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Dynamic Surveillance by Multiple Agents with Fuel Constraints

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SUMMARY The surveillance problem is to find optimal trajectories of agents that patrol a given area as evenly as possible. In this paper, we consider multiple agents with fuel constraints. The surveillance area is given by a weighted directed graph, where the weight assigned to each arc corresponds to the fuel consumption/supply. For each node, the penalty to evaluate the unattended time is introduced. Penalties, agents, and fuels are modeled by a mixed logical dynamical system model. Then, the surveillance problem is reduced to a mixed integer linear programming (MILP) problem. Based on the policy of model predictive control, the MILP problem is solved at each discrete time. In this paper, the feasibility condition for the MILP problem is derived. Finally, the proposed method is demonstrated by a numerical example.

key words: *fuel constraints, mixed integer programming, model predictive control, surveillance*

1. Introduction

Control technologies for realizing a smart city have attracted much attention [5]. In a smart city, it is important to apply these to many services such as transportation, energy distribution, healthcare, environmental monitoring, business, commerce, emergency response, and social activities. In this paper, we focus on persistent surveillance (monitoring, patrol), which is closely related to some of the above services. The surveillance problem is to find optimal trajectories of agents that patrol a given area as evenly as possible. This problem has been studied from several viewpoints (see, e.g., [1], [4], [6], [7], [9], [11], [12], [14]).

In [1], [9], a surveillance area is given by a graph. It is appropriate to model a complicated area as a graph by using discrete abstraction techniques [15]. In [1], the automaton-based method has been proposed. In [9], the optimization-based method has been proposed. In these methods, the penalty to evaluate the unattended time is assigned to each node. In [9], time evolutions of agents and penalties are modeled by a mixed logical dynamical (MLD) system model [2]. The surveillance problem is reduced to a mixed integer linear programming (MILP) problem. Based on the policy of model predictive control (MPC) [3], [13], the MILP problem is solved at each discrete time. The MPC-based methods enable us persistent surveillance, and can adapt to the change in environment (e.g., the surveillance area and the number of agents). In [11], [12], some methods to solve the surveillance problem faster have been proposed.

In this paper, we consider multiple agents with fuel constraints. In the case where a surveillance area is large, it is important to impose fuel constraints for each agent. Fuel constraints have been considered in the method in [1]. In this existing method, the fuel of each agent is decreased in a constant rate. In the proposed method, we use a weighted directed graph, where the weight assigned to each arc corresponds to the fuel consumption/supply. Hence, the problem setting is more flexible. In the case of using a weighted directed graph, we cannot use our previously method [9] in which the binary decision variable is not assigned to each arc. We propose a new method to assign the binary decision variable to each arc. Using this method, the surveillance problem is reduced to an MILP problem.

Furthermore, in MPC, it is important to guarantee the feasibility of the finite-time optimal control problem solved at each discrete time. Since in the method in [9], there are no fuel constraints, the feasibility of the MILP problem is always guaranteed. However, when fuel constraints are imposed, the feasibility of the MILP problem is not guaranteed generally. In this paper, we propose two conditions of the prediction horizon and the terminal constraint for guaranteeing the feasibility of the MILP problem.

Thus, by the proposed method, we can overcome several technical issues on fuel constraints in persistent surveillance by multiple agents.

This paper is organized as follows. First, a surveillance area given by a weighted directed graph is explained, and the penalty for each node and the fuel of each agent are defined. Then, the surveillance problem is formulated. Next, the reduction of the surveillance problem to an MILP problem is explained. Third, the feasibility conditions are derived. Finally, we present a numerical example to show the effectiveness of the proposed method.

Notation: Let $\mathcal{R}$ denote the set of real numbers. Let $\{0, 1\}^n$ denote the set of n-dimensional vectors, which consists of elements 0 and 1. Let $1_{m\times n}$ ($0_{m\times n}$) denote the $m \times n$ matrix whose elements are all one (zero). Let I_n denote the $n \times n$ identity matrix. For simplicity, we sometimes use the symbol 0 instead of $0_{m\times n}$, and the symbol I instead of I_n. For the matrix M, let $M^\top$ denote the transpose matrix of M. For the two matrices X and Y, let $X \otimes Y$ denote the Kronecker product of X and Y.

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2. Problem Formulation

In this section, we formulate the optimal surveillance problem with fuel constraints.

Let $k \in \{0, 1, 2, \dots\}$ denote the discrete time. A surveillance area is given by a strongly connected directed graph with weighted arcs $\mathcal{G} = (\mathcal{V}, \mathcal{E}, f)$, where $\mathcal{V} = \{1, 2, \dots, n\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of arcs, and $f : \mathcal{E} \to \mathcal{R} \setminus \{0\}$ is the function representing the fuel consumption/supply. Since $\mathcal{G}$ is strongly connected, there exists a path in each direction between each pair of nodes. For move of agents, we assume that when some agent is located at some node at time k, it can be located at one of adjacency nodes at time $k+1$. For fuel consumption/supply, we assume that when some agent moves from the node a to b (note that $(a, b) \in \mathcal{E}$), its fuel changes depending on $f((a, b))$. We also assume that $f((a, b)) < 0$, $a \neq b$. If the node i has the self-loop with $f((a, a)) > 0$, it is called a supply node. The number of agents is given by m. Let $q_j(k) \in \mathcal{R}$ denote the fuel of the agent $j \in \{1, 2, \dots, m\}$.

We present an example.

Example 1: Consider the case of a single agent. The surveillance area is given by the graph in Fig. 1, where the node 1 is the supply node. Suppose that the initial location and the initial fuel are given by the node 4 and $q_1(0) = 30$, respectively. Then, the candidates of the locations at the next time are constrained to the set $\{2, 3, 4, 5, 6, 7\}$. If the agent moves from the node 4 to 2, the fuel is updated as $q_1(1) = 30 - 1 = 29$. Next, if the agent moves from the node 2 to 1, the fuel is updated as $q_1(2) = 29 - 1 = 28$. Finally, if the agent stays at the node 1 (i.e., the self-loop $(1, 1)$ is chosen), the fuel is updated as $q_1(3) = 28 + 8 = 36$.

Next, we introduce the notion of the penalty for each node. For each node, the penalty $p_i(k) \in \mathcal{R}$, $i \in \{1, 2, \dots, n\}$ is defined as follows:

$$p_i(k+1) = \begin{cases} 0 & \text{if some agent is located} \\ & \text{on the node } i \text{ at time } k, \\ p_i(k) + 1 & \text{otherwise.} \end{cases} \tag{1}$$

When a node is not monitored by any agent, the penalty for this node is increased. Hence, the penalty for each node can be used for evaluating the performance of surveillance. Then, the optimal surveillance problem is formulated as follows.

Problem 1: For the weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, f)$ and the update rule (1) of the penalty, suppose that the current location of agents, the current penalty $p_i(t)$ (t is the current time), and the prediction horizon $N \geq 1$ are given. Then, find trajectories of m agents minimizing the following cost function:

$$J = \sum_{k=t}^{t+N} \sum_{i=1}^{n} Q_i p_i(k) \tag{2}$$

subject to the following fuel constraint for each agent $j \in$

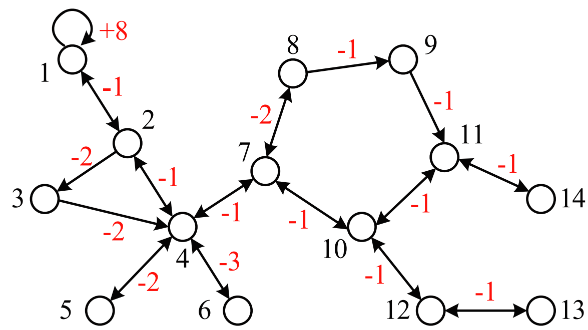

Fig. 1 Example of weighted directed graphs. The number assigned to each arc represents the fuel consumption/supply, where $f((a, b)) = f((b, a))$ holds for the bidirectional arc. Self-loops except for the arc $(1, 1)$ are omitted ($f((a, a)) = -1$, $a \neq 1$).

$\{1, 2 \dots, m\}$:

$$q_j(k) \geq c, \quad k \in \{t, t+1 \dots, t+N-1\}, \tag{3}$$

$$q_j(t+N) \geq c_f, \tag{4}$$

where $Q_i \geq 0$ is a given weight, and $c, c_f \geq 0$ are a given scalar.

The fuel constraint is closely related to the feasibility of this problem. In Sect. 4, we further discuss it.

3. Reduction of Problem 1 to an MILP Problem

In this section, we consider reducing Problem 1 to an MILP problem.

3.1 Preliminaries

The set of arcs $\mathcal{E}$ is denoted by $\mathcal{E} = \{1, 2, \dots, |\mathcal{E}|\}$, where each element corresponds to the pair of nodes. Let $\mathcal{I}_{\text{in}}(i)$ and $\mathcal{I}_{\text{out}}(i)$ denote the index sets of input-arcs and output-arcs at the node i, respectively.

We define binary variables as follows:

- $\delta_{i,j}(k)$: $\delta_{i,j}(k) = 1$ if the agent j is located on the node i at time k. Otherwise $\delta_{i,j}(k) = 0$.
- $\delta_i(k)$: $\delta_i(k) = 1$ if at least one agent is located on the node i at time k. Otherwise $\delta_i(k) = 0$.
- $\xi_{l,j}(k)$: $\xi_{l,j}(k) = 1$ if the agent j is located at the node a at time $k-1$, and is located at the node b at time k, where $l \in \mathcal{I}_{\text{out}}(a)$ and $l \in \mathcal{I}_{\text{in}}(b)$. Otherwise $\xi_{l,j}(k) = 0$.

From the definition

$$\xi_{l,j}(k) = \delta_{a,j}(k-1)\delta_{b,j}(k) \tag{5}$$

holds, where $l \in \mathcal{I}_{\text{out}}(a)$ and $l \in \mathcal{I}_{\text{in}}(b)$. In addition, we impose the following equality constraint:

$$\sum_{l=1}^{|\mathcal{E}|} \xi_{l,j}(k) = 1, \quad j \in \{1, 2, \dots, m\}.$$

If this constraint is satisfied, then $\sum_{i=1}^{n} \delta_{i,j}(k) = 1$ is also satisfied. The relation between $\delta_{i,j}(k)$ and $\delta_i(k)$ is given by the following linear inequalities [9]:

$$\delta_{i,1}(k) \le \delta_i(k) \le \sum_{r=1}^{m} \delta_{i,r}(k),$$
$$\delta_{i,2}(k) \le \delta_i(k) \le \sum_{r=1}^{m} \delta_{i,r}(k),$$
$$\vdots$$
$$\delta_{i,m}(k) \le \delta_i(k) \le \sum_{r=1}^{m} \delta_{i,r}(k),$$

which can be rewritten as

$$\begin{bmatrix} \delta_{1,j}(k) \\ \delta_{2,j}(k) \\ \vdots \\ \delta_{n,j}(k) \end{bmatrix} \le \begin{bmatrix} \delta_1(k) \\ \delta_2(k) \\ \vdots \\ \delta_n(k) \end{bmatrix} \le \begin{bmatrix} \sum_{r=1}^{m} \delta_{1,r}(k) \\ \sum_{r=1}^{m} \delta_{2,r}(k) \\ \vdots \\ \sum_{r=1}^{m} \delta_{n,r}(k) \end{bmatrix}, \tag{6}$$

where $j \in \{1, 2, \ldots, m\}$. We also define binary variable vectors as follows:

$$\bar{\delta}(k) := \begin{bmatrix} \delta_1(k) & \delta_2(k) & \cdots & \delta_n(k) \end{bmatrix}^\top,$$
$$\xi_j(k) := \begin{bmatrix} \xi_{1,j}(k) & \xi_{2,j}(k) & \cdots & \xi_{|\mathcal{E}|,j}(k) \end{bmatrix}^\top.$$

3.2 Modeling of Penalties

Using $\delta_i(k)$, the penalty $p_i(k)$ is modeled by

$$p_i(k+1) = (1 - \delta_i(k))(p_i(k) + 1). \tag{7}$$

The lower bound of $p_i(k)$ is 0, and the upper bound of $p_i(k)$ is given by $\bar{p} < \infty$ ($\bar{p}$ can be determined based on a given graph). Then, $z_i(k) := \delta_i(k)p_i(k) - 1$ is equivalent to the following linear inequalities [2]:

$$\begin{cases} -1 \le z_i(k) \le \bar{p}\delta_i(k) - 1, \\ p_i(k) - \bar{p}(1 - \delta_i(k)) - 1 \le z_i(k) \le p_i(k) - 1. \end{cases} \tag{8}$$

Hence, (7) can be represented by

$$p_i(k+1) = p_i(k) - z_i(k) - \delta_i(k)$$

and (8).

3.3 Modeling of Agents

Using $\xi_{l,j}(k)$, the input-output relation at the node i can be represented by

$$\sum_{l \in \mathcal{I}_{\rm out}(i)} \xi_{l,j}(k+1) = \sum_{l \in \mathcal{I}_{\rm in}(i)} \xi_{l,j}(k), \quad i \in \{1, 2, \ldots, n\},$$

which can be rewritten as the following implicit system:

$$E\xi_j(k+1) = F\xi_j(k), \tag{9}$$

where $E, F \in \{0, 1\}^{n \times |\mathcal{E}|}$ is derived from the graph $\mathcal{G}$ (see also

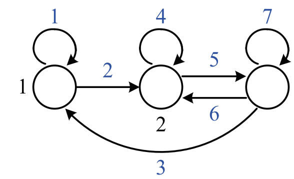

Fig. 2 Example of directed graphs. The number assigned to each arc represents the element in the arc set $\{1, 2, \ldots, |\mathcal{E}|\}$.

[10]). In the reduction to the MILP problem, the implicit system (9) can be rewritten as

$$\xi_j(k+1) = u_j(k),$$
$$0 \le Eu_j(k) - F\xi_j(k) \le 0.$$

Furthermore, from (5), $\delta_{i,j}(k) = \sum_{l \in \mathcal{I}_{\rm out}(i)} \xi_{l,j}(k+1)$ holds. Then, (6) can be rewritten as

$$Eu_j(k) \le \bar{\delta}(k) \le \sum_{r=1}^{m} Eu_r(k).$$

Hence, $\delta_{i,j}(k)$ may not be used.

We present a simple example.

Example 2: For the case of a single agent, consider the directed graph in Fig. 2, where $\mathcal{I}_{\rm out}(1) = \{1, 2\}$, $\mathcal{I}_{\rm out}(2) = \{4, 5\}$, $\mathcal{I}_{\rm out}(3) = \{3, 6, 7\}$, $\mathcal{I}_{\rm in}(1) = \{1, 3\}$, $\mathcal{I}_{\rm in}(2) = \{2, 4, 6\}$, and $\mathcal{I}_{\rm in}(3) = \{5, 7\}$. Focusing on the node 1, $\xi_{1,1}(k+1) + \xi_{2,1}(k+1) = \xi_{1,1}(k) + \xi_{3,1}(k)$ holds. Then, E, F in (9) can be derived as

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix},$$
$$F = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

In addition, from (5), $\xi_{1,1}(k+1) + \xi_{2,1}(k+1) = \delta_{1,1}(k)(\delta_{1,1}(k+1) + \delta_{2,1}(k+1))$ holds. Noting that if $\delta_{1,1}(k) = 1$, then either $\delta_{1,1}(k+1)$ or $\delta_{2,1}(k+1)$ holds, we see that $\delta_{1,1}(k) = \xi_{1,1}(k+1) + \xi_{2,1}(k+1)$ holds.

3.4 Modeling of Fuels

The fuel consumption/supply for each arc is defined by the function f given in advance. Let w_l denote the fuel consumption/supply for the arc $l \in \{1, 2, \ldots, |\mathcal{E}|\}$. Using $u_j(k)(= \xi_j(k+1))$, the time evolution of the fuel for the agent j can be modeled by

$$q_j(k+1) = q_j(k) + Wu_j(k),$$

where $W := [w_1\ w_2\ \cdots\ w_{|\mathcal{E}|}]$.

3.5 Reduction to an MILP Problem

From the obtained models of penalties, agents, and fuels, we can obtain the following MLD system model [2]:

$$\begin{cases} x(k+1) = Ax(k) + Bv(k), \\ Cx(k) + Dv(k) \le G, \end{cases} \tag{10}$$

where

$$\begin{aligned} x(k) &= [\, p_1(k)\ p_2(k)\ \cdots\ p_n(k)\ q_1(k)\ q_2(k)\ \cdots\ q_m(k) \\ &\qquad \xi_1^\top(k)\ \xi_2^\top(k)\ \cdots\ \xi_m^\top(k)\,]^\top \\ &\in \mathcal{R}^{n+m} \times \{0,1\}^{|\mathcal{E}|m}, \\ v(k) &= [z_1(k)\ z_2(k)\ \cdots\ z_n(k) \\ &\qquad \bar{\delta}^\top(k)\ u_1^\top(k+1)\ u_2^\top(k+1)\ \cdots\ u_m^\top(k+1)]^\top \\ &\in \mathcal{R}^n \times \{0,1\}^{n+|\mathcal{E}|m}. \end{aligned}$$

The matrices A, B, C, D, and G are derived as follows:

$$A = \begin{bmatrix} I_n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I_m & 0 \end{bmatrix},\quad B = \begin{bmatrix} -I_n & -I_n & 0 \\ 0 & 0 & I_{|\mathcal{E}|m} \\ 0 & 0 & I_m \otimes W \end{bmatrix},$$

$$C = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline I_n & 0 & 0 \\ -I_n & 0 & 0 \\ \hline 0 & 0 & I_n \otimes F \\ 0 & 0 & -I_n \otimes F \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & -I_m & 0 \end{array}\right],$$

$$D = \left[\begin{array}{ccc} -I_n & 0 & 0 \\ I_n & -\bar{p}I_n & 0 \\ \hline -I_n & \bar{p}I_n & 0 \\ I_n & 0 & 0 \\ \hline 0 & 0 & -I_n \otimes E \\ 0 & 0 & I_n \otimes E \\ \hline 0 & -I_{mn} & I_m \otimes E \\ 0 & I_{mn} & -1_{m\times m} \otimes E \\ \hline 0 & 0 & 0 \end{array}\right],$$

$$G = \left[\begin{array}{c} 1_{n\times 1} \\ -1_{n\times 1} \\ \hline (\bar{p}+1)1_{n\times 1} \\ -1_{n\times 1} \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline -c1_{m\times 1} \end{array}\right].$$

We remark that the terminal fuel constraint (4) is not included in the linear inequality in (10). From the state equation in (10), we can obtain

$$\bar{x} = \bar{A}x(t) + \bar{B}\bar{v}, \tag{11}$$

where

$$\begin{aligned} \bar{x} &= \begin{bmatrix} x^\top(t) & x^\top(t+1) & \cdots & x^\top(t+N) \end{bmatrix}^\top, \\ \bar{v} &= \begin{bmatrix} v^\top(t) & v^\top(t+1) & \cdots & v^\top(t+N-1) \end{bmatrix}^\top, \end{aligned}$$

$$\bar{A} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix},\quad \bar{B} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ AB & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ A^{N-1}B & \cdots & AB & B \end{bmatrix}.$$

From the linear inequality in (10) and the terminal fuel constraint (4), we can obtain

$$\bar{C}\bar{x} + \bar{D}\bar{v} \le \bar{G}, \tag{12}$$

where

$$\begin{aligned} \bar{C} &= \text{block-diag}(I_N \otimes C, [0_{m\times n}\ I_m\ 0_{m\times|\mathcal{E}|m}]), \\ \bar{D} &= \begin{bmatrix} I_N \otimes D \\ 0_{m\times(2n+|\mathcal{E}|m)N} \end{bmatrix},\quad \bar{G} = \begin{bmatrix} 1_{N\times 1} \otimes G \\ -c_f 1_{m\times 1} \end{bmatrix}. \end{aligned}$$

Thus, using (11) and (12), we can obtain the following theorem.

Theorem 1: Problem 1 is equivalently rewritten as the following MILP problem:

Problem 2:

$$\begin{aligned} \text{given} \quad & x(t) \\ \text{find} \quad & \bar{v} \in (\mathcal{R}^n \times \{0,1\}^{n+|\mathcal{E}|m})^N \\ \text{minimize} \quad & J = \bar{Q}\bar{A}x(t) + \bar{Q}\bar{B}\bar{v} \\ \text{subject to} \quad & (\bar{C}\bar{B} + \bar{D})\bar{v} \le \bar{G} - \bar{C}\bar{A}x(t) \end{aligned}$$

where $\bar{Q} = 1_{1\times(N+1)} \otimes [Q_1\ Q_2\ \cdots\ Q_n\ 0_{1\times m(1+|\mathcal{E}|)}]$.

The MILP problem obtained can be solved by using a suitable free/commercial solver.

4. Model Predictive Control and Guarantee of Feasibility

First, we present the procedure of MPC using the MILP problem derived in the previous section as follows.

MPC for Surveillance by Multiple Agents:

Step 1: Set $t = 0$, and give $p_i(0)$ (the initial penalty for each node) and the initial location for each agent.

Step 2: Solve Problem 2.

Step 3: Move agents based on the computation result.

Step 4: Update $t := t + 1$, and return to Step 2.

To realize persistent surveillance by multiple agents, we must guarantee the feasibility of Problem 2. In the case where there is no fuel constraint, Problem 2 is always solvable. In the case where there is fuel constraint, there is a possibility that Problem 2 is not solvable.

In this section, we clarify the conditions of N and c_f in (2) and (4) for guaranteeing the feasibility of Problem 2. In the conventional MPC method, setting the terminal constraint is important for guaranteeing the feasibility of the finite-time optimal control problem (see, e.g., [2], [3], [13]). Based on the existing results, we focus on the terminal constraint (4) and the prediction horizon N.

As preliminaries, we define the notation. For the graph $\mathcal{G}$, let $\mathcal{S} \subset \mathcal{V}$ denote the set of supply nodes. Next, consider the graph $\tilde{\mathcal{G}} = (\mathcal{V}, \mathcal{E}, \tilde{f})$ obtained from the graph $\mathcal{G}$, where $\tilde{f}(a,a) = 0$ and $\tilde{f}(a,b) = -f(a,b)$, $a \neq b$. That is, in the graph $\tilde{\mathcal{G}}$, the fuel supply is ignored, and the sign of the fuel consumption is changed. For the graph $\tilde{\mathcal{G}}$, let $P(v,s)$ denote the path from the node $v \in \mathcal{V}$ to the node $s \in \mathcal{S}$. Let $F(P(v,s))$ and $L(P(v,s))$ denote the total fuel consumption and the length of the path $P(v,s)$, respectively. Let $P^*(v,s)$ denote the path that $F(P(v,s))$ is minimum. Let $s^*(v) \in \mathcal{S}$ denote the supply point s such that $F(P^*(v,s))$ is minimum. We remark that s^* is different for each v. Finally, let $v^* \in \mathcal{V}$ denote the node v such that $F(P^*(v, s^*(v)))$ is maximum. We remark that s^* and v^* are not uniquely determined, but $F(P^*(v,s^*))$, $F(P^*(v^*,s^*))$, $L(P^*(v,s^*))$, and $L(P^*(v^*,s^*))$ are uniquely determined.

We present an example.

Example 3: Consider the surveillance area given by the graph in Fig. 1 again, where the node 1 is the supply node (i.e., $\mathcal{S} = \{1\}$). For example, the paths $P^*(13,1)$ and $P^*(9,1)$ are given by $(13,12,10,7,4,2,1)$ and $(9,11,10,7,4,2,1)$, respectively. From $\mathcal{S} = \{1\}$, we set $s^* = 1$. Then, we can obtain $F(P^*(v^*,s^*)) = 6$ and $L(P^*(v^*,s^*)) = 6$, where v^* is given by either 9 or 13 or 14.

Then, we can obtain two theorems.

Theorem 2: Problem 2 is always feasible if the following conditions are satisfied:

(i) $N \geq L(P^*(v^*,s^*)) + 1$,
(ii) Under the condition (i), Problem 2 at initial time is feasible.

Proof : For a feasible solution for Problem 2 at initial time, assume that at time N, the locations of all agents are the supply points, and all agents choose the self-loop. From the condition (i), there exists such a solution necessarily. One of the feasible solutions for Problem 2 at time 1 is that all agents stay at the supply points at times N and $N+1$. By repeating this procedure, the feasibility is guaranteed. □

The condition (ii) implies that there exist at least one solution under the condition (i) and a given initial condition. It does not imply that fuel constraints are satisfied regardless of the fuel consumption and supply values.

In the next theorem, c_f in the fuel constraint (4) depends on the agent $j \in \{1,2,\ldots,m\}$ and its location, and c_f is denoted by c_f^j. Let $v_j(k) \in \mathcal{V}$ denote the location of the agent j at time k.

Theorem 3: Problem 2 is always feasible if the following conditions are satisfied:

(i) $c_f^j \geq F(P^*(v_j(t+N), s^*)) + c$,
(ii) Under the condition (i), Problem 2 at initial time is feasible.

Proof : For any $N \geq 1$, it is guaranteed by the condition (i) that each agent has the remaining fuel that it can move from the node $v_j(t+N)$ to the supply point. Since the remaining fuel depends on the location of each agent, Problem 2 is always feasible under the condition (ii). □

In Theorem 3, the condition about the prediction horizon is not necessary by relaxing the fuel constraint (4).

By Theorem 2 and Theorem 3, we can easily check the feasibility. In addition, from the condition (i) in Theorem 3, the fuel constraint (4) becomes a constraint changing by the locations of agents. Problem 2 with this constraint can also be reduced to an MILP problem.

We present an example.

Example 4: We continue to Example 3. First, consider Theorem 2. From this theorem, the prediction horizon N must satisfy $N \geq 7$. Next, consider Theorem 3. For simplicity of discussion, c in (3) is given by $c = 0$. From $s^* = 1$, $F(P^*(v,s^*))$ can be obtained by

$$\begin{aligned}
F(P^*(1,s^*)) &= 0,\\
F(P^*(2,s^*)) &= 1,\\
F(P^*(4,s^*)) &= 2,\\
F(P^*(7,s^*)) &= 3,\\
F(P^*(v,s^*)) &= 4, \quad v \in \{3,5,10\},\\
F(P^*(v,s^*)) &= 5, \quad v \in \{6,8,11,12\},\\
F(P^*(v,s^*)) &= 6, \quad v \in \{9,13,14\}.
\end{aligned}$$

We suppose that the initial location and the fuel of the agent are given by the node 4 and $q_1(0) = 2$, respectively. Then, Problem 2 with $N = 1$ is always feasible by using the path $(4,2,1)$. Finally, consider the condition (i) in Theorem 3. Suppose that $m = 3$, $v_1(t+N) = 7$, $v_2(t+N) = 5$, and $v_1(t+N) = 12$. Then, the condition (i) implies that $c_f^1 \geq 3$, $c_f^2 \geq 4$, and $c_f^3 \geq 5$. Using $u_j(t+N-1)$, the fuel constraint (4) satisfying the condition (i) in Theorem 3 is given by e.g.,

$$\begin{aligned}
q_j(t+N) \geq{} & [\,0\ 1\ 4\ 2\ 4\ 5\ 3\ 5\ 6\ 4\ 5\ 5\ 6\ 6\,]\\
& \times E u_j(t+N-1),
\end{aligned} \tag{13}$$

where the row vector can be derived from the above $F(P^*(v,s^*))$.

We remark that in order to get the high control performance, a longer N must be set. In addition, we must consider the trade-off between the control performance and the computation time.

5. Numerical Example

To demonstrate the proposed method, we present a numerical example. The surveillance area and the fuel consumption/supply are given by the weighted directed graph in Fig. 1, where the number of nodes is 14 ($n = 14$). The number of agents is three ($m = 3$). The initial penalty $p_i(0)$, the initial location, the prediction horizon N, and the weight Q_i are set as $p_i(0) = 0$, the node 1 (all agents), $N = 8$, and

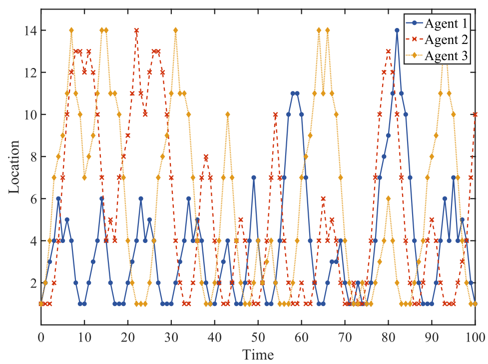

Fig. 3 Time response of the location in Case 1.

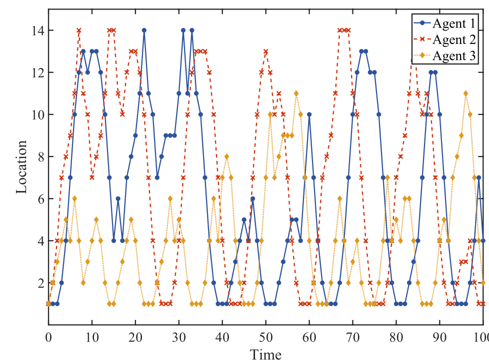

Fig. 5 Time response of the location in Case 2.

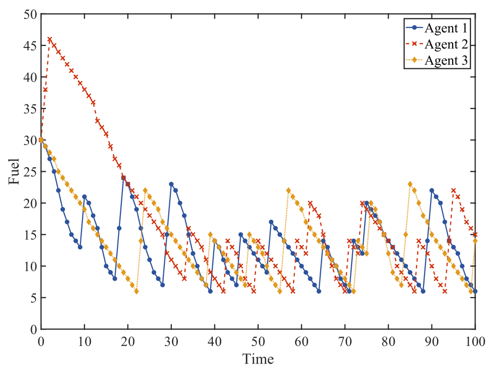

Fig. 4 Time response of the fuel in Case 1.

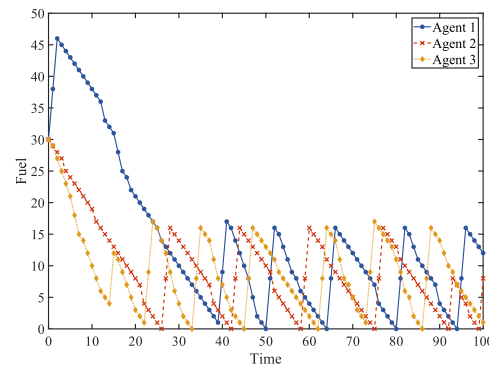

Fig. 6 Time response of the fuel in Case 2.

$Q_i = 1$, respectively. For the fuel constraints (3) and (4), we consider the following two cases:

Case 1: $c = c_f = 6$,

Case 2: $c = 0$ and $c_f = 6$.

To evaluate the control performance, we use the following performance index: $J_s^* := \sum_{k=0}^{100} J_s(k)$ and $J_s(k) := \sum_{i=1}^{14} p_i(k)$.

We present computation results. Figures 3 and 4 show time responses of the location and the fuel for each agent in Case 1. From Fig. 3, we see that the number of times that agents reach to the nodes 13 and 14 is small, because these nodes are far from the supply node (the node 1). From Fig. 4, we see that the fuel of each agent satisfies the constraint. We also see that the agent 2 charges the fuel at first.

Figures 5 and 6 show time responses of the location and the fuel for each agent in Case 2. From Fig. 5, we see that agents sometimes reach to the nodes 13 and 14, because the fuel constraint in Case 2 is relaxed.

Figure 7 shows the time response of $J_s(k)$. From this figure, we see that the performance in Case 2 is better than that in Case 1. In addition, J_s^* was derived as follows:

$$J_s^* = 8891 \text{ (Case 1)},\ 6432 \text{ (Case 2)}. \tag{14}$$

The mean computation time of Problem 2 was 1.84 sec (Case 1) and 1.42 sec (Case 2). The worst computation time of Problem 2 was 14.20 sec (Case 1) and 10.37 sec (Case 2).

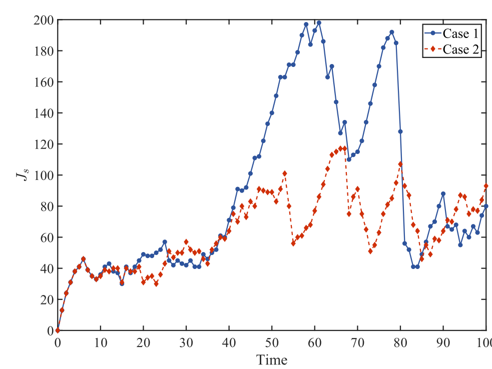

Fig. 7 Time response of J_s (the sum of penalties).

Here, we used the computer with CPU: Intel Core i7-6700K processor, Memory: 16GB, and used IBM ILOG CPLEX Optimizer 12.7.1 as an MILP solver. The difference in the computation time is small. We comment about the feasibility. The prediction horizon $N = 8$ satisfies the condition (i) in Theorem 2, i.e., $N \geq 7$. Since Problem 2 can be solved at initial time, the feasibility of Problem 2 is always guaranteed. If we set $N = 7$, Problem 2 can be always solved in both Case 1 and Case 2. If we set $N = 6$, then in both Case 1 and Case 2, Problem 2 at $t = 20$ cannot be solved.

Finally, consider utilizing the terminal fuel constraint (13) (in Example 4 and this numerical example, we consider the same graph (Fig. 1)). We set $c = 0$. Consider the cases of $N = 3, 4, \ldots, 8$. Other setting is the same as the

Table 1 Computation results in the case of utilizing (13).

N	J_s^*	Mean comp. time [sec]	Worst comp. time [sec]
3	11849	0.01	0.10
4	18360	0.03	0.19
5	20661	0.06	0.31
6	8421	0.19	1.86
7	5948	0.65	3.49
8	5769	1.82	8.09

above. Table 1 shows the computation results. From this table and (14), we see that for a shorter N, the performance is low (but, by (13), the feasibility for the case of the shorter N is guaranteed). We also see that in the case of $N = 8$, the performance is improved by using (13). The terminal fuel constraint (13) is more relaxed than that in the above Case 2, but the path to the supply node is restrictive. Then, there is possibility that the performance is decreased. We may adjust the terminal fuel constraint, depending on the required performance.

6. Conclusion

In this paper, we proposed a surveillance method by multiple agents with fuel constraints. The surveillance problem was reduced to an MILP problem. The feasibility conditions of the MILP problem were also derived. The proposed method is effective for surveillance of a large area.

In the proposed method, the upper bound of the fuel is not constrained. If the fuel capacity is equal to or larger than the fuel supply, then such a constraint can be easily imposed.

One of the future efforts is to apply the proposed method to practical applications. It is also significant to impose temporal logic constraints [8].

Acknowledgments

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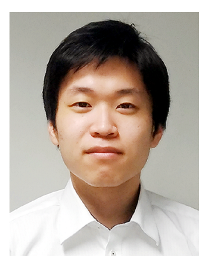

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