ability
  - Ability through informative updates
    - Maximal ability

Background

We assume given

a finite set $N = \{1, \ldots, n\}$ of agents

a countably infinite set of primitive propositions

let $\mathcal{GR} = \wp(N) \setminus \emptyset$ (the set of non-empty groups)

Models

A model is a tuple $M = \langle W, \sim_1, \dots, \sim_n, V \rangle$:

- W is a set of states
- $\sim_i$ is an accessibility relation
  - Assumed to be an equivalence relation (S5) when we model knowledge
  - Assumed have weaker properties when we talk about belief, e.g., transitive, euclidian and serial (KD45)
- V is a valuation function, assigning primitive propositions to each state

Epistemic/doxastic logic

Language $\mathcal{EL}$: $\phi ::= p \mid K_i\phi \mid \neg\phi \mid \phi_1 \wedge \phi_2$

Interpretation:

$$\begin{array}{rcl} M, s \models p & \text{iff} & p \in V(s) \\ M, s \models K_i\phi & \text{iff} & \text{for all } t \text{ s.t. } s \sim_i t,\ (M,t) \models \phi \\ M, s \models \neg\phi & \text{iff} & M, s \not\models \phi \\ M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \text{ and } M, s \models \psi \end{array}$$

For belief we often write B_i instead of K_i

Group Knowledge

General Knowledge (“everybody-knows”)

$G \in \mathcal{GR}$

$$\sim_G^E = \bigcup_{i \in G} \sim_i$$

$$M, s \models E_G\phi \text{ iff for all } t \text{ s.t. } s \sim_G^E t, (M,t) \models \phi$$

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Already expressible: $E_G\phi \equiv \bigwedge_{i \in G} K_i\phi$

Common Knowledge

$G \in \mathcal{GR}$

$$\sim_G^C = (\bigcup_{i \in G} \sim_i)^*$$

$$M, s \models C_G\phi \text{ iff for all } t \text{ s.t. } s \sim_G^C t,\ (M,t) \models \phi$$

Lewis, 1969

Aumann, 1976

Distributed Knowledge

Fagin et al., 1995

van der Hoek and Meyer, 1995

$\sim_G^D = \bigcap_{i \in G} \sim_i$ $\quad G \in \mathcal{GR}$

$M, s \models D_G\phi$ iff for all t s.t. $s \sim_G^D t$, $(M, t) \models \phi$

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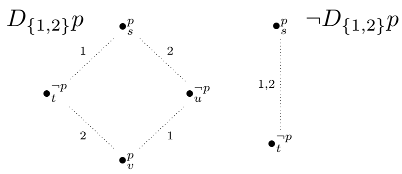

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"... the knowledge that would result of the agents could somehow 'combine' their knowledge" Roelofsen, 2006

Distributed Knowledge

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$D_{1,2}(p \wedge \neg K_1 p)$

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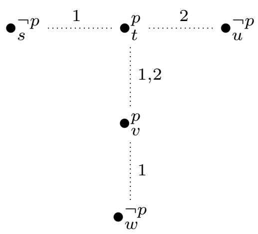

What Distributed Knowledge Actually Is

- Common interpretations of distributed knowledge:
  - Knowledge the group could obtain if they had unlimited means of communication
  - *"A group has distributed knowledge of a fact phi if the knowledge of phi is distributed among its members, so that by pooling their knowledge together the members of the group can deduce phi ..."*

What Distributed Knowledge Actually Is

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  - ~~*"A group has distributed knowledge of a fact phi if the knowledge of phi is distributed among its members, so that by pooling their knowledge together the members of the group can deduce phi ..."*~~

A group has distributed knowledge of a fact phi if after "pooling their knowledge together" ***the members of the group know that phi was true before they did that***

Group Knowledge

$G \in \mathcal{GR}$

$$\models C_G\phi \to E_G\phi$$
$$\models E_G\phi \to K_i\phi \qquad (i \in G)$$
$$\models K_i\phi \to S_G\phi$$
$$\models S_G\phi \to D_G\phi$$

Group Belief

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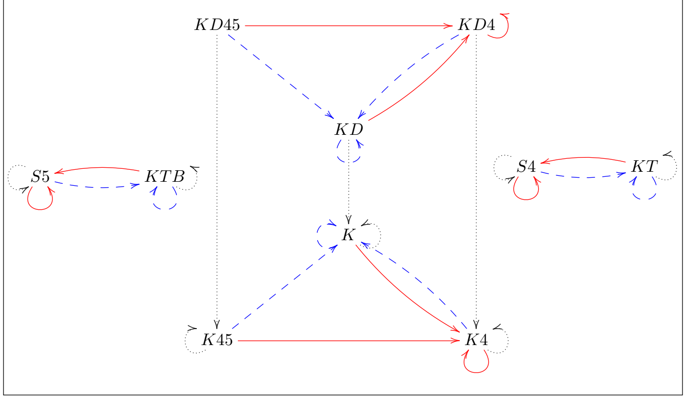
$G \in \mathcal{GR}$

$$\models C_G\phi \to E_G\phi$$
$$\models E_G\phi \to B_i\phi \qquad (i \in G)$$
$$\models B_i\phi \to S_G\phi$$
$$\models S_G\phi \to D_G\phi$$

Group Belief

Solid: C_G
Dashed: E_G
Dotted: D_G



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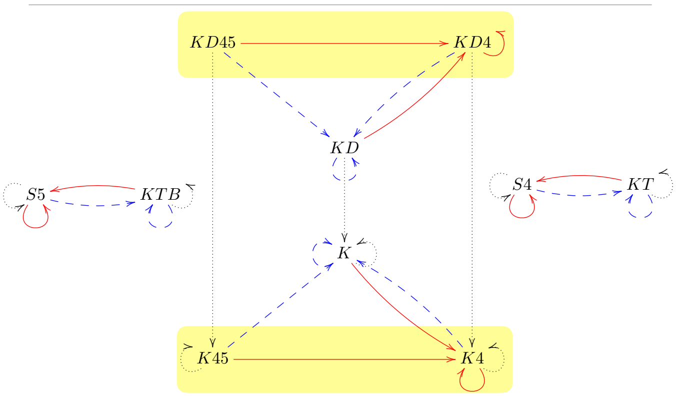

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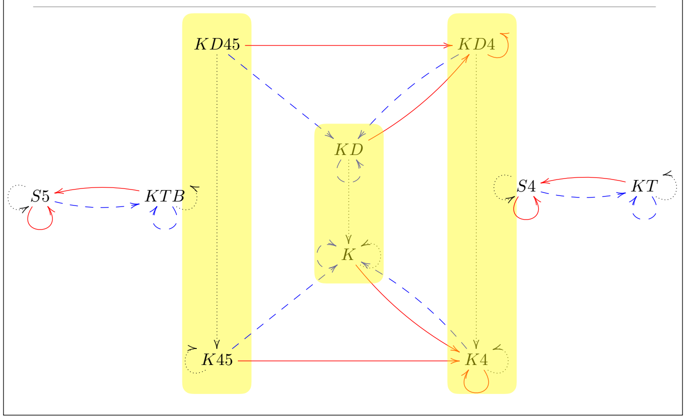

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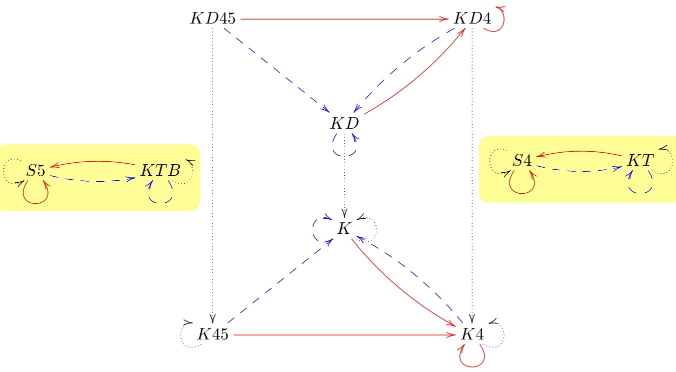

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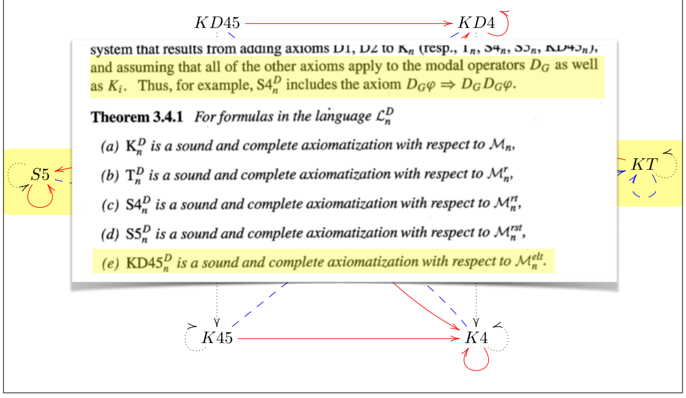

system that results from adding axioms D1, D2 to K_n (resp., T_n, $S4_n$, $S5_n$, $KD45_n$), and assuming that all of the other axioms apply to the modal operators D_G as well as K_i. Thus, for example, $S4_n^D$ includes the axiom $D_G\varphi \Rightarrow D_G D_G\varphi$.

Theorem 3.4.1 *For formulas in the language $\mathcal{L}_n^D$*

(a) K_n^D *is a sound and complete axiomatization with respect to* $\mathcal{M}_n$,

(b) T_n^D *is a sound and complete axiomatization with respect to* $\mathcal{M}_n^r$,

(c) $S4_n^D$ *is a sound and complete axiomatization with respect to* $\mathcal{M}_n^{rt}$,

(d) $S5_n^D$ *is a sound and complete axiomatization with respect to* $\mathcal{M}_n^{rst}$,

(e) $KD45_n^D$ *is a sound and complete axiomatization with respect to* $\mathcal{M}_n^{elt}$.

Group Belief

Solid: C_G
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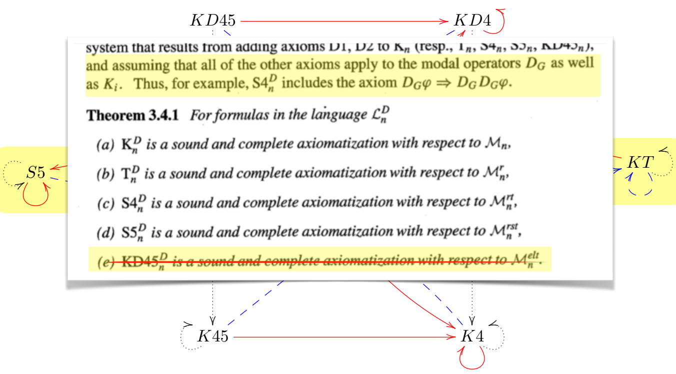

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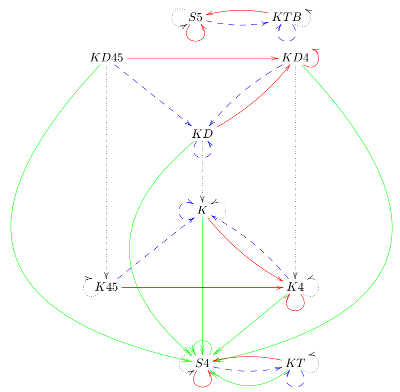
Alternative definition of common belief

Double arrow: alternative CK
Solid: C_G
Dashed: E_G
Dotted: D_G



Group Belief
Generalised Distributed Belief

Distributed belief

$$\sim_G^D = \bigcap_{i \in G} \sim_i$$

- The group considers a state
  - possible iff all the agents in the group considers it possible
  - impossible iff at least one member of the group considers it impossible
- For S5 agents this makes sense
  - If an S5 agent considers a state impossible, then it is impossible
    - .. and this is common knowledge

Distributed belief for non-S5 agents

$$\sim_G^D = \bigcap_{i\in G} \sim_i$$

- For non-S5 agents, in particular agents without T/reflexivity (e.g., KD45):
  - If one agent considers a state impossible, that agent might in fact be wrong
  - Ruling out a state based on the evidence of a single agent is then a very credulous group attitude
  - Curious asymmetry between the evidence need for possibility vs. impossibility
    - impossibility: every agent is a veto voter, possibility: unanimity

Generalised Distributed Belief

- The group considers a state
  - possible iff at least k agents in the group considers it possible
  - impossible iff not at least k agents in the group considers it impossible

The generalised distributed belief operator

$$M, s \models D_G^{+k}\phi \Leftrightarrow \forall (s,t) \in \sim_G^{+k} M, t \models \phi$$

$$\sim_G^{+k} = \bigcup_{H\subseteq G, |H|\geq k} \bigcap_{i\in H} \sim_i$$

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$$\text{E.g., } \sim_G^{maj} = \sim_G^{+\lceil(|G|+1)/2\rceil}$$

Generalised distributed belief: the extremes

$$\sim_G^{+k} = \bigcup_{H\subseteq G, |H|\geq k} \bigcap_{i\in H} \sim_i$$

Generalised distributed belief: the extremes

$$\sim_G^{+k} = \bigcup_{H\subseteq G, |H|\geq k} \bigcap_{i\in H} \sim_i$$

- k = |G|: the group considers a state
  - impossible iff at least one member of the group considers it impossible
  - possible iff all the agents in the group considers it possible

Generalised distributed belief: the extremes

$$\sim_G^{+k} = \bigcup_{H\subseteq G, |H|\geq k} \bigcap_{i\in H} \sim_i$$

- k = |G|: the group considers a state

$$\sim_G^{+|G|} = \sim_G^D$$

  - impossible iff at least one member of the group considers it impossible
  - possible if … ble

standard distributed belief

Generalised distributed belief: the extremes

$$\sim_G^{+k} = \bigcup_{H \subseteq G, |H| \geq k} \bigcap_{i \in H} \sim_i$$

- k = |G|: the group considers a state $\sim_G^{+|G|} = \sim_G^D$
  - impossible iff at least one member of the group considers it impossible
  - possible if[standard distributed belief]ble
- k = 1: the group considers a state
  - impossible iff all agents in the group considers it impossible
  - possible at least one agent in the group considers it possible

Generalised distributed belief: the extremes

$$\sim_G^{+k} = \bigcup_{H \subseteq G, |H| \geq k} \bigcap_{i \in H} \sim_i$$

- k = |G|: the group considers a state $\sim_G^{+|G|} = \sim_G^D$
  - impossible iff at least one member of the group considers it impossible
  - possible if[standard distributed belief]ble
- k = 1: the group considers a state $\sim_G^{+1} = \sim_G^E$
  - impossible iff all agents in the group considers it impossible
  - possib[general belief (everybody believes)]e

Generalised distributed belief: conclusions

- Between distributed and general belief
  - Intuitively two entirely different concepts
  - Difference between them can be explained quantitatively rather than qualitatively
  - Specific instances of the same concept, corresponding to which voting threshold is used
  - There is a scale of intermediate concepts between them

Adding dynamics

Public Announcement Logic with Distributed Knowledge

Wáng and Ågotnes, Synthese 190

$\mathcal{PAD}$: $\phi ::= p \mid K_i\phi \mid D_G\phi \mid [\phi]\phi \mid \neg\phi \mid \phi_1 \wedge \phi_2$

Public Announcement Logic with Distributed Knowledge

Wáng and Ågotnes, Synthese 190

$\mathcal{PAD}$: $\phi ::= p \mid K_i\phi \mid D_G\phi \mid [\phi]\phi \mid \neg\phi \mid \phi_1 \wedge \phi_2$

PC	All instances of propositional tautologies		
K_K	$K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$	T_K	$K_a\varphi \to \varphi$
K_D	$D_A(\varphi \to \psi) \to D_A\varphi \to D_A\psi$	T_D	$D_A\varphi \to \varphi$
5_K	$\neg K_a\varphi \to K_a\neg K_a\varphi$	5_D	$\neg D_A\varphi \to D_A\neg D_A\varphi$
DK1	$K_a\varphi \leftrightarrow D_a\varphi$, if $a \in N$	DK2	$D_A\varphi \to D_B\varphi$, if $A \subseteq B$
$\mathrm{R}_{[]p}$	$[\varphi]p \leftrightarrow (\varphi \to p)$	$\mathrm{R}_{[]\neg}$	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
$\mathrm{R}_{[]\wedge}$	$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$	$\mathrm{R}_{[]K}$	$[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$
$\mathrm{R}_{[]D}$	$[\varphi]D_A\psi \leftrightarrow (\varphi \to D_A[\varphi]\psi)$	$\mathrm{R}_{[][]}$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
MP	$\vdash \varphi$ & $\vdash \varphi \to \psi \Rightarrow \vdash \psi$	N_K	$\vdash \varphi \Rightarrow \vdash K_a\varphi$

Sound and complete: by reduction to $\mathcal{ELD}$.

Public Announcement Logic with Common Knowledge

$\mathcal{PAC}$: $\phi ::= p \mid K_i\phi \mid C_G\phi \mid [\phi]\phi \mid \neg\phi \mid \phi_1 \wedge \phi_2$

PC	All instances of tautologies	CK1	$C_A(\varphi \to E_A\varphi) \to \varphi \to C_A\varphi$
T_K	$K_a\varphi \to \varphi$	CK2	$C_A\varphi \to \varphi \wedge E_A C_A\varphi$
K_K	$K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$	K_C	$C_A(\varphi \to \psi) \to C_A\varphi \to C_A\psi$
5_K	$\neg K_a\varphi \to K_a\neg K_a\varphi$	N_C	$\vdash \varphi \Rightarrow \vdash C_A\varphi$
$R_{[]p}$	$[\varphi]p \leftrightarrow (\varphi \to p)$	$R_{[]C}$	$\vdash \chi \to [\varphi]\psi$ & $\vdash \chi \wedge \varphi \to E_A\chi$
$R_{[]\neg}$	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$		$\Downarrow$
$R_{[]\wedge}$	$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$		$\vdash \chi \to [\varphi]C_A\psi$
$R_{[]K}$	$[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$		
$R_{[][]}$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$		
MP	$\vdash \varphi$ & $\vdash \varphi \to \psi \Rightarrow \vdash \psi$		
N_K	$\vdash \varphi \Rightarrow \vdash K_a\varphi$		
$N_{[]}$	$\vdash \varphi \Rightarrow \vdash [\psi]\varphi$		

Sound and complete.

Public Announcement Logic with Common and Distributed Knowledge

$\mathcal{PACD}$: $\phi ::= p \mid K_i\phi \mid C_G\phi \mid D_G\phi \mid [\phi]\phi \mid \neg\phi \mid \phi_1 \wedge \phi_2$

PC	All instances of tautologies	T_K	$K_a\varphi \to \varphi$
K_K	$K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$	T_D	$D_A\varphi \to \varphi$
K_C	$C_A(\varphi \to \psi) \to C_A\varphi \to C_A\psi$	5_K	$\neg K_a\varphi \to K_a\neg K_a\varphi$
K_D	$D_A(\varphi \to \psi) \to D_A\varphi \to D_A\psi$	5_D	$\neg D_A\varphi \to D_A\neg D_A\varphi$
CK1	$C_A(\varphi \to E_A\varphi) \to \varphi \to C_A\varphi$	CK2	$C_A\varphi \to \varphi \wedge E_A C_A\varphi$
DK1	$K_a\varphi \leftrightarrow D_a\varphi$, if $a \in N$	DK2	$D_A\varphi \to D_B\varphi$, if $A \subseteq$
$R_{[]p}$	$[\varphi]p \leftrightarrow (\varphi \to p)$	$R_{[]\neg}$	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
$R_{[]\wedge}$	$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$	$R_{[]K}$	$[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi$
$R_{[]D}$	$[\varphi]D_A\psi \leftrightarrow (\varphi \to D_A[\varphi]\psi)$	$R_{[][]}$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
MP	$\vdash \varphi$ & $\vdash \varphi \to \psi \Rightarrow \vdash \psi$	N_K	$\vdash \varphi \Rightarrow \vdash K_a\varphi$
N_C	$\vdash \varphi \Rightarrow \vdash C_A\varphi$	$N_{[]}$	$\vdash \varphi \Rightarrow \vdash [\psi]\varphi$
$R_{[]C}$	$\vdash \chi \to [\varphi]\psi$ & $\vdash \chi \wedge \varphi \to E_A\chi \Rightarrow \vdash \chi \to [\varphi]C_A\psi$		

Sound and complete.

Public Announcement Logic with Common and Distributed Knowledge

$\mathcal{PACD}$: $\phi ::= p \mid K_i\phi \mid C_G\phi \mid D_G\phi \mid [\phi]\phi \mid \neg\phi \mid \phi_1 \wedge \phi_2$

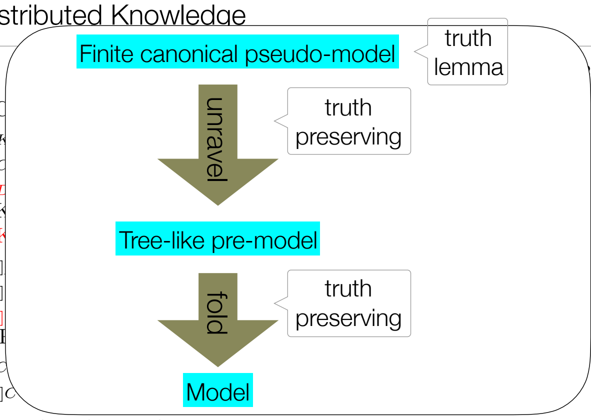
Complications for completeness proof:

- Distributed knowledge is not modally definable
- Model updates
- Common knowledge is not compact
- S5

N_C	$\vdash \varphi \Rightarrow \vdash C_A\varphi$	$N_{[]}$	$\vdash \varphi \Rightarrow \vdash [\psi]\varphi$
$R_{[]C}$	$\vdash \chi \to [\varphi]\psi$ & $\vdash \chi \wedge \varphi \to E_A\chi \Rightarrow \vdash \chi \to [\varphi]C_A\psi$		

Sound and complete.

Public Announcement Logic with Common and Distributed Knowledge



Sound and complete.

Public Announcement Logic with Common and Distributed Knowledge

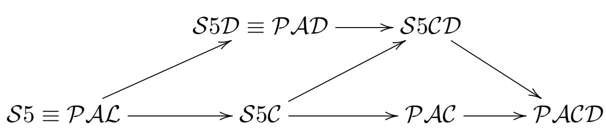
A note on completeness proofs for epistemic logic with distributed knowledge

Several claims about completeness for ELD can be found in the literature (Fagin et al. 1992, van der Hoek and Meyer 1992, Halpern and Moses 1992, Fagin et al. 1995, van der Hoek and Meyer 1997, Gerbrandy 1999). Most of them either

- only allow distributed knowledge operators for the grand coalition; and/or
- do not provide detailed proofs.

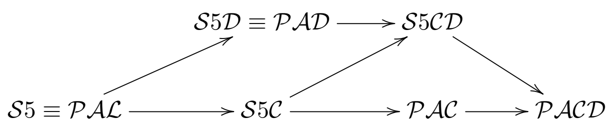
Sound and complete.

Public Announcement Logic with Common and Distributed Knowledge: expressivity



Public Announcement Logic with Common and Distributed Knowledge: expressivity

Wáng and Ågotnes, Synthese 190



Open problem: relax S5 assumptions

Some complexity results

Logic	Result	Reference
$\mathcal{EL}$	PSPACE-complete	Halpern and Moses 1992
$\mathcal{ELC}$	EXPTIME-complete	Fisher and Ladner 1977
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$\mathcal{PAC}$	EXPTIME-complete	Lutz 2006
$\mathcal{PACD}$	EXPTIME-complete	Wáng and Ågotnes 2013

Ability

Coalitional Ability Logics

- Logics with coalition operators. Typical notation:

$$\langle\!\langle C \rangle\!\rangle \phi \qquad \langle\langle C \rangle\rangle \phi \qquad [C]\phi$$

  - where C is a coalition (= set of agents, possibly empty)
  - intuitive meaning: C has the ability to make phi true

Coalitional Ability Logics

- Pauly's Coalition Logic (CL): $\phi ::= p \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \langle\!\langle C \rangle\!\rangle \phi$
  - extends propositional logic with coalition operators
  - interpreted in game structures: ability = the coalition can choose a joint action such that phi becomes true *no matter what the other agents do*
- Alur et al.'s Alternating-time Temporal Logic (ATL):

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\!\langle A \rangle\!\rangle \bigcirc \varphi \mid \langle\!\langle A \rangle\!\rangle \Box \varphi \mid \langle\!\langle A \rangle\!\rangle \varphi \mathcal{U} \varphi$$

  - can be seen as a combination of CL and CTL
  - ability = the coalition can choose a joint strategy such that phi becomes true no matter what the other agents do
- Seeing-to-it-that (STIT) logics
- van Benthem on forcing
- …

Coalitional ability: examples

$$\langle\!\langle Thomas, Meiyun \rangle\!\rangle \Diamond students\_happy$$

$$\langle\!\langle alibaba, tencent \rangle\!\rangle \neg applepay\_successful$$

Coalition Logic

Pauly, JLC 2004

$\phi ::= p \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \langle\!\langle C \rangle\!\rangle \phi \qquad C \subseteq N$

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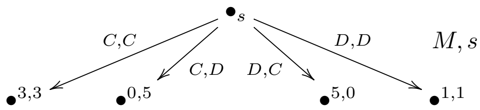

$M, s \models \langle\!\langle Ann \rangle\!\rangle jail_B$

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Pauly, JLC 2004

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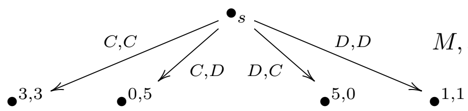

$M, s \models \langle\!\langle Ann \rangle\!\rangle jail_B$

Alternative: **neighbourhood semantics.**

$M, s \models \langle\!\langle C \rangle\!\rangle \Leftrightarrow \phi^M \in E_s(C)$

Coalition Logic

Pauly, JLC 2004

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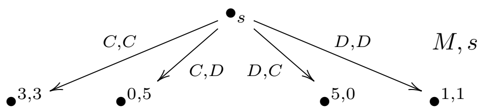

$M, s \models \langle\!\langle Ann \rangle\!\rangle jail_B$

Alternative: **neighbourhood semantics.**

Playable effectivity function in each state: $E_s : \wp(N) \to \wp(\wp(S))$

$M, s \models \langle\!\langle C \rangle\!\rangle \Leftrightarrow \phi^M \in E_s(C)$

1. $\forall C \subseteq N$: $\emptyset \notin E_s(C)$
2. $\forall C \subseteq N$: $S \in E_s(C)$
3. $\forall X \subseteq S$: $S \setminus X \notin E_s(\emptyset) \Rightarrow X \in E_s(N)$
4. $\forall C$: $\forall X \subseteq X' \subseteq S$: $X \in E_s(C) \Rightarrow X' \in E_s(C)$ (*outcome-monotonicity*)
5. $\forall C_1 \subseteq N$: $\forall C_2 \subseteq N$: $\forall X_1 \subseteq S$: $\forall X_2 \subseteq S$: ($C_1 \cap C_2 = \emptyset$ and $X_1 \in E_s(C_1)$ and $X_2 \in E_s(C_2)$) $\Rightarrow X_1 \cap X_2 \in E(C_1 \cup C_2)$ (*superadditivity*)

Coalition Logic

Pauly, JLC 2004

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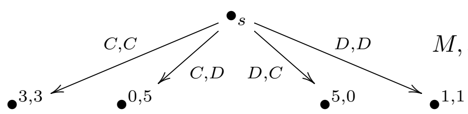

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Representation theorem: equivalent semantics.

Goranko et al., AAMAS 2010

1. $\forall C \subseteq N$: $\emptyset \notin E_s(C)$
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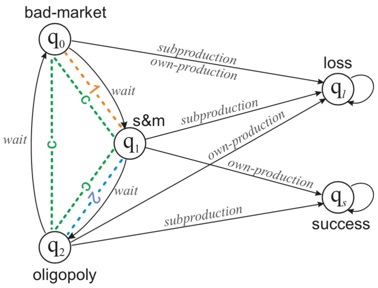
Epistemic Coalition Logic

$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\!\langle G \rangle\!\rangle \phi \mid K_i \phi \mid C_{G'} \phi \mid D_{G'} \phi \qquad G' \in \mathcal{GR},\ G \subseteq N$

Epistemic Coalition Logic

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\!\langle G \rangle\!\rangle \phi \mid K_i\phi \mid C_{G'}\phi \mid D_{G'}\phi \qquad \begin{matrix} G' \in \mathcal{GR} \\ G \subseteq N \end{matrix}$$



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$K_i\phi \rightarrow \langle\!\langle \{i\} \rangle\!\rangle K_j\phi$: i can communicate her knowledge of ϕ to j

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$\langle\!\langle G \rangle\!\rangle \psi \rightarrow D_G\phi$: distributed knowledge in G of ϕ is necessary for G to ensure that ψ

$D_G\phi \rightarrow \langle\!\langle G \rangle\!\rangle E_G\phi$: G can cooperate to make distributed knowledge explicit

Epistemic Coalition Logic

$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\!\langle G \rangle\!\rangle \phi \mid K_i\phi \mid C_{G'}\phi \mid D_{G'}\phi$

Ågotnes and Alechina, JLC 2016

Epistemic Coalition Logic

$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\!\langle G \rangle\!\rangle \phi \mid K_i\phi \mid C_{G'}\phi \mid D_{G'}\phi$

K $K_i(\phi \to \psi) \to (K_i\phi \to K_i\psi)$

T $K_i\phi \to \phi$

4 $K_i\phi \to K_iK_i\phi$

5 $\neg K_i\phi \to K_i\neg K_i\phi$

C $C_G\phi \to E_G(\phi \wedge C_G\phi)$

RN $\vdash_{CLC} \phi \Rightarrow \vdash_{CLC} K_i\phi$

RC $\vdash_{CLC} \phi \to E_G(\phi \wedge \psi) \Rightarrow \vdash_{CLC} \phi \to C_G\psi$

DK $D_G(\phi \to \psi) \to (D_G\phi \to D_G\psi)$

DT $D_G\phi \to \phi$

D4 $D_G\phi \to D_GD_G\phi$

D5 $\neg D_G\phi \to D_G\neg D_G\phi$

D1 $K_i\phi \leftrightarrow D_i\phi$

D2 $D_G\phi \to D_H\phi$, if $G \subseteq H$

Sound and complete
(all combinations of operators: $\mathcal{CLK}, \mathcal{CLD}, \mathcal{CLC}, \mathcal{CLCD}$)

Ågotnes and Alechina, JLC 2016

Epistemic Coalition Logic: adding interaction axioms

Property	Axiom	Completeness?
$s \sim_i t \Rightarrow E(s)(i) = E(t)(i)$	$\langle\!\langle i \rangle\!\rangle\varphi \to K_i\langle\!\langle i \rangle\!\rangle\varphi$	Yes
$s \sim^C_G t \Rightarrow E(s)(G) = E(t)(G)$	$\langle\!\langle G \rangle\!\rangle\varphi \to C_G\langle\!\langle G \rangle\!\rangle\varphi$	Yes
$s \sim^D_G t \Rightarrow E(s)(G) = E(t)(G)$	$\langle\!\langle G \rangle\!\rangle\varphi \to D_G\langle\!\langle G \rangle\!\rangle\varphi$	?

Open problem: completeness of ECL with the distributed knowledge axiom

Ågotnes and Alechina, JLC 2016

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$\mathcal{PAC}$	EXPTIME-complete	Lutz 2006
$\mathcal{PACD}$	EXPTIME-complete	Wáng and Ågotnes 2013
$\mathcal{CL}$	PSPACE-complete	Pauly 2002
$\mathcal{CLC}$	EXPTIME-complete	Ågotnes and Alechina 2016
$\mathcal{CLCD}$	EXPTIME-complete	Ågotnes and Alechina 2016
$\mathcal{CLD}$	PSPACE-complete	Ågotnes and Alechina 2016
$\mathcal{CLD}$+	PSPACE-complete	Ågotnes and Alechina 2016
$\mathcal{CLC}$+	**unknown**	

ATL with group knowledge

$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\!\langle A \rangle\!\rangle \bigcirc \varphi \mid \langle\!\langle A \rangle\!\rangle \Box \varphi \mid \langle\!\langle A \rangle\!\rangle \varphi \mathcal{U} \varphi \mid C_A\varphi \mid E_A\varphi \mid D_A\varphi$

- Plain ATL completely axiomatised
  Goranko and van Drimmelen, Th. Comp. Sci. 2007
- A lot of work on epistemic extensions, but no completeness proof yet
  van der Hoek and Wooldridge, Studia Logica 2003
  - Completeness claim with common knowledge only
    Goranko et al., LOFT 2014

Open problem: complete axiomatisation of ATL with group knowledge

Epistemic ATL: knowing that vs. knowing how (knowledge of ability *de dicto* vs. *de re*)

$C_G\langle\!\langle G \rangle\!\rangle\gamma$: in every G-reachable state G has a strategy that will ensure γ

??: G has a strategy that in every G-reachable state will ensure γ

Ågotnes, Goranko, van der Hoek, Wooldridge, Handb. of Epistemic Logic, 2015

Jamroga and van der Hoek 2004

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not expressible in Epistemic ATL

Group knowing how: who knows that the group strategy is winning?

- Common knowledge in the group: requires the least amount of coordination
- General knowledge in the group
- Distributed knowledge in the group: if they communicate they can identify a winning strategy
- A single agent (e.g., *the leader*)
- A subgroup (e.g., *the executive committee*)
- A disjoint group (e.g., *a consulting company)*
- …

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Jamroga and Ågotnes, 2007

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Constructive knowledge → $\mathbb{C}_G\langle\!\langle G\rangle\!\rangle\gamma$

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Jamroga and Ågotnes, 2007

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Constructive knowledge → $\mathbb{C}_G\langle\!\langle G\rangle\!\rangle\gamma$

$\mathbb{E}_G\langle\!\langle G\rangle\!\rangle\gamma$

$\mathbb{D}_G\langle\!\langle G\rangle\!\rangle\gamma$

$\mathbb{K}_i\langle\!\langle G\rangle\!\rangle\gamma$ $i \in G$

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- A single agent (e.g., *the leader*) $\mathbb{K}_i \langle\!\langle G \rangle\!\rangle \gamma$ $\quad i \in G$
- A subgroup (e.g., *the executive committee*) $\mathbb{C}_H \langle\!\langle G \rangle\!\rangle \gamma$ $\quad H \subseteq G$
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Constructive knowledge

Jamroga and Ågotnes, 2007

Group knowing how: who knows that the group strategy is winning?

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- ...

Constructive knowledge

Jamroga and Ågotnes, 2007

Constructive Knowledge

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\!\langle A \rangle\!\rangle \bigcirc \varphi \mid \langle\!\langle A \rangle\!\rangle \Box \varphi \mid \langle\!\langle A \rangle\!\rangle \varphi \mathcal{U} \varphi \mid C_A\varphi \mid E_A\varphi \mid D_A\varphi \mid \mathbb{C}_A\varphi \mid \mathbb{E}_A\varphi \mid \mathbb{D}_A\varphi.$$

Jamroga and Ågotnes, 2007

Open problems: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators

Constructive Knowledge

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$$M, q \models \mathbb{C}_G\varphi \Leftrightarrow M, [q]_{\sim^C_G} \models \varphi$$

Jamroga and Ågotnes, 2007

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$$M, q \models \mathbb{C}_G\varphi \Leftrightarrow M, [q]_{\sim^C_G} \models \varphi$$

$M, Q \models \langle\!\langle G \rangle\!\rangle \gamma \Leftrightarrow G$ has a joint strategy that will ensure that γ is true in all states in Q

Jamroga and Ågotnes, 2007

Open problems: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators

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$M, q \models \mathbb{C}_H \langle\!\langle G \rangle\!\rangle \varphi \Leftrightarrow G$ has a strategy that will ensure that γ is true, starting in any state H-reachable from q

Jamroga and Ågotnes, 2007

Open problems: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators

Constructive Knowledge

Jamroga and Ågotnes, 2007

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$M, q \models \mathbb{C}_G\varphi \Leftrightarrow M, [q]_{\sim_G^C} \models \varphi$

$M, Q \models \langle\!\langle G \rangle\!\rangle\gamma \Leftrightarrow G$ has a joint strategy that will ensure that γ is true in all states in Q

$M, q \models \mathbb{C}_H\langle\!\langle G \rangle\!\rangle\varphi \Leftrightarrow G$ has a strategy that will ensure that γ is true, starting in any state H-reachable from q

I.e., H knows how G can achieve γ

Open problems: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators

Ability

through publicly observed informational actions

What if we interpret group ability modalities directly on epistemic models, in terms of possible public announcements?

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$\langle\!\langle G \rangle\!\rangle\phi$: "Group G can make a joint announcement such that, no matter what the other agents announce, ϕ will be true"

What if we interpret group ability modalities directly on epistemic models, in terms of possible public announcements?

$\langle G \rangle\phi$: "Group G can make an announcement after which ϕ is true"

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Coalition Announcement Logic (GAL)

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle\!\langle G \rangle\!\rangle\phi$$

Ågotnes and van Ditmarsch, AAMAS 2008

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$\langle\!\langle G \rangle\!\rangle \phi$: "Group G can make a joint announcement such that, no matter what the other agents announce, ϕ will be true"

Group Announcement Logic (GAL)

$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G \rangle \phi$

Ågotnes et al., JAL 2010

Coalition Announcement Logic (GAL)

$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle\!\langle G \rangle\!\rangle \phi$

Ågotnes and van Ditmarsch, AAMAS 2008

What if we interpret group ability modalities directly on epistemic models, in terms of possible public announcements?

$\langle G \rangle \phi$: "Group G can make an announcement after which ϕ is true"

$\langle\!\langle G \rangle\!\rangle \phi$: "Group G can make a joint announcement such that, no matter what the other agents announce, ϕ will be true"

Group Announcement Logic (GAL)

$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G \rangle \phi$

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Ågotnes and van Ditmarsch, AAMAS 2008

Related: **Abritrary Public Announcement Logic**

$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \Diamond\phi$

Balbiani et al., TARK 2007

GAL: example (Russian Cards)

$$\langle Ann \rangle \langle Bill \rangle (one \wedge two \wedge three)$$

$$\langle \{Ann, Bill\} \rangle (one \wedge two \wedge three)$$

GAL: expressing knowing-how

Knowledge of ability, *de dicto*

$\forall s \sim_a t\ \exists\psi\ t \models \langle K_a\psi \rangle \phi$

⇕

$s \models K_a \langle a \rangle \phi$

Knowledge of ability, *de re*

$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi \rangle \phi$

⇕

??

GAL: expressing knowing-how

Knowledge of ability, *de dicto*

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⇕

??

$s \models \langle a \rangle K_a \phi$

⇕

$\exists\psi\ s \models \langle K_a\psi \rangle K_a\phi$

GAL: expressing knowing-how

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Knowledge of ability, *de re*

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$s \models \langle a \rangle K_a \phi$

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$\exists\psi\ s \models \langle K_a\psi \rangle K_a\phi$

GAL: expressing knowing-how

Knowledge of ability, *de dicto*

$\forall s \sim_a t \; \exists \psi \; t \models \langle K_a \psi \rangle \phi$

$\Updownarrow$

$s \models K_a \langle a \rangle \phi$

Depends on
(1) the fact that actions are *announcements*
(2) the S5 properties

Knowledge of ability, *de re*

$\exists \psi \; \forall s \sim_a t \; t \models \langle K_a \psi \rangle \phi$

$\Updownarrow$

$s \models \langle a \rangle K_a \phi$

$\Updownarrow$

$\exists \psi \; s \models \langle K_a \psi \rangle K_a \phi$

GAL: expressing knowing-how

$\exists \psi \; \forall s \sim_a t \; t \models \langle K_a \psi \rangle \phi \Longleftrightarrow s \models \langle a \rangle K_a \phi$

GAL: expressing knowing-how

$\exists \psi \; \forall s \sim_a t \; t \models \langle K_a \psi \rangle \phi \Longleftrightarrow s \models \langle a \rangle K_a \phi$

$\exists \{\psi_i : i \in G\} \; \forall (s,t) \in (\bigcap_{i \in G} \sim_i), \; t \models \langle \bigwedge_{i \in G} K_i \psi_i \rangle \phi \qquad s \models \langle G \rangle D_G \phi$

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$\exists \{\psi_i : i \in G\} \; \forall (s,t) \in (\bigcup_{i \in G} \sim_i), \; t \models \langle \bigwedge_{i \in G} K_i \psi_i \rangle \phi \not\Leftrightarrow s \models \langle G \rangle E_G \phi$

GAL: expressing knowing-how

$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi \Longleftrightarrow s \models \langle a\rangle K_a\phi$

$\exists\{\psi_i : i \in G\}\ \forall(s,t) \in (\bigcap_{i\in G} \sim_i),\ t \models \langle \bigwedge_{i\in G} K_i\psi_i\rangle\phi \Longleftrightarrow s \models \langle G\rangle D_G\phi$

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$\exists\{\psi_i : i \in G\}\ \forall(s,t) \in (\bigcup_{i\in G} \sim_i)^*,\ t \models \langle \bigwedge_{i\in G} K_i\psi_i\rangle\phi \qquad s \models \langle G\rangle C_G\phi$

GAL: expressing knowing-how

$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi \Longleftrightarrow s \models \langle a\rangle K_a\phi$

$\exists\{\psi_i : i \in G\}\ \forall(s,t) \in (\bigcap_{i\in G} \sim_i),\ t \models \langle \bigwedge_{i\in G} K_i\psi_i\rangle\phi \Longleftrightarrow s \models \langle G\rangle D_G\phi$

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GAL: expressing knowing-how

$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi \Longleftrightarrow s \models \langle a\rangle K_a\phi$

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Open problem: express common knowledge *de re*

GAL: infinitary axiomatisation

Propositional tautologies	$[\varphi]p \leftrightarrow (\varphi \to p)$
$K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
$K_a\varphi \to \varphi$	$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$
$K_a\varphi \to K_aK_a\varphi$	$[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$
$\neg K_a\varphi \to K_a\neg K_a\varphi$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
$\vdash \varphi \Rightarrow \vdash K_a\varphi$	$[G]\varphi \to [\psi_G]\varphi$
$\vdash \varphi \to \psi, \vdash \varphi \Rightarrow \vdash \psi$	$\vdash \varphi \Rightarrow \vdash [G]\varphi$
	$\forall\psi_G :\vdash \eta([\psi_G]\varphi) \Rightarrow \vdash \eta([G]\varphi)$

$\eta(\sharp) ::= \sharp \mid \varphi \to \eta(\sharp) \mid K_a\eta(\sharp) \mid [\varphi]\eta(\sharp)$

Sound and complete.

GAL: infinitary axiomatisation

Propositional tautologies	$[\varphi]p \leftrightarrow (\varphi \to p)$
$K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
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$K_a\varphi \to K_aK_a\varphi$	$[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$
$\neg K_a\varphi \to K_a\neg K_a\varphi$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
$\vdash \varphi \Rightarrow \vdash K_a\varphi$	$[G]\varphi \to [\psi_G]\varphi$
$\vdash \varphi \to \psi, \vdash \varphi \Rightarrow \vdash \psi$	$\vdash \varphi \Rightarrow \vdash [G]\varphi$
	$\forall\psi_G :\vdash \eta([\psi_G]\varphi) \Rightarrow \vdash \eta([G]\varphi)$

$\eta(\sharp) ::= \sharp \mid \varphi \to \eta(\sharp) \mid K_a\eta(\sharp) \mid [\varphi]\eta(\sharp)$

Sound and complete.

Open problem: finitary axiomatisation (same for APAL)

GAL-D: ability and distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

Detour: Distributed Knowledge and the Principle of Full Communication

van der Hoek et al., 1999

Full communication: $M, s \models D_G\phi \Rightarrow KS_G(M,s) \vdash \phi$

$KS_G(M,s) = \{\psi \in \mathcal{L}_{\mathcal{EL}} : M, s \models \bigvee_{i \in G} K_a\psi\}$

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Does not always hold.

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Does not always hold.

Roelofsen, 2006

Roelofsen gives a complete characterisation of the models in which it does.

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Roelofsen gives a complete characterisation of the models in which it does.

van Benthem, 2002

van Benthem: what about public communication?

GAL-D: ability and distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

GAL-D: ability and distributed knowledge

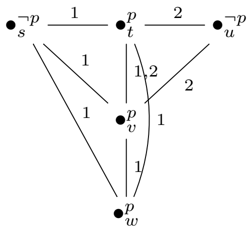
$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$$D_G\phi \to \langle G\rangle E_G\phi$$

GAL-D: ability and distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$$D_G\phi \rightarrow \langle G\rangle E_G\phi$$

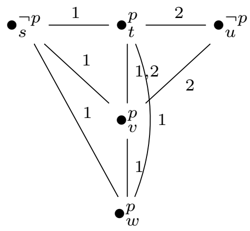


GAL-D: ability and distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$$D_G\phi \rightarrow \langle G\rangle E_G\phi$$

$M, t \models D_{\{1,2\}}(p \wedge \neg K_1 p)$



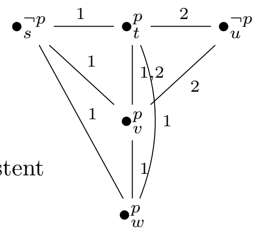
GAL-D: ability and distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$$D_G\phi \rightarrow \langle G\rangle E_G\phi$$

$M, t \models D_{\{1,2\}}(p \wedge \neg K_1 p)$

$E_{\{1,2\}}(p \wedge \neg K_1 p)$ not S5-consistent



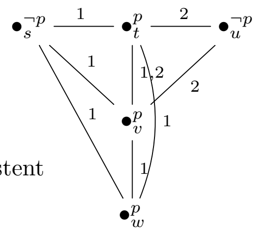
GAL-D: ability and distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$$\not\models D_G\phi \rightarrow \langle G\rangle E_G\phi$$

$M, t \models D_{\{1,2\}}(p \wedge \neg K_1 p)$

$E_{\{1,2\}}(p \wedge \neg K_1 p)$ not S5-consistent



Group announcement logic with distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$(A0)$	Propositional tautologies	$(A11)$	$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$
$(A1)$	$K_a(\varphi \rightarrow \psi) \rightarrow K_a\varphi \rightarrow K_a\psi$	$(A12)$	$[\varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[\varphi]\psi)$
$(A2)$	$K_a\varphi \rightarrow \varphi$	$(A13)$	$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$
$(A3)$	$K_a\varphi \rightarrow K_aK_a\varphi$	$(A14)$	$[\varphi]K_a\psi \leftrightarrow (\varphi \rightarrow K_a[\varphi]\psi)$
$(A4)$	$\neg K_a\varphi \rightarrow K_a\neg K_a\varphi$	$(A15)$	$[\varphi]D_G\psi \leftrightarrow (\varphi \rightarrow D_G[\varphi]\psi)$
$(A5)$	$D_G(\varphi \rightarrow \psi) \rightarrow D_G\varphi \rightarrow D_G\psi$	$(A16)$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
$(A6)$	$D_G\varphi \rightarrow \varphi$	$(A17)$	$[G]\varphi \rightarrow [\psi_G]\varphi$
$(A7)$	$D_G\varphi \rightarrow D_GD_G\varphi$	$(R0)$	$\vdash \varphi \rightarrow \psi, \vdash \varphi \Rightarrow \vdash \psi$
$(A8)$	$\neg D_G\varphi \rightarrow D_G\neg D_G\varphi$	$(R1)$	$\vdash \varphi \Rightarrow \vdash K_a\varphi$
$(A9)$	$D_a\varphi \leftrightarrow K_a\varphi$	$(R2)$	$\vdash \varphi \Rightarrow \vdash [G]\varphi$
$(A10)$	$D_G\varphi \rightarrow D_H\varphi$, if $G \subseteq H$	$(R3)$	$\forall\psi_G : \vdash \eta([\psi_G]\varphi) \Rightarrow \vdash \eta([G]\varphi)$

Sound and complete.

Galimullin, Ågotnes and Alechina, LORI 2019

Group announcement logic with distributed knowledge

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

$(A0)$	Propositional tautologies	$(A11)$	$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$
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Sound and complete. **Open problem**: add common knowledge

Galimullin, Ågotnes and Alechina, LORI 2019

Group announcement logic with distributed knowledge

Galimullin, Ågotnes and Alechina, LORI 2019

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \langle G\rangle\phi \mid D_G\phi$$

(A0) Propositional tautologies
(A1) $K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$
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(A12) $[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
(A13) $[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$
(A14) $[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$
(A15) $[\varphi]D_G\psi \leftrightarrow (\varphi \to D_G[\varphi]\psi)$
(A16) $[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
(A17) $[G]\varphi \to [\psi_G]\varphi$
(R0) $\vdash \varphi \to \psi, \vdash \varphi \Rightarrow \vdash \psi$
(R1) $\vdash \varphi \Rightarrow \vdash K_a\varphi$
(R2) $\vdash \varphi \Rightarrow \vdash [G]\varphi$
(R3) $\forall\psi_G : \vdash \eta([\psi_G]\varphi) \Rightarrow \vdash \eta([G]\varphi)$

Sound and complete.

Open problem: add common knowledge

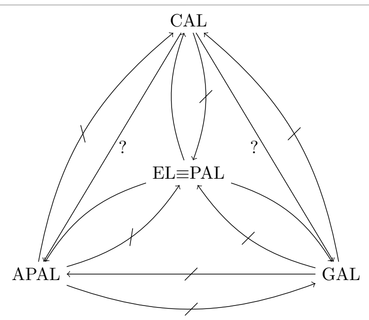
Open problem: axiomatisation for CAL

Relative expressivity of logics of quantified announcement

Ågotnes et al., JAL 2010

Galimullin and Alechina, AAMAS 2019

Galimullin PhD thesis 2019

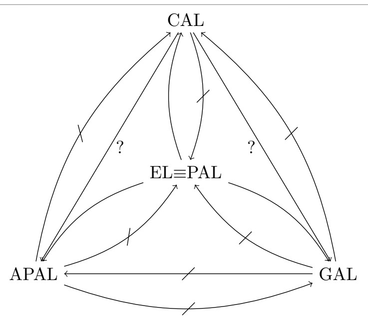


Relative expressivity of logics of quantified announcement

Ågotnes et al., JAL 2010

Galimullin and Alechina, AAMAS 2019

Galimullin PhD thesis 2019



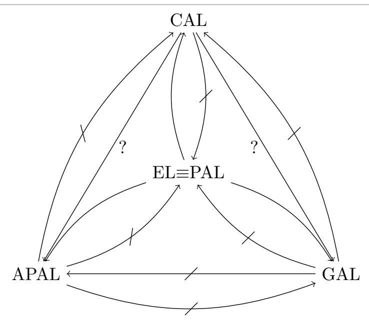
Open problem: can GAL express everything CAL can express?

Relative expressivity of logics of quantified announcement

Ågotnes et al., JAL 2010

Galimullin and Alechina, AAMAS 2019

Galimullin PhD thesis 2019



Open problem: can GAL express everything CAL can express?

Open problem: can APAL express everything CAL can express?

Complexity

Logic	Result	Reference
$\mathcal{EL}$	PSPACE-complete	Halpern and Moses 1992
$\mathcal{ELC}$	EXPTIME-complete	Fisher and Ladner 1977
$\mathcal{ELD}$ (D only for grand coal.)	PSPACE-complete	Halpern and Moses 1992
$\mathcal{ELCD}$ (C, D only for grand coal.)	EXPTIME-complete	Halpern and Moses 1992
$\mathcal{ELCD}$ (no restrictions)	EXPTIME-complete	Wáng and Ågotnes 2013
$\mathcal{PA}$	PSPACE-complete	(follows)
$\mathcal{PAD}$	PSPACE-complete	(follows)
$\mathcal{PAC}$	EXPTIME-complete	Lutz 2006
$\mathcal{PACD}$	EXPTIME-complete	Wáng and Ågotnes 2013
$\mathcal{CL}$	PSPACE-complete	Pauly 2002
$\mathcal{CLC}$	EXPTIME-complete	Ågotnes and Alechina 2016
$\mathcal{CLCD}$	EXPTIME-complete	Ågotnes and Alechina 2016
$\mathcal{CLD}$	PSPACE-complete	Ågotnes and Alechina 2016
$\mathcal{CLD}+$	PSPACE-complete	Ågotnes and Alechina 2016
$\mathcal{CLC}+$	**unknown**	
$\mathcal{ATL}$	EXPTIME-complete	Walther et al. 2005
$\mathcal{ATEL}$	EXPTIME-complete	Walther 2005
$\mathcal{APAL}$	undecidable	van Ditmarsch and French, 2008
$\mathcal{GAL}$	undecidable	Ågotnes, van Ditmarsch and French, 2016
$\mathcal{CAL}$	undecidable	Ågotnes,van Ditmarsch and French, 2016

Resolving Distributed Knowledge

Resolving distributed knowledge

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- In this work we introduce a new modality, saying that a formula is true after the group have shared their information - after their distributed knowledge has been resolved

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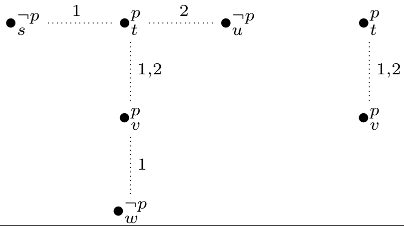

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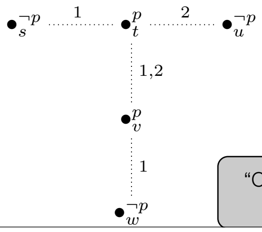

"Communication core" (van Benthem, 2011)

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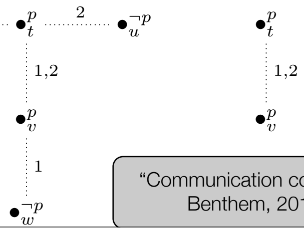

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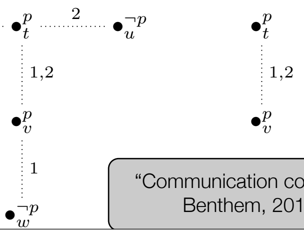

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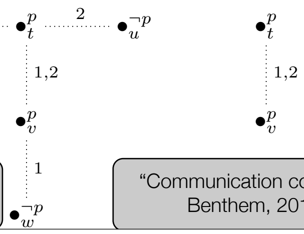

Similar to "tell all you know" logic, Baltag ESSLLI 2010

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What do other agents know about the fact that a group G resolve their knowledge?

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$M = (S, \sim_1, \ldots, \sim_n, V)$ (S5 model)

For a group of agents G, the *(global) G-resolved update of* M is the model $M|_G$ where $M|_G = (S', \sim'_1, \ldots, \sim'_n, V')$ and

- $S' = S$
- $\sim'_i = \begin{cases} \bigcap_{j \in G} \sim_j & i \in G \\ \sim_i & \text{otherwise} \end{cases}$
- $V' = V$

Resolving Distributed Knowledge: Logic

$$\mathcal{RD} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid R_G\phi$$
$$\mathcal{RCD} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid C_G\phi \mid R_G\phi$$

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$$M, s \models R_G\phi \quad \Leftrightarrow \quad M|_G, s \models \phi$$

Resolution: from distributed to common knowledge

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid C_G\phi \mid R_G\phi$$

$$D_G\phi \to R_G C_G\phi$$

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Resolution: from distributed to common knowledge

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$$\not\models D_G\phi \to R_G C_G\phi$$

$$\models D_G R_G\phi \leftrightarrow R_G C_G\phi$$

Resolving distributed knowledge: expressivity

$\mathcal{ELD} \equiv \mathcal{ELCD} \equiv \mathcal{RD} \equiv \mathcal{RCD} \equiv \mathcal{PAD} \equiv \mathcal{PACD}$

(a) $|\text{AG}| = 1$

(b) $|\text{AG}| = 2$ (c) $|\text{AG}| \geq 3$

$\mathcal{RD} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid R_G\phi$

$\mathcal{RCD} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid C_G\phi \mid R_G\phi$

Resolution reduction axioms

The following are valid:

- $R_G p \leftrightarrow p$
- $R_G(\phi \wedge \psi) \leftrightarrow R_G\phi \wedge R_G\psi$
- $R_G\neg\phi \leftrightarrow \neg R_G\phi$
- $R_G K_i\phi \leftrightarrow K_i R_G\phi$, when $i \notin G$
- $R_G K_i\phi \leftrightarrow D_G R_G\phi$, when $i \in G$
- $R_G D_H\phi \leftrightarrow D_H R_G\phi$, when $G \cap H = \emptyset$
- $R_G D_H\phi \leftrightarrow D_{G\cup H} R_G\phi$, when $G \cap H \neq \emptyset$

$\mathcal{RD} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid R_G\phi$

Axiomatisation: RD

(S5)	classical proof system for multi-agent epistemic logic
(DK)	characterization axioms for distributed knowledge
(RR)	reduction axioms for resolution
(N_R)	from ϕ infer $R_G\phi$

DK:

(K_D)	$D_G(\phi \to \psi) \to (D_G\phi \to D_G\psi)$
(T_D)	$D_G\phi \to \phi$
(5_D)	$\neg D_G\phi \to D_G\neg D_G\phi$
(D1)	$K_i\phi \leftrightarrow D_i\phi$
(D2)	$D_G\phi \to D_H\phi$, if $G \subseteq H$.

Proposition: sound and complete.

$\mathcal{RCD} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid D_G\phi \mid C_G\phi \mid R_G\phi$

Axiomatisation: RCD

(S5)	classical proof system for multi-agent epistemic logic
(CK)	axioms and rules for common knowledge
(DK)	characterization axioms for distributed knowledge
(N_R)	from ϕ infer $R_G\phi$
(RR)	reduction axioms for resolution
(RR_C)	from $\phi \to (E_H\phi \wedge R_{G_1} \cdots R_{G_n}\psi)$ infer $\phi \to R_{G_1} \cdots R_{G_n} C_H\psi$

CK:

(K_C)	$C_G(\phi \to \psi) \to (C_G\phi \to C_G\psi)$
(T_C)	$C_G\phi \to \phi$
(C1)	$C_G\phi \to E_G C_G\phi$
(C2)	$C_G(\phi \to E_G\phi) \to (\phi \to C_G\phi)$
(N_C)	from ϕ infer $C_G\phi$.

Theorem: sound and complete.

Resolution: some open issues

- Other assumptions about what other agents know about the resolution event
  - E.g., local updates
- Syntax vs. semantics and full communication
- Belief
- Expressive power:
  - compare to languages with relativised common knowledge
- Computational complexity

Discussion

What Distributed Knowledge Actually Is

- Common interpretations of distributed knowledge:
  - ~~Knowledge the group could obtain if they had unlimited means of communication~~
  - ~~*"A group has distributed knowledge of a fact phi if the knowledge of phi is distributed among its members, so that by pooling their knowledge together the members of the group can deduce phi ..."*~~

A group has distributed knowledge of a fact phi if after "pooling their knowledge together" ***the members of the group know that phi was true before they did that***

Interesting (and pretty much unexplored!) connection

$D_G\varphi$: after resolving G has common knowledge that φ was true before that

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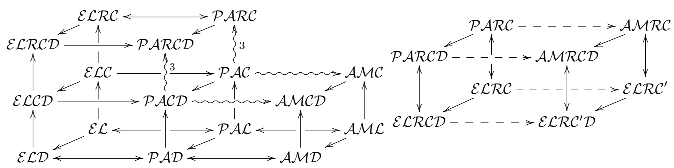

Issues with distributed knowledge and the literature

- What it is ("pooling")
- Distributed belief is not belief, under many common assumptions about what belief is
- Unsound axiomatisations
- Allowing the empty coalition (universal modality)
- "Not invariant under bisimulation"

Some related things I didn't talk about

- Deeper philosophical accounts of group belief - both reductionist and non-reductionist
  - Formalisations: see Gaudou et al., 2015
- Group knowledge in plausibility models (Baltag and Smets)
  - Belief merge (Baltag and Smets, MALLOW 2009)
  - Christoff et al., 2019 (under review) on *priority merge* (with resolution!)

The road ahead: group knowledge in social networks

- Parikh and Pacuit (2004): first steps towards analysing the information that can be shared by a group of agents restricted to a communication network
- Seligman et al. (TARK 2013): epistemics of network events
- On group formation in social networks
  - Smets and Velazquez-Quesada, LORI 2017: *social selection*
  - Xiong and Ågotnes, JoLLI 2019: on the logic of *balance* in social networks
  - Pedersen, Smets and Ågotnes, LORI 2019: on the formation of *echo chambers*

Summary

- Group belief is most often not actually belief
- There is a range of notions of group belief corresponding to different aggregation rules, the extremes being general and distributed belief
- We developed techniques for dealing with distributed knowledge in completeness proofs, used for PAL, CL, GAL, CAL, resolving, ..
- Epistemic coalition logic: general reasoning about group knowledge and group ability
  - Group ability and constructive knowledge: separating who knows how
- Group and coalition announcement logics: ability through announcement
- Resolving distributed knowledge
  - Captures exactly the relationship between distributed and common knowledge
    - Between ***group knowledge, dynamics and ability***