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Trade Policy, Lobbying and Heterogeneous Firms

Ahmed Waqar Qasim
# TABLE OF CONTENTS

1. **INTRODUCTION** ................................................................................................................... 1  
   1.1. The Recent Developments in International Trade Theory ............................................... 1  
   1.2. Heterogeneous Firms and Trade Policy ........................................................................... 3  
   1.3. Trade Policy under Lobbying ........................................................................................... 5  
   1.4. Heterogeneous Firms and Lobbying ................................................................................ 6  
   1.5. Trade Policy and Intermediate Inputs Trade .................................................................... 8  
   1.6. A Brief Road Map ............................................................................................................ 9  

2. **TRADE POLICY UNDER THE HETEROGENEOUS LOBBYING FIRMS** ..................... 12  
   2.3. The Model ....................................................................................................................... 15  
   2.3.1. The Trade Policy Choice ........................................................................................ 23  
   2.3.2. Benevolent Equilibrium .......................................................................................... 25  
   2.3.3. Political Equilibrium ............................................................................................... 26  
   2.4. A Quantitative Illustration .............................................................................................. 29  
   2.5. Conclusion ...................................................................................................................... 30  

   **Appendices** ....................................................................................................................... 30  

3. **THE HETEROGENEOUS FIRMS AND LOBBY PARTICIPATION DECISION** ............ 35  
   3.3. The Model ...................................................................................................................... 36  
   3.4. Second Stage .................................................................................................................. 40  
   3.5. First Stage ....................................................................................................................... 41  
   3.5.1. Selection into Lobbying .......................................................................................... 42  
   3.5.2. The Equilibrium Mass of Lobbying Firms ............................................................. 43  
   3.6. Conclusion ...................................................................................................................... 45  

   **Appendix** .......................................................................................................................... 45  

4. **TRADE POLICY WITH INTERMEDIATE INPUTS TRADE** ........................................... 48  
   4.3. The Model ...................................................................................................................... 51  
   4.3.1. Households .............................................................................................................. 51  
   4.3.2. Final Goods Producing Firms ................................................................................. 52  
   4.3.3. Intermediate Input Supplier .................................................................................... 53  
   4.3.4. Exit, Import/Export Decision of a Firm .................................................................. 54  
   4.3.5. Total Revenue and Welfare .................................................................................... 56
4.4. Stackelberg Equilibrium Tariff ................................................................. 57
4.5. Cooperative/Efficient Equilibrium Tariff ................................................ 58
4.6. Non-cooperative/Nash Equilibrium Tariff .............................................. 59
4.7. Political Equilibrium Tariff .................................................................... 60
4.8. Quantitative Illustration ........................................................................ 62
4.9. Conclusion ............................................................................................. 64

Appendices ...................................................................................................... 65

5. HETEROGENEOUS FIRMS AND TECHNOLOGY ADOPTION: THE ROLE OF
POLITICAL INSTITUTIONS AND MARKET SIZE ........................................ 73
5.3. Empirical Motivation ............................................................................. 75
5.4. The Model .............................................................................................. 77
   5.2.1. Household Sector ................................................................................ 78
   5.2.2. Production Sector ............................................................................... 79
   5.2.3. Firm’s Behavior .................................................................................. 79
5.3. Closed Economy Equilibrium .................................................................. 80
   5.3.1. Entry and Exit .................................................................................... 80
   5.3.2. Entrepreneur’s Problem ..................................................................... 82
5.4. Open Economy Equilibrium with Costly Trade ....................................... 83
   5.4.1. Trade Policy Making .......................................................................... 84
   5.4.2. Firm’s Behavior .................................................................................. 84
   5.4.3. Entry and Exit .................................................................................... 85
   5.4.4. Entrepreneur’s Problem ..................................................................... 87
   5.4.5. Elite’s Problem .................................................................................. 88
5.5. Technology Adoption: Decision to Adopt or Resist via Lobby for Trade Restrictions. 90
   5.5.1. Entry and Exit .................................................................................... 90
   5.5.2. Selection of Technological Up-Gradation ......................................... 91
   5.5.3. The Possibility of Block Technology Adoption by Lobbying .......... 92
5.6. Conclusion ............................................................................................. 96

Appendix ......................................................................................................... 96

6. DISCUSSION AND CONCLUSION ............................................................. 99
REFERENCES ............................................................................................... 103
1. INTRODUCTION

The recent developments in trade literature positioned firms as the central element for the analysis and emphasize that the firm-level decisions have macro-level implications. These developments render conceivable because of the availability of micro-level trade data in the late 1970s. The analysis based on these data sets raises questions on the relevance of traditional trade theories. Resultantly, new trade theories emerge to encompass these anomalies between neoclassical trade theory and observed trade patterns. These new trade theories bring new insights and challenges for trade policy. The developments of new trade theories comprise two stages. At the first stage, the focus was descended from the country-level to the industry-level and at the second stage, from the industry-level to the firm-level. These two stages also dubbed as “new trade theory” and “new-new trade theory” in the literature. The discussion below contains a brief overview of these developments and offers some insights that these theories propose on the policy front. This overview also enables us to outline the importance and need of the political economy of trade policy in the most recent trade theory, which is the aim of this research endeavor.

1.1. The Recent Developments in International Trade Theory

Traditional trade theories are based on the principle of comparative advantage. According to these theories, countries trade due to the comparative advantage of technology (Ricardo, 1817), or due to the differences in factor endowments (Heckscher, 1919 and Ohlin, 1933). Countries with greater technological differences or greater factor endowment differences have a greater volume of trade with each other and that leads to greater specialization. The benchmark two-country, two-good neoclassical trade models provide the basic descriptions of the welfare gains from specialization. These welfare gains are contingent upon mutual specialization, resultantly, the policy predictions emerge from these models induce the removal of trade barriers and promote free trade.

These neoclassical models are based on strong assumptions, such as; constant return to scale technology, perfect competition, two goods, and two countries. Furthermore, these theories adopt countries as the basic unit of analysis and trade between countries is assumed to be inter-industry. However, the empirical patterns that emerge in the early 1970s contradict particularly this prediction of the neoclassical trade models.¹ The trade patterns observed in most industrial countries are not inter-industry instead these patterns show the dominance of intra-industry trade. While trade between countries with the same level of technology and endowments comprises the

major share of international trade. Furthermore, the trade liberalization is leading towards the diversification instead of specialization. Besides, the difference in the estimated gains from trade liberalization by neoclassical trade theories and actual post liberalization higher growth of the world economy also highlights the need for a new framework on the theoretical front.

**From country-level to industry-level:** By making the industry as the unit of analysis instead of country, Krugman (1980) and Krugman & Helpman (1985) enable us to comprehend the then observed trade patterns. The model they develop features increasing returns to scale, imperfect competition, and product differentiation. Moreover, the preferences are based on the love of variety assumption, which provides the basic foundation to embed intra-industry trade. Consequently, countries with the same level of technology and endowments can trade in order to have a larger set of available varieties in the market. On the production side, the model assumes that all firms within an industry have the same technology but produce different varieties. The model also treats firms within an industry as homogeneous with respect to their decision of export participation and production.

This new trade theory offers an additional channel for welfare gains from trade liberalization, along with specialization. This channel identifies that trade liberalization leads to an increase in the available varieties of differentiated goods, which increases consumer welfare in the economy. However, this model also poses the possibility of a strategic trade policy, which is to provide a subsidy to some industries in order to exploit increasing return to scale through trade expansion (Ciuriak, et al., 2015). Nevertheless, if other country reacts to such policy and also adopt a strategic policy, then according to the model the predicted welfare gains from the trade are not very large.

The availability of firm-level data in the 1990s makes it possible to analyze trade data at the micro-level.² The empirical findings based on the firm-level data, as summarizes by Melitz & Redding (2012), challenge the industry-level model of homogeneous firms on three fronts. First, firms within an industry are significantly heterogeneous in their productivity level, export participation, revenue, and factor intensity. Within an industry, a few firms export and exporting firms behave differently than non-exporting firms. They are more productive, use capital more intensively and pay higher wages. Second, the composition of the industry changes after trade liberalization. Some firms exit the market, while new firms enter. The exiting firms are found to be less productive and pay lower wages. Third, the gains in the market don’t accrue to all firms equally, but rather to the more productive exporting firms. The firm-level data also reveals the role of productivity in the export decision and the price-setting behavior of the firms.

new entrants are more productive. Third, the firm-level decisions like; technology adoption, number of products supplies, markup, depends upon the trading environments and have macro-level implications.

**From Industry-level to Firm-level:** Driven by this empirical evidence, Melitz (2003) and Bernard, et al. (2003) develop frameworks that incorporate firm-level heterogeneity. The model develops by Melitz has structure closely related to Krugman (1980) except firms are heterogeneous with respect to their productivity level. The model envisages low export participation is due to the fixed and sunk cost associated with it. This model becomes the standard framework in trade literature and dubbed as a new-new trade theory. It offers another source of trade liberalization gain, which emerges from the increase in average productivity due to change in the composition of firms with the industry after liberalization. Therefore, the revolution from the neoclassical trade theory to new-new trade theory brings new channels of welfare gains into consideration and makes models more inclusive. Chapter 2 contains a detail characterization of the Melitz model of heterogeneous firms.

### 1.2. Heterogeneous Firms and Trade Policy

The theory of positive tariff has been presented by Johnson (1953), which shows that a country gains by imposing a tariff even in the event of retaliation from the other countries. However, Staiger & Tabellini (1987) argue that the optimal trade policy instrument depends upon the time dynamics. They demonstrate that if trade policy is time consistent, then the optimal instrument of trade policy is production subsidy. While the traditional positive tariff theory is only optimal in the case of time-inconsistent and unanticipated trade policy.

In the case of the Krugman model, the existence of a positive tariff rate is shown by Gros (1987) for a small country, where the size of the optimal tariff model depends upon the degree of differentiation and the country size. Likewise, Venables (1987) also concludes that the domestic country can increase her welfare by imposing a positive tariff or providing subsidy to domestic exporting firms within homogeneous firms model. Since, the imposition tariff by one country can provoke retaliation by other country, therefore, Ossa (2011) analyze the role of noncooperative tariff in Krugman model. He also provides the estimations by using the data of six main players of GATT/WTO negotiations. The case of cooperative tariff settings is again taken by Ossa (2014) along with noncooperative tariff settings. The results of the analysis show that in case of unilateral tariff setting game, a country can gain at the cost of others. While in the case of the trade war,
countries adopt the Nash tariff and no longer benefits at the cost of each other. However, a cooperative tariff that is imposed after the negotiation is welfare improving for both countries within homogeneous firms model settings.

In the case of heterogeneous firms model, Demidova & Rodriguez-Clare (2009) characterizes the optimal trade policy in the presence of mark-up and consumer surplus distortion. They conclude that either one of these instruments is optimal to overcome these distortions; a subsidy on domestic varieties consumption, an import tariff, or an export tax. The formulation of these instruments incorporates the degree of heterogeneity and the elasticity of substitution between varieties in heterogeneous firms model. They show that the productivity gain argument that suggests that an export subsidy to induce resource reallocation towards more productive firms may be dominated by negative effects of positive subsidy. Felbermayr et al. (2013) extended the results of the previous study to a general equilibrium model. Their formulation of optimal tariff internalizes terms-of-trade externality along with both mark-up and entry distortions. Their results suggest that the optimal tariff is large in case of large productive dispersion among heterogeneous firms. While the strategic tariff imposed by a small country is lower than a large country.

Mansfield et al. (2000) construct a strategic tariff setting between two countries on the basis of political institutions of the economy. They show that in the case of democracy, legislatures have the responsibility of ratification which leads to a lower tariff and more open trade among democratic pairs. Moreover, the tariff set by a country also get influenced by the constraints imposed by the World Trade Organization (WTO). As describe by Broda et al. (2008), the tariff set by a country without such a restriction is higher for the goods with lower export elasticities. Similarly, the market power also plays a crucial role in tariff setting without WTO restrictions. A large country always set a higher tariff on goods in which it has significant market power.

In contrast, Bilbiie et al. (2008) show that the competitive equilibrium in the case of dynamic stochastic general equilibrium heterogeneous firms model is efficient as long as the preferences are Dixit-Stiglitz type and labor supply is inelastic. Therefore, the possibility of any fiscal tool is ruled out provided these conditions are satisfied. Meanwhile, in the case of elastic labor supply, a leisure tax can restore the efficiency property of the competitive equilibrium. In addition, Cole et al. (2009) even present the case of a negative social optimal tariff rate in the presence of multinational firms. They promulgate the case of a negative tariff on the basis of productivity gain argument. Since trade makes the least productive firms exit the market and these productivity gains
are larger than the relative price distortion. Therefore, subsidizing imports is welfare improving worldwide. They have also shown the existence of a positive tariff in the case of the Nash tariff.

1.3. **Trade Policy under Lobbying**

In the context of endogenous trade policy formation, the political economy model “Protection for Sale” (PFS henceforth) developed by Grossman & Helpman (1994) becomes the standard framework. The PFS framework recognizes trade policy formation as a process of bargaining between the organized lobbying industries and the government. They portray lobbying as a common agency problem in which special interest groups from the organized industries of the country seek protection and offer political contributions to the incumbent policymaker. on the other hand, the policymaker maximizes his own welfare that depends upon the political contributions and social welfare. The outcome of this bargaining gives political equilibrium, where the protection received by an industry depends upon its political organization. As highlighted by Rausser et al. (2011), the PFS framework although provides some surprising results but also makes some sharp assumptions like; the policymaker assigns the same weight to all interest groups, the weight assigned to organized groups does not depends upon the size of the political contributions, the weight assigned to the active interest groups is more than the weight assigned to inactive interest groups, the special interest groups are given, industry overcome free-rider problem during lobbying.

Saha (2019) relaxes the assumption of the same weight assignment to all organized industries and modifies the PFS framework by assigning different weights to different industries and called it lobbying effectiveness. The modified framework is then tested for Indian panel data and sector-specific lobbying effectiveness is uncovered. While Mitra (1999) relaxes the assumption of the given information about the organized and unorganized industries and endogenize lobby formation or getting organized within an industry. Industries with attributes such as; capital intensive, fewer capital owners, inelastic labor demand, geographically concentrated, are more likely to be organized. Breton & Salanie (2003) show that some industries remain unorganized due to their inability to overcome free-rider problem. The relationship between lobbying expenditures and policy enactment has been explored by Kang (2015). By using the data of the energy sector during the 110th US Congress, the author shows that average returns are over 130 percent from lobbying expenditures. Goldberg & Maggi (1999) test the PFS framework empirically by using US protection data. They found the consistency among the framework predictions and observed
protection patterns. However, the weight assigned to special interest groups is much smaller than the weight assign to social welfare by the policymaker.

Since the protection prediction in the PFS framework depends upon either the industry is politically organized or not. Therefore, Alt et al. (1999) bring the discussion of the nature of industry into lobby formation debate. They empirical testify that firms with less mobile or specific assets are more likely to be politically organized. The surveyed firms from Norway and conclude that firms in an industry with fewer mobile assets are over twice as likely to be organized and lobby than firms in the mobile assets industry. Marceau & Smart (2003) also present a similar kind of analysis for the case of sunk investment where investors anticipate the government policies. They elaborate that firms with such investments are more inclined to lobby and lobby mitigates even reverses the capital levy problem.

Furthermore, another reason for politically activism of an industry is that the firms in a politically organized industry have competitive advantage (Schuler et al., 2002). They have also associate political activism with firms size, political setup and institutions, government contracts, and concentration within industry. Besides, Bonardi et al. (2005) characterize the political activism of firms as market-like process. Firms enter in the political market on basis of its attractiveness attributes like; defense of status quo, new issues creation to oppose a policy, benefits and costs etc. Yu (2000) extend the discussion of politically organized or unorganized with consumer interest. The firms in an industry can be organized, while the consumer interest in unorganized. Although unorganized consumer interest receives weight in the PFS model, higher political tariff compared to the optimal tariff reflects the imbalance between organized interest groups and unorganized interest groups. Feenstra & Bhagwati (1982) provides the rationale of tariff protection granted by the government on the basis of pressure created by the labor lobby. Since trade decrease the relative price, therefore, labor lobby seeks tariff protection. However, equilibrium with the tariff is welfare inferior in the case of the small economy and the government can restore the first best by compensating the labor through lump-sum transfers. Since the tariff itself is the source of revenue, therefore, the government will not be able to eliminate all lobby, but surely can change the number of lobbying activities and tariff that improve welfare.

1.4. **Heterogeneous Firms and Lobbying**

The case of firm-level lobbying instead of industry-level lobbying is empirically presented by Kim (2017). He argues that the political economy models of trade policy are based on inter-industry
differences of protections and policies. Whereas, the applied tariff rates variation within an industry is much higher than among industries. He describes this within industry tariff variation as a result of product differentiation and argues that product differentiation leads to firm-level lobbying. A higher level of product differentiation reduces the collective action problems faced by domestic exporting firms. The firm-level data construct in the study also confirms that highly differentiated products face lower tariff rates. Besides, Paltseva (2014) also shows that lobbying at the industry-level is less likely to happen when products are highly substitutable. In this regard, Deng & Kennedy (2010) provides a comprehensive analysis of individual firms as well as associations lobbying in case of public policymaking in China. Their survey results indicate that the lobbying strategy of firms differs significantly from associations and firms operate in a more competitive environment. The lobbying influence of industry is found unclear, while the large firms affect substantially on public policymaking in China.

The analysis of panel data conducted by Ker et al. (2011) indicates that within an industry only a few firms lobby and these lobbying firms are large in size. These facts emerge due to the existence of significant political market entry costs. Since the political market entrance contains a barrier in the form of lobbying cost, therefore, once a firm enters into the political market, she remains persistently active over a long time. Bombardini (2008) has also confirmed that in the presence of fixed cost of lobbying, it is efficient that only the largest firms in a sector participate in the lobby. She provides empirical evidence that an industry with large firms on average is more politically active. The analysis is conducted by introducing firm heterogeneity in the PFS framework along with endogenizing lobby formation. Richter et al. (2009) shows that lobbying expenditures dominates the firm’s political spending and a firm that spends high resources on lobbying faces lower effective tax next year in the case of the US. Their estimates show that increasing 1 percent of lobbying expenditures yield a lower effective tax rate ranges from 0.5 to 1.6 percentage points on average.

Harstad & Svensson (2011) brings the role of the development level of an economy into the lobbying discussion. They analyze the choice of a firm to follow a regulation or lobby the policymaker to relax it or bribe the enforcing officer to get around it. Their model shows that firms are more inclined to adopt a bribing strategy unless the level of development is sufficiently high. Only then they adopt lobbying. For developing countries, Weymouth (2012) also builds a model to determine the effectiveness of firm-level lobbying on policy outcomes. He argues that the
political institutions shape the incentives for the policymaker in response to heed lobbying firms. By utilizing the data of 20,000 firms from 42 developing economies, he testifies that firms have greater policy influence in democracies and that influence depends upon the size and market power of the firm. Karacaovali (2011) extend the debate of trade policy with productivity and show that more productive firms receive more protection in Colombia. Moreover, industries with higher productivity gains are less liberalized even in the presence of unilateral liberalization that affects all industries.

Gawande et al. (2006) argue that in the PFS framework, more attention is given to the interaction between domestic firms and the policymaker. The foreign firms are considered as non-participants of lobbying because their participation is harmful to the home country. By using the US data on foreign firms political activities, they find that foreign firms lobbying reduce tariffs and non-tariff barriers. Therefore, they assert that foreign firms lobbying participation in the home country are not harmful. Whereas, the presence of foreign firms lobbying favored by Aidt & Hwang (2014) on the basis of cross-national externalities. Their argument is that the policy made in the home country effects the welfare of firms and consumers in foreign countries. Therefore, by allowing foreign firms to lobby participation, these externalities can be internalized. However, the benefits of allowing foreign firms lobbying depend upon the alignment of interest of the unorganized group (e.g. consumers) and interest of the foreign firm. If both interests are aligned, then foreign lobby removes the pre-existing bias in the home country’s policymaking. Stoyanov (2009) traces a significant role of foreign firms lobbying on domestic trade policy in free trade agreements of Canada.

1.5. Trade Policy and Intermediate Inputs Trade
Trade of intermediate inputs accounted for a major share in international trade. The role of trade policy on the intermediate inputs trade is more significant than on the final goods trade. This holds due to the fact that the production process involves intermediate inputs and restricting or liberalizing intermediate inputs trade has direct as well as indirect effects. The concept of effective protection, which theoretically originated by Ruffin (1969), measures the domestic protection by considering the import tariff on intermediate inputs along with final goods. The effective tariff rate is crucial in conceptualizing the trade policy, because, the terms of trade advantage of tariff on final goods might be wiped out if effective tariff rate is zero. To make the comparison and what role intermediate inputs tariff can play, Dardis (1967) develops a model with intermediate inputs
and without intermediate inputs. By using the Garman livestock data of 1960, he measured the effective tariff rate is around 10 percent. He further shows that removing the final goods tariff hardly affect the relative cost, while removing intermediate inputs tariff lower relative cost substantially.

The outcome of tariff reduction on intermediate inputs and final goods on productivity gains disentangles by Amiti & Konings (2007). The final good tariff reduction enhances productivity by inducing tougher import competition. While reducing intermediate inputs tariffs raise productivity via; learning, variety, and quality effects. By using Indonesian data, they estimate that the productivity gains from intermediate inputs tariff reducing are twice as the productivity gains of final goods tariff reducing. Therefore, just focusing on competition effect underestimates the total effect on productivity gains of trade liberalization. Halpern et al. (2015) also conclude that about half of the productivity gain is due to the intermediate inputs trade in the case of Hungary. They performed a counterfactual analysis by using microdata and show that the productivity gain from a tariff cut is large in the presence of foreign intermediate inputs importers. The crucial role played by intermediate inputs in productivity gain is also highlighted by Goldberg et al. (2010) in case India. They attribute 31 percent of new products introduced in the market by the Indian firms to intermediate inputs trade liberalization.

In the case of monopolistic competitive models, monopoly power also provides another rationale for tariff imposition. However, the policymaker has two instruments to manipulate if intermediate inputs trade also involves. The first instrument is related to the term of trade between final goods and second involves term of trade between final and intermediate inputs Das (1983). Therefore, optimal levels of the tariff of final goods differ in the absence and presence of intermediate inputs in the model. In the case of missing intermediate inputs trade, the optimal tariff on final goods is larger and the same is true for intermediate inputs tariff.

1.6. A Brief Road Map

This study will elaborate on the trade policymaking in the case of heterogeneous firms model. Our starting point is the political economy of trade policy. In this regard, we first focus on the standard Grossman & Helpman (1994) PFS framework and introduce modification within this framework to make it appropriate for heterogeneous firms. Then we will discourse ways other than the political economy of trade policy formation such as; cooperative trade policy, non-cooperative
trade policy. In the end, we will present another model of political economy to show how the decisions made at the firm-level have implications of the macro-level.

The next chapter is built on the hypothesis that a lobbying firm can affect the trade policy selection by the policymaker. In this regard, we first explore the optimal policy in the absence of a lobbying firm and results show that the equilibrium policy selected by the policymaker includes a pair of uniform import tariffs and uniform export tax. However, once we allow the firm to participate in the lobby, then the policymaker adopts a discriminatory policy for the lobbying firm. Both import tariff and export tax are lower for lobbying firms at the political equilibrium. We utilize the Grossman and Helpman “protection for sale” framework into heterogeneous firms model to articulate lobbying by an individual firm.

The third chapter is an extension of the second chapter and endogenizes the lobby participation decision of a firm in the heterogeneous firms model. To endogenize the lobby participation decision at firm-level, an additional stage in the standard political economy of trade policy framework has been introduced. This additional stage entails information about the equilibrium mass of lobbying firms and the equilibrium contribution levels. By assuming fixed and sunk costs associated with lobby participation, we have shown that the firm with productivity above a threshold level can only afford to engage in the lobby.

In the second and third chapters, we assumed the foreign country in a two-country model does not impose any trade restriction in the form of import tariff or export tax. In the fourth chapter, we remove this assumption and allow the possibility of a reaction from the foreign country in response to the imposition of trade restriction from home country and we focused only on the import tariff. Moreover, we also consider the trade of intermediate inputs and that chapter characterizes the tariff policy for intermediate inputs as well. We developed a theoretical model to show how the tariff on final goods and intermediate inputs affect the welfare, productivity, and the entry of firms in a country. We formulate the tariff level selection choice available to the policymaker with respect to four policy experiments. These policy experiments include; unilateral tariff selection, cooperative tariff selection, non-cooperative tariff selection, and political tariff selection. Our results show that at the Stackelberg equilibrium, which results from the unilateral tariff selection, the policy level selected by the leader is higher compared to the rest of the experiments. While, in the case of cooperation, free trade will be the equilibrium outcome. At Nash equilibrium, which results in non-cooperative tariff policy selection, both countries select policy levels simultaneously.
and applied positive tariff rates for both intermediate inputs and final goods. Lastly, at political
equilibrium, which results after considering lobby by the heterogeneous firms, the policy level
selection diverges from benchmark unilateral level. To illustrate our tariff policy formulations
quantitively, we use the US import data to estimate the policy levels. These estimates are then
compared to the factual tariff rates to evaluate the degree of political interference of lobbying firms
in the policy level selection.

The fifth chapter analyzes the technology adoption decision of heterogeneous firms by building
another political economy model. This model assumes that the population is divided into elite,
middle-class and labor class. The political power is controlled by the elite who formulate trade
policy to consolidate power. While the middle-class access the production technology and labor
class provide labor inelastically. In a two-country, two-sector model, we assume technological
differences across countries. Therefore, firms from the technologically inferior country make the
decision about whether to adopt new technology or resist adoption, once trade opened. The model
shows that the technology adoption decision of a firm crucially depends upon the political
institutions and the market size of the country. Firms in a country with strong democratic
institutions adopt new technology more rapidly, compared to firms in a less democratic country.
In a weak democracy, firms successfully persuade elite policymakers to impose higher trade
restrictions and obtain higher protection from the foreign technologically advance importing firms.

The final chapter contains a discussion on the future research agenda related to the trade policy in
heterogeneous firms model. This chapter also offers a review of the shortfalls of this research
project.
2. TRADE POLICY UNDER THE HETEROGENEOUS LOBBYING FIRMS

This chapter characterizes the political economy of trade policy in the heterogeneous firms mode. We consider a simple two-country two-sector model, where one sector produces freely traded homogeneous good under perfect competition. While, the other sector produces differentiated varieties with heterogeneous firms under monopolistic competition, as in Melitz (2003). The differentiated sector features with free entry and Pareto productivity distribution. The consumer utility is represented by the quasi-linear function, where homogeneous good enters linearly and differentiated varieties enter in CES fashion. The imports and exports of a country are subject to trade barriers and trade policy is one of them. A country follows trade policy in order to maximize social welfare by taxing the imports and exports. The heterogeneous firms enter the market after the enactment of trade policy. Thus, the mass of active firms and their pricing decision preceded by trade policy selection. Nevertheless, the trade policy level selection can be influenced by the political interference of heterogeneous firms. Therefore, the primary objective of this chapter is to show how trade policy selection diverges from the optimal policy level once we allow the possibility of heterogeneous firms lobby participation in a country. In order to achieve this objective, we elaborate the trade policy formation in two scenarios; one without lobbying possibility and other with lobbying possibility. Therefore, the endogenous trade policy making in heterogeneous firms model is considered by utilizing the influential work of Grossman and Helpman (1994) “protection for sale (PFS henceforth)” with firm-level lobby game.

In heterogeneous firms model, besides the terms-of-trade, the rationale for trade policy intervention include the existence of markup and entry distortions (Felbermayr et al. 2013). Additionally, in the presence of perfectly competitive numeraire sector, the mark-up distortion leads to inefficient entry in the differentiated sector due to inefficient labor allocation in this sector (Melitz and Redding 2012). Consequently, this entry inefficiency provides another ground for corrective policy measures (Bagwell and Lee 2018). However, the enactment of trade policy affects the production and pricing decision of the firm active in the differentiated sector. Hence, a firm active in the market put efforts in order to persuade the policymaker to select a policy level that is handpicked for the firm but not socially optimal. This lobby engagement of the firm involves the monetary offerings to the policymaker in the form of political contribution. Thus, the lobbying firm incurs a fixed cost of lobbying in order to participate in the lobbying business. Nevertheless, not all firms active in the market can afford the lobbying cost and as a result of which not all active
firms participate in the lobby, as concluded in section 1.4. Moreover, as we have discussed that
the decision to lobby or not is the decision of an individual firm in the heterogeneous firms model.
Since the product differentiation and substitutability endorse firm-level lobbying instead of
industry-level lobbying (Kim 2017). Also, the probability to be organized and participate in
lobbying activities at the industry level is less when the product substitutability is high (Paltseva
2014). Furthermore, the existence of greater variations in intra-industry tariff rates compares to
inter-industry tariff rates also verify firm-level lobbying (Bombardini 2008). Empirical evidence
also shows that the participation in the lobby is not done by all firms in the industry and the
decision of a firm to participate in the lobby depends on the size and the productivity of the firm
(Dellis and Sondermann 2017; Kim 2017; Weymouth 2012; Kerr et al. 2011). As the environment
at which the individual firm lobby is highly competitive, hence more productive firms can only
participate in the lobby (Deng and Kennedy 2010). Resultantly, we consider lobbying as the
business of an individual firm and policymaker can devise the trade policy at the micro level.3
The model also features with the possibility for a foreign importing firm to participate in the lobby
along with domestic firms in an economy. The foreign firms lobbying participation consideration
in the model, which is discussed in detail in section 1.4, is based on the empirical evidence; like
Gawande et al. (2006) and Stoyanov (2009) brings the case of the existence of foreign firms lobby
in US trade policy formation into the discussion. Meanwhile, Aidt and Hwang (2014) legitimize
the involvement of foreign firms in lobbying on the bases of cross-national externalities argument.
As trade policy of the home country affects the welfare of firms from foreign as well, therefore
internalization of this externality would be welfare enhancing. Additionally, by allowing the
possibility of foreign firms lobby participation, the biases in the PFS framework towards export
subsidies, as emphasized by Rodrik (1995), can also be tackled.
Our results show that in the absence of lobbying firms, the policymaker selects a trade policy that
is optimal and maximizes the social welfare of the society. The policymaker imposes a uniform
import tariff on all foreign importing firms and a uniform export tax on all domestic exporting
firms. However, if we allow the lobbying possibility, then the selection of policy level diverts from

3Here we focused on the ability of an individual firm to influence the policy level through lobbying and do away
with the lobbying channel of the individual firm. The lobbying channel adopts by a firm to approach the
policymaker can either be direct or an indirect route. Richter et al. (2009) define the indirect route as hiring
services of a registered lobbyist who then coordinates the meeting with the policymaker. Whereas, Schuler et
al. (2002) design lobbying process as a combination of both routes and show that it depends on the size of the
firm and nature of the industry.
the optimal level. The policymaker adopts a lower policy rate for the lobbying firm, while the rest of non-lobbying active firms face the optimal policy rate. The results also indicate that both domestic exporting firms and foreign importing firms emerge successful in altering the policy level once engage in the lobby and bear the cost of it. The diversion of the policy rate at the political equilibrium from the optimal equilibrium depends negatively upon; the elasticity of substitution between the varieties of the differentiated good, the properties of the productivity distribution, and the social welfare weight. In addition, we have also shown that the implementation of trade policy reduces the mass of cross-border trading firms and lower the average productivity level in the economy.

The study by Chang (2005) is most related, which incorporates the Krugman model into the PFS framework and shows that the selection of a positive import tariff and maybe an export tax by the policymaker at the equilibrium. However, the selection of an export subsidy as equilibrium policy tool only emerges when the lobby constitutes only a small proportion of the population and policymaker ignores the revenue considerations. The study also demonstrates that a sector with low import penetration always receives high protection irrespective of being organized or not, as the equilibrium policy pair always involve a positive import tariff. The study by Ossa (2014) is also relevant and quantify the welfare impacts of trade policy in the model of homogeneous firms. The welfare implications by imposing the optimal tariff, non-cooperative tariff, and cooperative tariff was analyzed within the framework of Cobb-Douglas preferences. Another related study is Mitra (1999), that takes the initiative to endogenize lobby formation in the political economy of trade policy. The task to endogenize lobby formation along with trade policy is performed by introducing one additional stage in the PFS framework. At the first stage, the decision to be organized as a group or not takes place on the basis of the costs associated with being organized. While, the interaction between the policymaker and the organized lobby group takes place in the second stage. Bombardini (2008) introduced the firms’ heterogeneity and endogenizes lobby participation decision of individual firm on the basis of optimal lobby criterion. According to this criterion, the decision of the firm to participate or not in the lobby depends upon her marginal contribution and joint surplus. If the latter is higher in the event of a particular firm joining the lobby, then firm participates. The study by Demidova and Rodriguez-Clare (2009) is also closely related. They consider the case of a small open economy in the standard Melitz model and assert that in the presence of markup and entry distortions, the optimal policy to achieve the first best
allocation is either import tariff or export tax. Another study by Felbermayr et al. (2013) focuses on the terms-of-trade rationale for tariffs and elaborates the role of relative market size.

This chapter is also related to the strand of literature on quasi-linear demand in heterogeneous firms model. Melitz and Ottaviano (2008) extend the heterogeneous firms model with quasi-linear preferences. They have also endogenized the mark-up and offer an additional channel of resource allocation towards more productive firms. Nevertheless, the objectives of this chapter, which distinguish it from the previous work, are twofold. First, our objective is to show how an individual firm can affect the policy selection decision of the policymaker, and how much the selected policy level diverges from the optimal level in the presence of a lobbying firm. Secondly, our objective is to characterize endogenous policy formation in heterogeneous firms model of Melitz (2003) by considering firm-level lobbying under these settings.

The next section describes the basic settings of the model. While section 2.1 followed by two subsections contains the details about the interaction between the policymaker and lobbying firm. The subsections characterize trade policy equilibrium in case of a benevolent policymaker and in case of a non-benevolent policymaker, respectively. Section 2.2 holds a quantitative illustration to demonstrate the degree of divergence of the policy level selection, and section 3 concludes the chapter.

2.3. The Model

Consider a model of two countries, home country $i$ and foreign country $j$. The economy of each country consists of two sectors; a homogeneous good producing sector and a differentiated good producing sector. The homogeneous good producing sector is perfectly competitive, while differentiated good producing sector has monopolistic competition.

**Households:** The representative household derives utility from the consumption of the homogeneous good $Q_o$ and differentiated good $Q$. Labor is supplied inelastically and total labor supply in an economy is $L$. The utility function is quasi-linear with CES preferences for differentiated good.\(^4\) Therefore, the utility function of a representative agent in the home country $i$ is given by:

$$U_i = Q_o + \frac{1}{\theta} \left( \int_{\omega \in \Omega_n} (q_{ni}(\omega))^{(\sigma-1)/\sigma} d\omega \right)^{\theta/\sigma}, n \in \{i, j\}$$

\(^4\) see Helpman and Itskhoki (2010) for the quasi-linear utility function.
where $\omega$ is a particular variety of differentiated good that belongs to the set of continuum horizontally differentiated varieties $\Omega_n$. This set of differentiated good includes domestically produced varieties as well as the imported varieties. The elasticity of substitution among varieties of differentiated good is given by $\sigma = \frac{1}{1-\rho} > 1$, and $\theta \in (0,1)$ measures the substitution between the consumption of homogenous good and differentiated good. Given the total spending $Y_i$ and the price of an imported variety $\omega$ as $p_{ji}(\omega)$, the demand for the imported variety $\omega$ of differentiated good in the home country $i$ is:

$$q_{ji}(\omega) = P_i^{-\frac{\sigma(1-\theta)-1}{(1-\theta)}} (p_{ji}(\omega))^{-\sigma}$$

where $P_i$ is the ideal price index in the home country $i$;

$$P_i = \left( \int_{\omega \in \Omega_n} (p_{ni}(\omega))^{1-\sigma} d(\omega) \right)^{1/1-\sigma}, n \in \{i,j\}$$

The indirect utility function is $V_i = Y_i + \frac{1-\theta}{\theta} P_i^{-\frac{\theta}{1-\theta}}$, where the last term on the right-hand side specifies the consumer surplus. Since the demand for differentiated varieties depends only upon the prices, therefore, the demand of numeraire good bears the outcome of any change in the income level.

**Production:** Labor is the only factor of production with wage rate $w$, and all costs present in the model are in terms of labor units. The homogeneous good is produced under perfect competition with a unit input-output coefficient. Moreover, homogeneous good is freely traded and we treat it as the numeraire. Labor is movable across sectors and the wage in both countries is set to equal to one.

The differentiated varieties are produced by a continuum of heterogeneous firms and each firm produces a unique variety under monopolistic competition. The production technology in differentiated good producing sector exhibits the increasing returns to scale along with the free entry. The labor demand to produce a variety of differentiated good in the country $n \in \{i,j\}$ is given by $l_n = f_n + q_n/\varphi_n$, where $f_n$ is the overhead cost and $\varphi_n > 0$ is the productivity of the firm. The productivity of the firm $\varphi_n$ is drawn from cumulative distribution $G(\varphi_n)$ after incurring the sunk entry cost $f_n^e$. Moreover, the fixed overhead cost $f_n^e$ depends on the market location, such that firms from the home country $i$ that serve domestic market incur the fixed cost $f_{ii}$ and when
access the foreign market then incur the fixed cost $f_{ij}$, where $f_{ii} < f_{ij}$. Firms with the same productivity level behave symmetrically and we can index them with respect to their productivity. Furthermore, two types of trade frictions also exist in the model. The first trade friction exists in the form of transport cost, which is the iceberg type. Hence, in order to send one unit of differentiated good to foreign market $j$ the domestic firm ship $\tau_{ij} > 1$ units of the variety, while $\tau_{ii} = 1$. The second trade friction appears in the form of trade taxes. The home country $i$ imposes a tax $\eta_{ji} = (1 + t_{ji})$ on the imports from foreign country $j$ and provides a subsidy $\gamma_{ij} = (1 + s_{ij})$ to domestic firms that exports to foreign country. Whereas, $\eta_{ji} > 1$ indicates an import tariff and $\eta_{ji} < 1$ indicates an import subsidy, while $\gamma_{ij} > 1$ indicates an export subsidy and $\gamma_{ij} < 1$ indicates an export tax. Furthermore, it is possible to revise the tariff and subsidy at the micro level and different varieties of the differentiated good face different tariff or subsidy rates. This implies that the policymaker can treat heterogeneous firms in the differentiated sector discriminatorily. Such kind of discriminatory tax treatment would yield optimal policy level compare to uniform treatment in case of import tariff (Costinot et al. 2016). Following Costinot et al. (2016), we also assume that the home country $i$ is strategic and impose taxes in order to maximize welfare. While the foreign country $j$ is passive and does not impose taxes. In this regard, the trade policy precedes the entry of firms in the differentiated sector.

**Lobbying Mechanism:** As we will see later, the policymaker in the home county $i$ adopts a uniform trade policy in the absence of lobbies and all importing firms face the same tariff rate and all exporting firms face the same subsidy rate. However, the possibility for a firm to lobby and change the policy level governing her product also exists in the model, as the policymaker inclines to accept the political offerings from firms to change trade policy level. In this case, the discriminatory tax level will be selected for the lobbying firm and rest of non-lobbying firms face the same benchmark tax level. A firm that participates in lobby endures a fixed cost of the lobby $c_l$. The cost of the lobby $c_l$ presents in the form of contribution schedule, which lobbying firm devise against all viable policy level options of the policymaker. Due to the existence of lobbying cost, not all firm active in the market can participate in the lobby. Therefore, the more productive firms which can bear the cost of the lobby only participate in the lobby. We can classify the firms from home country $i$ into three types; firms that serve only the domestic market, firms that also serve the foreign market, firms that also participate in the lobby along with serving the foreign market. Similarly, we can also classify foreign firms into three types as well. Since serving foreign
market and lobby participate involves fixed costs, therefore, firms that serve only the domestic market are least productive firms among the mass of active firms. While firms that exports and participate in the lobby are the most productive firms among the active mass.

The lobbying game we consider here is different compared to the original PFS framework. As discussed above, here we assume lobbying at firm-level instead of industry-level. In order to design the lobby game, first, we have to identify the potential lobby candidates in the model. As there are three types of firms active in the home country $i$; the first type of firms consists of the least productive firms that are from home country $i$ and serve only the domestic market without participating in the export business. The second type of firms are also from home country $i$ and not only serve the domestic market but also export to foreign country $j$. The third type of firms that are active in the home country $i$ are the foreign importing firms from country $j$. Therefore, we can classify two potential lobby sets; one consists of only domestic market serving firms and other consists of domestic exporting firms and foreign importing firms. These two sets lobby in opposite directions and a domestic market serving firm prefers more trade restrictions in the form of high tariff, while domestic exporting firm and foreign importing firm prefers more trade openness in the form of lower trade barriers [as also shown by Gawande et al. (2006)]. A foreign importing firm lobby for a lower import tariff that applies to her product. Similarly, a domestic exporting firm lobby for the higher export subsidy if $\gamma_{ij} > 1$ or a lower export tax if $\gamma_{ij} < 1$. So, we have three possible candidates that can lobby and the decision to actually participate in the lobby depends upon the cost and benefits of lobbying, where the benefits of the lobby present in the form of change of policy level.

**Profit Maximization:** Since the heterogeneous firms face a residual demand curve with constant elasticity $\sigma$, therefore, charge a constant markup $\left(\frac{\sigma}{\sigma - 1}\right)$. A domestic firm in the home country charges the price $p_{i}(\varphi_{i}) = \left(\frac{\sigma}{\sigma - 1}\right)\frac{1}{\varphi_{i}}$, and earns revenue $r_{i}(\varphi_{i}) = \frac{\sigma(1-\theta)^{-1}}{(1-\theta)}\left(\left(\frac{\sigma - 1}{\sigma}\right)\varphi_{i}\right)^{\sigma - 1}$, and make a profit;

$$\begin{align*}
\pi_{i}^{w}(\varphi_{i}) &= \frac{r_{i}(\varphi_{i})}{\sigma} - f_{i} & \text{Without lobbying} \\
\pi_{i}^{l}(\varphi_{i}) &= \frac{r_{i}(\varphi_{i})}{\sigma} - f_{i} - c_{i} & \text{With lobbying}
\end{align*}$$

While a firm from foreign country $j$ at the market $i$ charges the price $p_{ji}(\varphi_{ji}) = \left(\frac{\sigma}{\sigma - 1}\right)\frac{\eta_{ji}\tau_{ji}}{\varphi_{ji}}$, and makes a profit;
\[
\begin{align*}
\pi^w_{ji}(\varphi_{ij}) &= \frac{r^w_{ji}(\varphi_{ij})}{\sigma} - f_{ji} \quad \text{Without lobbying} \\
\pi^l_{ji}(\varphi_{ij}) &= \frac{r^l_{ji}(\varphi_{ij})}{\sigma} - f_{ji} - c_i \quad \text{With lobbying}
\end{align*}
\]

where \( r^w_{ji}(\varphi_{ij}) = (\eta^w_{ji})^{-\sigma} \gamma_{ji}(\varphi_{ij})^{\sigma-1} \) and \( r^l_{ji}(\varphi_{ij}) = (\eta^l_{ji})^{-\sigma} \gamma_{ij}(\varphi_{ij})^{\sigma-1} \).

The comparison between profit levels with and without lobbying indicates the firms will lobby only when the benefits of lobbying in the form of change in the policy level, i.e. \( \eta^w_{ji} \) to \( \eta^l_{ji} \), are more than the cost of lobbying. Similarly, a domestic firm when serving foreign country charges the price at the market \( j \) \( p_{ij} = \left( \frac{\sigma}{\sigma-1} \right) \gamma_{ij} \) and earns a profit;

\[
\begin{align*}
\pi^w_{ij}(\varphi_{ij}) &= \left( \frac{\sigma}{\sigma-1} \right) \gamma_{ij}^{\sigma} - f_{ij} - c_i \quad \text{Without lobbying} \\
\pi^l_{ij}(\varphi_{ij}) &= \left( \frac{\sigma}{\sigma-1} \right) \gamma_{ij}^{\sigma} - f_{ij} - c_i \quad \text{With lobbying}
\end{align*}
\]

with \( r^w_{ij}(\varphi_{ij}) = (\gamma^w_{ij})^{\sigma} \left( \frac{\sigma-1}{\sigma} \right) \gamma_{ij}^{\sigma-1} \) and \( r^l_{ij}(\varphi_{ij}) = (\gamma^l_{ij})^{\sigma} \left( \frac{\sigma-1}{\sigma} \right) \gamma_{ij}^{\sigma-1} \).

The pricing rule indicates price inversely related to productivity \( \varphi \). Thus, a more productive firm will charge a lower price and earns higher sale revenues.

**Firm Entry, Exit, and Export:** The decision of a firm to serves only domestic market or both domestic and foreign market depends on the realized productivity level of the firm. The minimum productivity level \( \varphi^* \) to serve a market determines by the zero-profit condition. Therefore, the zero-profit conditions for a foreign importing firm is given by;

\[
\left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \left( \frac{\sigma-1}{\sigma} \right) \gamma_{ij}^{\sigma} \left( \frac{\sigma-1}{\sigma} \right) \gamma_{ij}^{\sigma-1} = \sigma f_{ij}
\]

In addition to this import productivity cutoffs, there also exists domestic productivity cutoff \( \varphi^*_{jj} \) for serving the domestic market only. Therefore, firms with a productivity level \( \varphi < \varphi^*_{jj} \) will quit the market right after realizing their productivity levels. However, firms with productivity \( \varphi^*_{jj} \leq \varphi < \varphi^*_{ij} \) will continue to serve the domestic market of foreign country \( j \). Whereas, firms with productivity \( \varphi^*_{ij} \leq \varphi \) will also operate in the market of the home country \( i \). We can also define the productivity cutoffs for firms from the home country in the same fashion.

In a particular market, domestic firms compete with foreign firms. Therefore, the domestic productivity cutoff to serve domestic market \( \varphi_{ii} \) is inversely related to import productivity cutoff.
\( \varphi_{ji} \) to serve home country \( i \). By considering the ratio of revenues of domestic and foreign firms that are competing in the home country \( i \), it is straightforward to show that;

\[
\varphi^*_{ii} \cdot 1^{-\sigma} = B \varphi^*_{ji} \cdot 1^{-\sigma}
\]

where \( B \equiv -\left( \frac{1-\kappa}{\kappa} \right) \eta_{ji}^{-\sigma} \tau_{ji}^{-1} \) with \( \kappa \) as the share of expenditures on domestic varieties of the differentiated good out of total expenditures. The nature of the relationship between the domestic and import productivity cutoffs indicates that an increase in import cutoff productivity lowers the domestic productivity cutoff. Resultantly, least productive domestic firms prefer a higher import tariff, as import tariff and import productivity cutoff are positively related.

**Proposition-1**: A unilateral change in the trade policy of the home country affects the productivity cutoffs not only in the home country but also in the foreign country \( j \). Therefore;

\[
\frac{\partial \varphi^*_{ji}}{\partial \eta_{ji}} \cdot \frac{\partial \varphi^*_{ij}}{\partial \eta_{ji}} > 0 > \frac{\partial \varphi^*_{ii}}{\partial \eta_{ji}} \cdot \frac{\partial \varphi^*_{ji}}{\partial \eta_{ji}}
\]

\[
\frac{\partial \varphi^*_{ii}}{\partial \gamma_{ij}} \cdot \frac{\partial \varphi^*_{ij}}{\partial \gamma_{ij}} < 0 < \frac{\partial \varphi^*_{ii}}{\partial \gamma_{ij}} \cdot \frac{\partial \varphi^*_{ji}}{\partial \gamma_{ij}}
\]

Proof: see appendix

The proposition shows that an increase in import tariff in the home country increases the import productivity cutoff, which leads to a decrease in the mass of importing firms in the economy. Due to the trade balance condition, this increase in import productivity cutoff also increases the export productivity cutoff for firms from home country \( i \). Hence, the mass of exporting firms from home

**Figure 1**: Productivity Cutoffs and Export Tax
country $i$ also decreases. This indicates that an increase in trade barrier either in the form of an increase in import tariff or an increase in export tax will reduce international trade participation. Therefore, all firms, irrespective of country of origin, that engage in cross-border trade prefer more trade openness and lobby for trade barrier reduction.

The figure 1 contains a demonstration of the productivity cutoffs for domestic firms of home country $i$. From the profit function described above, we know that the profit is a linear function of the productivity $\varphi^{\sigma-1}$ of the firm. Since $\tau_{ij}^{1-\sigma} < 1$, therefore, the slope of the profit function $\pi_{ij}$ of the domestic exporting firm is flatter than the profit function $\pi_{ii}$ of the only domestic market serving firm. The figure also illustrates how an increase in the export tax changes the slope of the profit function. Suppose the export tax increase from $\gamma'_{ij}$ to $\gamma''_{ij}$, as a result of it, the profit function becomes flatter and move downward as shown in the figure by $\pi_{ij}(\varphi_{ij}^{*'})$ and $\pi_{ij}(\varphi_{ij}^{*''})$. This leads to an increase in the productivity cutoff to participate in the export business and the export productivity cutoff move from $\varphi_{ij}^{*'}$ to $\varphi_{ij}^{*''}$.

The proposition 1 also advocates the existence of two opposing sets of lobbying firms and endorses the lobbying game we have described above. Out of these two sets of lobbying firms, the first set consists of only domestic market serving firms from home country $i$ and the second set consists of domestic exporting firms and foreign importing firms. Chang and Willman (2006) has also identified these opposing lobby forces within the firms from home country $i$ and assert that the empirical results obtained on PFS imply an upward bias for the weight on social welfare because of these opposing forces. Therefore, lobbies play a bigger role than what has been empirically estimated. However, they have ignored the third player in the lobby that is foreign importing firms.

While the mass of active firms in foreign country $j$ is $M_j = \left(1 - G(\varphi_{jj}^{*})\right)M^e_j$, where $M^e_j$ is the mass of potential entrants. The cumulative productivity distribution function is assumed Pareto distribution with shape parameter $\alpha$ and $\alpha > \sigma - 1$ (Axtell 2001; Helpman et al. 2004; Melitz and Redding 2012). Therefore, $G(\varphi_j) = 1 - \left(\frac{\varphi_j}{\varphi}\right)^\alpha$, with $\varphi_j$ being the lowest possible productivity draw. The mass of firms that export to country $i$ and do not lobby is $M_{ji} = m_{ji}M_j$, where $m_{ji} = \frac{1 - G(\varphi_{ji}^{*})}{1 - G(\varphi_{jj}^{*})}$ is the export participation rate.
Considering the minimum productivity cutoff, expected profit of a firm from foreign country \( j \) serving market \( i \) can be expressed as \( \bar{\pi}_{ji} = \psi - 1) m_{ji}(f_{ji} + c_i) \) where \( \psi = \alpha / (\alpha - \sigma + 1) \). So, the free entry condition for foreign country \( j \) is;

\[
(\psi - 1)(\varphi_{ji}^\ast) - \alpha \left( f_{ji} + m_{ji}(f_{ji} + c_i) \right) = f_j^e \left( \varphi_j^\ast \right)^{-\alpha}
\]

(4)

Taking the optimal pricing rule and Pareto productivity distribution into consideration, the ideal price index can also transform as;

\[
P_i^{1-\sigma} = \psi \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \sum_{n \in [i,j]} m_n \varphi_{ni} \varphi \left( \frac{\varphi_{ni}}{\varphi_i} \right)^{\sigma-1}
\]

(5)

We can utilize the labor market clearing condition in order to determine the mass of active firms in an economy. Therefore, the mass of firms in home country \( i \) is (see appendix);

\[
M_i = \frac{L_i}{\sigma \psi (f_{ii} + m_{ij}(f_{ii} + c_i))} = \frac{(\psi - 1) L_i \varphi_i^\ast}{\psi \sigma f_i^e} \left( \frac{\varphi_i}{\varphi_i^\ast} \right)^{\alpha}
\]

(6)

While, the mass of potential entrants is proportional to the labor supply of one country, and in the home country this mass is given by \( M_i^e = \frac{(\psi - 1) L_i}{\psi \sigma f_i^e} \).

**Income and Net Tariff Revenue:** The total income of an individual consists of the labor income and the lump sum transfer from the government \( Y_i = w_i L_i + R_i \). The lump sum transfer from the government is made up of net tariff revenue;

\[
R_i = (\eta_{ji} - 1) M_j \bar{r}_{ji} + (1 - \gamma_{ij}) M_i \bar{r}_{ij}
\]

where \( \bar{r}_{ji} \) is the expected revenue of foreign importing firm from market \( i \) and \( \bar{r}_{ij} \) is the expected revenue of domestic exporting firm from market \( j \). These revenues are;

\[
\bar{r}_{ji} = \psi P_i \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \tau_{ji} \eta_{ji}^{-\sigma} \varphi_{ji}^{-\sigma-1}
\]

\[
\bar{r}_{ij} = \psi P_j \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \tau_{ij} \gamma_{ij}^{-\sigma} \varphi_{ij}^{-\sigma-1}
\]

The change in net tariff revenue due to the change in import tariff and export subsidy can be shown as;

\[
\frac{dR_i}{\eta_{ji}} = M_j \bar{r}_{ji} \left( 1 + \left( \frac{1}{\eta_{ji}} - 1 \right) \frac{\alpha \sigma}{\sigma - 1} \right)
\]

\[
\frac{dR_i}{\gamma_{ij}} = M_i \bar{r}_{ij} \left( 1 - \frac{1}{\gamma_{ij}} \frac{\alpha \sigma}{1-\sigma} - 1 \right)
\]
The terms inside the brackets of above equations are decreasing functions of import tariff and export tax for \( \eta_{ji} > 1 \) and \( \gamma_{ij} < 1 \), respectively. Therefore, holding the assumptions \( \frac{a\sigma}{\sigma-1} > 1 \), there is a tariff-maximizing tariff rate \( \hat{\eta}_{ji} \) and a subsidy-maximizing tax rate \( \hat{\gamma}_{ij} \) such that \( \frac{dR_i}{dT_i} = 0 \), \((\eta_{ji}, \gamma_{ij}) \in T_i \). for \( T_i = \hat{T}_i \). Finally, the balance trade condition requires that the expenditures in the home country \( i \) on imported varieties must equate the revenue earned through exports. This condition can be expressed as;

\[
\frac{\varphi^{\alpha L_i}}{f^e} \left( \varphi^{*}_{ij} \right)^{-\alpha} \frac{(f_{ij}+c_i)}{y_{ij}} = \frac{\varphi^{\alpha L_j}}{f^e} \left( \varphi^{*}_{ji} \right)^{-\alpha} (f_{ji} + c_i)
\]

### 2.3.1. The Trade Policy Choice

Now we characterize the interaction between the policymaker and lobbying firms, the outcome of which is the political equilibrium. To elaborate interaction, we utilize the PFS framework that articulates it as a “common agency problem”. In these settings, the policymaker acts as an agent and lobbying firm act as principal. The bargaining process can be considered a two-stage non-cooperative game. At the first stage, the firm moves first and offers a political contribution schedule to the policymaker. Firm devise this schedule of political contribution by keeping in mind the schedules of other lobbying firms. This schedule contains the political contribution offers to correspond to different policy levels that policymaker can select. In the second stage, the policymaker selects a policy level and receives political contributions from lobbying firms. The underlying condition for the equilibrium is that the equilibrium pair, policy level and political contributions, is jointly efficient. The solution of the game can be figured out by solving backwardly. To proceed, we have to express the objective functions of the policymaker and lobbying firm.

The policymaker of the home country \( i \) wants to maximize the utility function, which is based on single-peaked preferences with respect to trade policy \( T_i \), with \((\eta_{ji}, \gamma_{ij} \in T_i) \). where \( \eta_{ji} \) and \( \gamma_{ij} \) are the vectors of the import tax and export subsidy. The policymaker’s objective function is;

\[
U_i(T_i, c_i) = bSW_i(T_i) + \sum_{n,n'=\{(i,j)\}} m_{nn'} M_{n} c_i(T_i)
\]

(8)

The utility of the policymaker depends on the vector of political contributions \( c_i \) and weighted social welfare \( SW_i \). The political contributions link positively with policymaker utility and a higher level of contribution is always preferable. While the social welfare enters into the utility function with a weight \( b \). The weight assign to social welfare depends upon the degree of the benevolence
of the policymaker. A higher weight indicates a higher preference towards social welfare or a higher degree of the benevolence of the policymaker. The social welfare comprises producers surplus (which is zero due to free entry condition), consumer surplus, and net transfer;
\[
SW_i(T_i) = CS_i + R_i(T_i)
\]  
(9)
As consumer surplus is \(CS_i = \frac{1-\theta}{\theta} P_i^{\theta-1} \), and change in consumer surplus due to change in change in net tariff is;
\[
\frac{\partial CS_i}{\partial \eta_{ji}} = -\frac{1}{P_i^{(\theta-1)}} \frac{\partial P_i}{\partial \eta_{ji}}
\]
Since \(\frac{\partial P_i}{\partial \eta_{ji}} > 0\), therefore, the increase in tariff will decrease the consumer surplus in the home country \(i\). In this regard, the change in trade policy affects the price index through three channels. The appearance of trade policy instruments in the price index formulation, as shown in equation (5), is the first direct effect. The second effect comes through productivity cutoff. The change in trade policy will affect the cutoff productivities as discussed in Proposition 1. The final channel is through the mass of domestic and foreign firms. Since, an increase in import tariff increases import cutoff, which reduces the mass of imported varieties. While, domestic productivity cutoff fall, which increases the mass of domestic varieties. However, the overall productivity will fall in the country and price index rise.

Compared to the objective function of the policymaker, firms want to maximize profits. Due to homothetic production technology and a constant markup, the gross profits are constant proportion to the revenues (Melitz and Redding 2012). Subsequently, the objective function of the firms includes operating profits \(\hat{\pi}_{ij}\) rather than net profits \(\pi_{ij}\). Hence, the objective function of a lobbying firm from the home country \(i\) is given as;
\[
\mathcal{V}_i(T_i, c_i) = \sum_{n \in \{i, j\}} \hat{\pi}^l_{in}(T_i) - c_i
\]  
(10)
Following Grossman and Helpman (2001), the contribution schedules are;
**Assumption 1:** The contribution schedules are differentiable around the equilibrium.

**Assumption 2:** The contribution schedules are truthful, that is;
\[
c_i(T_i) = \max\{0, \hat{\pi}^l_{nn'}(T_i) - B_n\}, (n, n') \in \{i, j\}
\]
where \(B_n\) is the welfare anchor of the firm. As in the PFS, we focus only on those equilibria where the lobbying firm makes a positive contribution;
\[
c_i(T_i) = \left(\hat{\pi}^l_{nn'}(T_i) - B_n\right), (n, n') \in \{i, j\}
\]
The change in the operating profit due to a change in the trade policy expresses as;

\[ M_i \frac{\partial \bar{\pi}_{ij}}{\partial y_{ij}} = M_i \frac{\bar{\pi}_{ij}}{\gamma_{ij}} \left( \frac{\alpha \sigma}{\sigma-1} \right) \]

\[ M_j \frac{\partial \bar{\pi}_{ji}}{\partial \eta_{ji}} = -M_j \frac{\bar{\pi}_{ji}}{\eta_{ji}} \left( \frac{\alpha \sigma}{\sigma-1} \right) \]

As the change in trade policy transmits to the profit of the firm through the change of cutoff productivity. A change in trade policy changes productivity cutoff, as shown in Proposition 1. Thus, the change in productivity cutoff changes the mass of exporters and importers in the economy. More explicitly, an increase in import tariff is negatively related to the operating profit of the foreign importing firms and an increase in export subsidy is positively related to the operating profit of exporting firms.

### 2.3.2. Benevolent Equilibrium

For the sake of benchmark, let’s first illustrate the equilibrium trade policy without political intervention by lobbying firm. In this case, the policymaker makes social welfare as his priority and assign a weight equal to one to social welfare and ignore all political contributions offers. Consequently, we call this equilibrium as the benevolent equilibrium. The objective function of the policymaker in this case is;

\[ W_i(T_i, 0) = SW_i(T_i) = CS_i + R_i(T_i) \]

The unconstraint maximization will give the following first order condition;

\[ \frac{\partial W_i(T_i, 0)}{\partial T_i} = \frac{\partial CS_i}{\partial T_i} + \frac{\partial R_i}{\partial T_i} = 0 \]

This first order condition gives the following pair of benevolent tariff and subsidy level (see Appendix);

\[ \eta_{ji}^w = \frac{\alpha}{\alpha - 1} \]

\[ \gamma_{ij}^w = \frac{\alpha \sigma}{\alpha \sigma + \sigma - 1} \]

As \( \eta_{ji}^w > 1 \) and \( \gamma_{ij}^w < 1.5 \). Therefore, the optimal policy choice entails a pair of import tariff and export tax. The selection of import tariff rate by the policymaker at benevolent equilibrium depends solely on Pareto shape parameter. This parameter \( \alpha \) measures the productivity dispersion among heterogeneous firms. Meanwhile, the selection of export tax depends not only on Pareto

---

5 Where \( w \) in the superscript indicates the policy-level selected by the policymaker without lobbying. While equilibrium policy-level with lobbying is denoted by the superscript \( l \).
shape parameter but also on the elasticity of substitution among the varieties of differentiated good. 
Since;

$$\frac{\partial \eta_i^w}{\partial \alpha} = -\frac{1}{(\alpha-1)^2} < 0, \quad \frac{\partial \eta_i^w}{\partial \sigma} = 0$$

$$\frac{\partial \gamma_{ij}^w}{\partial \alpha} = \frac{\sigma(\sigma-1)}{(\alpha \sigma + \sigma-1)^2} > 0, \quad \frac{\partial \gamma_{ij}^w}{\partial \sigma} = -\frac{\alpha}{(\alpha \sigma + \sigma-1)^2} < 0$$

The above comparative statistics state a negative relationship between import tariff and Pareto shape parameter, while the relationship between export tax and Pareto shape parameter is positive. The inverse relationship between import tariff and Pareto shape parameter can be explained as follow. Since a high value of Pareto shape parameter means a low productivity dispersion among the heterogeneous firms. Therefore, this low productivity dispersion makes the market selection by the heterogeneous firms more sensitive to the variation in import tariff. As a result of this market selection sensitivity, the policymaker will select a lower tariff in the case of higher Pareto shape parameter. Besides, the formulation of an import tariff in case of benevolent equilibrium analogous to previous work on heterogeneous firms trade policy. For example, we can interpret the import tariff formula, as in Demidova and Rodriguez-Clare (2009), by distinguishing between the markup distortion neutralizing $$\left(\frac{1}{\rho}\right)$$ and entry distortion neutralizing $$\left(\frac{\alpha \rho}{\alpha-1}\right)$$ terms.

In case of export tax, the positive relationship with Pareto shape parameter means a lower ad-valorem export tax when $$\alpha$$ is high, since $$\gamma_{ij}^w < 0$$. The economic rationale for imposing a lower ad-valorem export tax given low productivity dispersion is the same as in the case of import tariff. However, the export tax also depends negatively with the elasticity of substitution among the varieties of the differentiated good. A higher elasticity of substitution means that the domestic varieties are more substitutable to foreign varieties of the differentiated good. As proposition 1 indicates that the implementation of a higher export tax will reduce the mass of exporters and importers in the economy. Therefore, imposing a higher export tax will not affect the welfare drastically in the home country when $$\sigma$$ is high.

2.3.3. Political Equilibrium

After describing the benchmark benevolent equilibrium policy levels, let’s now characterize the equilibrium policy levels in the environment described in section 2.1. We call this equilibrium, the political equilibrium. The following proposition characterizes the political equilibrium policy level (for details see Bombardini 2008).
Proposition-2: \((c_i^*, T_i^*)\) is a sub-game Nash equilibrium of the trade-policy game in the home country \(i\) if and only if;

1. \(c_i^*\) is feasible to all firms in the market \(i\).
2. \(T_i^*\) maximizes \((bSW_i(T_i) + \sum_{n,n' \in \{i,j\}} m_{nn'} M_n c_i(T_i))\) on \(T_i\), given \(\eta_{ji}^*, \gamma_{ij}^* \in T_i^*\)
3. \(T_i^*\) maximizes \((\hat{c}^l_{mm'}(T_i^*) - c_i^*(T_i^*) + bSW_i(T_i^*) + \sum_{n,n' \in \{i,j\}} m_{nn'} M_n c_i^*(T_i^*))\) on \(T_i\) for every firm in the market \(i\).
4. For every firm \(k \in \{i,j\}\) there exists \(T_i^{-k} \in T_i\) that maximizes \((bSW_i(T_i) + \sum_{n,n' \in \{i,j\}} m_{nn'} M_n c_i^*(T_i))\) on \(T_i\) such that \(c_i^{-k}(T_i^{-k}) = 0\).

The first condition places the feasibility restriction on the contribution schedule of each firm, and condition (2) states that the policymaker maximizes his own utility given the contribution schedules offered by lobbying firms. The condition (3) elaborates the fact that the equilibrium policy vectors must maximize the joint objective functions of the policymaker and lobbying firm. Whereas, the last condition indicates that at the equilibrium, some firms pay no political contribution due to policy choice made by the policymaker.

As condition (3) elaborates the fact that \(T_i^*\) maximizes \(\sum_{n \in \{i,j\}} V_n(T_i, c_i) + W_i(T_i, c_i)\). The first order condition is given as;

\[
\frac{\hat{c}^l_{mm'}(T_i^*)}{\partial T_i^*} - \frac{\partial c_i^*(T_i^*)}{\partial T_i^*} + \sum_{n,n' \in \{i,j\}} m_{nn'} M_n \frac{\partial c_i^*(T_i^*)}{\partial T_i^*} + b \frac{\partial SW(T_i^*)}{\partial T_i^*} = 0
\]  (11)

However, condition (2) states that the first order condition of the policymaker’s utility maximization is;

\[
\sum_{n,n' \in \{i,j\}} m_{nn'} M_n \frac{\partial c_i^*(T_i^*)}{\partial T_i^*} + b \frac{\partial SW(T_i^*)}{\partial T_i^*} = 0
\]  (12)

Taking together the equations (11) and (12) (summing over all domestic and foreign firms);

\[
\sum_{n,n' \in \{i,j\}} m_{nn'} M_n \frac{\hat{c}^l_{mm'}(T_i^*)}{\partial T_i^*} = \sum_{n,n' \in \{H,F\}} m_{nn'} M_n \frac{\partial c_i^*(T_i^*)}{\partial \eta_{ji}^*}
\]  (13)

Substitute the equation (13) into the equation (12);

\[
\sum_{n,n' \in \{i,j\}} m_{nn'} M_n \frac{\hat{c}^l_{mm'}(T_i^*)}{\partial T_i^*} + b \frac{\partial SW(T_i^*)}{\partial T_i^*} = 0
\]  (14)

The above condition described the equilibrium policy outcome. The political equilibrium tariff and subsidy level selected by the policymaker will be (see appendix);
\[ \eta_{ij}^l = \frac{\alpha (b\sigma - 1)}{b\sigma (a-1)} = \frac{\alpha}{(a-1)} - \frac{\alpha}{b\sigma (a-1)} \]

\[ \gamma_{ij}^l = \frac{\alpha (b\sigma + 1)}{b(a\sigma + \sigma - 1)} = \frac{\alpha \sigma}{a\sigma + \sigma - 1} + \frac{\alpha}{b(a\sigma + \sigma - 1)} \]

The second terms on the right-hand side of the above equations specify the divergences of the political policy levels from the benevolent policy levels. Since \( \frac{\alpha}{b\sigma (a-1)} \), \( \frac{\alpha}{b(a\sigma + \sigma - 1)} > 0 \), therefore, the political import tariff and export tax selected by the policymaker is lower than the benevolent import tariff and export tax. In the case of political equilibrium, import tariff depends upon the elasticity of substitution among differentiated varieties and social welfare weight as well along with the productivity parameter. The comparative statistics in this case state:

\[ \frac{\partial \eta_{ij}^l}{\partial \sigma} = \frac{-b\sigma (b\sigma - 1)}{(b\sigma (a-1))^2} < 0, \quad \frac{\partial \eta_{ij}^l}{\partial \alpha} = \frac{ab(a-1)}{(b\sigma (a-1))^2} > 0, \quad \frac{\partial \eta_{ij}^l}{\partial b} = \frac{a\sigma (\sigma - 1)}{(b\sigma (a-1))^2} > 0 \]

\[ \frac{\partial \gamma_{ij}^l}{\partial \sigma} = \frac{(b\sigma + b)(\sigma - 1)}{(b(a\sigma + \sigma - 1))^2} > 0, \quad \frac{\partial \gamma_{ij}^l}{\partial \alpha} = \frac{-ab(b + a + 1)}{(b(a\sigma + \sigma - 1))^2} < 0, \quad \frac{\partial \gamma_{ij}^l}{\partial b} = \frac{-a(a\sigma + \sigma - 1)}{(b(a\sigma + \sigma - 1))^2} < 0 \]

These derivatives again confirm a negative relationship between import tariff and Pareto shape parameter and a positive relationship between export tariff and Pareto shape parameter. Now import tariff depends positively with the elasticity of substitution and social welfare weight, while the export tax has a negative relationship with both. In this regard, an increase in the elasticity of substitution leads to a reduction in the divergence term present in both import tariff and export tax formula. This shows that even though the lobbying firm is successful in shifting the policy level selection, but the degree of divergence from the benevolent level is conditional on the elasticity of substitution. The political policy level selected by the policymaker in case of political equilibrium will be closer to the optimal level if the elasticity of substitution among varieties of differentiated good is high. The same is true in case of social welfare weight, and an increase in social welfare weight also moves the policy selection towards benevolent level. The social welfare weight measures the degree of the benevolence of the policymaker and an increase in the social welfare weight implies that the policymaker becomes more benevolent. Which in turn reduce the divergence from the optimal level. Similarly, the Pareto shape parameter also has the same impact on the degree of divergence. A higher value of \( \alpha \) leads to a lower degree of divergence between benevolent and political equilibrium policy levels.
By making a comparison between the benevolent and political equilibrium it is apparent that both domestic exporting lobbying firms and foreign importing lobbying firms are successful in lobbying. While the only domestic market serving firms remains out of lobby participation. Some intuition for no participation in the lobby by only domestic market serving firms can be developed as follow. First, the recent trade patterns involve global production and offshoring, either through FDI or at arm’s length trading. Firms involve in global production always prefer to abolish trade barriers across countries and struggle for it. At the same time, only productive firms can have production facilities across countries. Therefore, our results are quite intuitive and conform with empirical findings, as discussed in the introduction section, that only more productive firms engage in lobby business. Moreover, we must also keep in consideration the difference between the characteristics of developed and developing markets. This enables us to understand why in less developed markets the least productive firms mostly lobby and gain high protection from foreign competitions, as evident by the protectionist policies adopted by less developed countries. While in case of more developed markets the protection from foreign competition for the least productive domestic firms is not certain and markets are more competitive even in term of lobby participation. Therefore, least productive firms cannot afford the cost of lobbying and refrain from lobby participation.

2.4. A Quantitative Illustration

As shown in the above section that the success of lobbying firm to alter policy level in her favor depends upon; the elasticity of substitution, Pareto shape parameter, and the social welfare weight. To portray the degree of divergence of the policy level selection in the presence of lobbying firms, we consider a quantitative illustration of the policy levels in both benevolent and political equilibrium cases. Following Bernard et al. (2003), we start by assuming the elasticity of substitution $\sigma$ is equal to 3.8 and the shape parameter $\alpha$ is equal to 4. With $(t^b, s^b)$ and $(t^p, s^p)$ ad-valorem policy pairs at benevolent and political equilibrium, the table explains how the change in parameters affects the policy selection.

<table>
<thead>
<tr>
<th>$b$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$\tau^w$</th>
<th>$\theta^w$</th>
<th>$t^l$</th>
<th>$s^l$</th>
</tr>
</thead>
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<td>4</td>
<td>0.33</td>
<td>-0.15</td>
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<td>-0.13</td>
</tr>
<tr>
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<td>3.80</td>
<td>6</td>
<td>0.20</td>
<td>-0.10</td>
<td>0.31</td>
<td>-0.14</td>
</tr>
<tr>
<td>30</td>
<td>3.80</td>
<td>4</td>
<td>0.33</td>
<td>-0.15</td>
<td>0.32</td>
<td>-0.14</td>
</tr>
<tr>
<td>10</td>
<td>4.5</td>
<td>4</td>
<td>0.33</td>
<td>-0.16</td>
<td>0.16</td>
<td>-0.08</td>
</tr>
<tr>
<td>20</td>
<td>4.5</td>
<td>6</td>
<td>0.20</td>
<td>-0.10</td>
<td>0.18</td>
<td>-0.09</td>
</tr>
<tr>
<td>30</td>
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<td>4</td>
<td>0.33</td>
<td>-0.16</td>
<td>0.19</td>
<td>-0.10</td>
</tr>
</tbody>
</table>


2.5. Conclusion

By considering a two-country two-sector model, we characterize trade policy in the presence of lobbying firms. The existence of mark-up distortion among homogeneous and differentiated sectors leads to inefficient entry in the latter sector. So, policymaker implements trade policy to overcome these distortions. The heterogeneous firms react to the enactment of trade policy and engage in lobbying to tilt the policy level selection in their favor. The main objective of this is to show how the policy selection process influenced by an individual lobbying firm. In this regard, we explore the classical Grossman and Helpman (1994) protection for sale framework and consider lobbying participation decision at firm-level. Our results show that in the absence of lobbying firms, the policymaker imposes a uniform import tariff on all importing firms and a uniform export tax on all exporting firms. However, when we allow the lobbying possibility, the selection of policy level diverts from the optimal level. The policymaker adopts a lower policy rate for the lobbying firm. The diversion of the policy rate depends upon; the elasticity of substitution, Pareto shape parameter, and the social welfare weight that measures the degree of the benevolence of the policymaker. In addition, we have also shown that the implementation of trade policy reduces the mass of cross-border trading firms and lower the average productivity level in the economy.

Appendices

Appendix-A: Ideal Price Index in Market-i

\[
P_{i}^{1-\sigma} = \frac{M_{i}}{(1 - \alpha)(\varphi_{i}^{*})} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \int_{\varphi_{i}}^{\infty} (\varphi_{i}^{*})^{\sigma-1} dG(\varphi) + \frac{m_{j}M_{j}}{(1 - \alpha)(\varphi_{j}^{*})} \left( \frac{\sigma}{\sigma - 1} \right) \eta_{ji} \tau_{ji} \int_{\varphi_{j}}^{\infty} (\varphi_{j}^{*}) dG(\varphi)
\]

\[
P_{i}^{1-\sigma} = \frac{M_{i}}{(1 - \alpha)(\varphi_{i}^{*})} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} a_{\varphi_{i}} \int_{\varphi_{i}}^{\infty} (\varphi_{i}^{*})^{\sigma-2} dG(\varphi) + \frac{m_{j}M_{j}}{(1 - \alpha)(\varphi_{j}^{*})} \left( \frac{\sigma}{\sigma - 1} \right) \eta_{ji} \tau_{ji} \int_{\varphi_{j}}^{\infty} (\varphi_{j}^{*})^{\sigma-2} dG(\varphi)
\]

\[
P_{i}^{1-\sigma} = \psi M_{i} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\varphi_{i}^{*})^{\sigma-1} + \psi m_{j}M_{j} \left( \frac{\sigma}{\sigma - 1} \right) \eta_{ji} \tau_{ji} \left( \varphi_{j}^{*} \right)^{\sigma-1}
\]

Appendix-B: The Mass of Firms and Potential Entrants in the Home Country
The aggregate labor demand in the home country $i$ is:

$$L_i = M^e_i f^e_i + M_i \sum_{n \in \{i,j\}} m_n f_{in} + m_{ij} M_i c_i + M_i \sum_{n \in \{i,j\}} m_n \int_{\varphi_{in}}^{\infty} \frac{\tau_n q_{in}(\varphi_{in})}{\varphi_{in}} \frac{dG(\varphi)}{(1-G(\varphi_{in}))}$$

As $M_i = (1 - G(\varphi^*_i))M^e_i$. By using (2) and (4):

$$L_i = \frac{M_i}{(1-\alpha(\varphi_i))} f^e_i + \sum_{n \in \{i,j\}} m_n M_i (f_{in} + c_i) + \sum_{n \in \{i,j\}} \frac{m_{in} M_i}{(1-\alpha(\varphi_{in}))} \tau_n \gamma_{in}^{\frac{\sigma(1-\theta)-1}{\sigma-1}} \psi \left( \frac{\tau_n}{\varphi_{in}} \right)^{\frac{1}{\gamma_{in}}} \frac{dG(\varphi)}{(1-G(\varphi_{in}))}$$

As $\bar{\pi}_i = \bar{\pi}_{ii} + \bar{\pi}_{ij} = (\psi - 1) f_{ii} + (\psi - 1) m_{ij} (f_{ij} + c_i) = \frac{f^e_i}{1-\alpha(\varphi_i)}$

$$L_i = M_i \bar{\pi}_i + M_i \sum_{n \in \{i,j\}} m_n (f_{in} + c_i) + M_i \sum_{n \in \{i,j\}} m_n \tau_n^{\frac{\sigma(1-\theta)-1}{\sigma-1}} \psi \left( \frac{\tau_n}{\varphi_{in}} \right)^{\frac{1}{\gamma_{in}}} \left( \sigma(f_{in} + c_i) \right)^{1-\sigma}$$

From zero profit conditions, $\varphi^*_i = \left( \frac{\sigma}{\sigma-1} \right) \tau_n^{\frac{\sigma(1-\theta)-1}{\sigma-1}} \eta_{in}^{-\frac{\sigma}{1-\sigma}} \left( \sigma(f_{in} + c_i) \right)^{1-\sigma}$

$$L_i = M_i \bar{\pi}_i + \sum_{n \in \{i,j\}} m_n M_i (f_{in} + c_i) + \frac{\sum_{n \in \{i,j\}} m_n M_i \psi(\sigma - 1) w_i (f_{in} + c_i)}{\psi(\sigma - 1) w_i (f_{in} + c_i)}$$

The average profit is, $\bar{\pi}_i = (\psi - 1) \left( f_{ii} + m_{ij} (f_{ij} + c_i) \right) + (\psi - 1) m_{ij} (f_{ij} + c_i)$

$$L_i = M_i \sigma \psi \left( f_{ii} + m_{ij} (f_{ij} + c_i) \right)$$

Solving for $M_{ji}$:

$$M^e_i = \frac{(\psi-1)L_i}{\psi \sigma f^e_i}$$

**Appendix-C: Proposition-1**

To prove the proposition 1, we follow Felbermayr et al. (2013). From the zero profit conditions, the relative productivity cutoffs of firms competing in the home country $i$;

$$\eta_{ji}^{-\sigma} \left( \frac{\varphi_{ij}}{\varphi_{ii}} \right)^{\sigma-1} = \frac{(f_{ij} + c_i)}{f_{ii}}$$

By differentiating after taking the log, holding transport cost constant, gives;
\[
\left(\frac{\sigma-1}{\sigma}\right)\left(\dot{\phi}_{ji} - \dot{\phi}_{ii}\right) = \dot{\eta}_{ji}
\]

where dot above the variable denotes the percentage change in the variable. This expression indicates that any change in tariff rate will affect both cutoff productivities in the market \(i\). The variation in tariff rate is positively related with import cutoff and negatively with domestic cutoff. However, the trade balance condition dictates a positive association between \(\varphi_{ij}^*\) and \(\varphi_{ji}^*\), which is given by:

\[
\varphi_{ij}^* = A\varphi_{ji}^* \text{ where } A = \frac{\varphi_i}{\varphi_j} \left(\frac{(f_{ji}+c_i)}{(f_{ij}+c_i)/\gamma_{ij} f_i^e f_j^e}\right)^\alpha > 0
\]

So, this positive relationship between both productivities indicates that if productivity cutoff of foreign firms to serve market \(i\) falls, then productivity cutoff for domestic firms to serve foreign market \(j\) also falls.

The negative relationship between \(\varphi_{ij}^*\) and \(\varphi_{ii}^*\) is given by the equation (7):

\[
\varphi_{ij}^{1-\sigma} = B\varphi_{ji}^{1-\sigma} \text{ where } B \equiv -\left(\frac{1-\kappa}{\kappa}\right) \left(\frac{1}{\eta_{ji}}\right)^\sigma \tau_{ji}^{1-\sigma}
\]

Therefore, the fall of imported productivity cutoff for foreign firms in the home country due to decrease in tariff rate will increase productivity cutoff of domestic firms to serve domestic market. On the other hand, this also decrease import productivity cutoff in foreign country \(j\), which increase domestic productivity cutoff \(\varphi_{ji}^*\).

Similarly, in case of export subsidy, the relative productivities cutoffs in the foreign country \(j\) leads to:

\[
\frac{(\sigma-1)}{\sigma} (\dot{\phi}_{jj} - \dot{\phi}_{ij}) = \dot{\gamma}_{ij}
\]

Thus, any change in export subsidy rate of the home country \(i\) will affect exporting cutoff negatively and foreign country’s domestic cutoff negatively. While we can complete the rest of the analysis for export subsidy by following the above steps.

**Appendix-D:**

**The Benevolent Tariff**

Here we derive tariff rate, however, the derivation for subsidy can be done in the same fashion. Taking the derivation in the equation (17) with respect to import tariff yield the following condition:

\[
\frac{\partial W_i(T_i,0)}{\partial \eta_{ji}} = \frac{\partial C_S}{\partial \eta_{ji}} \frac{\partial P_i}{\partial \eta_{ji}} + M_j \tilde{r}_{ji} + (\eta_{ji} - 1) M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}} = 0
\]
Solving for \((\eta_{ji} - 1)\):

\[
(\eta_{ji} - 1) = \frac{\partial P_i}{\partial \eta_{ji}} = \frac{M_j^f \partial P_i}{M_j^f \partial \eta_{ji}} - \frac{M_j^f \partial (\eta_{ji})}{M_j^f \partial \eta_{ji}}
\]

We can write the price index, the average sale, and the operating profit of foreign firm from domestic market in terms of model’s parameters as:

\[
P_i = \left[ M^e_i \psi_i^\alpha \left( \frac{w_i}{P} \right) \left( \phi_i^* \right)^{[\alpha-(\sigma-1)]} + M^e_j \psi_j^\alpha \left( \frac{\eta_{ji}(\tau_j \omega_j)}{\rho} \right)^{1-\sigma} \left( \phi_j^* \right)^{[\alpha-(\sigma-1)]} \right]^{1-\sigma}
\]

\[
m_{ji} M_j^f \tilde{r}_{ji} = M_j^f \psi_P \left( \frac{\rho}{\tau_j \omega_j} \right)^{1-\sigma} \eta_{ji}^\sigma \phi_j^* \left( \phi_j^* \right)^{[\alpha-(\sigma-1)]}
\]

The change in the price index due to change in the import tariff is given by:

\[
\frac{\partial P_i}{\partial \eta_{ji}} = \left( M_i^f \psi_i^\alpha \left( \frac{w_i}{P} \right) \left( \phi_i^* \right)^{[\alpha-(\sigma-1)]} \right) + M_j^f \psi_j^\alpha \left( \frac{1}{\eta_{ji}} \right)^{1-\sigma} \eta_{ji}^\sigma \phi_j^* \left( \phi_j^* \right)^{[\alpha-(\sigma-1)]}
\]

The change in the expected revenue will be:

\[
m_{ji} M_j^f \frac{\partial (\tau_{ji} \gamma_j i)}{\partial \eta_{ji}} = M_j^f \psi_P \left( \frac{\tau_{ji} \omega_j}{\rho} \right)^{1-\sigma} \eta_{ji}^\sigma \phi_j^* \left( \phi_j^* \right)^{[\alpha-(\sigma-1)]}
\]

Put (V. II) - (V. IV) into (V.I),

\[6\] The average productivity of foreign importing firms is;

\[
\phi_j^* = \left( \frac{\tau_{ji} \gamma_j i}{\eta_{ji} \sigma \left( \frac{1}{\sqrt{\sigma-1}} \right)} \right)^{1-\sigma} \left( \frac{\alpha \sigma}{\alpha - 1} \right) \phi_i^*
\]

The ratio of productivities of the average domestic and foreign firms competing in the market \(i\) will be;

\[
\frac{\partial \ln \phi_{ji}^*}{\partial \eta_{ji}} = \frac{\alpha}{\sigma - 1} < 0
\]

\[7\] As the mass of entrants depends only on the parameters \(\frac{\partial \ln M_i^e}{\partial \ln \eta_{ji}} = 0\)

\[8\] From the productivity cut-off ratio of the individual domestic and foreign firms;

\[
\phi_{ji}^* = \frac{\tau_{ji} \gamma_j i}{\eta_{ji} \sigma \left( \frac{1}{\sqrt{\sigma-1}} \right)} \phi_i^*
\]

\[
\frac{\partial \ln \phi_{ji}^*}{\partial \ln \eta_{ji}} = \frac{\alpha}{\sigma - 1}
\]
\[ (\eta_{ji} - 1) = \frac{\sigma_{(1-\theta)^{-1}}}{M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)} \frac{1}{\alpha} \frac{\eta_{ji} - 1}{\eta_{ji}} - \frac{\sigma_{(1-\theta)^{-1}}}{M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)} \frac{1}{\alpha} \frac{\eta_{ji} - 1}{\eta_{ji}} - \frac{\sigma_{(1-\theta)^{-1}}}{M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)} \frac{1}{\alpha} \frac{\eta_{ji} - 1}{\eta_{ji}} \]

\[ \eta_{ji} = \frac{\alpha}{\alpha - 1} \]

The Political Tariff

Taking the derivative of the equation (21) with respect to tariff is;

\[ m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}} + b \left( \frac{\partial C_{si}}{\partial P_i} \frac{\partial P_i}{\partial \eta_{ji}} + (\eta_{ji} - 1) m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}} + m_j M_j \tau_{ji} \right) = 0 \]

Solving for \( \eta_{ji} \);

\[ m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}} = \frac{\frac{\partial C_{si}}{\partial P_i} \frac{\partial P_i}{\partial \eta_{ji}}}{m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}}} - \frac{m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}}}{m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}}} \]

where the operating profit in terms of parameter is;

\[ m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}} = M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)^{\sigma_{(1-\theta)^{-1}}} \eta_{ji}^{\sigma_{-1}} \left( \frac{\alpha}{\alpha - 1} \right) \]

The change in operating profits due to change in tariff will be;

\[ m_j M_j \frac{\partial \tau_{ji}}{\partial \eta_{ji}} = M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)^{\sigma_{(1-\theta)^{-1}}} \eta_{ji}^{\sigma_{-1}} \left( \frac{\alpha}{\alpha - 1} \right) \]

Plug (D, II)- (D, IV) and (D, VI) into (D, V)

\[ (\eta_{ji} - 1) = \frac{\sigma_{(1-\theta)^{-1}}}{M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)} \frac{1}{\alpha} \frac{\eta_{ji} - 1}{\eta_{ji}} - \frac{\sigma_{(1-\theta)^{-1}}}{M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)} \frac{1}{\alpha} \frac{\eta_{ji} - 1}{\eta_{ji}} - \frac{\sigma_{(1-\theta)^{-1}}}{M_j^\alpha \psi_{ii} \left( \frac{\rho}{\tau_{ji}} \right)} \frac{1}{\alpha} \frac{\eta_{ji} - 1}{\eta_{ji}} \]

\[ \eta_{ji} = \frac{\alpha}{\alpha - 1} \]

\[ \eta_{ji}^I = \frac{\alpha (b\sigma - 1)}{b\sigma (a - 1)} \]

34
3. THE HETEROGENEOUS FIRMS AND LOBBY PARTICIPATION DECISION

The previous chapter characterizes the political economy of trade policy in Melitz (2003) model of heterogeneous firms. The modified Grossman and Helpman (1994) PFS framework utilize to elaborate the policy level selection in the political equilibrium. However, the mass of lobbying firms is assumed as given during the analysis. This chapter is developed on relaxing this exogeneous lobbying firms mass assumption and endogenize lobbying participation decision of an individual firm along with trade policy. The basic setup of the model is same as in the previous chapter, however, here we further extend PFS framework and introduce another stage, which not only elaborates the lobbying participation decision of an individual firm but also determines equilibrium level of political contribution and mass of lobbying firms. Since the implementation of trade policy affects the production and participation decision of firms, therefore, firms exert efforts to change the policy level selection. Such policy effecting activities of the firm are model as lobbying. These lobbying activities involve the lobbying costs, and here we consider two types of lobbying costs; political contribution and fixed lobbying cost. Nevertheless, due to the existence of costs associated with lobby participation, not all firms active in the market can afford to participate. Empirical evidence also settles that the lobby participation decision of a firm depends on the size and productivity of the firm (see, inter alia, Dellis and Sondermann 2017, Kim 2017, Kerr et al. 2011, and Weymouth 2012). Yet most theoretical studies, with some exceptions,9 assume the lobby participation decision of a firm as exogenously given. Hence, this study aims to endogenize the lobby participation of a firm along with trade policy formation. In this regard, we extend the Melitz (2003) model by introducing the protection for sale (PFS) framework of Grossman and Helpman (1994) in these settings. The decision to participate in lobbying activities or not is the decision of an individual firm in case of heterogeneous firms. Since the product differentiation endorses firm-level lobbying instead of industry-level lobbying (Kim 2017). Also, the probability to be organized and participate in lobbying activities at the industry level is less when the product substitutability is high (Paltseva 2014). In addition, the existence of greater variations in intra-industry tariff rates compare to inter-industry tariff rates also corroborate firm-level lobbying (Bombardini 2008). In this model, on the basis of empirical observations, we also consider the possibility of foreign firms lobbying in the

9 For Example; Bombardini (2008), Mitra (1999)
home country along with domestic firms. Aaidt and Hwang (2014) legitimize the involvement of foreign firms in the lobby on the bases of cross-national externalities argument, which associate the welfare of foreign firms with the home country’s trade policy. Moreover, by allowing the possibility of foreign firms lobby participation, the biases in the PFS framework towards export subsidies, as emphasized by Rodrik (1995), can also be tackled.

The initiative to endogenize lobby formation decision of an industry in the PFS framework has been taken by Mitra (1999). He performs this task by introducing an additional stage in the PFS framework, where the decision to get organized or remain unorganized takes place on the basis of associated costs. While the policymaker and organized group interact at the second stage. Another study by Bombardini (2008) introduces the heterogeneity aspect of firms in the analysis and formulates the optimal lobby criterion that regulates the lobby participation decision of firms. According to this criterion, it is optimal for a firm to join the lobby if the joint surplus of the industry increases after the participation of that particular firm. Otherwise, the optimal decision for the firm is no-participation in the lobby group.

In order to endogenize the lobby participation decision of a firm, we utilize the framework of Mitra (1999) and contemplate it at the firm-level. Unlike Mitra (1999), we assume a constant fixed cost of lobby participation. Our analysis indicates that the decision to participate in the lobby depends upon three factors; the market size, the benefits of lobbying in the form of policy level change, and the cost of lobbying. More productive firms can afford the cost associated with lobbying activities, therefore, more inclined to lobby.

3.3. The Model

Consider a model of two countries, home country $i$ and foreign country $j$. The economy of each country consists of two sectors; a homogeneous good producing sector and a differentiated good producing sector. The utility function of a representative agent in the home country is given by:

$$U_i = Q_0 + \frac{1}{\theta} \left( \int_{\omega \in \Omega_n} (q_{ni}(\omega))^{(\sigma-1)/\sigma} d\omega \right)^{\theta\sigma}, \ n \in \{i, j\}$$

where $\Omega_n$ is the set of a continuum horizontally differentiated good and $\sigma = \left( \frac{1}{1-\rho} \right) > 1$ is the elasticity of substitution between the varieties. $Q_0$ is the consumption of homogeneous good and $q_{ni}(\omega)$ is the consumption of $\omega$ variety of differentiated goods. $\theta \in (0,1)$ measures the

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10 As Gawande et al. (2006) and Stoyanov (2009) presents the case of foreign firms participation in the lobby for trade policy formation of the US.
substitution between the consumption of the homogenous good and the differentiated varieties. Given the total spending $Y_i$ and price of an imported variety $\omega$ as $p_{ji}(\omega)$, the above utility function generates the following demand of the imported variety $\omega$ of differentiated good in the home country:

\[
q_{ji}(\omega) = P_i^{\sigma(1-\theta)-1} (p_{ji}(\omega))^{-\sigma}
\]

where $P_i$ is the ideal price index in home country $i$;

\[
P_i = \left( \int_{\omega \in \Omega_n} (p_{ni}(\omega))^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, n \in \{i, j\}
\]

Labor is the only factor of production with the wage rate $w$ and homogenous good is produced under perfect competition with a unit input-output coefficient. Labor is movable among sectors and homogenous good trade freely and acts as numeraire in the model. However, differentiated varieties are produced by a continuum of heterogeneous firms, as in Melitz (2003). These heterogeneous firms produce under monopolistic competition with production technology,

\[
l_n = f_n + \frac{q_n}{\varphi_n}, n \in \{i, j\}
\]

The productivity of the firm $\varphi_n$ is drawn from cumulative distribution $G(\varphi_n)$ after incurring the sunk entry cost $f_n^e$. Besides the entry cost, the production process also involves an overhead fix cost $f_n$ that depends on the market location. Domestic firms that serve domestic market incur $f_{ii}$ fixed cost and when access foreign market bear $f_{ij}$ fixed cost, with $f_{ii} < f_{ij}$.

Trade among countries involve the transport cost and trade taxes. The transport cost is iceberg type and in order to send one unit of the good to the foreign market, domestic firm ship $\tau_{ij} > 1$ units, where $\tau_{ii} = 1$. The trade taxes are imposed by country such that the home country $i$ imposes tax $\eta_{ji} = (1 + t_{ji})$ on imports from the foreign country and provides a subsidy $\gamma_{ij} = (1 + s_{ij})$ to domestic exporting firms. Where, $\eta_{ji} > 1$ indicates import tariff and $\eta_{ji} < 1$ indicates import subsidy, while $\gamma_{ij} > 1$ indicates an export subsidy and $\gamma_{ij} < 1$ indicates an export tax. Furthermore, it is possible to devise the tariff and subsidy at the micro-level and different varieties of differentiated goods face different tariff or subsidy rate. This implies that the policymaker can treat heterogeneous firms in the differentiated sector dissimilatory. This kind of discriminatory tax treatment would yield optimal policy level compare to uniform treatment in case of import tariff (Costinot et al. 2016). Following Costinot et al. (2016), we also assume that the home country $i$ is
strategic and imposed taxes in order to maximize welfare. While the foreign country $j$ is passive and does not impose taxes.

The policymaker in the home county adopts a uniform tax policy in the absence of lobbies and all importing firms face the same level of import tariff, while all exporting firms face the same subsidy level. However, the policymaker inclines to accept the political offerings from firms to change the trade policy level. In this case, the discriminatory tax level will be selected for a lobbying firm and rest of non-lobbying firms face the same benchmark tax level. Therefore, the possibility to lobby for the trade policy also exists in the home country. A firm that participates in lobby endures fixed and sunk costs of the lobby, such that $\tilde{c}_i = f^l_i + c_i$. The fixed cost of the lobby $f^l_i$ occurs while channelizing lobbying strategy like hiring a professional lobbyist, as suggested by Richter et al. (2009). While the sunk cost $w_n c_i$ presents in the form of contribution schedule, which is the monetary contribution offered by the firm to the policymaker.

Since the heterogeneous firms face a residual demand curve with constant elasticity, therefore, charged a constant markup $\left(\frac{\sigma}{\sigma - 1}\right)$. The optimal price charged by the foreign importing firm in the $i$-market is $p_{ji} (\varphi_{ji}) = \left(\frac{\sigma}{\sigma - 1}\right) \eta_{ji} \frac{\tau_{ji}}{\varphi_{ji}}$. Similarly, the price charged by the domestic exporting firm in $j$-market is $p_{ij} (\varphi_{ij}) = \left(\frac{\sigma}{\sigma - 1}\right) \frac{\tau_{ij}}{\gamma_{ij} \varphi_{ij}}$. The pricing rule indicates that the price charged by a firm is inversely related to the productivity of the firm. However, a firm’s decision to serve another country’s market depends on her productivity level. The minimum productivity level of a firm from foreign country $j$ wants to serve home country $i$ is determined by zero-profit condition. Therefore, the zero-profit condition for the foreign importing firm is given as;

$$\eta_{ji} - \sigma P_i \frac{\sigma (1 - \theta)^{-1}}{(1 - \theta)} \left(\frac{\sigma}{\sigma - 1}\right)^{1 - \sigma} \left(\frac{\varphi_{ji}}{\tau_{ji}}\right)^{\sigma - 1} = \sigma f^l_{ji}$$

This condition gives the cutoff productivity to serve the home country $\varphi_{ji}^*$. The productivity parameter is restricted, such that $\varphi_{ij}^* < \varphi_{ji}^*$, where $\varphi_{ij}^*$ is the domestic market cutoff productivity level. Thus, only more productive firms engage in exports business. If the mass of potential entrants in foreign country $j$ is $M^e_j$, then the mass of active firms would be $M_j = \left(1 - G(\varphi_{ji}^*)\right) M^e_j$.

Therefore, the mass of exporters to the country $i$ is $M_{ji} = m_{ji} M_j$, where $m_{ji} = \frac{\left(1 - G(\varphi_{ji}^*)\right)}{\left(1 - G(\varphi_{ij}^*)\right)}$ is the
The productive distribution is assumed to follow Pareto Distribution. The productivity distribution in country \( j \) is \( G(\varphi_j) = 1 - \left( \frac{\varphi_j}{\varphi} \right)^\alpha \), with \( \varphi_j \) being the lowest possible productivity draw and \( \alpha \) is the shape parameter and represents the dispersion of productivity, with \( \alpha > \sigma - 1 \). The productivity cutoffs for home country \( i \) can also be defined in the same fashion.

By considering the minimum productivity cutoff, expected profit of a firm from foreign country \( j \) serving the market \( i \) can be expressed as \( \pi_{ji} = (\psi - 1)(f_{ij} + c_i) \), where \( \psi = \alpha / (\alpha - \sigma + 1) \). So, the free entry condition can be expressed as (see online appendix);

\[
(\psi - 1)\left( \varphi_{ij}^* \right)^{-\alpha} \left( f_{ij} + m_{ji} \left( f_{ij} + c_i \right) \right) = f_j^e \left( \varphi_j \right)^{-\alpha}
\]

From this equation, we can determine the unique value of \( \varphi_{ij}^* \) that depends only on the parameters of the model. By taking the optimal pricing rule and Pareto distribution of productivity into consideration, the ideal price index transforms as;

\[
P_i^{1-\sigma} = \psi \left( \frac{\sigma}{\sigma-1} \right) \sum_{n \in (i,j)} m_{ni} M_n \left( \frac{\varphi_{ni}}{\varphi_{ni}^*} \right)^{\sigma-1}
\]

Since the mass of active firms in home country \( i \) can be measured by using the labor market clearing condition. Therefore, the mass of active firms in home country \( i \) is;

\[
M_i = \frac{(\sigma - 1) L_i}{\psi \sigma f_i^e} \left( \frac{\varphi_i}{\varphi_{ii}^*} \right)^{\alpha}
\]

From the utility function, the per worker welfare can be expressed as;

\[
U_i = Y_i + \frac{1-\theta}{\theta} P_i^{-\sigma} - \frac{1}{1-\theta}
\]

where the last term on the right-hand side specifies the consumer surplus and \( Y_i \) is the income level at home country \( i \). The income consists of labor income and the lump-sum transfer from the government, \( Y_i = w_i L_i + R_i \). Theses lump-sum transfers are made up of net tax revenue;

\[
R_i = (\eta_{ji} - 1) m_{ji} M_j \bar{r}_{ji} + (1 - \eta_{ij}) m_{ij} M_i \bar{r}_{ij}
\]

Where \( \bar{r}_{ji} \) is the expected revenue accrued by the foreign importing firms in market \( i \) and given as;

\[
\bar{r}_{ji} = \psi P_i^{\frac{\sigma(1-\theta)-1}{(1-\theta)}} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \tau_{ji}^{1-\sigma} \eta_{ji}^{-\sigma} \varphi_{ji}^{\sigma-1}
\]

---

11 See Axtell (2001), Helpman et al. (2004), and Melitz and Redding (2012).
The utility function of the policymaker in home country $i$ is assumed to be based on single-peaked preferences with respect to trade policy $T_i$ with $(\eta_{ji}, \gamma_{ij} \in T_i)$, where $\eta_{ji}$ and $\gamma_{ij}$ are the vectors of import and export tax. The policymaker’s objective function is;

$$ W_i(T_i, c_i) = bSW_i(T_i) + \sum_{n,n' \in (i,j), n \neq n'} m_{nn'} M_{n} c_i(T_i) \quad (11) $$

The utility of the policymaker depends on the vector of political contributions $w_n c_i$ and weighted social welfare $SW_i$. The political contributions link positively with policymaker utility and a higher level of contributions is preferable. While the social welfare enters into the utility function with weight $b$. The weight assign to social welfare depends upon the degree of the benevolence of the policymaker. The social welfare comprises producer surplus (which is zero due to free entry condition), consumer surplus, and net transfer;

$$ SW_i(T_i) = CS_i + R_i(T_i) $$

Compared to this, firms want to maximize profits. Since the gross profits are constant proportion to revenues and the margin is also constant, therefore, firms are more concerned with operating profits $\hat{\alpha}$. The objective function of a lobbying firm from home country $i$ is given as;

$$ V_i(T_i, c_i) = \sum_{n \in (i,j)} \left( \hat{\alpha}_{in}^{lb} (T_i) - c_i \right) \quad (12) $$

Following Mitra (1999) and Bombardini (2008), we devise this game of endogenous trade policy in two-stages. At the first stage, the firm decides whether to participate in the lobby and bears the associated fixed and sunk costs or remains politically inactive. While, at the second stage, the interaction between the policymaker and lobbying firms takes place. The solution of the game can be figured out by solving backwardly.

3.4. Second Stage

As in the PFS, the interaction between the policymaker of home country $i$ and lobbying firms articulate as “common agency problem”. The policymaker acts as agent and lobbying firms as principals. Firm moves first and offers a political contribution schedule to the policymaker keeping in view the political contributions offered by other lobbying firms. Later, the policymaker chooses policy level and receives the political contribution. The underlying condition at this stage’s sub-game Nash equilibrium is that the optimal equilibrium pair of policy level and contribution is jointly efficient for both.

Assumption 1: contribution schedules are differentiable around the equilibrium.

Assumption 2: Contribution Schedules are truthful;
\[ c_i(T_i) = \max\{0, \hat{p}_{n\pi}(T_i) - B_n\}, \ (n,n') \in \{i,j\}, n \neq n' \]

where, \(B_n\) is the welfare anchor of the firm. We focus only on those equilibria where lobbying firm make positive contributions;

\[ c_i(T_i) = (\hat{p}_{n\pi}(T_i) - B_n), \ (n,n') \in \{i,j\}, n \neq n' \tag{13} \]

With the mass of lobbying firm given, the optimal policy level can be determined by the following condition (see appendix);

\[ \sum_{n,n' \in \{i,j\}, n \neq n'} m_{nn'} M_n \frac{\partial \pi_{n\pi}(T'_i)}{\partial T'_i} + b \frac{\partial SW(T'_i)}{\partial T'_i} = 0 \tag{14} \]

From the above condition, the equilibrium import tariff and export tax level selected by the policymaker formulate as;

\[ \eta_{ji} = \frac{a(b\sigma - 1)}{b\sigma(a-1)} = \frac{a}{(a-1)} - \frac{\alpha}{b\sigma(a-1)} \]

\[ \gamma_{ij} = \frac{a(b\sigma + 1)}{b(a\sigma + \sigma - 1)} = \frac{aa}{a\sigma + \sigma - 1} + \frac{\alpha}{b(a\sigma + \sigma - 1)} \]

The above formulations indicate that the policymaker will select a pair import tariff and export tax in order to maximize welfare, since, \(\eta_{ji} > 1\) and \(\gamma_{ij} < 1\). Taking together the optimal pricing rule in the equation (4) and above equilibrium policy levels, it is apparent that the targeted policy instrument for the domestic firm is the export tax. As reasoned by Grossman & Helpman (2001, pp. 240-243) targeting the subsidy level instead of the tariff would be more welfare enhancing for the domestic lobbying firm as well as for the aggregate welfare. Similarly, the foreign lobbying firm always targets import tariff, which enters directly into the operating profit of the firm. If we call the equilibrium policy level in the absence of lobbying firms as the benevolent equilibrium and equilibrium in the presence of lobbying firms as the political equilibrium. Then, both benevolent import tariff and export tax will be higher than the political levels. Following Bernard et al. (2003), by taking the estimates of the elasticity of substitution \(\sigma\) equal to 3.8 and the shape parameter \(\alpha\) equal to 4, with \(b = 10\), we have \(\eta_{ji}^w = 1.33\) and \(\eta_{ij}^w = 1.29\). Thus, import tariff at political equilibrium is less than at benevolent equilibrium \(\eta_{ji}^l < \eta_{ji}^w\). While with same parametric values, \(\gamma_{ij}^w = 0.84\) and \(\gamma_{ij}^l = 0.87\), hence, \(\gamma_{ij}^l > \gamma_{ij}^w\) (see online appendix).
3.5. First Stage

To proceed further, let’s define the operating profits with lobbying and without lobbying for a domestic firm serving foreign country \( j \) given the mass of domestic lobbying firm is \( M_{ij}^l \);

\[
V_{ij}^l(M_{ij}^l) = \hat{\pi}_{ij}^l(M_{ij}^l) \quad \text{and} \quad \bar{V}_{ij}^w(M_{ij}^l) = \hat{\pi}_{ij}^w(M_{ij}^l)
\]

Similarly, the cost of production faced by a domestic exporting firm with and without lobby is;

\[
CP_{ij}^l = w_i \left( \frac{\tau_{ij}q_{ij}}{\varphi_{ij}} + f_{ij} + \check{c}_i \right) \quad \text{and} \quad CP_{ij}^w = w_i \left( \frac{\tau_{ij}q_{ij}}{\varphi_{ij}} + f_{ij} \right)
\]

Therefore, the profit function without and with lobbying can be expressed as;

\[
\pi_{ij}^l(\varphi_{ij}) = \left( \frac{1}{\sigma} \right) y_{ij}^{\sigma} P_j^{\sigma(1-\theta)-1} \left( \frac{\varphi_{ij}}{\sigma - 1} \right)^{1-\sigma} \left( \frac{\varphi_{ij}}{\tau_{ij} w_i} \right)^{\sigma - 1} - \left( f_{ij} + \check{c}_i \right) w_i = \hat{\pi}_{ij}^l - w_i \left( f_{ij} + \check{c}_i \right) \quad (15)
\]

\[
\pi_{ij}^w(\varphi_{ij}) = \left( \frac{1}{\sigma} \right) y_{ij}^{\sigma} P_j^{\sigma(1-\theta)-1} \left( \frac{\varphi_{ij}}{\sigma - 1} \right)^{1-\sigma} \left( \frac{\varphi_{ij}}{\tau_{ij} w_i} \right)^{\sigma - 1} - f_{ij} w_i = \hat{\pi}_{ij}^w - w_i f_{ij} \quad (16)
\]

3.5.1. Selection into Lobbying

**Proposition-1:** Due to the costs associated with lobby participation, only most productive domestic exporting firms find it worthwhile to bear the cost of lobbying and to benefit from a lower export tax rate. Therefore, lobbying is the ideal choice only for the domestic exporting firm with a productivity \( \varphi_{ij} > \hat{\varphi}_{ij}^l \), given the benefits of lobbying are more than the cost of lobbying.

From the cost of production, we know that \( CP_{ij}^w < CP_{ij}^l \). This indicates that not all exporting firms might able to recover the costs of lobbying. Firms with low productivity level might find that \( \pi_{ij}^w(\varphi_{ij}) > \pi_{ij}^l(\varphi_{ij}) \) and the optimal strategy for the firm is to not participate in the lobby. However, a firm with a sufficiently high productivity level may find it optimal to engage in the lobby. Nevertheless, the benefits of lobbying come in the form of a decrease in export tax \((\gamma_{ij}^l > \gamma_{ij}^w)\). The figure shows that due to a decrease in export tax the profit function with lobbying
has a steep slope. Moreover, at point A, $\pi^w_{ij}(\varphi_{ij}) = \pi^l_{ij}(\varphi_{ij})$. Using the definitions from (15) and (16), the lobby participation condition will be:

$$V^l_{ij} - V^w_{ij} - c_i = f^l_{ij}$$

(17)

The above equation states that firm does not participate unless the net benefits of lobbying are equal to the fixed cost of lobbying. Resultantly, the lobby participation is an optimal choice for a domestic exporting firm with productivity $\varphi_{ij} > \phi^l_{ij}$, where $\phi^l_{ij}$ is the productivity level at which firm earns the same level of profit with lobbying and without lobbying and given as (see appendix):

$$\phi^l_{ij} = \left(\frac{1}{B}\left[\varphi_{ij}^l - \varphi_{ij}^w \right]^\frac{1}{\sigma-1}\right)$$

where $B = \left(\frac{1}{\sigma}\right)\left(\frac{\sigma-1}{\sigma}\right)^{1-\sigma} P_j^{\sigma(1-\theta)-1} \tau_{ij}^{\sigma-1}$ (18)

This equation also specifies the determining factors of lobby participation which are; the cost of lobbying, benefits of lobbying in terms of change in policy level, and the market size measured by $B$. The benefits of lobbying and the market size have a negative relationship with the lobby participation productivity level $\phi^l_{ij}$, while the cost of lobby has a positive relationship. This shows that firms in a large market with larger benefits of lobbying are more inclined to lobby, whereas a higher cost of lobby discourages firm to participate in the lobby.

3.5.2. The Equilibrium Mass of Lobbying Firms

All domestic exporting firms with productivity level $\phi^l_{ij}$ or more can lobby in the home country $i$. Let’s define the net benefits of domestic lobbying firms at the aggregate level;
\[ NB(M_{ij}^l) = GB(M_{ij}^l) - M_{ij}^l w_i c_i(M_{ij}^l) \]  

where \( GB(M_{ij}^l) = M_{ij}^l \bar{V}_i^l(M_{ij}^l) - M_{ij}^l \tilde{V}_i^w(M_{ij}^l) \) are gross benefits of the domestic lobbying firms.

With mass of lobbying firms \((M_{ij}^l)\), the truthful contribution in (14) will be;

\[ c_i(M_{ij}^l) = \bar{V}_i^l(M_{ij}^l) - B_i(M_{ij}^l) \]  

Where \( B_i(M_{ij}^l) = \bar{V}_i^l(M_{ij}^l) - w_i c_i(M_{ij}^l) \) is the net welfare of a domestic lobbying firm. Let \( SW_i^* \) be the welfare generated under free trade. Then, \( D_p(M_{ij}^l) = SW_i^* - SW_i^p(M_{ij}^l) \) and \( D_b(M_{ij}^l) = SW_i^* - SW_i^b(M_{ij}^l) \) are the deadweight losses created at the political and benevolent equilibrium, respectively. Using these descriptions, we can write truthful contribution (see Online Appendix) as;

\[ c_i(M_{ij}^l) = b\Delta D(M_{ij}^l) - GB(M_{ij}^l) \]  

Where \( \Delta D(M_{ij}^l) = D_p(M_{ij}^l) - D_b(M_{ij}^l) \). This indicates that a firm’s contribution level at equilibrium depends upon the gross benefits, which is the difference in operating profits at benevolent and political policy levels, as well as the change of deadweight loss. Now, the question arises, how the equilibrium contribution level design by one firm depends upon the contribution schedules of other firms. To see this, we observe the changes in the contribution level by a firm due to the change in the mass of lobbying firms. This can be shown by differentiating (21) with respect to \( M_{ij}^l \);

\[ c_i'(M_{ij}^l) = b\Delta D'(M_{ij}^l) - GB'(M_{ij}^l) \]

\[ \Delta D'(M_{ij}^l) = SW_i^{b'} - SW_i^{b''} = (SW_i^{p'} - SW_i^{p''}) < 0 \]

\[ GB'(M_{ij}^l) = (1 - M_{ij}^l) \left( \bar{V}_i^l(M_{ij}^l) - \tilde{V}_i^w(M_{ij}^l) \right) < 0 \]

As in the equation (22), the mass of lobbying firm affects the equilibrium contribution level of an individual firm through two channels; change in deadweight loss and change in gross benefits. Both channels have a negative relationship with the mass of lobbying firms. This indicates as the mass of lobbying firms increases, the equilibrium contribution level of the firm will decrease due to both; declines in gross benefits and drop of deadweight losses. By plugging (21) into (19) and differentiating with respect to \( M_{ij}^l \), we have the net benefits;

\[ NB'(M_{ij}^l) = 2GB'(M_{ij}^l) - b\Delta D'(M_{ij}^l) < 0 \]
Once again, the negative relationship between the mass of lobbying firms and benefits achieved by an individual firm is shown by the above equation. The intuition behind this negative relation is that as more domestic firms engage in lobbying to get lower export tax for their product, the less will be the comparative price advantage lobbying firms will have over non-lobbying firms. In extreme case, if all firms engage in lobbying, then the export tax level decreases from benevolent to political for all firms. Compare to the case, where only one firm lobby and enjoy political export tax level for her product, while the rest of the firms have the benevolent level. Therefore, as the mass of lobbying firms increases, the net benefits to individual firm diminish. Since, $NB'(M_{ij}) < 0$ and $f_i^l(M_{ij})$ is constant in $M_{ij}$. As shown in the figure, there is a unique mass of the lobbying firm at the equilibrium, where $NB(M_{ij}^{o}) = f_i^l(M_{ij}^{o})$ determined endogenously.

3.6. Conclusion

This chapter analyzes the lobby participation decision of a firm in the heterogeneous firms model. To endogenize the lobby participation decision at firm-level, an additional stage in the standard political economy of trade policy framework has been introduced. This additional stage entails information about the equilibrium mass of lobbying firms and the equilibrium contribution levels. By assuming fixed and sunk costs associated with lobby participation, we have shown that the firm with productivity above a threshold level can only afford to engage in the lobby. Moreover, the lobby participation decision depends not only on the participation cost but also on the market size and benefits of lobbying. Our results conform to empirical findings related to the lobbying participation behavior of firms.

Figure 3: Mass of Lobbying Firms

![Figure 3: Mass of Lobbying Firms](image)
**Appendix**

**Appendix-F: derivation of condition 17**

As firm will not lobby until the net benefits of lobbying are equal to the fixed cost of lobbying.

\[
\pi_{i \mid j}^w(\varphi_{i \mid j}) = \pi_{i \mid j}^l(\varphi_{i \mid j})
\]

\[
\left( \frac{1}{\sigma} \right) \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma(1-\theta)-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma-1} - f_{i \mid j}w_i = \left( \frac{1}{\sigma} \right) \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma(1-\theta)-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma-1} - (f_{i \mid j} + \bar{c}_i)
\]

\[
\left( \frac{1}{\sigma} \right) P_j \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma(1-\theta)-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left[ \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma-1} \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma} \right] = \bar{c}_i
\]

\[
\varphi_{i \mid j}^{l_j} = \left( \frac{1}{B} \left[ \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma-1} \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma} \right] \right) \frac{1}{\sigma-1}, \text{ with } B = \left( \frac{1}{\sigma} \right) P_j \left( \frac{\varphi_{i \mid j}}{\tau_{i \mid j}} \right)^{\sigma(1-\theta)-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left( \frac{1}{\tau_{i \mid j} w_i} \right)^{\sigma-1}
\]

\[
\Delta M_{i \mid j} = \frac{\bar{c}_i}{\varphi_{i \mid j}^{l_j}}
\]

**Appendix-G: The Truthful Contribution**

In order to calculate the equilibrium contribution by a domestic firm, consider the situation where a small number of the domestic firms of a measure \( \Delta M_{i \mid j} \) decides to defect. Then, the welfare that policymaker will obtain is given by;

\[
W_i^P(M_{i \mid j}, M_{j \mid i}) = bS W_i(M_{i \mid j} - \Delta M_{i \mid j}) + (M_{i \mid j} - \Delta M_{i \mid j})[p_i^\prime(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})] + M_{i \mid j}[p_i^\prime(M_{i \mid j}) - B_i(M_{i \mid j})] \quad \text{(G.I)}
\]

However, at political equilibrium the policymaker’s welfare is;

\[
W_i^P(M_{i \mid j}, M_{j \mid i}) = bS W_i(M_{i \mid j} - \Delta M_{i \mid j}) + \sum_{n \neq n'} \left[ \frac{\bar{M}_{i \mid j}^n(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})}{\Delta M_{i \mid j}} \right] \quad \text{(G.II)}
\]

The above equations will be equal for a small measure of \( \Delta M_{i \mid j} \). Hence, by equating both equations and taking the limit \( \Delta M_{i \mid j} \to 0 \), we have the welfare anchor of the firm as;

\[
bS W_i(M_{i \mid j} - \Delta M_{i \mid j}) = bS W_i(M_{i \mid j}) + (M_{i \mid j} - \Delta M_{i \mid j})[p_i^\prime(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})] + M_{i \mid j}[p_i^\prime(M_{i \mid j}) - B_i(M_{i \mid j})]
\]

\[
\Delta M_{i \mid j} B_i(M_{i \mid j}) = bS W_i(M_{i \mid j} - \Delta M_{i \mid j}) - \sum_{n \neq n'} \left[ \frac{\bar{M}_{i \mid j}^n(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})}{\Delta M_{i \mid j}} \right]
\]

\[
B_i(M_{i \mid j}) = \frac{\Delta M_{i \mid j}}{\bar{M}_{i \mid j}^n(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})} - \frac{\bar{M}_{i \mid j}^n(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})}{\Delta M_{i \mid j}}
\]

\[
B_i(M_{i \mid j}) = \frac{\Delta M_{i \mid j}}{bS W_i(M_{i \mid j} - \Delta M_{i \mid j}) + (M_{i \mid j} - \Delta M_{i \mid j})[p_i^\prime(M_{i \mid j} - \Delta M_{i \mid j}) - B_i(M_{i \mid j})] + M_{i \mid j}[p_i^\prime(M_{i \mid j}) - B_i(M_{i \mid j})]}
\]

The equation (20) in the text will become;
The above expression means that the equilibrium contribution by a domestic lobbying firm compensates for the reduction in the social welfare and the welfare of other domestic firms. We can also write the equation (G.III), with \((T_i^p, T_i^b)\) trade policy level at benevolent and political equilibrium, as;

\[
c_i(M_{ij}^1) = \bar{V}_i(M_{ij}^1) - \Omega_i(M_{ij}^1)
\]

\[\text{(G.V)}\]

Where \(\Omega_i(M_{ij}^1) = aSW_i(M_{ij}^1) + M_{ij}^1\bar{V}_i(M_{ij}^1)\)

\(\Omega_i(T_i^{p^0} - T_i^{b^0}, M_{ij}^1) = \max_{T_i^p, T_i^b} bSW_i(T_i^p, T_i^b, M_{ij}^1) + M_{ij}^1\bar{V}_i(T_i^p, T_i^b, M_{ij}^1)\)

\[\text{(G.VI)}\]

Using envelope theorem;

\[
\frac{\partial \Omega_i(M_{ij}^1)}{\partial M_{ij}^1} = b \frac{\partial SW_i(T_i^{p^0}, T_i^{b^0}, M_{ij}^1)}{\partial M_{ij}^1} + M_{ij}^1 \frac{\partial \bar{V}_i(T_i^{p^0}, T_i^{b^0}, M_{ij}^1)}{\partial M_{ij}^1} + \bar{V}_i(T_i^{p^0}, T_i^{b^0}, M_{ij}^1)
\]

Plug into equation (G.III);

\[
w_i c_i(M_{ij}^1) = b(SW_i^b - SW_i^p) + M_{ij}^1(\bar{V}_i^w - \bar{V}_i^l)
\]

\[\text{(G.VI)}\]

Let \(SW_i^*\) be the welfare generated under free trade. Then, \(D_p(M_{ij}^1) = SW_i^* - SW_i^p(M_{ij}^1)\) and \(D_b(M_{ij}^1) = SW_i^* - SW_i^b(M_{ij}^1)\) are the deadweight losses created at the political and benevolent equilibrium, respectively. Using these descriptions, the equation (G.V) will become;

\[
w_i c_i(M_{ij}^1) = bD(M_{ij}^1) - GB(M_{ij}^1)
\]

\[\text{(21)}\]
4. TRADE POLICY WITH INTERMEDIATE INPUTS TRADE

In the previous chapter the model we develop consider only unilateral trade policy formation and assume a passive foreign country that does not impose any trade policy. In this chapter, the passive foreign country assumption is relaxed, and different scenarios of trade policy formulation are considered. In this regard, we focus only on import tariff as trade policy instrument. Moreover, the recent trade pattern indicates the dominance of the intermediate inputs trade.\(^\text{12}\) Analogous to the trade of final goods, trade of intermediate inputs also administers through trade barriers. In this regard, the most preferable trade barrier for the sake of protection is the import tariff (Staiger and Tabellini 1987). However, the presence of intermediate inputs tariff wipes out the conventionally perceived advantages of the final goods tariff (Ruffin 1969). Therefore, the gains from trade are more contingent on intermediate inputs tariff than final goods tariff.\(^\text{13}\) Against this backdrop, this chapter also intends to characterize the intermediate inputs tariff in the model of heterogeneous firms along with final goods tariff.

We develop a theoretical model of heterogeneous firms to show how the tariff on intermediate inputs and final goods affect the welfare, productivity, and entry of firms in a country. We also formulate the tariff level selection choice available to the policymaker with respect to four policy experiments. These policy experiments include; unilateral tariff selection, cooperative tariff selection, non-cooperative tariff selection, and political tariff selection.

We develop a model of two-country two-sector with quasi-linear preferences, where one sector produces homogenous good and other produce differentiated goods. The production of differentiated good requires the acquisition of intermediate inputs along with labor. The heterogeneous firms that produce differentiated goods can also employ imported intermediate inputs along with domestic intermediate inputs. However, cross-border trade involves the transport cost and import tariffs. Our primary focus is on the description of import tariff on intermediate inputs and final goods. In this regard, we explore the tariff policy option available to the policymaker. We first describe the unilateral tariff implementation by one country in order to maximize its own welfare and act as a leader in the Stackelberg tariff selection game. Then, we


\(^{13}\) As indicated by the empirical evidences, for example, the productivity gains are twice for intermediate inputs tariff reduction compare to final goods tariff reduction in case of Indonesia (Amiti and Konings 2007), and a quarter of productivity growth attributed to intermediate inputs trade liberalization for Hungry (Halpern et al. 2015). Dardis (1967) also bears similar results for German agriculture sector. For the product variety gains of intermediate inputs tariff reduction, see Goldberg et al. (2010).
illustrate the situation in which both countries collaborate with each other and select the efficient level of tariff rate that maximizes the joint welfare. At the third step, we discussed the situation of a non-cooperative tariff policy selection, where both countries select policy level simultaneously. This non-cooperative policy game offers the Nash equilibrium policy outcome. Lastly, we explore the possibility of lobbying by the heterogeneous firms, and the implications of the lobby on the tariff policy outcome that results in political equilibrium. In order to quantify our policy experiments and to validate our tariff characterizations, we use US trade statistics to estimates the elasticity of substitution and elasticity import demand with respect to the tariff. These estimates provide us a quantitative illustration of tariff levels.

Caliendo et al. (2017) presents a similar type of model and quantify the welfare impacts of Uruguay Round, preferential agreements, and free trade. They primarily focus on the entry effect of the tariff reduction and asserts that even if the reduction in tariff deteriorates the terms-of-trade of the country, still the movement of the entry towards the optimal level increases welfare. Therefore, liberalization is always welfare enhancing. Our model differs from this study in terms of framework and scope. We focused on quasi-linear preferences in order to get a neat presentation of consumer welfare and then articulate the tariff selection. In the case of homogeneous firms, Ossa (2014) studies the welfare implications of implementing the optimal tariff, non-cooperative tariff, and cooperative tariff. The study indicates that the welfare gains of one country at the expense of other countries are possible in case of the optimal/unilateral tariff, but not in the event of the non-cooperative tariff. Moreover, cooperative tariff brings significant welfare gains for all countries, like WTO negotiations. Another study by Kasahara and Lapham (2008) also develops a stochastic heterogeneous firms model for Chilean manufacturing industry. They demonstrate that the usage of imported intermediate inputs increases the productivity of the firms and large firms mostly participate in import/export business. Therefore, the importing intermediate inputs can also be an important channel of resource allocation, like exporting channel. The study by Bagwell and Lee (2018) is also related, which characterizes the tariff rates of final goods in the event of unilateral, cooperative, and non-cooperative policy selection. However, the trade of intermediate inputs and input-output linkage of production is missing in both Ossa (2014) and Bagwell and Lee (2018). While, Chakraborty (2003) formalize asymmetric countries model in which the intermediate inputs are export by capital-rich countries and final goods by labor-rich countries. In these asymmetric settings, he assessed the distributional effects of trade liberalization. The study
by Demidova and Rodriguez-Clare (2009) consider the case of a small open economy in the standard Melitz model and assert that in the presence of markup and entry distortions, the optimal policy to achieve the first best allocation is either import tariff or export tax in case of final goods trade. Another study by Felbermayr et al. (2013) focuses on the terms-of-trade rationale for final good tariffs and elaborates the role of relative market size. This chapter extends the analysis of intermediate inputs and final goods tariff policy in the heterogeneous firms model and contributes in three respects. First, we formulate the role of intermediate inputs trade policy on productivity and welfare with quasi-linear preference. By unfolding the channel through which tariff policy affect the total factor productivity makes the relationship between productivities of domestic and foreign firms operating in a market more perceptible. Secondly, the extent to which tariff policy effected by lobbies has also been explored in the case of heterogeneous firms. We analyze changes in policy level selection cause by lobbying activities of intermediate inputs importing and final good exporting firms. Lastly, we illustrate the equilibrium outcomes that can be resulted in case of different tariff policy experiments, for example, in the case of unilateral tariff selection, the Stackelberg equilibrium has been portrayed.

Our results show that in the event of unilateral tariff selection, the leader will have the first mover advantage and policy level selected by the follower will be lower than the leader for intermediate inputs. The follower either adopt the policy of a positive tariff rate or provision of a subsidy if allowed. However, the reaction of the follower depends critically upon the elasticity of substitution between the intermediate inputs. But, in the case of final goods tariff selection, the follower will select a higher tariff rate compared to the leader. While in the event of cooperation, free trade will be the equilibrium outcome. Since, the welfare gains of one country come at the cost of others, therefore, zero tariff is the optimal strategy for both countries. Comparatively, in the event of non-cooperation and when both countries move simultaneously to impose a tariff. Then, both countries select positive tariff level at this symmetric Nash equilibrium. In the event of political tariff selection, the selection of a lower tariff level compare to benchmark-unilateral level highlights the role played by lobbying firms. However, the extent of the role of lobby depends upon the degree of the benevolence of the policymaker. In our last step, we measure the elasticities required for quantification of tariff formulations by using trade data of US for the period of 2000-2006. Then, we use factual data of the US tariffs on intermediate input and final goods to compare our estimates.
The result of the comparison between factual tariffs and estimated tariffs describes that the policymaker assigns three times more weight to social welfare than political contributions.

The rest of the chapter is organized as follow. Section 2 describes the basic setup of the model and also holds the discussion about the impact of the tariff on welfare and productivity. Section 3 characterizes the tariff selection in case of four policy experiments. Section 4 presents a quantitative illustration on the tariff level selection in case of previously described policy experiments. Section 5 concludes.

4.3. The Model

Consider a two-country two-sector model, where one sector produces freely traded homogeneous good under perfect competition. While, the other sector produces differentiated goods under monopolistic competition. The differentiated good producing sector has a continuum of heterogeneous firms and each firm produces a different variety of the final good, as in Melitz (2003). The production of differentiated goods involves labor and intermediate inputs. The intermediate inputs are produced by a continuum of firms with the constant return to scale technology. The final good producing firms can employ either domestic or both domestic and imported intermediate inputs in the production process. Both home country \( i \) and foreign country \( j \) have similar economic structure except with respect to the trade policy level.

4.3.1. Households

The representative household derives utility from the consumption of homogeneous goods \( Q_o \) and the differentiated goods \( Q \), and supply labor inelastically. The total population in one country provides \( L \) hours of labor. The preferences in the home country \( i \) are given by;

\[
U_i = Q_o + \frac{1}{\theta} \left( \int_{\omega \in \Omega_n} (q_{nt}(\omega))^{(\sigma-1)/\sigma} d\omega \right)^{\sigma \theta \sigma}, \quad \sigma > 1, n \in \{i,j\} \tag{1}
\]

where \( Q_o \) is the consumption of numeraire good, and \( \omega \) is the particular variety that belongs to the set of continuum horizontally differentiated goods \( \Omega_n \). The elasticity of substitution between the different varieties is given by \( \sigma > 1 \), and \( \theta \in (0,1) \) measures the substitution between the consumption of homogenous good and differentiated goods. Given the total spending \( Y \) and price of a variety \( \omega \) denoted by \( p(\omega) \), the above utility function generates the following demand of variety \( \omega \) of differentiated good imported from foreign country \( j \) in home country \( i \) as;

---

14 see Helpman and Itskhoki (2010) and Bagwell and Lee (2018) for the usage of this type of utility function.
where $P_l$ is the ideal price index and given by;

$$P_l = \left[ \int_{\omega \in \Omega_n} p_{ni}(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}, \ n \in \{i,j\} \tag{3}$$

### 4.3.2. Final Goods Producing Firms

The homogeneous good production technology requires only labor as input. This sector is perfectly competitive with the unit input-output coefficient. Furthermore, the homogeneous good trades freely and serves as the numeraire in the model. While, differentiated final good producing firms are monopolistically competitive, and each firm is producing one particular variety of the differentiated final good. The production of differentiated final goods involves intermediate inputs along with labor. The heterogeneous firm can use domestic as well as imported intermediate inputs. The technology in differentiated final good producing sector exhibit increasing returns to scale along with the free entry. This sector is our main focus in the rest of the discussion.

The entry in the differentiated sector requires a fixed entry cost $f_i^e$. After incurring this entry cost, the final producing firm draws productivity $\varphi_i$ from the cumulative productivity distribution $G(\varphi)$. Besides the fixed entry cost, the production process also involves an overhead cost $f_i^o$ and a cross-border market access cost $f_{ij}$. Hence, the total fixed cost of a differentiated final good producing firm in home country $i$ that employ imported intermediate inputs from foreign country $j$ and also export final good is given by;

$$F_i(z) = f_i^e + z f_{ij} \tag{4}$$

where $z \in \{0,1\}$ indicates the decision of the differentiated final good producing firm to engage in foreign trade, with $z = 0$ implies that the firm does not engage in import/export. The production function of the differentiated final good is;

$$q_i(\varphi_i, z) = \varphi_i l_i^a \left[ \int_0^1 x_{ii}(s)^{\frac{\gamma-1}{\gamma}} \, ds + z \int_0^1 x_{ij}(s)^{\frac{\gamma-1}{\gamma}} \, ds \right]^{\frac{\gamma(1-\sigma)}{\gamma-1}} \tag{5}$$

with $\gamma > 1$ as the elasticity of substitution between any two intermediate inputs. While $l_i$ is labor input with share $0 < \alpha < 1$, $x_{ii}(s)$ is the domestic variety of intermediate inputs and $x_{ij}(s)$ is the imported variety of intermediate inputs. To simplify the analysis, we fixed the measure of intermediate inputs produced in one country at one.
4.3.3. Intermediate Input Supplier

In the intermediate inputs production sector, the entry and access to the blueprint of production technology are free. The continuum intermediate good producing firms are identical and produced only with labor under perfect competition. The underlying constant return to scale technology is identical for all suppliers and marginal productivity of labor is one. These considerations allow the domestic price for the intermediate good to set equal to one.

However, there are two types of trade frictions exist in cross-border intermediate inputs trade. The first friction presents in the form of iceberg type transport cost. Resultantly, \( \tau_{ji} > 1 \) units of an intermediate input required to be imported from foreign country \( j \) in order to receive one unit in home country \( i \). The second trade friction considered here is the import tariff. The home country \( i \) imposes an import tariff \( t_{ji} \) on all varieties of intermediate imported inputs and \( \bar{t}_{ji} \) on the import of final goods. Therefore, the price of intermediate input imported in the home country \( i \) will be \( t_{ji}\tau_{ji} \). In this regard, we assume the trade policy selection precedes the firms’ entry decision.

**Cost Minimization:** To simplify the analysis, let’s assume that the final good producing firm chooses the same level of employment of all intermediate input varieties. The solution of the cost minimization problem of the final good producing firm from home country \( i \) yields following conditional factor demand and variable cost function;

\[
x_{ii}(\varphi_i, z) = \frac{q_i(\varphi_i)}{\varphi_i} \left( \frac{1-\alpha}{\alpha} \right) \alpha \left[ 1 + z(t_{ji}\tau_{ji})^{1-\gamma} \right]^{\frac{\alpha-\gamma}{(\gamma-1)}}
\]

\[
x_{ji}(\varphi_i, z) = \frac{q_i(\varphi_i)}{\varphi_i} \left( \frac{1-\alpha}{\alpha} \right) \alpha \left( t_{ji}\tau_{ji} \right)^{-\gamma} \left[ 1 + z(t_{ji}\tau_{ji})^{1-\gamma} \right]^{\frac{\alpha-\gamma}{(\gamma-1)}}
\]

\[
l_i(\varphi_i, z) = \frac{q_i(\varphi_i)}{\varphi_i} \left( \frac{1-\alpha}{\alpha} \right) \alpha^{-1} \left[ 1 + z(t_{ji}\tau_{ji})^{1-\gamma} \right]^{\frac{(\alpha-1)}{(\gamma-1)}}
\]

\[
C_i(\varphi_i, z) = \frac{q_i(\varphi_i)}{\varphi_i} (1 - \alpha)^{\alpha-1} (\alpha)^{-\alpha} \left[ 1 + z(t_{ji}\tau_{ji})^{1-\gamma} \right]^{\frac{(\alpha-1)}{(\gamma-1)}}
\]

(6)

However, by applying duality we can write the production function as;

\[
q_i(\varphi_i, z) = A_i(\varphi_i, z) l_i^{\alpha} [x_{ii} + z(t_{ji}\tau_{ji})x_{ji}]^{1-\alpha}
\]

(7)

where;

\[
A_i(\varphi_i, z) = \varphi_i \xi_i^{\varphi_i}, \text{ with } \xi_i^{\varphi_i} = \left[ 1 + z(t_{ji}\tau_{ji})^{1-\gamma} \right]^{\frac{1-\alpha}{(\gamma-1)}}
\]

The term \( A_i(\varphi_i, z) \) measures the total factor productivity. This expression, as emphasizes by Kasahara and Lapham (2008), also shows the final good producing firm that employs imported
intermediate inputs has higher productivity compared to the firm that employs only domestic intermediate inputs, since \( A_i(\varphi_i, 0) < A_i(\varphi_i, 1) \).

**Profit Maximization:** By considering fixed cost in equation (4) and variable cost in equation (6), the optimal domestic pricing rule for the final good producing firm will be;

\[
p_{il}(\varphi_{ij}, z) = \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1}{A_i(\varphi_{ij}, z)\Gamma} \right) \tag{8}
\]

where \( \Gamma = \alpha^\alpha(1 - \alpha)^{1 - \alpha} \). While, the price charged at foreign market incorporate transport cost \( \tau_{ij} \) and tariff \( \tilde{t}_{ij} \). The optimal pricing rule at foreign market \( j \) changed by the final good producer from home country \( i \) will be;

\[
p_{ij}(\varphi) = \tau_{ij}\tilde{t}_{ij}p_{il}(\varphi_{ij}, z)
\]

The domestic and foreign revenue of the differentiated final good producing firm from the home country that also participate in import/export will be:

\[
R_{ii}(\varphi_{ij}, z) = P_i^{\frac{\sigma(1-\theta)-1}{\sigma-1}} \left( \left( \frac{\sigma - 1}{\sigma} \right) A_i(\varphi_{ij}, z)\Gamma \right)^{\sigma - 1}
\]

\[
R_{ij}(\varphi_{ij}, z) = \tau_{ij}^{1-\sigma}\tilde{t}_{ij}^{-\sigma} P_j^{\frac{\sigma(1-\theta)-1}{\sigma-1}} \left( \left( \frac{\sigma - 1}{\sigma} \right) A_i(\varphi_{ij}, z)\Gamma \right)^{\sigma - 1} \tag{9}
\]

### 4.3.4. Exit, Import/Export Decision of a Firm

The differentiated final good producing firm’s decisions; either to produce or quit the market, produce with only domestic intermediate inputs or with imported intermediate inputs as well, export the final product or not, depend upon the productivity level of the firm. For the sake of simplicity, we assume only two types of heterogeneous firms in both countries. The first type consists of those firms that produce with domestic intermediate inputs and sell in the domestic market only. The second type of firms include firms that produce with both domestic and imported intermediate inputs and also export final good to other country. The threshold productivity level of a firm from home country \( i \) to produce final goods with imported intermediate inputs and sell in the foreign market can be determined by utilizing zero-profit condition. The zero-profit condition will read;

\[
\tau_{ij}^{1-\sigma}\tilde{t}_{ij}^{-\sigma} P_j^{\frac{\sigma(1-\theta)-1}{\sigma-1}} \left( \left( \frac{\sigma - 1}{\sigma} \right) A_i(\varphi_{ij}, z)\Gamma \right)^{\sigma - 1} = \sigma(f_{ii} + zf_{ij}) \tag{10}
\]

The above condition gives the cutoff productivity, \( \varphi_{ij}^* \), of final good producer from home country \( i \) with import/export engagement. Besides this productivity cutoff, there is the productivity level
for producing with domestic inputs and selling at the domestic market $\phi_{ii}^*$. More explicitly, these productivity cutoffs are given as:

$$\phi_{ii}^* = \tilde{e}_{ij}^{\sigma-1} p_{i}^{(1-\theta)(1-\sigma)} \frac{\sigma}{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \psi (f_{iil} + zf_{ij})$$

$$\phi_{ij}^* = \frac{\tilde{e}_{ij}^{\sigma-1} p_{i}^{(1-\theta)(1-\sigma)} \frac{\sigma}{\sigma-1}}{\sigma (\sigma-1)} (\sigma f_{iil})^{1-\sigma}$$

(11)

The associated fixed costs of production dictates $\phi_{ii}^* < \phi_{ij}^*$. Hence:

- The firm with the productivity level $\phi < \phi_{ii}^*$ will quit the market right after the realization of the productivity level after paying the fixed entry cost $f_{i}^e$.
- The firms with productivity level $\phi_{ii}^* \leq \phi < \phi_{ij}^*$ will decide to produce with domestic intermediate input and serve the domestic market only.
- While the firms with productivity level $\phi_{ij}^* \leq \phi$ will export the final product to foreign country and produce with imported intermediate inputs.

The total mass of active firms $M_i$ in home country is $[1 - G(\psi_i)]M_{i}^e$, where the potential entrants are denoted by $M_{i}^e$. The cumulative productivity distribution function $G(\psi_i)$ is assumed Pareto with shape parameter $\beta$ and $\beta > \sigma - 1$. Therefore, $G(\psi_i) = 1 - \left( \frac{\psi}{\varphi} \right)^\beta$. Hence, the mass of firms producing with imported inputs will be, $M_{ij} = m_{ij}M_i$, where $m_{ij} = \frac{1 - G(\psi_{ij})}{1 - G(\psi_{ii})} = \left( \frac{\psi_{ii}}{\varphi_{ij}} \right)^\beta$ is the import/export participation rate.

The expected profit of a final good producing firm by serving the foreign country market is,

$$\bar{\pi}_{ij}(\varphi_{ij}, z) = (\psi - 1)(f_{iil} + zf_{ij})$$

(12)

where $\psi = \frac{\beta}{(\beta - \sigma + 1)}$. Therefore, the free entry condition reads;

$$(\psi - 1)(2f_{iil} + zf_{ij})(\varphi_{ii})^{-\beta} = f_{i}^e \varphi^{-\beta}$$

(13)

By considering the mass of firms and productivity cutoff, the price index in the equation (3) can be transformed as;

$$P_{i}^{1-\sigma} = \psi M_{i} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} (I \varphi_{ii}^*)^{-\sigma-1} + \psi \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \sum_{n,n' \in (i,j)} m_{nn}M_{n} \left( \frac{\xi_{nn}}{\varphi_{nn}} \right)^{\sigma-1}$$

(14)

As all the costs present in the form of per unit labor cost. Therefore, we can use the labor market clearing condition to determine the equilibrium mass of firms active in the home country. The mass of firms in a country is given by the following equation;
\[ M_i = L_i \left( \frac{\psi^{\psi-1} \varphi_\psi (\varphi_i)^{-\psi}}{\sigma \psi f_i^\psi} \right) \quad (15) \]

The above equation along with free entry condition can also determine the mass of potential entrants \( M_i^e = \frac{L_i (\psi-1)}{\sigma \psi f_i^\psi} \) in the economy.

### 4.3.5. Total Revenue and Welfare

The total expenditures on imported intermediate inputs and on the imported final goods in the home country \( i \) can be expressed as:

\[
E_i^{\text{int}} = m_{ij} M_i (t_{ji} \tau_{ji}) \bar{x}_{ji} \\
E_i^{\text{final}} = m_{ji} M_j \bar{t}_{ji} \bar{R}_{ji}
\]

Therefore, the net tariff revenue will be:

\[
TR_i = (t_{ji} - 1) m_{ij} M_i \tau_{ji} \bar{x}_{ji} + (\bar{t}_{ji} - 1) m_{ji} M_j \bar{R}_{ji} \quad (16)
\]

Given the quasi-linear utility function, the welfare per worker in home country will be, \( W_i = 1 + TR_i + \frac{1-\theta}{\theta} P_i \frac{\theta}{1-\theta} \) \( (17) \)

**Proposition-1:** The consumer surplus depends negatively on both intermediate inputs and final goods tariffs. Therefore, a tariff reduction either on intermediate inputs or final good increases the consumer surplus;

\[
\frac{\partial CS_i}{\partial t_{ji}}, \frac{\partial CS_i}{\partial \bar{t}_{ji}} < 0
\]

**Proof:** From equation (20), we know that \( CS = \frac{1-\theta}{\theta} P_i \frac{\theta}{1-\theta} \). Hence,

\[
\frac{\partial CS_i}{\partial t_{ji}} = -P_i \frac{1}{1-\theta} \frac{\partial P_i}{\partial t_{ji}} \\
\frac{\partial CS_i}{\partial \bar{t}_{ji}} = -P_i \frac{1}{1-\theta} \frac{\partial P_i}{\partial \bar{t}_{ji}}
\]

The last term in the equations, change in the price index due to change in tariff, is positive. This positive relationship is partly because of the increase in average productivity in the market due to tariff reduction. A reduction in tariff increases the mass of firms engage in import/export and decreases the mass of domestic firms, which leads to an increase in the average productivity in the market. Since, price is inversely related to the productivity of the firm, therefore, prices will fall with a fall of the tariff. The same phenomenon will happen in the foreign market as well.
Next, we characterize the equilibrium tariff rates for the intermediate inputs and final goods in case of four scenarios. In this regard, we start with sequential tariff rate selection and then move to simultaneous selection.

4.4. Stackelberg Equilibrium Tariff

First, we consider the case of sequential tariff selection in which one country moves first and select the policy levels without the fear of retaliation from another country. Therefore, the country that moves first maximizes her own welfare without considering the reaction of the other country. In this sequential tariff level selection, we assume home country \(i\) acts as leader and foreign country \(j\) as the follower.\(^{15}\) Since, the welfare is comprised producer surplus (which is zero due to free entry condition), consumer surplus, and tariff revenue. Therefore, the maximization problems of the home country (leader) \(i\) is;

\[
\max_{\tilde{t}_{ji}, t_{ji}} W_i(\tilde{t}_{ji}, t_{ji}, \tilde{t}_{ij}(\tilde{t}_{ji}, t_{ji}), t_{ij}(\tilde{t}_{ji}, t_{ji})) = 1 + CS_i(\tilde{t}_{ji}, t_{ji}, \tilde{t}_{ij}(\tilde{t}_{ji}, t_{ji}), t_{ij}(\tilde{t}_{ji}, t_{ji})) + TR_i(\tilde{t}_{ji}, t_{ji})
\]

However, the policymaker of the foreign country, which acts as follower, solves following maximization problem;

\[
\max_{\tilde{t}_{ij}, t_{ij}} W_j(\tilde{t}_{ij}, t_{ij}, \tilde{t}_{ij}, t_{ij}) = CS_j(\tilde{t}_{ij}, t_{ij}) + TR_j(\tilde{t}_{ij}, t_{ij})
\]

\[s.t.
IM_j^{int} - EX_j^{int} = m_{ji}M_j t_{ij} \tau_{ij} x_{ij} - m_{ij}M_i t_{ji} \tau_{ji} x_{ji} = 0
\]

\[IM_j^{final} - EX_j^{final} = m_{ij}M_i \tilde{t}_{ij} \tilde{R}_{ij} - m_{ji}M_j \tilde{t}_{ji} \tilde{R}_{ji} = 0
\]

where \(IM\) stands for imports and \(EX\) stands for exports. With \(f_{ii} = f_{jj}\) and \(\tau_{ij} = \tau_{ji} = \tau\), the tariff rates selected by foreign country (follower) for intermediate inputs and final goods are given by (see appendix);

\[
\tilde{t}_{ij}^{SB} = \frac{(t_{ij})^{(\sigma-1)/\sigma}}{(t_{ij})^{(\sigma-1)/\sigma} + (t_{ij})^{(\sigma-1)/\sigma}}
\]

Now, from the unconstraint maximization problem of the leader, the tariff rates selected by the home country are given as;

\[
t_{ji}^{SB} = \frac{\zeta(\sigma-1)+\sigma}{\zeta(\sigma-1)+1}
\]

\(^{15}\) For timing of trade policy selection and endogenous selection of leader and follower, see Supasri (2007).
\[ \tilde{\tau}_{ji}^{SB} = \frac{\beta}{\beta - 1} \]

where \( \zeta = \frac{\partial x_{ji}^*}{\partial t_{ji}^*} \) is the elasticity of import demand with respect to tariff. The tariff rate selection for intermediate inputs, in case of leader, depends upon elasticities; the elasticity of substitution between the varieties of final goods and the intermediate inputs import tariff elasticity. A higher elasticity of substitution between final goods varieties means that the domestic variety is close substitute to imported variety, therefore, imposing a higher import tariff on the intermediate inputs will not affect the consumer welfare greatly. Resultantly, the policymaker in the home country will select a higher intermediate inputs tariff in case of having a higher substitutability between the varieties of final goods. The same is true for the follower, however, the policy selection is more sensitive to the substitutability compare to the case of the leader. While the intermediate inputs import tariff elasticity is negatively related to the policy selection for both leader and follower policymakers. The economic intuition is simple, since having a higher imported intermediate inputs elasticity with respect to intermediate inputs tariff leads a greater reduction in intermediate inputs import due an increase in intermediate inputs tariff. Hence, the optimal strategy is to select a lower tariff rate. In case of final goods, the specifications above indicate that leader care about only the productivity dispersion of the final good producers and follower also consider the elasticity of substitution. The economic rational for negative relationship between productivity dispersion and final good tariff is same as in chapter 2.

**Proposition-2:** Consider the above described Stackelberg unilateral tariff equilibrium and suppose \( \gamma = 2 \). Then the tariff rate selected by the follower will always be lower than the leader in case of intermediate inputs. If we allow the case of negative tariff (subsidy), then the retaliation options available to follower also include the possibility to select negative tariff rate. However, the tariff rate of final goods will be higher than the leader.

**Proof:** see the quantitative illustration section.

**4.5. Cooperative/Efficient Equilibrium Tariff**

Now, we assume cooperation among countries related to tariff level selection. Since, the welfare of each country depends upon own tariff policy and foreign tariff policy. Therefore, in the case of cooperation between the countries, both countries will maximize the joint welfare \( AW \);

\[
AW(\tilde{\tau}_{ji}, t_{ji}, \tilde{\tau}_{ij}, t_{ij}) = W_i(\tilde{\tau}_{ji}, t_{ji}, \tilde{\tau}_{ij}, t_{ij}) + W_j(\tilde{\tau}_{ji}, t_{ji}, \tilde{\tau}_{ij}, t_{ij})
\]

\[
AW(\tilde{\tau}_{ji}, t_{ji}, \tilde{\tau}_{ij}, t_{ij}) = 2 + CS_i(\tilde{\tau}_{ji}, t_{ji}, \tilde{\tau}_{ij}, t_{ij}) + TR_i(\tilde{\tau}_{ji}, t_{ji}) + CS_j(\tilde{\tau}_{ij}, t_{ij}, \tilde{\tau}_{ji}, t_{ij}) + TR_j(\tilde{\tau}_{ij}, t_{ij})
\]
The first order conditions entail:

$$\frac{\partial CS_i}{\partial P_i} \frac{\partial P_i}{\partial t_{ij}} + \frac{\partial CS_j}{\partial P_j} \frac{\partial P_j}{\partial t_{ij}} = \left( \frac{m_{ij}M_t \tau_{ij} \bar{x}_{ij}}{t_{ij}^2} \right) \left( (t_{ij} - 1) \zeta - 1 \right) - \frac{(t_{ij} - 1)}{t_{ij}} \left( m_{ij}M_t \tau_{ij} \bar{x}_{ij} + m_{ij}M_t \tau_{ij} \bar{x}_{ij} \frac{\partial \bar{x}_{ij}}{\partial t_{ij}} \right)
$$

$$\frac{\partial CS_i}{\partial P_i} \frac{\partial P_i}{\partial t_{ij}} + \frac{\partial CS_j}{\partial P_j} \frac{\partial P_j}{\partial t_{ij}} = \left( \frac{m_{ij}M_t \epsilon_{ij} R_{ij}}{t_{ij}^2} \right) \left( (t_{ij} - 1) \zeta - 1 \right) - \frac{(t_{ij} - 1)}{t_{ij}} \left( m_{ij}M_t \epsilon_{ij} R_{ij} + m_{ij}M_t \epsilon_{ij} R_{ij} \frac{\partial R_{ij}}{\partial t_{ij}} \right)
$$

By applying symmetry assumption and trade balance conditions, we end up with:

$$\frac{\partial CS_i}{\partial P_i} \frac{\partial P_i}{\partial t_{ij}} + \frac{\partial CS_j}{\partial P_j} \frac{\partial P_j}{\partial t_{ij}} = \frac{\partial CS_i}{\partial P_i} \frac{\partial P_i}{\partial t_{ij}} + \frac{\partial CS_j}{\partial P_j} \frac{\partial P_j}{\partial t_{ij}} = - \left( \frac{m_{ij}M_t \epsilon_{ij}}{t_{ij}^2} \right) \left( (t_{ij} - 1) \zeta - 1 \right) - \frac{(t_{ij} - 1)}{t_{ij}} \left( m_{ij}M_t \epsilon_{ij} + m_{ij}M_t \epsilon_{ij} \frac{\partial R_{ij}}{\partial t_{ij}} \right)
$$

The Proposition-1 indicates that the change in consumer surplus is negatively related to the change in the tariff rate. The imposition of a positive tariff by either country causes a fall in the joint welfare. Therefore, the cooperative tariff rates selected by both countries will be zero. Furthermore, as argued by Caliendo et al. (2017), the imposition of a positive tariff by either country reduces the entry and this reduction in entry causes a contraction of output and raises the price level. However, the tariff redistribution unable to offset entry reduction effect entirely. Therefore, free trade will be the outcome in the event of cooperation.

$$t_{ij}^E = t_{ij}^E = 0$$

4.6. Non-cooperative/Nash Equilibrium Tariff

Next, we formulate tariff selection in case of non-cooperation among countries. The difference of this policy experiment from the first case is that here each country set the tariff rates keeping in mind the retaliation from the other country. Both countries move simultaneously and select tariff levels in a non-cooperative manner. The Nash equilibrium is the outcome of this policy experiment and policymaker in each country solves the following problem;

$$\max_{\tilde{t}_{nm}, \tilde{t}_{nn}} W_n = 1 + CS_n(\tilde{t}_{nn}, \tilde{t}_{nn}, \tilde{t}_{nn}, \tilde{t}_{nn}) + TR_n(\tilde{t}_{nn}, \tilde{t}_{nn}), nn' \in \{i, j\}, n \neq n'$$

s.t. $IM_n^{int} - EX_n^{int} = m_{nn}M_n t_{nn} \tau_{nn} \bar{x}_{nn} - m_{nn}M_n t_{nn} \tau_{nn} \bar{x}_{nn} = 0, nn' \in \{i, j\}, n \neq n'$
\[ IM_{n}^{final} - EX_{n}^{final} = m_{nn}M_{n}N_{n}R_{nn} - m_{nn}M_{n}N_{n} = 0, \forall nn' \in \{i, j\}, n \neq n' \]

The first order conditions are:

\[
\frac{\partial W_{n}}{\partial t_{nn}} = \frac{\partial CS_{n}}{\partial t_{nn}} + \frac{t_{nn}(m_{nn} - 1)m_{nn}M_{n}N_{n}R_{nn}}{t_{nn}} \frac{\partial R_{nn}}{\partial t_{nn}} = 0, \forall nn' \in \{i, j\}, n \neq n',
\]

As we are seeking for symmetric Nash equilibrium\(^{16}\), therefore, \(t_{nn} = t_{nn'} = t\) and \(\bar{t}_{nn} = \bar{t}_{nn'} = \bar{t}\). The application of symmetry assumption gives following equilibrium tariff level:

\[
t^{N} = \frac{\zeta(\sigma-1)+2\sigma-1}{\zeta(\sigma-1)+\sigma} \quad \bar{t}^{N} = \frac{\sigma\beta+\sigma-1}{\sigma\beta-1}
\]

The intermediate inputs tariff specification again depends upon intermediate inputs import tariff elasticity and elasticity of substitution. Yet, the final goods tariff specification depends on productivity parameter and elasticity of substitution. The relationships of these parameters with policy rate selection is same as in case of unilateral tariff selection.

### 4.7 Political Equilibrium Tariff

Finally, we depict the case when the tariff selection process can be influenced by lobbying firms. Therefore, here we allow the participation in the lobby by the heterogeneous firms in order to make the policymaker to select a lower tariff level compare to the unilateral tariff level.\(^{17}\) The lobbying firms offer monetary benefits to the policymaker in response to the change of policy level selection. We assume lobbying possibility only exist in home country \(i\) and call the tariff level selected by policymaker at the home country \(i\) after lobby as the political tariff rate. We not only allow participation in lobbying activities by domestic firms but also by foreign firms as well.\(^{18}\) To articulate this political economy of tariff policy, we utilize the Grossman and Helpman (1994)

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\(^{16}\) Ogawa (2012) has identified the condition of symmetric price elasticity of numeraire good across all nonnumeraire for a uniform Nash tariff rate in the case of two countries. He has also characterized the case when these conditions do not satisfy.

\(^{17}\) For endogenous lobby formation see Mitra (1999) and in case of heterogeneous firms see Bombardini (2008).

\(^{18}\) See Gawande et al. (2006) and Stoyanov (2009) for foreign firms lobbying participation in the case of US.
“protection for sale (PFS)” framework. In order to proceed, assuming the policymaker in the home country \( i \) is willing to accept the political contribution \( C(t_{ji}) \) offered by the import/export participating firms. So, these political contributions appear along with fixed costs \( f_{ii} + z f_{ij} \) in the profit function of the heterogeneous firms.

**Firm’s Objective Function:** The objective functions of the firm from home country \( i \) and from foreign country \( j \) are given as;

\[
V_i(t_{ji}, C) = \hat{\pi}_i(t_{ji}, \tilde{t}_{ij}) - C(t_{ji})
\]
\[
V_j(\tilde{t}_{ji}, C) = \hat{\pi}_{ji}(\tilde{t}_{ij}) - C(\tilde{t}_{ij})
\]

where \( \hat{\pi}_i(t_{ji}, \tilde{t}_{ij}) = \hat{\pi}_{ii}(t_{ji}) + \hat{\pi}_{ij}(t_{ji}, \tilde{t}_{ij}) \) is the operating profit of the firm. The political contributions are assumed to be;

Assumption-1: The political contribution schedule is differentiable, at least around the equilibrium.

Assumption-2: The political contribution schedules are truthful, that is, given the welfare scalar \( B \);

\[
C(t_{ji}) = \max\{0, \hat{\pi}_i(t_{ji}, \tilde{t}_{ij}) - B\}
\]
\[
C(\tilde{t}_{ji}) = \max\{0, \hat{\pi}_{ji}(\tilde{t}_{ij}) - B\}
\]

**Policymaker’s Objective Function:** the utility function of the policymaker depends upon the social welfare and political contributions. The political contribution is positively related to the utility level of the policymaker. Thus, the single peaked preferences of the policymaker with respect to trade policy \( T_i \), with \( (t_{ji}, \tilde{t}_{ij} \in T_i) \) can be represented by the following utility function;

\[
\mathcal{U}(T_i, C) = a W_i(T_i) + \sum_{n,n' \in \{i,j\}} m_{nn'} M_{n' \rightarrow n} C(T_i)
\]

where \( a \in (0,1) \) is the weight that assigned to social welfare. This weight also indicates the degree of the benevolence of the policymaker, higher the policymaker valued social welfare, higher will be the value of the coefficient \( a \). Following the PFS framework, the conditions below describe the equilibrium tariff level on the intermediate inputs and final goods that will be selected by the policymaker (see appendix);

\[
a \frac{\partial W_i(T_i)}{\partial t_{ji}} + m_{ij} M_i \frac{\partial \tilde{\pi}_i(T_i)}{\partial t_{ji}} = 0
\]
\[
a \frac{\partial W_i(T_i)}{\partial \tilde{t}_{ji}} + m_{ji} M_j \frac{\partial \tilde{\pi}_j(T_i)}{\partial \tilde{t}_{ji}} = 0
\]

Therefore, the equilibrium tariff levels given by the above conditions are;
\[ t_{ji}^P = \frac{a(\varsigma(\sigma-1)+\varsigma)+(\sigma-\beta-1)}{(a+c+1)(\sigma-1)+a} \]
\[ t_{ji}^p = \frac{\beta(a\sigma-1)}{a\sigma(\beta-1)} \]

**Proposition-3:** Consider the above described policy experiments, the unilateral tariff rate will be highest compared to non-cooperative and political rates, and

\[ \frac{\partial t_{ji}^{SB}}{\partial \sigma} > 0, \frac{\partial t_{ji}^{N}}{\partial \sigma} > 0, \frac{\partial t_{ji}^{P}}{\partial \sigma} > 0 \]
\[ \frac{\partial t_{ji}^{SB}}{\partial \varsigma} < 0, \frac{\partial t_{ji}^{N}}{\partial \varsigma} < 0, \frac{\partial t_{ji}^{P}}{\partial \varsigma} < 0 \]
\[ \frac{\partial t_{ji}^{SB}}{\partial \beta} = \frac{\partial t_{ji}^{N}}{\partial \beta} = 0, \frac{\partial t_{ji}^{P}}{\partial \beta} < 0 \]
\[ \frac{\partial t_{ji}^{SB}}{\partial a} = \frac{\partial t_{ji}^{N}}{\partial a} = 0, \frac{\partial t_{ji}^{P}}{\partial a} > 0 \]

Proof: see the quantitative illustration section.

### 4.8. Quantitative Illustration

To make a comparison between these tariff formulas and to elaborate on how the policy level selection differs in a different framework, consider a quantitative illustration. In all formulations, the tariff rate specification contains the elasticity of substitution of differentiated goods \( \sigma \), import tariff elasticity \( \varsigma \), Pareto shape parameter \( \beta \), and political weight. In order to measure the elasticity of substitution and import tariff elasticity, we applied Feenstra (1994) approach by using US import data for the period of 2000 to 2006. The disaggregated data at HS 10-digit code is obtained from the center for international data.\(^{19}\) In order to differentiate between the trade of final goods and intermediate inputs, we use the concordance of BEC and HS.\(^{20}\) Additionally, in the case of followers intermediate inputs tariff formulation, the transport cost also present and the data on the transport cost obtained from ESCAP-World Bank International Trade Cost Database. While, to obtain the estimate for the Pareto shape parameter \( \beta \), we use the estimates of Bernard and Jensen (1999). Our elasticities estimation results yield \( \sigma = 4.55 \) and \( \varsigma = 8.08 \) and transport cost in case of US-Canada is around 28.5% for the period of analysis with Pareto parameters ranges from 8 to 9 as in Bernard and Jensen (1999). Given the estimates of all parameters and assuming the

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\(^{19}\) The data is available at https://cid.econ.ucdavis.edu/

elasticity of substitution between intermediate inputs $\gamma = 2$ for the sake of lucid description, the table below contains the quantitative illustration of tariff rates in case to all policy experiments.

The most prominent feature in the above table is that the tariff rate of the final goods is higher than the tariff on intermediate goods in all specifications. We assume the political weight equal to 2.85 in order to match our estimates with the factual data. The factual data on the tariff rates of intermediate inputs and final consumption goods for the US is collected from the World Bank’s World Integrated Trade Solution (WITS) for the years 2000-2006. We treated the MFN applied tariff rates on intermediate inputs and final consumption goods as the political tariff rate that results from the political equilibrium. For the period of analysis, the weighted average political tariff rate for the intermediate good is 1.88 and in case of final goods is 4.33. While, our estimation with political weight equal to 2.85 indicates the intermediate input tariff equal to 1.78 and final good tariff equal to 4.26, which are very close to the factual data.

The tariff rates are dependent on the variations of the parameters. The following table contains a representation of how the tariff rates depend upon the parameters.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\varsigma$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\tau$</th>
<th>$t_{ij}^{SB}$</th>
<th>$t_{ij}$</th>
<th>$t^{N}$</th>
<th>$\tilde{t}_{ij}^{P}$</th>
<th>$\tilde{t}_{ij}$</th>
<th>$\tilde{t}_{ij}^{N}$</th>
<th>$\tilde{t}_{ij}$</th>
<th>$\tilde{t}_{ij}^{N}$</th>
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<tr>
<td>2</td>
<td>4.55</td>
<td>8.08</td>
<td>8.50</td>
<td>1.119</td>
<td>1.106</td>
<td>1.106</td>
<td>0.977</td>
<td>1.023</td>
<td>1.133</td>
<td>1.193</td>
<td>1.120</td>
<td>1.008</td>
</tr>
<tr>
<td>4</td>
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<td>8.08</td>
<td>8.50</td>
<td>1.121</td>
<td>1.120</td>
<td>1.108</td>
<td>1.045</td>
<td>1.069</td>
<td>1.133</td>
<td>1.138</td>
<td>1.119</td>
<td>1.062</td>
</tr>
<tr>
<td>2</td>
<td>4.55</td>
<td>10.0</td>
<td>8.50</td>
<td>1.097</td>
<td>1.088</td>
<td>1.086</td>
<td>1.082</td>
<td>1.133</td>
<td>1.138</td>
<td>1.119</td>
<td>1.097</td>
<td>1.086</td>
</tr>
<tr>
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<td>4.55</td>
<td>8.08</td>
<td>10</td>
<td>1.119</td>
<td>1.106</td>
<td>1.106</td>
<td>0.953</td>
<td>1.007</td>
<td>1.111</td>
<td>1.176</td>
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<td>0.989</td>
</tr>
<tr>
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<td>4.55</td>
<td>10.0</td>
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<tr>
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<td>1.111</td>
<td>1.176</td>
<td>1.102</td>
<td>0.989</td>
</tr>
</tbody>
</table>

Table 2: The Tariff Rate in Case of Policy Experiments

Table 3: Parameters and the Tariff Rates
The table shows that in the case of unilateral tariff intermediate tariff selection, the leader will select a tariff rate that is higher than all other policy levels. While, in the case of unilateral final goods tariff, the follower selects a higher tariff rate. Another feature is shown in the table that the political tariff rates of intermediate inputs and final good are less than the unilateral tariff rates, which indicates the role that lobbying firms have played. However, the divergence of political tariff rate from the unilateral rate depends upon the degree of the benevolence of the policymaker. As the policymaker assign higher weights to social welfare, the political tariffs converge toward the unilateral tariff rates. Furthermore, the political tariff also depends negatively upon the productivity parameter $\beta$ of the Pareto distribution. As a high value of $\beta$ translates into low productivity dispersion among firms. When the productivity dispersion is low among heterogeneous firms, then tariff rate variations make the market selection more sensitive. Hence, the tariff rate will be low, if the productivity dispersion among heterogeneous firms is low.

Moreover, the elasticity of substitution between the differentiated varieties $\sigma$ is positively associated in all the tariff formulation, except political final good tariff. While, the elasticity of imported intermediate inputs has a negative relationship with intermediate tariffs. As the varieties of the final differentiated good become more substitutable, the incentive to have more varieties of differentiated good that used imported inputs will reduce. Therefore, apply a higher tariff will not affect the welfare much, against the case where the elasticity of substitution of final differentiated varieties is low. On the other hand, when the demand for imported intermediate inputs is more elastic with respect to the tariff, the policymaker will select a lower tariff level.

4.9. Conclusion

In this chapter, we have analyzed the trade policy of intermediate inputs and final goods in a two-country two-sector model. The model we develop indicates that only more productive firms use imported inputs in the production process. In case of intermediate inputs; the imposition of tariff affects the welfare negatively, as it erects a trade barrier that lowers the average productivity in the market. In this regard, we focus on the channel through which the productivity gets affected by intermediate input tariff. Then, we characterize the policymaker’s tariff selection process in the event of four policy experiments. First, we assume country behave unilaterally and select a tariff rate that maximizes her own welfare. The tariff level selected by the policymaker, in case of intermediate inputs, is high and the country enjoys first mover advantage. While, in the case of final goods, the follower will select a higher tariff compared to the leader. Second, the policy
experiment incorporates cooperation between the countries and both countries maximize the joint welfare. The outcome of this policy experiment is free trade. In the third scenario, we consider non-cooperative simultaneous tariff selection context and characterize symmetric Nash equilibrium. In our last step, we bring the political economy of tariff policy in the discussion and allow the possibility of lobbying.

The analysis can be extended in different dimensions. One particular dimension would be the introduction of asymmetry and analysis of the distributional effects of intermediate inputs change. Another dimension to extend this analysis would be to incorporate more than two types of firms along with vertical production process.

**Appendices**

**Appendix-A: Stackelberg Equilibrium Tariff:**

The Home Country’s Case (Leader)

**Intermediate Inputs Tariff:** the first order conditions for home country $i$ for the intermediate inputs tariff can be expressed explicitly as;

$$
\frac{\partial c_s(i, t_{ji}, x_{ji})}{\partial p_i} \frac{\partial p_i}{\partial t_{ji}} + m_{ij} M_i(t_{ji} - 1) t_{ji} \frac{\partial \bar{x}_{ji}}{\partial t_{ji}} + m_{ij} M_i t_{ji} \bar{x}_{ji} = 0
$$

Solving for $(t_{ji} - 1)$, with defining the elasticity of import $(-\zeta = \frac{\partial x_{ji}}{\partial t_{ji} \bar{x}_{ji}})$

$$(t_{ji} - 1) = \frac{t_{ji}}{\zeta} \left( \frac{\partial c_s(i, t_{ji}, x_{ji})}{\partial p_i} \frac{\partial p_i}{\partial t_{ji}}}{m_{ij} M_i t_{ji} \bar{x}_{ji}} + 1 \right)$$

(A.I)

Where;

$$m_{ij} M_i t_{ji} \bar{x}_{ji} = \frac{m_{ij} M_i}{1 - G(\varphi_{ij})} \int_{\varphi_{ij}}^{\infty} q_i(\varphi_i, z) \left( (1 - \alpha) \varphi_i \right)^{\gamma} \left( t_{ji} \varphi_i \right)^{1 - \gamma} \left[ 1 + \left( t_{ji} \varphi_i \right)^{1 - \gamma} \right]^{\frac{\sigma - \gamma}{\gamma - 1}} dG(\varphi_{ij})$$

(A.II)

However;

$$\frac{\partial c_s(i, t_{ji}, x_{ji})}{\partial p_i} \frac{\partial p_i}{\partial t_{ji}} = -P_i \frac{1}{1 - \theta} \frac{\partial p_i}{\partial t_{ji}}$$

The price index in terms of parameters of the model is given by;
\[ P_i = \left[ M_i^f \left( \left( \frac{\sigma}{\rho} \right) \Gamma \right)^{\sigma-1} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} + M_i^f \left( \frac{\sigma}{\rho} \right)^{\sigma-1} \Gamma^{\sigma-1} \left[ 1 + (t_{ji} \tau_{ji})^{1-\gamma} \right] \right]^{(1-\alpha)(\sigma-1)/(\gamma-1)} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} + \]

\[ M_j^f \left( \left( \frac{\sigma}{\rho} \right) \Gamma \right)^{\sigma-1} \Gamma^{\sigma-1} \left[ 1 + (t_{ji} \tau_{ji})^{1-\gamma} \right]^{(1-\alpha)(\sigma-1)/(\gamma-1)} \psi \phi \phi_{j} \phi_{i}^{\sigma-\beta-1} \]

\[ \frac{\partial P_i}{\partial t_{ji}} = \left[ M_i^f \left( \left( \frac{\sigma}{\rho} \right) \Gamma \right)^{\sigma-1} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} + M_i^f \left( \frac{\sigma}{\rho} \right)^{\sigma-1} \Gamma^{\sigma-1} \left[ 1 + (t_{ji} \tau_{ji})^{1-\gamma} \right] \right]^{(1-\alpha)(\sigma-1)/(\gamma-1)} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} + \]

\[ (t_{ji} \tau_{ji})^{1-\gamma} \left[ 1 + (t_{ji} \tau_{ji})^{1-\gamma} \right]^{(1-\alpha)(\sigma-1)/(\gamma-1)} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} \]

\[ \frac{\partial P_i}{\partial t_{ji}} = \left[ M_i^f \left( \left( \frac{\sigma}{\rho} \right) \Gamma \right)^{\sigma-1} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} + M_i^f \left( \frac{\sigma}{\rho} \right)^{\sigma-1} \Gamma^{\sigma-1} \left[ 1 + (t_{ji} \tau_{ji})^{1-\gamma} \right] \right]^{(1-\alpha)(\sigma-1)/(\gamma-1)} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} + \]

\[ (t_{ji} \tau_{ji})^{1-\gamma} \left[ 1 + (t_{ji} \tau_{ji})^{1-\gamma} \right]^{(1-\alpha)(\sigma-1)/(\gamma-1)} \psi \phi \phi_{i} \phi_{i}^{\sigma-\beta-1} \]

Hence;

\[ \frac{\partial C S_i}{\partial P_i} \frac{\partial p_i}{\partial t_{ji}} = -M_i^f \phi_i \phi_{i}^{\sigma-\beta-1} = \sigma (f_{ii} + zf_{ij}) \]

\[ \phi_i = \left( t_{ji} \sigma \phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1} = \sigma (f_{ii} + zf_{ij}) \]

From the ratio of conditions;

\[ \frac{\sigma \phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1}}{\phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1}} = \sigma (f_{ii} + zf_{ij}) \]

\[ \frac{\partial \phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1}}{\partial t_{ji}} = \frac{(1-\alpha)(\sigma-1)}{(\sigma-\beta-1)} \phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1} \]

\[ \frac{\partial \phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1}}{\partial t_{ji}} = \frac{(1-\alpha)(\sigma-1)}{(\sigma-\beta-1)} \phi_{i} \phi_{i}^{\sigma-\beta-1} \phi_{i} \phi_{i}^{\sigma-\beta-1} \]

By plugging equation (A.III) and (A.II) into (A.I)

\[ (t_{ji} - 1) = \frac{t_{ji}}{\sigma} \left( \frac{(1-\alpha)}{(\sigma-1)} + 1 \right) \]

\[ t_{ji} = \frac{\sigma}{(\sigma-1)} \left( \frac{(1-\alpha)}{(\sigma-1)} + 1 \right) \]

---

21 By considering the Mass of importers and Pareto distribution, the zero-profit conditions are;
**Final Goods Tariff:** the first order condition with respect to final goods tariff can also be expressed as:

\[
\frac{\partial C_S(t_{ij},t_{ij},t_{ij},t_{ij})}{\partial P_i} \frac{\partial P_i}{\partial t_{ij}} + m_{ij} M_j \left( \tilde{t}_{ij} - 1 \right) \frac{\partial R_{ji}}{\partial t_{ij}} + m_{ij} M_j \tilde{R}_{ji} = 0
\]

Which gives,

\[
\left( \tilde{t}_{ij} - 1 \right) = - \frac{m_{ij} M_j \tilde{R}_{ji}}{m_{ij} M_j \frac{\partial R_{ji}}{\partial t_{ij}}} - \frac{\frac{\partial C_S(t_{ij},t_{ij},t_{ij},t_{ij})}{\partial P_i} \frac{\partial P_i}{\partial t_{ij}}}{m_{ij} M_j \frac{\partial R_{ji}}{\partial t_{ij}}} \tag{A.IV}
\]

Where;

\[
m_{ij} M_j \tilde{R}_{ji} = M_j^e \tilde{t}_{ij} \frac{\sigma(1-\theta-1)}{(1-\theta)} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \zeta_j \Gamma \right)^{\sigma-1} \psi_{\bar{p}}^\theta \psi_{j_i}^{*-\beta-1}
\]

\[
m_{ij} M_j \frac{\partial R_{ji}}{\partial t_{ij}} = M_j^e \tilde{t}_{ij} \frac{\sigma(1-\theta-1)}{(1-\theta)} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \zeta_j \Gamma \right)^{\sigma-1} \psi_{\bar{p}}^\theta \psi_{j_i}^{*-\beta-1} \left( -\sigma + (\sigma - \beta - 1) \frac{\partial \ln \psi_{j_i}^*}{\partial \ln \tilde{t}_{ji}} \right) \tag{A.V}
\]

\[
\frac{\partial C_S l}{\partial t_{ij}} = -M_j^e \frac{\sigma(1-\theta-1)}{(1-\theta)} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \zeta_j \Gamma \right)^{\sigma-1} \Gamma_{\sigma-1} \left( \tau_{ji} \right)^{1-\sigma} \tilde{t}_{ij}^{*-\sigma} \psi_{\bar{p}}^\theta \psi_{j_i}^{*-\beta-1} \left( \frac{1}{1-\sigma} \right) \tag{A.VI}
\]

By plugging A.V-A.VII into A.IV, we will have;

\[
\left( \tilde{t}_{ij} - 1 \right) = \frac{\tilde{t}_{ij}}{\beta} + \frac{\tilde{t}_{ji}^{(\sigma-1)}}{\sigma \beta}
\]

**Foreign Country’s Case (Follower)**

**Intermediate Inputs Tariff:** the first order condition will be;

\[
\frac{\partial C_S(\tilde{t}_{j},\tilde{t}_{j}^{-1},\tilde{t}_{j}^{-1},\tilde{t}_{j}^{-1})}{\partial P_j} \frac{\partial P_j}{\partial t_{ij}} + \frac{m_{ij} M_j t_{ij}^{(1-\theta-1)} t_{ij}^{(1-\theta-1)} t_{ij}^{(1-\theta-1)} t_{ij}^{(1-\theta-1)}}{(\tau_{ij})^2} = 0
\]

By using the analogy of equation (A.II) and (A.III);

\[
\psi_{j_i}^* = \tilde{t}_{ji}^{\frac{\sigma}{\sigma-1}} \Gamma_{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right) \left( \sigma f_{ji} + z f_{ji} \right)^{1-\sigma}
\]

\[
\psi_{j_i}^* = M_j^e \frac{\sigma(1-\theta-1)}{(1-\theta)} \Gamma_{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right) \left( \sigma f_{ji} + z f_{ji} \right)^{1-\sigma}
\]

In terms of ratio

\[
\psi_{j_i}^* = \frac{\tilde{t}_{ji}^{\frac{\sigma}{\sigma-1}}}{\frac{\tau_{ji}}{\tau_{ji}^{\sigma-1}}} t_{ji}^{\frac{\sigma}{\sigma-1}} \left( \frac{\tilde{t}_{ji}^{(1-\sigma)} + z f_{ji}}{f_{ji}} \right)^{1-\sigma} \psi_{j_i}^*
\]

\[
\frac{\partial \ln \psi_{j_i}^*}{\partial \ln \tilde{t}_{ji}} = \frac{\sigma}{\sigma-1}
\]

---

22 As the productivity cutoffs are;

\[
\psi_{j_i}^* = \tilde{t}_{ji}^{\frac{\sigma}{\sigma-1}} \Gamma_{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right) \left( \sigma f_{ji} + z f_{ji} \right)^{1-\sigma}
\]

\[
\psi_{j_i}^* = M_j^e \frac{\sigma(1-\theta-1)}{(1-\theta)} \Gamma_{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right) \left( \sigma f_{ji} + z f_{ji} \right)^{1-\sigma}
\]

In terms of ratio

\[
\psi_{j_i}^* = \frac{\tilde{t}_{ji}^{\frac{\sigma}{\sigma-1}}}{\frac{\tau_{ji}}{\tau_{ji}^{\sigma-1}}} t_{ji}^{\frac{\sigma}{\sigma-1}} \left( \frac{\tilde{t}_{ji}^{(1-\sigma)} + z f_{ji}}{f_{ji}} \right)^{1-\sigma} \psi_{j_i}^*
\]

\[
\frac{\partial \ln \psi_{j_i}^*}{\partial \ln \tilde{t}_{ji}} = \frac{\sigma}{\sigma-1}
\]

---

67
\[
M_j^\alpha P_j^{\alpha(1-\theta)-1} \left( \frac{\sigma-1}{\sigma} \right)^{\sigma-1} \Gamma^{\sigma-1} \left[ 1 + (t_{ij} \tau_{ij})^{1-\gamma} \right]^{\alpha-\gamma+\sigma(1-\alpha)(1-\theta)} \psi^{\alpha} \varphi_j^{\sigma-\beta-1} t_{ij}^{-\gamma-1} \tau_{ij}^{-1-\gamma} (1 - \alpha) (t_{ij} - 1) = \frac{M_i^\alpha P_i^{\alpha(1-\theta)-1} (1-\alpha)(t_{ij}\tau_{ji})^{1-\gamma} \left[1 + (t_{ji} \tau_{ji})^{1-\gamma}\right]^{\alpha-\gamma+\sigma(1-\alpha)(1-\theta)} \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \Gamma^{\sigma-1} \psi^{\alpha} \varphi_i^{\sigma-\beta-1} \tau_{ji}^{-\gamma-1} \tau_{ji}^{-1-\gamma}}{t_{ij}^{\gamma-1}}
\]

By using the definition of the productivity cutoffs and \( f_{ij} = f_{jj} \) and \( \tau_{ij} = \tau_{ji} \),

\[
t_{ij}^{1-\gamma} (t_{ij}-1) \frac{(t_{ij})^{1-\gamma}}{1+(t_{ji} \tau_{ji})^{1-\gamma}} = \frac{(t_{ji})^{1-\gamma}}{(t_{ji})^{1-\gamma} - (t_{ji} \tau_{ji})^{1-\gamma}}
\]

Which gives;

\[
t_{ij} = \frac{(t_{ji})^{(\sigma-1)/\sigma} \Gamma^{1-\gamma}}{(t_{ji})^{1-\gamma} - (t_{ji} \tau_{ji})^{1-\gamma}}
\]

**The final goods Tariff:** The first order condition is given as;

\[
\frac{\partial CS_j(t_{ij}, t_{ji}, t_{ij}, t_{ji})}{\partial t_{ij}} + \frac{m_j M_j t_{ij} \bar{R}_{ji}}{t_{ij}^2} = 0
\]

\[
P_j^{\frac{\sigma(1-\theta)-1}{\sigma}} \left( \frac{\sigma-1}{\sigma} \right)^{\sigma-1} \Gamma^{\sigma-1} \xi_i^{\sigma-1} (t_{ij})^{1-\sigma} \tau_{ij}^{2-\sigma} \varphi_{ij}^{\sigma-1} \left( \frac{1}{1-\sigma} \right) = \bar{t}_{ij}^{1-\sigma} P_i^{\frac{\sigma(1-\theta)-1}{\sigma}} \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \varphi_{ji}^{\sigma-1}
\]

\[
\varphi_{ij}^* = \bar{t}_{ij}^{\frac{\sigma}{\sigma-1}} P_j^{\frac{\sigma(1-\theta)-1}{\sigma}} \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \xi_i^{\sigma-1} \Psi_j^{\sigma-1} \left( \frac{1}{1-\sigma} \right) \Gamma^{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \varphi_{ji}^{\sigma-1}
\]

Appendix-B: Non-Cooperative/Nash Equilibrium Tariff;

\[
\max_{\varphi_{nn}, \varphi_{nm}} W_n = 1 + CS_n(\varphi_{nn}, \varphi_{nm}, \varphi_{nn}, \varphi_{nm}) + TR_n(\varphi_{nn}, \varphi_{nm}), n \neq n'
\]

s.t.

\[
IM_{n}^{\text{int}} - EX_{n}^{\text{int}} = m_{nn} M_n \varphi_{nn} \varphi_{nm} \varphi_{nn} - m_{nn} M_n \varphi_{nn} \varphi_{nm} \varphi_{nn} = 0, n \neq n'
\]

68
\( IM_n^{t \text{inal}} - EX_n^{t \text{inal}} = m_{n'n'} M_{n'} \tilde{t}_{n'n'} \tilde{R}_{n'n'} - m_{n'n'} M_n \tilde{t}_{n'n'} \tilde{R}_{n'n'} = 0 \), \( nn' \in \{i, j\} , nn' \neq n' \)

The first order conditions are;

\[
\frac{\partial W_n}{\partial t_{nn'}} = \frac{\partial C_{nn'}}{\partial t_{nn'}} + t_n \left( (t_{nn'}-1) m_{n'n'} M_{n'n'} \frac{\partial R_{n'n'}}{\partial t_{n'n'}} + m_{n'n'} M_{n'n'} \frac{\partial R_{n'n'}}{\partial t_{n'n'}} \right)
\]

\[
= 0 , \quad nn' \in \{i, j\} , nn' \neq n' \]

\[
\frac{\partial w_n}{\partial t_{nn'}} = \frac{\partial C_{nn'}}{\partial t_{nn'}} + \tilde{t}_{nn'} \left( (\tilde{t}_{nn'}-1) m_{n'n'} M_{n'n'} \frac{\partial R_{n'n'}}{\partial \tilde{t}_{n'n'}} + m_{n'n'} M_{n'n'} \frac{\partial R_{n'n'}}{\partial \tilde{t}_{n'n'}} \right)
\]

\[
= 0 , \quad nn' \in \{i, j\} , nn' \neq n' \]

\[
\{ i, j \} , nn' \neq n' \]

Symmetric Nash equilibrium, therefore, \( t_{ij} = t_{ji} = t \) and \( \tilde{t}_{ij} = \tilde{t}_{ji} = \tilde{t} \) . Hence, \( \tilde{x}_{ij} = \tilde{x}_{ji} \) and \( \tilde{R}_{ij} = \tilde{R}_{ji} \)

\[
\frac{\partial \psi_i}{\partial t_{ij}} = \frac{\partial C_i}{\partial t_{ij}} + \frac{t_{ij} \left( (t_{ij}-1) m_{ij} M_{ij} \frac{\partial R_{ij}}{\partial t_{ij}} + m_{ij} M_{ij} \frac{\partial R_{ij}}{\partial t_{ij}} \right) - (t_{ij}-1) m_{ij} M_{ij} t_{ij} \tilde{x}_{ij}}{t} = 0
\]

\[
\frac{\partial \psi_i}{\partial t_{ij}} = \frac{\partial C_i}{\partial t_{ij}} + \frac{t_{ij} \left( (t_{ij}-1) m_{ij} M_{ij} \frac{\partial R_{ij}}{\partial t_{ij}} + m_{ij} M_{ij} \frac{\partial R_{ij}}{\partial t_{ij}} \right) - (t_{ij}-1) m_{ij} M_{ij} t_{ij} \tilde{x}_{ij}}{t} = 0
\]

\[
\{ i, j \} , nn' \neq n' \]

\[
\{ i, j \} , nn' \neq n' \]

Appendix-C: Political Equilibrium Tariff;

\( (C^*, T_i^*) \) is the sub-game Nash equilibrium of trade policy game between policymaker and the lobbying firms if and only if;

i) \( C^* \) is feasible to all firms that employ imported intermediate inputs.

\[
(t - 1) = \frac{-\sigma (t - 1)}{\varsigma (\sigma - 1)} + \frac{1}{\varsigma}
\]

\[
(\tilde{t} - 1) = \frac{\tilde{t}}{\sigma \beta} + \frac{(\sigma - 1)}{\varsigma (\sigma - 1) + \sigma}
\]

\[
t = \frac{\varsigma (\sigma - 1) + 2 \sigma - 1}{\varsigma (\sigma - 1) + \sigma}
\]

\[
\tilde{t} = \frac{\sigma \beta + \sigma - 1}{\sigma \beta - 1}
\]
ii) \( T_i^* \) maximized \( aW_i(T_i) + \sum_{n,n' \in (i,j)} m_{nn'} M_n C(T_i) \) on \( T_i \), given \((t_{ji}, \tilde{t}_{ji} \in T)\).

iii) \( T_i^* \) maximizes \( \tilde{F}_{nn}(T_i^*) - C^*(T_i^*) + aW_i(T_i^*) + \sum_{n,n' \in (i,j)} m_{nn'} M_n C^*(T_i^*) \) on \( T \) for every firm in the market \( i \).

iv) For every firm \( h \in m_{nn'}.M_n \) in the market \( i \) there exists a \( T_i^{-h} \in T_i \) that maximizes \( aW_i(T_i) + \sum_{n,n' \in (i,j)} m_{nn'} M_n C^*(T_i) \) on \( T \) such that \( C_h(T_i^{-h}) = 0 \).

The condition (i) restricts the contribution schedule for each firm that participates in the lobby is feasible. While, condition (ii) indicates that the policymaker maximizes his own welfare given the contribution schedules offered by the lobbying firms, and (iii) states the equilibrium tariff level must maximizes the joint welfare of both. The last condition (iv) describes that at equilibrium the policy choice may yields no political contribution and hence no lobby participation by some firms.

Hence, from condition (iii) the first order condition will be:

\[
\frac{\partial \tilde{F}_{nn}(T_i^*)}{\partial T_i^*} - \frac{\partial C^*(T_i^*)}{\partial T_i^*} + a \frac{\partial W_i(T_i^*)}{\partial T_i^*} + \sum_{n,n' \in (i,j)} m_{nn'} M_n \frac{\partial C^*(T_i^*)}{\partial T_i^*} = 0
\]

C.I

However, the policymaker’s maximization requires:

\[
a \frac{\partial W_i(T_i^*)}{\partial T_i^*} + \sum_{n,n' \in (i,j)} m_{nn'} M_n \frac{\partial C^*(T_i^*)}{\partial T_i^*} = 0
\]

C.II

Taking the above conditions together yields:

\[
a \frac{\partial \tilde{F}_{nn}(T_i^*)}{\partial T_i^*} + \sum_{n,n' \in (i,j)} m_{nn'} M_n \frac{\partial \tilde{F}_{nn}(T_i^*)}{\partial T_i^*} = 0
\]

C.III

By summing over all lobbying firms and substituting (C.III) into (C.I) gives the equation that characterizes the equilibrium tariff level, which is:

\[
a \frac{\partial W_i(T_i^*)}{\partial T_i^*} + \sum_{n,n' \in (i,j)} m_{nn'} M_n \frac{\partial \tilde{F}_{nn}(T_i^*)}{\partial T_i^*} = 0
\]

More explicitly:

\[
a \frac{\partial W_i(T_i^*)}{\partial \tilde{t}_{ji}} + m_{ij} M_i \frac{\partial \tilde{F}_i(T_i^*)}{\partial \tilde{t}_{ji}} = 0 \quad \text{C.IV}
\]

\[
a \frac{\partial W_i(T_i^*)}{\partial \tilde{t}_{ji}} + m_{ij} M_j \frac{\partial \tilde{F}_j(T_i^*)}{\partial \tilde{t}_{ji}} = 0 \quad \text{C.V}
\]

**Intermediate Inputs Tariff:** Now, consider the condition (C.IV).

\[
a \left( \frac{\partial CS(t_{ji}, \tilde{t}_{ji})}{\partial p_i} \right) \frac{\partial p_i}{\partial t_{ji}} + m_{ij} M_i \left( t_{ji} - 1 \right) \tau_{ji} \frac{\partial \tilde{e}_{ji}}{\partial t_{ji}} + m_{ij} M_i \tau_{ji} \tilde{x}_{ji} + m_{ij} M_i \frac{\partial \tilde{e}_{ji}}{\partial t_{ji}} = 0
\]

Solving for \( (t_{ji} - 1) \):
\[
(t_{ji} - 1) = \frac{t_{ji}}{s} \left( \frac{\partial c(t_{ji}, \tilde{t}_{ji}) \partial p_i}{\partial \tilde{t}_{ji}} - \frac{\partial \pi_i(t_{ji})}{\partial \tilde{t}_{ji}} + 1 \right) C.VI
\]

The new term in the above equation compare to the equation in Appendix-A, is the operating profit only. Which is give in terms of parameters as;

\[
\hat{\pi}_i(t_{ji}) = \frac{1}{\sigma} P_i \left( \frac{\sigma(1-\theta)-1}{(1-\theta)} \left( \frac{1}{\sigma} \right) \Gamma \right) \left[ 1 + (\tau_{ji} t_{ji})^{1-\gamma} \right] \left[ \frac{(1-a)(\sigma-1)}{(1-\gamma)(\sigma-1)} \psi \phi^\beta \phi_i^* \sigma^{\beta-1} \right]
\]

Therefore,

\[
\frac{\partial \hat{\pi}_i(t_{ji})}{\partial \tau_{ji}} = \frac{\sigma(1-\theta)-1}{(1-\theta)} \left( \frac{1}{\sigma} \right) \Gamma \left[ 1 + (\tau_{ji} t_{ji})^{1-\gamma} \right] \left[ \frac{(1-a)(\sigma-1)}{(1-\gamma)(\sigma-1)} \psi \phi^\beta \phi_i^* \sigma^{\beta-1} \right]
\]

\[
(t_{ji} - 1) = \frac{1}{\sigma} P_i \left( \frac{\sigma(1-\theta)-1}{(1-\theta)} \left( \frac{1}{\sigma} \right) \Gamma \right) \left[ 1 + (\tau_{ji} t_{ji})^{1-\gamma} \right] \left[ \frac{(1-a)(\sigma-1)}{(1-\gamma)(\sigma-1)} \psi \phi^\beta \phi_i^* \sigma^{\beta-1} \right] + \frac{1}{\alpha} \left( \frac{(1-\sigma)(t_{ji}+\sigma-\beta-1)}{\sigma-1} \right) \frac{\partial \ln \phi_i^*}{\partial \tau_{ji}} C.VII
\]

Now, by plugging equation (A.III), (A. II), and (C.VII) into the equation (C.VI) yields;

\[
(t_{ji} - 1) = \frac{t_{ji}}{\sigma} \left( \frac{\sigma(1-\theta)-1}{(1-\theta)} \left( \frac{1}{\sigma} \right) \Gamma \right) \left[ 1 + (\tau_{ji} t_{ji})^{1-\gamma} \right] \left[ \frac{(1-a)(\sigma-1)}{(1-\gamma)(\sigma-1)} \psi \phi^\beta \phi_i^* \sigma^{\beta-1} \right] + \frac{1}{\alpha} \left( \frac{(1-\sigma)(t_{ji}+\sigma-\beta-1)}{\sigma-1} \right) \frac{\partial \ln \phi_i^*}{\partial \tau_{ji}} C.VII
\]

Then solve for \( t_{ji} \);

\[
t_{ji}^p = \frac{a(c(\sigma-1)+\sigma+\alpha-\gamma-1)}{(\alpha c+1)(\sigma-1)+\alpha}
\]

**The final goods Tariff:** The condition (C.V) gives the political tariff rate of the final goods;

\[
\alpha \left( \frac{\partial c(t_{ji}, \tilde{t}_{ji}) \partial p_i}{\partial \tilde{t}_{ji}} + m_{ji} M_{j} \left( \tilde{t}_{ji} - 1 \right) \frac{\partial R}{\partial \tilde{t}_{ji}} + m_{ji} M_{j} \tilde{R}_{ji} \right) + m_{ji} M_{j} \frac{\partial R_{ji}}{\partial \tilde{t}_{ji}} = 0 C.VIII
\]

\[
(t_{ji} - 1) = \frac{t_{ji}}{\sigma} \left( \frac{\sigma(1-\theta)-1}{(1-\theta)} \left( \frac{1}{\sigma} \right) \Gamma \right) \left[ 1 + (\tau_{ji} t_{ji})^{1-\gamma} \right] \left[ \frac{(1-a)(\sigma-1)}{(1-\gamma)(\sigma-1)} \psi \phi^\beta \phi_i^* \sigma^{\beta-1} \right] + \frac{1}{\alpha} \left( \frac{(1-\sigma)(t_{ji}+\sigma-\beta-1)}{\sigma-1} \right) \frac{\partial \ln \phi_i^*}{\partial \tau_{ji}} C.VII
\]

\[23\] From the productivity cutoff ratio;

\[
\phi_i^* = \left[ 1 + (\tau_{ji} t_{ji})^{1-\gamma} \right] \left[ \frac{(1-a)(\sigma-1)}{(1-\gamma)(\sigma-1)} \psi \phi^\beta \phi_i^* \sigma^{\beta-1} \right] + \ln D
\]

\[
\frac{\partial \ln \phi_i^*}{\partial \tilde{t}_{ji}} = \frac{\tau_{ji} t_{ji}^{1-\gamma} \psi \phi_i^* \sigma^{\beta-1}}{1 + (\tau_{ji} t_{ji})^{1-\gamma}} C.VII
\]
As, the operating profit function in terms of the model’s parameters is;

\[ \hat{\pi}_{ji}(\hat{t}_{ji}) = \frac{1}{\sigma} M_j^\sigma \ell_{ji}^{-\sigma} P_i^{\sigma(1-\theta)\sigma-1} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \xi \right)^{\sigma-1} \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \]

Therefore,

\[ \frac{\partial \hat{\pi}_{ji}(\hat{t}_{ji})}{\partial \ell_{ji}} = M_j^\sigma \ell_{ji}^{-\sigma} P_i^{\sigma(1-\theta)\sigma-1} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \xi \right)^{\sigma-1} \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( -1 + (\sigma - \beta - 1) \frac{\partial \ln \varphi_{ji}^*}{\partial \ln \ell_{ji}} \right) \]

\[ \frac{\partial \hat{\pi}_{ji}(\hat{t}_{ji})}{\partial \tau_{ji}} = M_j^\sigma \ell_{ji}^{-\sigma} P_i^{\sigma(1-\theta)\sigma-1} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \xi \right)^{\sigma-1} \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( - \frac{\beta}{\sigma-1} \right) \]

Hence,

\[ (\hat{t}_{ji} - 1) = \frac{M_j^\sigma \ell_{ji}^{-\sigma} P_i^{\sigma(1-\theta)\sigma-1} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \xi \right)^{\sigma-1} \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( \frac{1}{1-\sigma} \right)}{M_j^\sigma \ell_{ji}^{-\sigma} P_i^{\sigma(1-\theta)\sigma-1} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \xi \right)^{\sigma-1} \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( \frac{1}{1-\sigma} \right) \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( - \frac{\sigma \beta}{\sigma-1} \right)} + \]

\[ aM_j^\sigma \ell_{ji}^{-\sigma} P_i^{\sigma(1-\theta)\sigma-1} \left( \tau_{ji}^{-1} \left( \frac{\sigma-1}{\sigma} \right) \xi \right)^{\sigma-1} \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( \frac{1}{1-\sigma} \right) \psi \phi^\theta \varphi_{ji}^{\sigma-\beta-1} \left( - \frac{\sigma \beta}{\sigma-1} \right) \]

\[ (\hat{t}_{ji} - 1) = \frac{\ell_{ji} - 1}{\alpha \sigma} + \frac{\ell_{ji} \left( \sigma - 1 \right)}{\alpha \beta} \]

\[ \frac{\tau_{ji}^p}{\ell_{ji}^p} = \frac{\beta (a \sigma - 1)}{\alpha \sigma (\beta - 1)} \]
5. HETEROGENEOUS FIRMS AND TECHNOLOGY ADOPTION: THE ROLE OF POLITICAL INSTITUTIONS AND MARKET SIZE

Many economists have underlined the importance of political institutions and policies for the adoption and diffusion of new technologies.\(^{24}\) At the same time, the decision to adopt new technology is the decision of an individual firm of the economy. Thus, the technology adoption in an economy critically depends upon the behavior of firms towards the adoption.\(^{25}\) However, there is still a lack of a framework to analyze, how political institutions impact the behavior of an individual firm towards technology adoption. In particular, if new technology makes firms more productive and enhance welfare, why do firms resist to adopt it in some societies? Furthermore, what role the market size plays, along with the political orientation of the society, in the technology adoption decision of a firm? These are the key points that this chapter seeks to address.

Acemoglu (2007) shows that inefficient institutions generate inefficient policies and the existence of inefficient institutions is due to the induce preferences of power groups. Therefore, in a society where the elite controls political power, the policy formation always intends either to extract revenue, manipulate factor prices, or consolidate political power. One particular illustration of policymaking aimed for power consolidation is the oligarchic society, where power maintenance is ensured by having entry barriers and full property right enforcements. However, these entry barriers cause economic losses, in the long run (Acemoglu, 2008). Nevertheless, the adoption of new technology could potentially create political losers and contains a political threat to the elite. Accordingly, the incumbent political power holding elite erect barriers against technology adoption (Acemoglu and Robinson, 2000). Parente and Prescott (1994) provide empirical evidence that the technology adoption barriers are the primary elements in explaining economic performances on the front of income disparity among countries. The technology adoption in the economies with large adoption barriers is slow as firms have to make larger investments for the adoption. Besides, the technology adoption decision also rests upon the market size. For instance, the technology adoption is very responsive to the trade openness as trade enhances the market size that a firm can serve, Atkeson and Burstein (2008). Therefore, the welfare gains from trade


\(^{25}\) For discussion see, Ludema and Takeno (2007), Liu et al. (2001), Comin and Hobijn (2009), Weymouth (2012), or Atkeson and Burstein (2008).
openness originate not only from the productivity gains but also from the rapid technology adoption by firms.

The purpose of this chapter is to develop a political economy model with heterogeneous firms at the helm of the technology adoption decision making. The two-country two-factor model assumes that the population is divided into three groups: the elite political power holding group, middle-class entrepreneur group, and labor group. The policy option available to the elite involves only trade policy and elite formulate the trade policy in order to maximize their own welfare. The middle class have the access to the production technology that involves labor and capital as the factors of production. While labor class supply labor inelastically. Furthermore, the model assumes technological difference among countries and one country has superior technology compared to the other. The firms with inferior technology face the critical problem either to adopt the superior technology or not. The adoption is costly, and firms have to incur a fixed cost of adoption in the form of R&D. On the other hand, the model also assumes that firms can resist the adoption and lobby for higher trade restriction whereby the foreign importing firms are excluded from the competition in the domestic market.

The contributions of this work to the literature are threefold. First, it develops a political economy model with production sector comprises upon heterogeneous firms as in Melitz (2003). The model elaborates the trade policy formation explicitly and how trade policy affects the entry and exit conditions of the firm. Second, the response of firms with respect to the trade openness in case of having an inferior production technology has been explored. As the model assumes a limited set of available varieties, therefore, domestic firm resists trade openness not only on technological inferiority base but also on the anti-competition basis. Last and most important, the model seeks to characterize the decision of an individual firm to adopt new technology or not. The role of the political institution and the market size in the adoption decision of a firm has been elaborated.

The theoretical excursion shows that the technology adoption decision of a firm in an economy is contingent upon the market size. Firms in a large market adopt new technology more rapidly than the firms in a small market. Moreover, the decision of firms to adopt new technology also depends upon the political orientation of the country. Since policymaker select import tariff and export tax as the trade policy tools in the equilibrium. Therefore, in the case of a weak democracy, where policymaking is not exclusively conditional on political consensus, firms lobby and persuade elite policymakers to impose a higher import tariff. By having higher protection from the foreign
technological advance importing firms, domestic firms shield themselves from the competition in a small market. However, in case of having a strong democracy, where policy amendments are done via due process, firms refrain from lobbying and adopt new technology. Another important result emerges from the model is that firms adopt technology when the productivity gain from the adoption are relatively larger and new technology is way much superior than the current state of firm’s technology. This is intuitive as the technology adoption decision of the firm also involves the cost and benefit analysis; a firm compares the relative benefits of the adoption with the cost of adoption and if the latter is greater than the former, then the firm will never adopt new technology.

5.3. **Empirical Motivation**

The model presented here is driven by the empiric on the relative performances of Indian and Pakistani auto sector. The auto sector in both countries has similar starting circumstances, but the current state of its progress and growth is far asunder. The auto sector of Pakistan represents 16 percent of total manufacturing and contributes merely 2.8 percent to GDP and provides 200,000 direct employment opportunities (Bari et al. 2016). Moreover, the auto sector embodies 5.27 percent of value-added in total manufacturing in Pakistan, (Qadir, 2016). According to the International Organization of Motor Vehicle Manufacturers (OICA) data, Pakistan ranked 30th in the world ranking of motor vehicle producers and has the lowest level of motor vehicles production i.e. 1.7 per 1000 people. The existing composition of the auto sector of Pakistan has only 3 passenger car manufacturers (Honda, Toyota, and Suzuki) and 4 commercial vehicle producers (Hino, Nissan, Master, and Isuzu). Furthermore, the market size in Pakistan is small (according to the United Nations, Pakistan rank 25th in the market size measured by the households final consumption expenditure) and consumer choice is limited due to high market concentration.

In contrast, the auto sector of India comprises 49 percent of national manufacturing and contributes 7.1 percent to GDP with a growth rate of 14.5 percent in the year 2019. India is the 4th largest motor vehicle producer and 7th largest commercial vehicle producer in the world, according to OICA. Currently, the passenger car market has 14 manufacturers with an average growth rate of 9.5 percent. According to the Department for Promotion of Industry and Internal Trade Statistics (DPIIT) India, the auto sector of India has received $21.38 billion foreign direct investment.

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26 For a historical review see, Pasha and Ismail (2012) for Pakistan and Tiwari and Kalogerakis (2017) for India.  
27 For details see, Pakistan Automotive Manufacturers Association (PAMA) and Pakistan Association of Auto Parts and Accessories Manufacturers (PAAPAM).  
between 2000-2019. Furthermore, during 2018-19 Indian auto sector exports 46,29,054 units of automobiles. In summary, the table below contains a brief snapshot of the production of the auto sector in both countries.

Table 4: The Production of Automotive Sector (2018)

<table>
<thead>
<tr>
<th></th>
<th>Pakistan</th>
<th></th>
<th>India</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Production</td>
<td>Domestic Market Share</td>
<td>Production</td>
<td>Domestic Market Share</td>
</tr>
<tr>
<td>Cars</td>
<td>217,774</td>
<td>9.6%</td>
<td>4,064,774</td>
<td>13%</td>
</tr>
<tr>
<td>Commercial Vehicles</td>
<td>115,701</td>
<td>5.1%</td>
<td>1,112,176</td>
<td>4%</td>
</tr>
<tr>
<td>Motorcycles /Three Wheelers</td>
<td>1,928,757</td>
<td>85%</td>
<td>25,771,809</td>
<td>80%</td>
</tr>
</tbody>
</table>

As the above discussion indicates that the Indian auto sector has relatively outperformed Pakistani auto sector in every aspect. This study envisions this outperformance of the Indian auto sector as the potential result of the existence of a strong democracy and the large market size. The political arena in India features continuous democracy since independence. The continuation of democracy ensures the continuation of development policies, which is crucial to realize the development objectives. While the political history of Pakistan stained with frequent military coups (1958-1971, 1977-1988, 199-2008) and almost half of her political history (36 years out of 72 independent years) ruled by marshal laws. These frequent regime changes bring discontinuation of policies and cause dis-alignment with development goals which were once envisioned through political consensus. Furthermore, India also has a large market which ranks 6th as per the United Nations data on the household final consumption expenditures and six-time larger than the market size of Pakistan. Consequently, having a weak democracy with a small market size retains the auto sector of Pakistan underdeveloped.

Outwardly the current policies related to the auto sector in both countries are protectionist and India provides the highest effective rate of protection to the auto sector via tariffs among all regional countries, (Bari et al., 2016). The current auto sector policy of Indian government outlined in Automotive Mission Plan 2016-26. The mission plan is aimed to promote the auto sector with an average growth rate of 7.5 percent for the period 2016-26. The key objectives set in the mission plan include; raising the GDP share of the auto sector to 12 percent with 40 percent share in manufacturing, creating over 65 million jobs, ensuring safety-efficiency-comfortability, enhancing export up to 35-40 percent of output. The mission plan also ensures policy stability and asserts special focus on policy predictability and sustainability. While in the case of Pakistan, the Auto
Development Policy 2016-21 provides the basic policy guidelines for the auto sector. The policy plan aims to increase the GDP contribution of the auto sector to 3.8 percent by attracting more investment in the auto sector and facilitating new entrants. Consumer welfare is the also key focal point and plan emphasize on quality, safety, and standards.

By comparing the policy plans of both countries, we can attribute the dismal performance of Pakistani auto sector to the stability and predictability of the policies. As Pakistan lags behind on both vital factors, business environment, and reliable trade flow; which are crucial for the development of the auto sector. However, the policy plan does not outline any policy measure to address these issues. Furthermore, the policy is highly unpredictable in Pakistan and often effective policies vary from the announced policies. One reason for these variations is the Statutory Rules and Orders (SROs). The drawback of these SROs is that they amend the effective policy rate and do not require a consensus among legislators to be effective. The rampant use of SROs is evident by as per the Federal Board of Revenue of Pakistan, currently, there are 103 active SROs related to imports and 29 for exports. Bari et al. (2016) report a 0.34 percent loss of GDP due to the exemption that has been granted by currently active SROs. In the year 2018-19, the Ministry of Commerce issued 11 SROs and 3 of which are related to the auto sector. In short, policies in Pakistan are less reliable because of the way policymaking is done is less democratic. This policy unreliability creates commitment problems, which is another source of economic inefficiency and known as the hold-up problem in the literature (see, Acemoglu, 2007).

The rest of the paper structured as follows. Section 2 presents the basic model and describes the behavior of firms in the economy. Section 3 characterizes the close economy equilibrium and discuss capital accumulation in autarky. Section 4 holds the discussion on costly trade openness and trade policy formation. Section 5 describes the decision of the individual firm to adopt new technology or block new technology. Section 6 concludes.

5.4. The Model

The world economy consists of two countries, home country $h$ and foreign country $f$. Population in home country $h$ is divided into three social classes; elite class denoted by $e$ with total agents $\theta^e$, middle-class with total agents $\theta^m$, and labor class with total agents $\theta^l = 1$. The elite control the

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29 For example, in case of doing business India rank 77th and Pakistan rank 136th out of 190 countries according to the World Bank’s ease of doing business report 2019. Similarly, in the case of reliable trade flow, The logistic performance report 2018 of the World Bank ranked India 44th and Pakistan 122nd out of 166 countries.
political power and makes policy decisions, while the middle-class is the entrepreneur and has access to production technology. However, the labor class provides labor inelastically in the economy and total labor endowment $\bar{L}_h(t)$ is normalized to 1 at time $t$. Individuals in the society are unable to change their class/group association over the time and the set of elite and middle-class is denoted by $S^e$ and $S^m$. Moreover, the foreign country is assumed to have a superior technology compared to the home country. Therefore, foreign firms are more productive than domestic firms. The superscript $i$ is used to denote the individuals or groups and subscript indicates the countries.

To begin with, we first assume that the technology adoption by firms from the home country is not possible and characterizes the equilibrium that determines the number of firms operating in the country. Then, we consider the case of technology adoption and discuss the determining factors of technology adoption by firms in the home country.

5.2.1. Household Sector

The utility of an agent $i$ in home country $h$ at time $t = 0$ is given by;

$$
E_0 \sum_{t=0}^{\infty} \beta^t C^i_h(t)
$$

where $C^i_h(t)$ is the consumption of agent $i$ and $E_t$ is the expectations operator that is conditional on the available information at time $t$. The preferences are Dixit and Stiglitz type and based on the consumption of the finite number of differentiated varieties;

$$
C^i_h(t) = \sum_{v \in V} \left( q_h(v, t) \right)^{\frac{\sigma-1}{\sigma}}
$$

The set of available varieties is represented by $V$ and $v$ represents an individual variety with the elasticity of substitution $\sigma > 1$. Following Yang and Heijdra (1993), we also assume the effect of an individual firm’s price-setting behavior on the aggregate price index of the economy during solving the above utility function. This effect emerges because of the set of differentiated varieties $V$ is assumed not extremely large, contrary to the assumption in Dixit and Stiglitz (1977). Another outcome of assuming a small set of varieties is that the elasticity of substitution between the differentiated varieties ($\sigma$) and the price elasticity of demand ($\epsilon$) are not the same, the point will become clearer in section 2.3. The solution of utility maximization of agent $i$ gives the demand of an individual variety at time $t$, which is;

$$
q_h(v, t) = Y^i_h(t) P_h(t)^{\sigma-1} p_h(v, t)^{-\sigma}
$$
where $Y_h^i(t)$ denotes the income of an agent $i$ at time $t$ and $P_h^i(t)$ is the aggregate price index at time $t$ that is given as:

$$P_h(t) = \left( \sum_{v \in V} p_h(v, t)^{1-\sigma} \right)^{1/1-\sigma} \quad (4)$$

### 5.2.2. Production Sector

The production function involves capital and labor as the factors of production. Each firm in the economy produces a unique variety of differentiated good. The capital is provided by the middle-class and labor comes inelastically from the labor class. The production technology at time $t$ that entrepreneur can access is:

$$q_h^m(\varphi_h, t) = \varphi_h \left( \frac{q_h(\varphi_h, t)}{\varphi_h} + f_h \right) \left( \frac{\delta}{1-\delta} \right)^{1-\delta} \left( \frac{w_h(t)}{r_h(t)} \right)^{\delta-1} \quad \forall m \in S^m \text{ at each } t. \quad (5)$$

where $\varphi_h$ indicates the productivity of the firm, which realizes after paying entry cost $f_h^{ent}$. In the meanwhile, $f_h$ denotes the fixed cost of production and depends upon the market location. The factor intensity of the fixed and entry costs is assumed to be same. The share of labor and capital in production function are given by $\delta$ and $(1 - \delta)$, respectively. The total capital $K_h(t)$ at time $t$ depends on the capital stock in period $(t - 1)$ and investment along with depreciation rate $\psi$. The aggregate capital stock at time $t$ in the economy is $K_h(t) = (1 - \psi)K_h(t - 1) + I_h(t - 1)$. The conditional demand for labor by an individual firm with the wage rate $w_h(t)$ and the rate of return $r_h(t)$ can be represented as:

$$L_h^m(t) = \left( \frac{q_h(\varphi_h, t)}{\varphi_h} + f_h \right) \left( \frac{\delta}{1-\delta} \right)^{1-\delta} \left( \frac{w_h(t)}{r_h(t)} \right)^{\delta-1} \quad (6)$$

Additionally, assume that the individual heterogeneous firm can employ the maximum $\bar{L}_h$ number of workers and $L_h^m(t) \in [0, \bar{L}_h]$, as in Acemoglu (2009). To ensure no-unemployment, further assume that all entrepreneurs employ the same number of workers, so that:

$$L_h^* = \min \left\{ \bar{L}_h, \frac{1}{\theta m} \right\}$$

By assuming $\theta m \bar{L}_h > 1$, the full employment is ensured and $L_h^* = \frac{1}{\theta m}$.

### 5.2.3. Firm’s Behavior

From the production function, we can derive the cost function of a firm with productivity level $\varphi_h$ at time $t$ as:

$$Z_h(\varphi_h, t) = \left( \frac{q_h(\varphi_h, t)}{\varphi_h} + f_h \right) \mu r_h(t)^{1-\delta} w_h(t)^\delta, \text{ with } \mu = \left( \frac{\delta}{1-\delta} \right)^{1-\delta} + \left( \frac{\delta}{1-\delta} \right)^{-\delta} \quad (7)$$

Given the demand for each variety in equation (3), the optimal pricing rule for the firm is;
\[ p_h(\varphi_h, t) = \left( \frac{\epsilon}{\epsilon-1} \right) \frac{1}{\varphi_h} \zeta_h(t), \text{ with } \zeta_h(t) = \mu r_h(t)^{1-\delta} w_h(t)^{\delta} \] (8)

where \( \left( \frac{\epsilon}{\epsilon-1} \right) \) is the markup of the firm and \( \epsilon \) is the price elasticity of demand. In Dixit and Stiglitz characterization of the monopolistic competition, this price elasticity of demand is equal to the elasticity of substitution between differentiated varieties \( \epsilon = \sigma \), while here we have a limited number of varieties and the price elasticity of demand is given as 

\[ \epsilon = \sigma - \frac{(\sigma-1)}{\frac{P_h(t)}{p_{h}(\varphi_{h}, t)}} = \sigma - \frac{(\sigma-1)}{V}. \]

This shows that as the number of differentiated varieties increases the price elasticity of demand also increases because the consumer has more varieties to choose from. The optimal pricing rule also indicates that the price charged by a firm is inversely related to the productivity of the firms. A more productive firm charged a lower price and capture a larger share of the market as the markup is constant for all firms.

The revenue and profit earned by a firm from home country \( h \) eat time \( t \) is given as;

\[ R_h(\varphi_h, t) = Y_{hi}(t) P_h(t)^{\sigma-1} \left( \left( \frac{\epsilon}{\epsilon-1} \right) \frac{1}{\varphi_h} \zeta_h(t) \right)^{1-\sigma} \] (9)

\[ \pi_h(\varphi_h, t) = \frac{1}{\epsilon} Y_{hi}(t) P_h(t)^{\sigma-1} \left( \left( \frac{\epsilon}{\epsilon-1} \right) \frac{1}{\varphi_h} \zeta_h(t) \right)^{1-\sigma} - \zeta_h(t) f_h \] (10)

5.3. **Closed Economy Equilibrium**

We start with the characterization of a closed economy equilibrium in order to explain some simple features of the model. Then in the next section, we will consider the case of costly trade.

5.3.1. **Entry and Exit**

Firms realize their productivity after incurring the sunk entry cost \( \zeta_h(t) f_{h}^{ent} \). The productivity is drawn from cumulative distribution \( G(\varphi_h) \) and cumulative productivity distribution is assumed to be a Pareto distribution 

\[ G(\varphi_h) = 1 - \left( \frac{\varphi_h}{\varphi_{h_\text{min}}} \right)^{\alpha} \]

with \( \varphi_{h_\text{min}} \) as the lowest possible productivity that a firm can draw in home country \( h \). The firm decides either to produce and serve the market or to exit the market once the productivity is realized. In this regard, the minimum productivity level \( \varphi_{h}^* \), which is required to produce and remain active in the market, can be determined by zero-profit condition. The zero-profit condition states as;

\[ Y_{hi}(t) P_h(t)^{\sigma-1} \left( \left( \frac{\epsilon}{\epsilon-1} \right) \frac{1}{\varphi_h} \zeta_h(t) \right)^{1-\sigma} = \epsilon \zeta_h(t) f_{h}^{ent} \] (11)
Hence, firms with realized productivity level \( \varphi_h < \varphi_h^* \) quit the market and firms with realized productivity \( \varphi_h^* \leq \varphi_h \) participate in the production and remain active in the market. Meanwhile, the decision of a firm to enter the market and bears the entry cost depends upon the expected revenue that a firm can accrue. The expected revenue of entering the home market with a successful entry is:

\[
\tilde{R}_h(t) = \int_{\varphi_h^*}^{\infty} R_h(\varphi_h, t) \frac{dG(\varphi)}{1-G(\varphi_h^*)} = \epsilon \chi \zeta_h(t) f_h, \quad \text{where} \quad \chi = \frac{\alpha}{\alpha-\sigma-1} \quad (12)
\]

In the same way, the expected profit is \( \tilde{\pi}_h(t) = (\chi - 1) \zeta_h(t) f_h \). Next, the free entry condition dictates that the expected ex-ante profit \( \tilde{\pi}_h(t) \) that include entry cost must be equal to zero in the equilibrium, that is;

\[
(\chi - 1)f_h \varphi_h^{\ast -\alpha} = f_h^{\text{ent}} \varphi_h^{\ast -\alpha} \quad (13)
\]

In the above condition, the factor reward term \( \zeta_h(t) \) has been canceled due to the assumption of the same factor intensity requirement for the fixed overhead production cost and the entry cost. From this condition, we can determine the unique value of \( \varphi_h^* \) that depends only on the parameters of the model. In fact, the mass of entrants in the economy is \( M_h^{\text{ent}} \) and the mass of active firms in the home country is defined as \( M_h = [1 - G(\varphi_h^*)]M_h^{\text{ent}} \). Given the optimal pricing rule and productivity distribution, we can transform the aggregate price index as;

\[
P_h(t)^{1-\sigma} = M_h \left( \frac{\epsilon}{\sigma-1} \right)^{1-\sigma} \zeta_h(t)^{1-\sigma} \chi \varphi_h^{\ast \sigma - 1} \quad (14)
\]

The price index is inversely related to the mass of active firms and the productivity cutoff. While it is positively related to the markup of the firms and factor rewards. Now, the next step in the characterization of a closed economy equilibrium is to determine the equilibrium factor prices. As the total payments to the factors of production must be equal to the difference between the aggregate revenue and aggregate profit, therefore, the factors market equilibrium condition is given as;

\[
w_h(t) + r_h(t) \bar{K}_h(t) = \overline{AR}_h(t) - \overline{\Pi}_h(t) + M_h^{\text{ent}} f_h^{\text{ent}} \quad (15)
\]

where \( \overline{AR}_h(t) = M_h \tilde{R}_h(t) \) and \( \overline{\Pi}_h(t) = M_h \tilde{\pi}_h(t) \) are the aggregate revenue and profit in the economy at time \( t \). Note that the free entry condition ensures that the aggregate expected profit is equal to the aggregate entry cost, so the above condition becomes; \( w_h(t) + r_h(t) \tilde{R}_h^{\text{ent}}(t) = \overline{AR}_h(t) \).

From the labor and capital market-clearing conditions, we can determine the equilibrium wage rate and return, which are given as;
The wage depends positively on the share of labor in the production function, the mass of active firms, and the price elasticity of demand. The positive relationship between the wage rate and the price elasticity of demand is because of the fact that an increase in the varieties leads to an increase in price elasticity. As a result, the firm will charge a lower markup and earn higher revenue, which generates a demand for wage increment. Accordingly, the mass of active firms contingent upon the aggregate revenue and the average firm size;

\[ M_h = \frac{\bar{AR}_h(t)}{R_h} = \frac{w_h(t) + r(t)\bar{R}_h(t)}{\epsilon(\bar{h}_r(t) + \zeta_h(t)f_h)} \]  

(17)

5.3.2 Entrepreneur’s Problem

Due to linear preferences, the value of the discounted sum of consumption of the entrepreneur is given as;

\[ U_h^m = \sum_{s=1}^{\infty} \beta^{s-1} \left[ q_h^m(\varphi_h, t) - \left( K_h^m(s) + (1 - \psi)K_h^m(s) \right) - w(s)L_h^m(s) \right] \]

The first-order condition of the above maximization problem gives the capital stock for the next period;

\[ \varphi_h(1 - \delta)(L_h^m(t + 1))^{\delta} \left( K_h^m(t + 1) \right)^{-\delta} + (1 - \psi) = 1 \]

Or, in capital-labor ratio form;

\[ k_h^m(t + 1) = \left( \frac{1 - \beta(1 - \psi)}{\beta \varphi_h(1 - \delta)} \right)^{\delta} \]

(18)

Finally, the equilibrium in the case of a closed economy can be characterized by the zero-profit productivity cutoff, the factor prices, the aggregate price index, and the aggregate revenue \(\{\varphi_h^*, P_h(t), QR_h(t), w(t), r(t)\}\). These quantities are determined by the free entry condition (equation (13)), the optimal pricing formula (equation (8)), and the factor market clearing condition (equation (16)). Given the distribution of capital stock at time \(t\) among the middle-class \([K_h^m(t)]\) and the sequence of capital stock for each entrepreneur by equation (18), we can define all the endogenous variables in the model in term of \(\{\varphi_h^*, P_h(t), QR_h(t), w(t), r(t)\}\).

**Proposition-1:** The number of varieties that an economy can support is proportional to the market size (in terms of population) and a larger market with larger number of varieties has a higher price elasticity of demand.

Following Desmet and Parente (2014), reconsider the labor market clearing condition as;
\[ M_h = \frac{w_h L_h}{e \delta \chi \zeta_h(t) f_{hh}} \]

While deriving wage rate in (16), we assumed \( \bar{L}_h = 1 \). The above equation shows that the mass of active firms increases as the labor supply increases. Since each firm produces a single variety of differentiated goods, therefore, the number of varieties also increases. Now, reconsider the price elasticity of demand as, \( \epsilon = \sigma - \frac{\sigma - 1}{M_h} \), which shows that an increase in the number of active firms will increase the elasticity of demand as well. Thus, a larger economy will have a higher price elasticity. By replacing price elasticity formulae into the above equation, we can present the positive relationship between the labor supply and mass of active firms straightforwardly as;

\[ M_h = \frac{w_h \bar{L}_h + \delta \chi \zeta_h(t) f_{hh}(\sigma - 1)}{\delta \sigma \chi \zeta_h(t) f_{hh}} \]

Furthermore, the increase of the price elasticity of demand leads to a fall in the markup \( \left( \frac{\epsilon}{\sigma - 1} \right) \) of the firm. The optimal pricing rule in equation (8) indicates a negative relationship between markup and price changed by the firm. Resultantly, the price charged by the firm also reduces and the firm earns more revenue and capture larger market share. For this reason, a large economy not only holds a larger number of varieties, but the average size of firms is also large.

5.4. Open Economy Equilibrium with Costly Trade

Now, we consider the case of trade between home country \( h \) and foreign country \( f \) and firms from the foreign country are technologically superior to firms from the home country at time \( t \). In this section, the possibility for firms from the home country to up-grade technology is not allowed. We assume this possibility and discuss the implications of technology adoptions by the firms from the home country in the next section.

The trade among countries involves the transport cost and trade taxes. The transport cost is iceberg type and in order to send one unit of the differentiated good to foreign market \( f \), the domestic firm ship \( \tau_{hf} > 1 \) units of the variety, with \( \tau_{hh} = 1 \). The trade taxes are defined by the elite policymaker such that the tax \( \eta_{fh}(t) = \left( 1 + tr_{fh}(t) \right) \) imposes on all imports from the foreign country and the subsidy \( \gamma_{hf}(t) = \left( 1 + sb_{hf}(t) \right) \) provides to all domestic firms that exports to the foreign country. Whereas, \( \eta_{fh}(t) > 1 \) indicates an import tariff and \( \eta_{fh}(t) < 1 \) indicates an import subsidy, while \( \gamma_{hf}(t) > 1 \) indicates an export subsidy and \( \gamma_{hf}(t) < 1 \) indicates an export tax. Following Costinot et al. (2016), we also assume that the elite in the home country \( h \) are
strategic and impose taxes in order to maximize their own welfare. Whereas, foreign country $f$ is passive and does not impose taxes. In this regard, the trade policy precedes the entry of firms in the economy.

**5.4.1. Trade Policy Making**

The policy options available to the elite policymakers in the home country involves only the trade policy and no other tools of taxation are available. The revenue generated from trade taxation at time $t$ is used for the lump-sum transfers to labor-class $T^l(t) \geq 0$, entrepreneurs $T^e(t) \geq 0$ and elite $T^e(t) \geq 0$. The lump-sum transfer assumption also indicates that a negative transfer (lump-sum tax) is not possible. The budget constraint of the government at time $t$ is:

$$\theta^e T^e(t) + \theta^m T^m(t) + T^l(t) \geq \{(\eta_{fh} - 1)\bar{AR}_{fh} + (1 - \gamma_{hf})\bar{AR}_{fh}\} \quad (19)$$

The timing of the trade policymaking is such that the elite policymakers announce the import tax $\eta_{fh}(t + 1)$ and export subsidy $\gamma_{hf}(t + 1)$ that will apply at the next date at time period $t$. Hence, trade policy precedes the decision of entrepreneurs and they choose the capital stocks for the next period $[k^m(t + 1)]$ and decide how much labor to hire $[L^m(t + 1)]$ after observing the announced trade policy for the next period. Since the entrepreneurs are fully informed about the next period’s trade policy rates, therefore, the hold problem will not be an issue in these settings. Furthermore, let $F^t = \{\eta_{fh}(s), \gamma_{hf}(s), T^l(s), T^m(s), T^e(s)\}_{s=t}^{\infty}$ denotes a feasible sequence of policies.

**5.4.2. Firm’s Behavior**

Given the transport cost $\tau_{hf}$ and export subsidy $\gamma_{hf}(t)$, the price charged by a firm that belongs to home country $h$ at; the domestic market and the foreign market at time $t$ is given as:

$$p_{hh}(\varphi_{h}, t) = \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{1}{\varphi_{h}} \zeta_{h}(t)$$

$$p_{hf}(\varphi_{h}, t) = \frac{\tau_{hf}}{\gamma_{hf}(t)} \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{1}{\varphi_{h}} \zeta_{h}(t) \quad (20)$$

Similarly, a foreign importing firm charged price $p_{fh}(\tilde{\varphi}_{f}, t) = \tau_{fh} \eta_{fh}(t) \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{1}{\tilde{\varphi}_{f}} \zeta_{f}(t)$ in home country $h$. Nonetheless, $\tilde{\varphi}_{f} > \varphi_{h}$, as foreign importing firms are more productive than the domestic firms. Moreover, the revenue and profit of a firm from home country $h$ at time $t$ is;

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30 Here, we exclude the revenue extraction motives of the policymakers and assume that policymakers have full capacity to raise and redistribute trade tax revenues. For revenue extraction motive of taxation see Acemoglu (2009).
$$R_h(\varphi_h, t) = \begin{cases} \frac{R_{hh}(\varphi_h, t)}{\varepsilon} = Y_h^i(t)P_h(t)\sigma^{-1}\left(\frac{1 - \varphi_h}{\varepsilon - 1}\right)^{1 - \sigma} & \text{if does not exports} \\ R_{hh}(\varphi_h, t)\left(1 + \tau_{hf}y_{hf}(t)\sigma^{-1}\right) = Y_{hf}^i(t)\left(\frac{p_f(t)}{P_h(t)\sigma^{-1}}\right)^{1 - \sigma} & \text{if exports.} \end{cases} \quad (21)$$

$$\pi_h(\varphi_h, t) = \begin{cases} \frac{1 - R_{hh}(\varphi_h, t)}{\varepsilon} = Y_h^i(t)P_h(t)\sigma^{-1}\left(\frac{1 - \varphi_h}{\varepsilon - 1}\right)^{1 - \sigma} & \text{if does not exports} \\ \frac{1}{\varepsilon}R_{hh}(\varphi_h, t)\left(1 + \tau_{hf}y_{hf}(t)\sigma^{-1}\right) = Y_{hf}^i(t)\left(\frac{p_f(t)}{P_h(t)\sigma^{-1}}\right)^{1 - \sigma} - (f_{hh} + f_{hf})\zeta_h(t) & \text{if exports.} \end{cases} \quad (22)$$

where $f_{hh} < f_{hf}$ indicates that the fixed overhead cost of production is higher in case of serving the foreign market than serving the domestic market. A firm exports to the foreign country $h$ at time $t$ only if $\frac{R_{hh}(\varphi_h, t)}{\varepsilon} > \zeta_h(t)f_{hf}$, which shows that the revenue accruing in the foreign market must cover the additional fixed cost of production.

5.4.3. **Entry and Exit**

Due to the costs associated with serving other country’s market, not all firms active in the domestic market of a country would be able to participate in the export business. Therefore, in case of costly trade, there are two minimum productivity cutoffs: the productivity cutoff to serve the domestic market only $\varphi_{hh}^*$ (zero-profit cutoff) and the productivity cutoff to serve the foreign market as well $\varphi_{hf}^*$ (export cutoff). Similar to equation (11), the productivity cutoffs of firms from the home country are determined by the zero-profit conditions and given as:

$$\begin{cases} Y_h^i(t)P_h(t)\sigma^{-1}\left(\frac{1 - \varphi_h}{\varepsilon - 1}\right)^{1 - \sigma} = \varepsilon\zeta_{hf}(t)f_{hf} & \text{for Domestic Market} \\ Y_{hf}^i(t)\left(1 + \tau_{hf}y_{hf}(t)\sigma^{-1}\right) = Y_{hf}^i(t)\left(\frac{p_f(t)}{P_h(t)\sigma^{-1}}\right)^{1 - \sigma} - (f_{hh} + f_{hf})\zeta_{hf}(t) & \text{for foreign market.} \end{cases} \quad (23)$$

Hence, firms with productivity level $\varphi_{hh}^* \leq \varphi < \varphi_{hf}^*$ serve the only domestic market of the home country and firms with productivity level $\varphi_{hf}^* \geq \varphi$ serve both domestic as well as the foreign market. We can also define the zero-profit cutoff $\tilde{\varphi}_{ff}^*$ and export cutoff $\tilde{\varphi}_{fh}^*$ for foreign firms in the same fashion. Although the foreign country does not pursue any trade policy, due to the presence of transport cost the zero-profit cutoff is less than the export cutoff $\tilde{\varphi}_{ff}^* < \tilde{\varphi}_{fh}^*$. In a particular market, for instance, the home country’s market at time $t$, domestic firms with minimum productivity $\varphi_{hh}^*$ compete with foreign importing firms with minimum productivity $\varphi_{hf}^*$. It is straightforward to show that these two productivity cutoffs in an individual market are inversely related. By considering the ratio of revenues of domestic and foreign importing firms, we have;
\[ \phi_{hh}^* = E \bar{\phi}_{fh} \]  
\[ (24) \]

where \( E \equiv -\left(\frac{1-\varsigma}{\varsigma}\right)^{\sigma-1} \eta_{fh}(t)^{-\frac{\sigma}{1-\sigma}} \tau_{fh} \) with \( \varsigma \) as the share of expenditure on the domestic varieties out of the total expenditures. The nature of the relationship between productivity cutoffs indicates that in the event moving from autarky to trade, the zero-profit cutoff \( \phi_{hh}^* \) for domestic firms raises. This rise in zero-profit cutoff makes marginal domestic firms to quit the market. Thus, domestic firms especially firms on the margin, prefer higher trade restrictions and that is import tariff in this model. The proposition below describes the relationship between productivity cutoffs and the trade policy.

**Proposition-2:** A change in the trade policy of the home country affects the productivity cutoffs in both countries, such that:

\[ \frac{\partial \phi_{fh}^*}{\partial \eta_{fh}(t)} \frac{\partial \phi_{hh}^*}{\partial \eta_{fh}(t)} > 0, \quad \frac{\partial \phi_{fh}^*}{\partial \gamma_{hf}(t)} \frac{\partial \phi_{hh}^*}{\partial \gamma_{hf}(t)} < 0, \]

**Proof.** See Appendix. \( \blacksquare \)

The proposition shows that an increase in import tariff by home country \( h \) leads to an increase of import cutoff \( \phi_{fh}^* \), since the import tariff and import cutoff are positively related. This increase in import cutoff makes less foreign importing firms to participate in import business in home country \( h \). However, due to the trade balance condition, the increase in import cutoff for the home country also increases the export cutoff for domestic firms. Resultantly, a higher trade barrier by home country leads to a reduction in international trade participation. The same is true in the case of export tax.

The expected revenue of a firm that serves both markets is now;

\[ \bar{R}_h(t) = \chi \epsilon \zeta_h(t)(f_{nh} + m_{hf}f_{hf}) \]

where \( m_{hf} = \frac{1-\chi}{1-\epsilon} \left( \frac{\phi_{hh}}{\phi_{hf}} \right) \) is the export participation rate. Moreover, the free entry condition again requires that the expected profit to be equal to the entry cost, which states as;

\[ (\chi - 1)(f_{nh} + m_{hf}f_{hf})\phi_{hh}^* \alpha = f_{h}^{ent} \phi_{h}^{-\alpha} \]

While the aggregate price index can now transform as;

\[ P_h(t)^{1-\sigma} = M_h \left( \frac{\epsilon}{\epsilon-1} \right)^{1-\varsigma} \zeta_h(t)^{1-\sigma} \chi \phi_{hh}^* \sigma^{-1} + m_{fh}M_f \left( \frac{\epsilon}{\epsilon-1} \right)^{1-\sigma} \zeta_f(t)^{1-\sigma} \tau_{fh}^{1-\sigma} \eta_{fh}(t)^{1-\sigma} \chi \hat{\phi}_{f}^{-1} \]

\[ (25) \]
The total factor payment is again determined by the equation (15) and factor prices are determined by the market-clearing conditions. The factor prices are now:

\[
\begin{align*}
    w_h(t) &= \delta M_h X e \xi_h(t) (f_{hh} + m_{hf} f_{hf}) \\
    r_h(t) &= \left(1 - \delta \right) M_h X e \xi_h(t) (f_{hh} + m_{hf} f_{hf})
\end{align*}
\]  

(26)

Finally, the trade balance requires imports of a country must equal to the exports of the country. The trade balance condition for the home country \( h \) is given as:

\[
\frac{1}{\gamma_{hf}(t)} m_{hf} M_{hf} f_{hf} \xi_h(t) = m_{fh} M_f f_{fh} \xi_f(t)
\]

(27)

**Proposition-3**: Firms from the home country resists the trade openness due to;

- the negative relationship between firm’s markup and the number of varieties available in the market
- the foreign importing firms are more productive.

However, trade openness increases the welfare of consumers due to the pro-competition effect.

The proof of the first part of the proposition is in the text above. However, the welfare in the economy after trade enhances due to two effects, as discussed by Edmond et al. (2012); the pro-competitive effect and the Ricardian effect. The former, pro-competitive effect captures the effect of a reduction in the aggregate price index due to the fall of the price of domestic varieties. Trade openness increases the number of varieties available in the market and domestic firms compelled to reduce their markups and reduce prices of domestic varieties. The later, Ricardian effect encompasses the traditional arguments of welfare increase due to productivity gain. Moving from autarky to trade, makes the least productive domestic firms to quit the market, and raises the average productivity in home country \( h \). The increase in average domestic productivity reduces the aggregate price index in the country. Consequently, the aggregate price index will fall because of both channels and welfare will increase.

### 5.4.4. Entrepreneur’s Problem

The entrepreneur’s problem can be described as, provided \([k^m_h(s)], F^t \) and \( w(t) \) are given at the equilibrium and factor markets are clear, \([\{k^m_h(s+1), L^m_h(s)\}]_{s=t}^{\infty} \) maximizes the utility of the entrepreneur, which is:

\[
U^m_h(\{K^m_h(s), L^m_h(s)\}_{s=t}^{\infty} | F^t, w(t)) = \sum_{s=t}^{\infty} \beta^{s-t} \left[ (q^m_h(\varphi_h, t) + (1 - \gamma_{hf}(s))q^m_f(\varphi_h, t)) - \right. \left. \left( K^m_h(s+1) - (1 - \psi) K^m_h(s) \right) - w(s) L^m_h(s) + T^m(s) \right]
\]
Now, the first-order condition that gives the capital stock for the next period is:

\[
\beta \left\{ \varphi_{hf}(1-\delta)(L_h^m(t+1))^\delta (K_h^m(t+1))^{1-\delta}(2-\gamma_{hf}(t+1))+(1-\psi) \right\} = 1 \tag{28}
\]

In term of capital-labor ratio;

\[
k_h^m(t+1) = \left( \frac{1-\beta(1-\psi)}{\varphi_{hf}(1-\delta)(2-\gamma_{hf}(t+1))} \right)^\delta \tag{29}
\]

By comparing the above equation with equation (18), we can see that in case of the open economy the capital level selected by the entrepreneur for the next period depends upon the export tax as well. If \( \gamma_{hf}(t+1) < 1 \) that is the case of export tax, then the capital stock selected by the entrepreneur for the next period is less than capital stock in case of autarky. While in the case of export subsidy, \( \gamma_{hf}(t+1) > 1 \), the capital stock in equation (25) is higher than the autarky.

5.4.5. **Elite’s Problem**

The primary objective of the trade policymaking by the elite is to keep political power with themselves and maximizes their utility by transferring the maximum amount of trade tax revenue to themselves. Acemoglu (2007) rationalizes such behavior of the elite on the revenue extraction and political consolidation basis. Resultantly, the elite transfer all revenue to themselves with \( \theta^m T^m(t) = 0 \) and \( T^l(t) = 0 \). The consumption function of the elite is given as;

\[
C_h^e(t) = \max \left\{ T^e(t) \right\}
\]

The government budget constraint holds in equality;

\[
T^e(t) = \frac{1}{\theta^e} (\eta_{fh}(t) - 1) A R_h(t) + \frac{1}{\theta^e} (1 - \gamma_{hf}(t)) A \bar{R}_{hf}(t)
\]

The maximization problem of the elite can then be written recursively;

\[
V_h^e(\eta_{fh}(t), \gamma_{hf}(t), [K_h(t)]) = \eta_{fh}(t+1), \gamma_{hf}(t+1)} \max \left\{ T^e(t) + \beta V_h^e(\eta_{fh}(t+1), \gamma_{hf}(t+1), [K_h(t+1)]) \right\}
\]

To characterize the equilibrium trade policy sequence, note that \( T^e(t) \) depends only on the trade policy at time \( t \). The utility-maximizing tariff and subsidy rates for the elite are given by the first-order conditions;

\[
\beta \left( \frac{1}{\theta^e} \left( \eta_{fh}(t+1) - 1 \right) \frac{\partial A R_{hf}(t+1)}{\partial \eta_{fh}(t+1)} + \frac{1}{\theta^e} A \bar{R}_{hf}(t+1) \right) = 0
\]

\[
\beta \left( \frac{1}{\theta^e} \left( 1 - \gamma_{hf}(t+1) \right) \frac{\partial A R_{hf}(t+1)}{\partial \gamma_{hf}(t+1)} + \frac{1}{\theta^e} A \bar{R}_{hf}(t+1) \right) = 0
\]

These conditions give (see appendix);

\[
\eta_{fh}(t+1) = \frac{\alpha \sigma}{\alpha \sigma - \sigma + 1} \tag{30}
\]
The equations above indicate that equilibrium trade policy pair selected by the elite involves an import tariff \( \eta_{fh}(t + 1) > 1 \) and an export tax \( \gamma_{hf}(t + 1) < 1 \). Furthermore, the elasticity of substitution between the varieties of differentiated goods and the shape parameter of Pareto distribution are emerged as crucial elements to determine the level of the policy rate. The comparative statistics indicates that:

\[
\frac{\partial \eta_{fh}(t+1)}{\partial \alpha} = -\frac{\sigma(\sigma-1)}{(\alpha\sigma-\sigma+1)^2} < 0, \quad \frac{\partial \gamma_{hf}(t+1)}{\partial \sigma} = \frac{\alpha}{(\alpha\sigma-\sigma+1)^2} > 0
\]

The derivatives in the above equations indicate an opposite relationship of import tariff and export tax with Pareto shape parameter. The negative relationship between import tariff and Pareto shape parameter is due to the market selection sensitivity. A large value of \( \alpha \) indicates a lower productivity dispersion, which makes heterogeneous firms more sensitive to the variations of import tariff and market selection. Resultantly, due to the existence of high market selection sensitivity, elite select a lower tariff in case of having a high value of \( \alpha \). Similarly, a positive relationship between export tax and Pareto parameter also means a lower ad-valorem export tax in case of having a high value of \( \alpha \), since \( \gamma_{hf}(t + 1) < 1 \).

However, import tariff links positively on the elasticity of substitution between the differentiated varieties. As a higher elasticity means domestic varieties are close substitutes of imported varieties. Therefore, applying a higher level of import tariff would not affect so badly the consumer welfare.

Similarly, having a negative relationship with export tax also shows a higher level of ad-valorem tax in case of the high value of \( \sigma \).

The Markov perfect equilibrium (MPE) in case of the open economy can be characterized by the cutoff productivity, the factor prices, the aggregate price index, and the aggregate revenue, import tariff, and export tax \( \{ \varphi_{hh}^*, \varphi_{hf}^*, \varphi_{ff}^*, \varphi_{fh}^*, P_h(t), P_f(t), \bar{AR}_h(t), w(t), r(t), \eta_{fh}(t + 1), \gamma_{hf}(t + 1) \} \). These quantities are determined by the free entry condition (equation (23)), optimal pricing formula (equation (20)), and factor market clearing condition (equation (26)). The sequence of capital stock for each entrepreneur is now determined by equation (29), import tariff by equation (30), and export tax by equation (31).
5.5. **Technology Adoption: Decision to Adopt or Resist via Lobby for Trade Restrictions**

Now assume the possibility that a firm from the home country can adopt new technology that improves her marginal product by \((1 + \lambda)\) factors, which implies that the productivity with new technology is \(\varphi_h = \varphi_h(1 + \lambda)\). However, the adoption involves a fixed cost \(\Gamma\) which reflects the R&D cost of the adoption. The firm that uses new technology produces with the production function:

\[
\tilde{q}_h(\varphi_h, t) = \varphi_h \left( \left( I_h(t) \right)^{1 - \delta} \left( K_h^m(t) \right)^{1 - \delta} - f_h - \Gamma \right)
\]

The price charged by the firm is \(\tilde{p}_h(\varphi_h, t) = \left( \frac{\ddot{\varepsilon}}{\varepsilon - 1} \right) \frac{1}{\varphi_h} \zeta_h(t)\), where \(\ddot{\varepsilon}\) is the price elasticity of demand and in case of technology adoption by one firm is given as:

\[
\ddot{\varepsilon} = \sigma - (\sigma - 1) \frac{\tilde{p}_{hh}(v, t)}{(v - 1)(p_{hh}(v, t))^{1 - \sigma} + (\tilde{p}_{hh}(v, t))^{1 - \sigma})^{\frac{1}{1 - \sigma}}
\]

The revenue and profit accrue by a firm that adopts new technology is:

\[
\tilde{R}_h(\varphi_h, t) = \begin{cases} 
\tilde{R}_{hh}(\varphi_h, t) - (f_{hh} + \Gamma)\zeta_h(t) & \text{if does not exports} \\
\tilde{R}_{hh}(\varphi_h, t) \left( 1 + \tau_{hf}^{1 - \sigma} \gamma_{hf}(t)^{\sigma} \frac{\gamma_f(t)}{\gamma_h(t)} \left( \frac{p_f(t)}{p_h(t)} \right)^{\sigma - 1} \right) - (f_{hh} + f_{hf} + \Gamma)\zeta_h(t) & \text{if exports.}
\end{cases}
\]

\[
\tilde{\pi}_h(\varphi_h, t) = \begin{cases} 
\frac{1}{\varepsilon} \tilde{R}_{hh}(\varphi_h, t) \left( 1 + \tau_{hf}^{1 - \sigma} \gamma_{hf}(t)^{\sigma} \frac{\gamma_f(t)}{\gamma_h(t)} \left( \frac{p_f(t)}{p_h(t)} \right)^{\sigma - 1} \right) - (f_{hh} + f_{hf} + \Gamma)\zeta_h(t) & \text{if exports.}
\end{cases}
\]

**5.5.1. Entry and Exit**

Analogous to zero-profit and export cutoffs, we can also develop a zero-profit condition for the firm to adopt new technology. Firms with productivity above that technology adoption cutoff are able to adopt new technology in the home country. The technology adoption cutoff is given as:

\[
\begin{align*}
\varphi_h \geq \varphi_h^* & \Rightarrow \frac{Y_h(t)P_h(t)^{\sigma - 1} \left( \frac{\ddot{\varepsilon}}{\varepsilon - 1} \frac{1}{\varphi_h} \zeta_h(t) \right)^{1 - \sigma}}{Y_f(t)P_f(t)^{\sigma - 1} \gamma_{hf}(t)^{\sigma} \tau_{hf}^{1 - \sigma} \left( \frac{\ddot{\varepsilon}}{\varepsilon - 1} \frac{1}{\varphi_h} \zeta_h(t) \right)^{1 - \sigma}} = \ddot{\varepsilon}(f_{hh} + \Gamma)\zeta_h(t) & \text{for Domestic Market} \\
\varphi_h \leq \varphi_h^* & \Rightarrow \frac{Y_h(t)P_h(t)^{\sigma - 1} \left( \frac{\ddot{\varepsilon}}{\varepsilon - 1} \frac{1}{\varphi_h} \zeta_h(t) \right)^{1 - \sigma}}{Y_f(t)P_f(t)^{\sigma - 1} \gamma_{hf}(t)^{\sigma} \tau_{hf}^{1 - \sigma} \left( \frac{\ddot{\varepsilon}}{\varepsilon - 1} \frac{1}{\varphi_h} \zeta_h(t) \right)^{1 - \sigma}} = \ddot{\varepsilon}(f_{hh} + f_{hf} + \Gamma)\zeta_h(t) & \text{for foreign market.}
\end{align*}
\]

Firms with the productivity level \(\varphi \geq \varphi_h^*\) can adopt new technology and firms with the productivity level \(\varphi_h^* < \varphi \) are unable to bear the technology adoption cost and keep operating with old technology.
5.5.2. Selection of Technological Up-Gradation

The adoption of an updated technology involves a fixed cost $\Gamma$ and the fact that $\zeta_h f_h < \zeta_h (f_h + \Gamma)$, ensures that for sufficient low levels of productivity, we have $\bar{\pi}_h(\varphi_h) < \pi_h(\varphi_h)$, and updating technology is not a viable option when keep operating with old technology is more profitable than adopting new technology, i.e. whenever:

$$\varphi_h^* < \varphi_h < \bar{\varphi}_h$$

From zero profit condition:

$$\varphi_h^* = \frac{Y_h^{(\sigma-1)}}{P_h} \left( \frac{\epsilon}{\epsilon - 1} \right) \zeta_h (\epsilon \zeta_h f_{hh})^{\frac{1}{\sigma-1}}$$

$$\bar{\varphi}_h^* = \frac{Y_h^{(\sigma-1)}}{P_h} \left( \frac{\bar{\epsilon}}{\bar{\epsilon} - 1} \right) \zeta_h (\bar{\epsilon} \zeta_h (f_{hh} + \Gamma))^{\frac{1}{\sigma-1}}$$

Therefore,

$$\left( \frac{\epsilon}{\bar{\epsilon}} \left( \frac{\epsilon}{\epsilon - 1} \right)^{\sigma-1} - \left( \frac{\bar{\epsilon}}{\bar{\epsilon} - 1} \right)^{\sigma-1} \right) f_{hh} < \Gamma$$

The above equation indicates that given the cost of technology adoption is greater than the relative benefits (in terms of markup) of technology adoption, firms will not adopt more productive technology. The relative benefits of adopting new technology again link with the price elasticity of demand. In the case of large markets, the relative benefits of adopting new technology will be higher and firms prefer to adopt new technologies. Furthermore, as shown in the figure the profit increase linearly with productivity and more productivity technology increases productivity by $(1 + \lambda)$ factor. This means the slope of $\bar{\pi}(\bar{\varphi}_h, t)$ is greater than $\pi(\varphi_h, t)$. However, at point A, we
have $\pi_h(\varphi_h, t) = \bar{\pi}_h(\bar{\varphi}_h, t)$. By utilizing the definitions of the profit function and eliminating common term, we have;

$$\bar{\varphi}_h = \left(\frac{\zeta_h}{\lambda D} \right)^{\frac{1}{\sigma - 1}} \text{ with } D = \frac{1}{\epsilon} \left( \frac{\epsilon}{\epsilon - 1} \right)^{1-\sigma} Y_h^i P_h^{-\sigma} (\zeta_h)^{1-\sigma} \quad (32)$$

where $D$ measures the size of the market. The above equation explicitly shows the critical variables in determining the technology adoption in an economy are; the cost of adoption, the market size, and the level of productivity increment. The cost of technology adoption $\zeta_h \Gamma$ has a positive link technology cutoff. An increase in the cost of technology adoption increases the technology adoption cutoff and fewer firms operating in home country $h$ enable to adopt new technology. While market size effects negatively to technology adoption cutoff. An increase in market size encourages more firms to adopt new technology. As we have seen in proposition 1 that a large market has a large number of varieties and firms. The availability of a large number of varieties in the market makes demand more elastic with respect to price. The high elasticity of demand induces firms to adopt new technology in order to increase productivity. As the productivity and price charged by the firm are inversely related. Therefore, having higher productivity ensures a lower price for the differentiated variety of firms. Besides, the existence of large firms in a large market also supports the rapid technology adoption due to the fact that large firms can bear the fixed cost of adoption more smoothly than the small firms. The last variable that plays a critical rule is the factor by which productivity increases after paying adoption cost. We can comprehend this factor straightforwardly with the analogy of rungs of a ladder. If paying adoption cost and adopting new technology leads the firm to a higher rung on the technology ladder, then firms prefer to up-grade technology. However, if adoption leads to the next rung of the ladder and that rung is not far from the rung where the firm is standing, then firms might want to stay on the initial rung and avoid the cost of adoption. Comin and Hobijn (2009) have also shown that technology diffusion is slower when new technology has close predecessors.

**Proposition-4:** The technology adoption decision of the firms also depends upon the market size; firms in a large market adopt new technologies more rapidly than firms in a small market.

**Proof,** In the text above.  

5.5.3. **The Possibility of Block Technology Adoption by Lobbying**  
Now, consider the possibility of lobbying by heterogeneous firms for trade policy in the home country $h$. Two fundamental rationales for considering the possibility of lobbying by the firms are
markup motivation and anti-competition motivation. Since the markup of firms is dependent upon the number of varieties in the market as discussed in section 2.3. Therefore, lobbying for a higher trade restriction in the form of a higher trade tariff on imports keeps the number of varieties available in the domestic market low. Thus, in order to maintain their markup and shares in the market, lobby by domestic firms is a natural outcome in these settings. Secondly, in the event of trade openness, the less productive domestic firms have to compete with higher productive foreign importing firms in the domestic market. This competition favors foreign importing firms as they charged lower prices. Hence, domestic firms also try to avoid such kind of competition and lobby to place higher trade barriers.

To what extent firms are capable to lobby and influence elite policymaker in the policy selection, rests on the degree of democracy and the size of the total industry of the home country. Firms in a weak democratic country are more prone to lobby for higher regulations, which yield a slow technology diffusion, Comin and Hobijn (2009). While a small industry with a small number of firms is more effective to slow down technology diffusion via lobbying, Bridgman, et al. (2007). In short, firms will not adopt new technology and lobby for the trade restrictions when firms are small and there is weak democracy in the economy, Weymouth (2012).

The lobbying mechanism considers here is based on classical Grossman and Helpman (1994), which involves monetary offerings by the firm to the elite policymakers in the form of bribe. The individual firm pays a fixed cost of lobby $\zeta_h(t)b_h$ and industry overcome the free-rider problem by punishing the firm who fails to pay the bribe. Firms in the home country $h$ offer a bribe $B = M_h\zeta_h(t)b_h$ to the elite policymakers at time $t$ in order to get maximum trade protection from the foreign importing firm at time $t + 1$. Elite devise trade policy and receive a bribe in case of implementing policy according to the desire of firms. Acemoglu and Robinson (2006) show that the elite policymaker also intends to block new technology due to incumbency advantage erosion. Hence, the objective function of the elite is now;

$$C^e_h(t) = \max \{T^e(t) + B\}$$

While the firm’s objective function is;

$$V^m_h(t) = \max \{\pi^m_h(\varphi_h, t) - \zeta_h(t)b\}$$

---

Where $\hat{R}_h(\varphi_h, t)$ is the operating profit. We can define the equilibrium trade policy and bribe level as;

**Lemma-1:** a Markov perfect equilibrium involves $\{\eta_{f}^*(t), \gamma_{h}^*(t)\}$, $\{B^*\}$ such that;

1. $\zeta_h(t)b^*$ is feasible for all firms in the home country $h$
2. $\{\eta_{f}^*(t), \gamma_{h}^*(t)\}$ maximizes $\{T^e(t) + B\}$ on $F^t$, given $\eta_{f}^*(t), \gamma_{h}^*(t) \in F^t$
3. $\{\eta_{f}^*(t), \gamma_{h}^*(t)\}$ maximizes $\{\hat{R}_h(\varphi_h, t) - \zeta_h(t)b^* + T^e(t) + B^*\}$ on $F^t$ for every firm
4. For every firm $k$, there exists $F_{b}^e \in F^t$ that maximizes $\{T^e(t) + B\}$ on $F^t$ such that $\zeta_h(t)b^* = 0$

The first condition places the feasibility restriction on the bribe for each firm in the industry, and condition (2) indicates that elite maximizes their own utility given the amount of bribe offered. The third condition elaborates the fact that equilibrium policy vector must maximize the joint objective functions and last condition is about the non-payment of bribe conditional on the policy level choice of the elite. From condition (3), the first-order conditions are;

$\frac{\partial \eta_{f}^*(t)}{\partial \eta_{f}^*(t)} \frac{\partial \zeta_h(t)b}{\partial \eta_{f}^*(t)} + M_h \frac{\partial \gamma_{h}^*(t)}{\partial \eta_{f}^*(t)} \frac{\partial \tau^e(t)}{\partial \eta_{f}^*(t)} = 0$ \hspace{1cm} (33)

$\frac{\partial \eta_{f}^*(t)}{\partial \gamma_{h}^*(t)} \frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} + M_h \frac{\partial \gamma_{h}^*(t)}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = 0$

From condition (2), the first-order condition of the elite is;

$M_h \frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = M_h \frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = 0$ \hspace{1cm} (34)

By summing over all firms (33) will become;

$\frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = M_h \frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = 0$ \hspace{1cm} (35)

Substitute (35) into (34);

$\frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = 0$ \hspace{1cm} (36)

$\frac{\partial \zeta_h(t)b}{\partial \gamma_{h}^*(t)} \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = \frac{\partial \tau^e(t)}{\partial \gamma_{h}^*(t)} = 0$

Compare to the first-order conditions of the elite’s problem in section 4.5, the first terms of the above equations are not present there. These terms indicate that the trade policy at this political equilibrium differs from section 4.5. The proposition 3 states that an increase in the tariff revenue will lead to a low variety in the market that enables domestic firms to charge higher markup. Accordingly, the change in operating profits of the firms from home country due to change in the
tariff is positive, i.e. \( \frac{\partial \hat{\eta}_{hf}(\varphi, t)}{\partial \eta_{hf}(t)} = M_h \hat{\eta}_{hf}(\varphi, t) \eta_{hf}^* (t)((\sigma - 1)) > 0 \). By denoting the political equilibrium tariff by \( \eta_{hf}^p(t) \), we know that \( \eta_{hf}^p(t) > \eta_{hf}(t) \). Similarly, \( \frac{\partial \hat{\eta}_{hf}(\varphi, t)}{\partial \gamma_{hf}(t)} = \frac{M_h \hat{\eta}_{hf}(\varphi, t)}{\gamma_{hf}(t)} \left( (\sigma - 1) + \frac{\alpha}{\sigma - 1} \right) > 0 \). Therefore, at the political equilibrium \( \gamma_{hf}^p(t) > \gamma_{hf}(t) \), which indicates the ad-valorem export tax is lower than in section 4.5.

**Proposition-5:** In case of a small market with weak democracy, the heterogeneous firms can influence the trade policymaking and lobby for a higher import restriction to maintain their market shares. However, in the event of large markets with strong democracy where influencing trade policy by lobbying is difficult to achieve, firms refrain from lobbying and adopt new technology more rapidly.

The decision to adopt advance technology or block technology diffusion via lobby depends upon the relative costs of both in a small market. In the event when the net benefits of lobbying are more than the net benefits of technology adoption, firms will adopt lobbying. The net benefits of lobbying are the difference in operating profit without lobbying and operating profit with lobbying minus the lobby cost. A firm’s level the net benefits are \( \{ \hat{\pi}_{hf}^{lb}(\varphi, t) - \hat{\pi}_{hf}^{wl}(\varphi, t) - \zeta_h(t) b_h \} \) where \( lb \) and \( wl \) in the superscript indicate operating profits with lobby and without a lobby, respectively. However, the net benefits of adopting new technology are \( \{ \hat{\pi}_{hf}(\varphi, t) - \hat{\eta}_{hf}(\varphi, t) - \zeta_h(t) \Gamma \} \). The cost of new technology adoption \( \zeta_h(t) \Gamma \) is fixed, while the cost of lobby i.e. the amount of bribe \( \zeta_h(t) b_h \) hinges upon how much political power the policymaker pedals. In weak democracy, the policymaker can change policy level without facing any strong opposition. Thus, the cost of the lobby will be lower than the cost of lobby in strong democracy where policymaker face backlash of the opposition for the policy decisions. Also, in weak democracy, the institutional mechanism for legislation is not so effective and bending orders and legislations are easy, for example, the statutory regulatory orders (SRO) that we have discussed in section 1.1. Hence, the cost of technology adoption is much higher than the cost of lobby \( \Gamma > b_h \) in a weak democracy. Moreover, the size of firms is also small in small economies, and firms in small economy might not be able to bear the adoption cost. Resultantly, they are more prone to lobby.
5.6. **Conclusion**

This chapter develops a political economy model of heterogeneous firms. The model consists of
two countries and two factors of production. The population is divided into elite, entrepreneur and
labor groups. Elite holds the political power, entrepreneurs engage in production, and labor is
provided by the third group. One country has superior technology compared to the other. Therefore,
firms in the less technological advance country face the critical question, either to adopt or resist
new technology once trade opens. The results emerge from the discussion indicate that the market
size and the political institutions of the country play a critical role in this regard. If the market size
is large, then firms will adopt new technology more rapidly. Similarly, in the event of having a
strong democracy, adoption will be rapid. In the event of a weak democracy, firms will resist the
adoption and lobby for higher trade restrictions. Another important result emerges from the model
is that firms adopt technology when the productivity gains from adoption are relatively large and
new technology is much superior to obsoleted technology firm is using.

**Appendix**

**Appendix-A: Proof of proposition-2**

To prove the proposition 2, we follow Felbermayr et al. (2013). From the zero-profit conditions,
the relative productivity cutoffs of firms competing in the home country $h$;

$$
\frac{\eta_{fh}^{-\sigma} \left( \frac{\varphi_{fh}}{r_{fh}} \right)^{\sigma-1}}{(\varphi_{hh})^{\sigma-1}} = \frac{f_{fh}}{f_{hh}}
$$

By differentiating after taking the log and holding transport cost constant gives;

$$
\left( \frac{\sigma-1}{\sigma} \right) \left( \dot{\varphi}_{fh} - \dot{\varphi}_{hh} \right) = \dot{\eta}_{fh}
$$

where dot above the variable denotes the percentage change in the variable. This expression
indicates that any change in tariff rate affects both productivity cutoffs in the market $h$. The
variation in tariff rate is positively related with import cutoff and negatively with domestic cutoff.
However, the trade balance condition dictates a positive association between $\varphi_{hf}^*$ and $\varphi_{fh}^*$, which
is given by;

$$
\varphi_{hf}^* = Q \varphi_{fh}^* \text{ where } Q = \frac{\varphi_h}{\varphi_f} \left( \frac{f_{fh}}{f_{fh}/y_f f_{ent}} \right)^{\alpha} > 0
$$
So, this positive relationship between both productivities indicates that if import cutoff of foreign firms to serve market $h$ falls, then export cutoff for domestic firms to serve foreign market $f$ also falls.

The negative relationship between $\phi_{fh}^*$ and $\phi_{hf}^*$ is given by the equation (24);

$$\phi_{fh}^* = E \phi_{fh}^* \eta_{fh}(t + 1) = -\frac{(1-c)}{\zeta} \sigma - 1 - \frac{\sigma}{\eta_{fh}(t + 1)} \frac{1}{\tau_{fh}}$$

Therefore, the fall of import cutoff for foreign firms in the home country due to decrease in tariff rate increases the zero-profit cutoff of domestic firms to serve domestic market. On the other hand, this also decrease import productivity cutoff in foreign country $f$, which increase domestic productivity cutoff $\phi_{ff}^*$.

Similarly, in case of export subsidy, the relative productivity cutoffs in the foreign country $f$ leads to;

$$\frac{(\sigma-1)}{\sigma} (\phi_{ff}^* - \phi_{hf}^*) = \gamma_{hf}$$

Thus, any change in export subsidy rate of the home country $h$ affects exporting cutoff negatively and foreign country’s domestic cutoff positively. While we can complete the rest of the analysis for export subsidy by following the above steps.

**Appendix-B: Derivation of Import tariff and Export Subsidy:**

From the maximization problem, the first-order conditions are given as;

$$\frac{\partial v_h^e}{\partial \eta_{hf}(t+1)} = \beta \left( \frac{1}{\theta} \eta_{hf}(t + 1) - 1 \right) \frac{\partial Ar_{fh}(t+1)}{\partial \eta_{hf}(t+1)} + \frac{1}{\theta} \frac{\partial Ar_{fh}(t+1)}{\partial \eta_{hf}(t+1)} = 0$$

$$\frac{\partial v_h^e}{\partial \gamma_{hf}(t+1)} = \beta \left( \frac{1}{\theta} (1 - \gamma_{hf}(t + 1)) \right) \frac{\partial Ar_{hf}(t+1)}{\partial \gamma_{hf}(t+1)} + \frac{1}{\theta} \frac{\partial Ar_{hf}(t+1)}{\partial \gamma_{hf}(t+1)} = 0$$

Solving for import tariff and export subsidy yields;

$$\left( \eta_{hf}(t + 1) - 1 \right) = -\frac{\partial Ar_{fh}(t+1)}{\partial \eta_{hf}(t+1)}$$

$$\left( 1 - \gamma_{hf}(t + 1) \right) = -\frac{\partial Ar_{hf}(t+1)}{\partial \gamma_{hf}(t+1)}$$

We can write the aggregate revenues in terms of parameters of the model explicitly as;

$$Ar_{fh}(t + 1) = M_{fh}^e \chi \phi_{fh}^* \alpha Y_f(t + 1) P_f(t + 1) \sigma - 1 \left( \frac{\epsilon}{\epsilon - 1} \right)^{1-\sigma} \zeta_f(t + 1) \sigma - 1 \eta_{fh}(t + 1) \sigma - 1 \phi_{ff}^* \sigma - 1$$

$$Ar_{hf}(t + 1) = M_{hf}^e \chi \phi_{hf}^* \alpha Y_f(t + 1) P_f(t + 1) \sigma - 1 \left( \frac{\epsilon}{\epsilon - 1} \right)^{1-\sigma} \zeta_h(t + 1) \sigma - 1 \eta_{ff}(t + 1) \sigma - 1 \phi_{ff}^* \sigma - 1$$

First, we will solve for import Tariff.
\[
\frac{\partial A_R^{f_{h(t+1)}}}{\partial \eta_{h(t+1)}} = M_{hf}^e \chi_{\eta_{f_{h(t+1)}}}^i Y_{h(t+1)} P_{h(t+1)}^{\sigma-1} \left( \frac{e}{\epsilon-1} \right)^{1-\sigma} \zeta_f(t+1)^{1-\sigma} \tau_f h(t+1)^{1-\sigma} \eta_{h(t+1)}^{1-\sigma-1} \eta_f h(t+1)^{1-\sigma-1} \phi_f^* \sigma^{-a-1} \left( \frac{\alpha \eta_f}{\sigma-1} \right)
\]

\[
\frac{\partial A_R^{h_{(t+1)}}}{\partial \eta_{h(t+1)}} = M_{hf}^e \chi_{\eta_{h(t+1)}}^i Y_{h(t+1)} P_{h(t+1)}^{\sigma-1} \left( \frac{e}{\epsilon-1} \right)^{1-\sigma} \zeta_f(t+1)^{1-\sigma} \tau_{h(t+1)}^{1-\sigma} \eta_{h(t+1)}^{1-\sigma} \phi_f^* \sigma^{-a-1} \left( \frac{\alpha \eta_f}{\sigma-1} \right)
\]

\[
\frac{\eta_{h(t+1)} - 1}{\eta_{h(t+1)}} = \frac{\sigma-1}{a \sigma}
\]

\[
\eta_{h(t+1)} = \frac{\alpha a}{a \sigma - \sigma + 1}
\]

Similarly, we can also solve for import subsidy as;

\[
\frac{\partial A_R^{h_{(t+1)}}}{\partial \eta_{h(t+1)}} = M_{hf}^e \chi_{\eta_{h(t+1)}}^i Y_{f(t+1)} P_f(t+1)^{\sigma-1} \left( \frac{e}{\epsilon-1} \right)^{1-\sigma} \zeta_h(t+1)^{1-\sigma} \tau_{h(t+1)}^{1-\sigma} \eta_{h(t+1)}^{1-\sigma-1} \phi_f^* \sigma^{-a-1} \left( \frac{\alpha \eta_f}{\sigma-1} \right)
\]

\[
\frac{\partial A_R^{h_{(t+1)}}}{\partial \eta_{h(t+1)}} = M_{hf}^e \chi_{\eta_{h(t+1)}}^i Y_{h(t+1)} P_{h(t+1)}^{\sigma-1} \left( \frac{e}{\epsilon-1} \right)^{1-\sigma} \zeta_h(t+1)^{1-\sigma} \tau_{h(t+1)}^{1-\sigma} \eta_{h(t+1)}^{1-\sigma} \phi_f^* \sigma^{-a-1} \left( \frac{\alpha \eta_f}{\sigma-1} \right)
\]

\[
\frac{\sigma - 1}{\sigma a \sigma - \sigma + 1}
\]

\[
\gamma_{h(t+1)} = \frac{\alpha a \sigma}{a \sigma + \sigma - 1}
\]

---

32 By considering the mass of importers in the country and the Pareto distribution productivity from zero profit condition;

\[
\phi_f^* = \frac{\gamma_f(t+1)^{\sigma-1}}{p_{f(t+1)}} \eta_{f(t+1)} \sigma \tau_f(t+1)^{1-\sigma} \left( \frac{\epsilon}{\epsilon-1} \right)^{1-\sigma} \zeta_f(t+1)^{1-\sigma} \tau_f h(t+1)^{1-\sigma} \eta_f h(t+1)^{1-\sigma-1} \eta_f h(t+1)^{1-\sigma-1} \phi_f^* \sigma^{-a-1} \left( \frac{\alpha \eta_f}{\sigma-1} \right)
\]

\[
\phi_h^* = \frac{\gamma_h(t+1)^{\sigma-1}}{\gamma_{h(t+1)}} \zeta_h(t+1)^{1-\sigma} \tau_h(t+1)^{1-\sigma} \eta_h(t+1)^{1-\sigma} \phi_h^* \sigma^{-a-1} \left( \frac{\alpha \eta_f}{\sigma-1} \right)
\]

\[
\ln \eta_f^* = \frac{\sigma}{\sigma-1} \ln \eta_{f(t+1)} + \frac{1}{\sigma-1} \ln Z
\]

\[
\ln \phi_f^* = \frac{1}{\sigma-1} \ln \phi_{f(t+1)} + \frac{1}{\sigma-1} \ln Z
\]

98
The current research agenda of international trade revolves around decision making at the firm-level. Correspondingly, the recent evidence show that firm-level decisions have the implications of macro-level, e.g. the decision of an individual firm in an industry to adopt new technology affects the technological level of the whole industry, or the pricing decision of an individual firm affects the average national price. Therefore, the behavior of an individual firm in an economy is now considered as the key to unraveling the dynamics of trade. In this regard, the model of heterogeneous firms developed by Melitz (2003) has almost become the standard framework for the analysis. On the other hand, international trade literature presents overwhelming arguments in favor of free trade. However, the real-world trade is far from free and still face a lot of trade barriers and mostly regulated with trade policy. The implementation of trade policy has both direct and indirect effects on the decision making of the firm within an industry. Therefore, understanding the trade policy formation within the heterogeneous firms’ framework of Melitz (2003) is essential to broaden the insights of the firm’s behavior. The study in hand takes a step towards this direction and characterized trade policy formation in the heterogeneous firms model. Furthermore, it also develops a theoretical model to illustrate how trade policy affects the decision of an individual firm, particularly the decision of technology adoption.

To elaborate on the process of trade policy formation within the heterogeneous firms model, the study proceeds in two steps. In the first step, we consider the case of unilateral trade policy with the possibility of having import tariffs, import subsidy, export tax, and export subsidy as potential trade policy instruments. The results suggest that the instruments selected by a welfare-maximizing policymaker at the equilibrium are import tariff and export tax. In the second step, we consider the case of bilateral trade policy formation and the possibilities of having cooperation and no-cooperation between two countries in policy formation are also explored. Meanwhile, we have also characterized the political economy of trade policy at both steps.

In the event of unilateral trade policy formation, we pay special attention to endogenous policy formation and elaborate on the role that lobbying firms can play. This elaboration has also been done in two stages. As in chapter 2, we first endogenize trade policy formation and does not consider the lobbying participation decision of the firm. While in the second step, we endogenize this decision as well. The model we developed consists of the simple environment of two countries and two sectors. One sector is assumed produces homogeneous goods with perfect competition.
and the other sector produces heterogeneous goods with monopolistic competition. Therefore, the inefficient entry in the heterogeneous sector provides an additional reason for the government to implement corrective trade policy in this framework. However, the heterogeneous firms react to the enactment of trade policy and engage in lobbying to tilt the policy level selection in their favor. In this regard, we explore the classical Grossman and Helpman (1994) protection for sale framework and consider lobbying participation decisions at firm-level. Our results show that in the absence of lobbying firms, the policymaker imposes a uniform import tariff on all importing firms and a uniform export tax on all exporting firms. However, when we allow the lobbying possibility, the selection of policy level diverts from the optimal level. The policymaker adopts a lower policy rate for the lobbying firm. The diversion of the policy rate depends upon; the elasticity of substitution, Pareto shape parameter, and the social welfare weight that measures the degree of the benevolence of the policymaker. In addition, we have also shown that the implementation of trade policy reduces the mass of cross-border trading firms and lower the average productivity level in the economy.

At the second stage of the endogenous unilateral trade policy formation, we analyze the lobby participation decision of a lobbying firm. This has been achieved by introducing an additional stage into the PFS framework, where the decision to participate in the lobbying business or not takes place. Hence, the equilibrium mass of lobbying firms and political contributions offered by firms are determined at this stage. Chapter 3 shows that due to the associated fixed and sunk cost with lobbying participation, only most productive firms can afford to participate in the lobbying business. Besides, the lobby participation decision depends not only on the participation cost but also on the market size and benefits of lobbying. Our results conform to empirical findings related to the lobbying participation behavior of firms.

Next, we characterize trade policy formation with two additional features. These features include an active foreign country and intermediate inputs trade. In case of having an active foreign country, we explore trade policy formation in case of four policy experiments, which include unilateral trade policy, cooperative trade policy, non-cooperative trade policy, and political equilibrium trade policy. However, introducing intermediate inputs makes the model more intriguing and the model indicates that only more productive firms use imported inputs in the production process. In the case of intermediate inputs; the imposition of tariff affects the welfare negatively, as it erects a trade barrier that lowers the average productivity in the market. The intermediate inputs tariff level
selected by the policymaker at unilateral equilibrium is high and the country enjoys first the mover-
advantage. While, in the case of final goods, the follower will select a higher tariff compared to
the leader. The outcome of a cooperative policy formation experiment is free trade, as any kind of
trade restriction reduces the aggregate welfare of both countries. In the third non-cooperative
simultaneous tariff selection context, the equilibrium is characterized by symmetric Nash
equilibrium. In our last step, we bring the political economy of tariff policy in the discussion and
allow the possibility of lobbying.

Finally, we develop another political economy model that considers the population in the home
country is divided into elite, entrepreneur and labor groups. Elite holds the political power,
entrepreneurs engage in production, and labor is provided by the third group. The foreign country
has superior technology compared to the home country. Therefore, firms in the less technological
advance home country face the critical question, either to adopt or resist new technology once
trade opens. The results emerge from the discussion indicate that the market size and the political
institutions of the country play a critical role in this regard. If the market size is large, then firms
will adopt new technology more rapidly. Similarly, in the event of having a strong democracy,
adoption will be rapid. In the event of a weak democracy, firms will resist the adoption and lobby
for higher trade restrictions.

This research endeavor to characterize trade policy in heterogeneous firms model is not without
limitations. These limitations not only highlight the shortcomings of our results but also provide
the future research agenda. The first limitation of this study is that the individual heterogeneous
firm is considered to produce a single product that is unique. In reality, the manufacturing firms
are multi-product, therefore, the extension of the analysis with multiple products producing
heterogeneous firms is needed. This extended analysis will also enhance the understandings of the
decision of the firm related to which product produced and to which product export and for which
product lobby for trade barrier reduction. Similarly, the models consider trade between the two
countries only and analysis of trade and political economy of trade policy with many countries is
also crucial.

Another shortfall of this study is the explicit assumption of the productivity distribution, which is
assumed Pareto distribution. As argue by Nigai (2017) that the firm-level data show that the
distribution of productivity is not Pareto distribution for entire support. Therefore, consider the
other distributions like mixed distribution (as suggested by Nigai) or log-normal will also enhance
over understandings. Since the distribution parameter plays a critical role in defining trade policy, therefore, incorporating the possibilities of having another productivity distribution is also among the top items of my future research.

Furthermore, by assuming a competitively produce and freely traded numeraire homogeneous good, the wage rate was normalized to one in both countries. Although this normalization makes the analysis more plausible but misses the impact of trade policy implementation on the factor rewards. Hence, by incorporating the factor rewards explicitly we will be able to see either the implementation of trade policy has a positive effect or not for the domestic factors. On the same note, the utility function consider is quasi-linear, which ignores the income effect on consumer decision making. This assumption restricts the interpretation and application of the results. Therefore, trade policy formation with income effect would also be an interesting research venture. Moreover, the political trade policy formation has been done here on the basis of the PFS framework, where lobbying through monetary contribution is the way to insert the influence. As highlighted by Harstad and Svensson (2007) that trade policy meddling is also possible by bribing instead of lobbying. Hence, one particular dimension that would be interesting is the characterization of equilibrium where firms use the bribe to bypass the rules.

One particular limitation of the analysis conducted in chapter four is that it only considers import tariff as the trade policy instrument. The extension of intermediate trade analysis with other trade policy instruments like; import subsidy, export tax, or export subsidy would also possible. Similarly, the analysis ignores the bad shock as in the basic model of Melitz (2003). Therefore, introducing a bad shock that leads the exit of the firm from the market is also potential.
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# TABLE OF CONTENTS

1.  INTRODUCTION ................................................................................................................... 1  
   1.1. The Recent Developments in International Trade Theory ............................................... 1  
   1.2. Heterogeneous Firms and Trade Policy ........................................................................... 3  
   1.3. Trade Policy under Lobbying ........................................................................................... 5  
   1.4. Heterogeneous Firms and Lobbying ................................................................................ 6  
   1.5. Trade Policy and Intermediate Inputs Trade .................................................................... 8  
   1.6. A Brief Road Map ............................................................................................................ 9  

2.  TRADE POLICY UNDER THE HETEROGENEOUS LOBBYING FIRMS ..................... 12  
   2.3. The Model ...................................................................................................................... 15  
      2.3.1. The Trade Policy Choice ........................................................................................ 23  
      2.3.2. Benevolent Equilibrium .......................................................................................... 25  
      2.3.3. Political Equilibrium ............................................................................................... 26  
   2.4. A Quantitative Illustration .............................................................................................. 29  
   2.5. Conclusion ...................................................................................................................... 30  
      Appendices ............................................................................................................................ 30  

3.  THE HETEROGENEOUS FIRMS AND LOBBY PARTICIPATION DECISION ............ 35  
   3.3. The Model ...................................................................................................................... 36  
   3.4. Second Stage .................................................................................................................. 40  
   3.5. First Stage ....................................................................................................................... 41  
      3.5.1. Selection into Lobbying .......................................................................................... 42  
      3.5.2. The Equilibrium Mass of Lobbying Firms ............................................................. 43  
   3.6. Conclusion ...................................................................................................................... 45  
      Appendix ............................................................................................................................... 45  

4.  TRADE POLICY WITH INTERMEDIATE INPUTS TRADE ........................................... 48  
   4.3. The Model ...................................................................................................................... 51  
      4.3.1. Households .............................................................................................................. 51  
      4.3.2. Final Goods Producing Firms ................................................................................. 52  
      4.3.3. Intermediate Input Supplier .................................................................................... 53  
      4.3.4. Exit, Import/Export Decision of a Firm .................................................................. 54  
      4.3.5. Total Revenue and Welfare .................................................................................... 56
4.4. Stackelberg Equilibrium Tariff ................................................................. 57
4.5. Cooperative/Efficient Equilibrium Tariff ................................................... 58
4.6. Non-cooperative/Nash Equilibrium Tariff .................................................. 59
4.7. Political Equilibrium Tariff ........................................................................ 60
4.8. Quantitative Illustration ............................................................................ 62
4.9. Conclusion .................................................................................................. 64
Appendices ............................................................................................................ 65

5. HETEROGENEOUS FIRMS AND TECHNOLOGY ADOPTION: THE ROLE OF POLITICAL INSTITUTIONS AND MARKET SIZE ......................................................... 73
5.3. Empirical Motivation .................................................................................. 75
5.4. The Model .................................................................................................. 77
  5.2.1. Household Sector .................................................................................. 78
  5.2.2. Production Sector .................................................................................. 79
  5.2.3. Firm’s Behavior ..................................................................................... 79
5.3. Closed Economy Equilibrium ..................................................................... 80
  5.3.1. Entry and Exit ....................................................................................... 80
  5.3.2. Entrepreneur’s Problem ......................................................................... 82
5.4. Open Economy Equilibrium with Costly Trade .......................................... 83
  5.4.1. Trade Policy Making ............................................................................ 84
  5.4.2. Firm’s Behavior ................................................................................... 84
  5.4.3. Entry and Exit ....................................................................................... 85
  5.4.4. Entrepreneur’s Problem ......................................................................... 87
  5.4.5. Elite’s Problem ...................................................................................... 88
5.5. Technology Adoption: Decision to Adopt or Resist via Lobby for Trade Restrictions .................................................................................................................. 90
  5.5.1. Entry and Exit ....................................................................................... 90
  5.5.2. Selection of Technological Up-Gradation .............................................. 91
  5.5.3. The Possibility of Block Technology Adoption by Lobbying .............. 92
5.6. Conclusion .................................................................................................. 96
Appendix ............................................................................................................... 96

6. DISCUSSION AND CONCLUSION .................................................................. 99
REFERENCES ........................................................................................................... 103
Trade Policy, Lobbying and Heterogeneous Firms

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