Abstract of Doctoral Dissertation

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Title of Doctoral Dissertation

Mathematical Study on Fluorescence Diffuse Optical Tomography
— Recovering the Distribution of Fluorophores Using Cuboid Approximation
（蛍光拡散トモグラフィの数理的研究
— 直方体近似を用いた蛍光体分布同定）

In this thesis the time-domain fluorescence diffuse optical tomography (FDOT) is theoretically and numerically investigated based on analytical expressions for a three space dimensional diffusion equation model (DE model). Physically the radiative transfer equation model (RTE model) is a better model to describe the physical process behind the measurement of the FDOT. We carefully analyzed the derivation of the DE model from RTE model to consider about the modelling error. Since the distance between the source and detectors are short, the initial boundary value problem for the DE can be considered in the half space. Here there are two diffusion equations coupled in one of its source term. Each of them describes the emission of angularly averaged excited photon density (i.e. excited light) and that of emitted photon density (i.e. emitted light). Usually for the excited light the distribution of fluorophores in biological tissue is ignored and have the so called linearized DE model. The emission light is analytically calculated by solving an initial boundary value problem for coupled diffusion equations in the half space. Based on the analytic expression of the solution to this initial boundary value problem, we establish an error estimate for linearizing the DE model.

Our FDOT is to recover the distribution of fluorophores in biological tissue based on the linearized DE model by using the time-resolved measurement data on the boundary surface. We theoretically analyzed the identifiability of this inverse absorption problem.

Aiming a fast and robust algorithm for our FDOT inverse problem, we identify the location of a fluorescence target by assuming that it has a cuboidal shape neglecting its precise shape. We proposed and verified our inversion strategy which is a combination of theoretical arguments and numerical arguments for an inversion, which enables to obtain a stable inversion and accelerate the speed of convergence. Its effectivity and performance were tested numerically using simulated data and experimental data obtained from ex vivo beef phantoms.

We summarize each thesis chapter as follows:

Chapter 1. Introduction to FDOT. We briefly explained the light propagation in tissue and the difficulty of the problem.

Chapter 2. Linearized Diffusion Equation Model. We established the error estimate rigorously for the excitation field due to ignoring the effect of fluorescence absorption and the corresponding error estimate for recovering the distribution of fluorophores in biological tissue from the linearized DE model.

Chapter 3. Parameter Identification for FDOT. Based on the analytic expressions of excitation and emission, we analyzed rigorously the identifiability of the absorption coefficient of fluorophores for absorption coefficient in special form. Also, we analyzed the long-time behavior of emission light, by which we could estimate the fluorescence lifetime.

Chapter 4. FDOT Using Cuboid Approximation. We considered the FDOT based on linearized DE model and introduced the idea of cuboid approximation, i.e., we identify the location of a fluorescence target by assuming that it has a cuboidal shape, which is reasonable and fast in computation by numerically testing.

Chapter 5. Local Analysis for FDOT Using Cuboid Approximation. From the view of mathematics, we described our FDOT inverse problem using cuboid approximation and showed its local analysis including local solvability and local Lipschitz stability, which is also essential for the convergence of iteration methods for example Levenberg-Marquardt (LM) iteration scheme.

Chapter 6. Inversion Strategy and Numerical Inversions Using Simulation Data. Based on the simulation and property of measurement in Chapter 4, we proposed an inversion strategy based on the idea of cuboid approximation for linearized DE model to accelerate the speed of convergence of LM iteration scheme, followed by two numerical examples illustrating the performance of the proposed strategy.

Chapter 7. Numerical Inversions Using Experimental Data. We further validated our inversion strategy using experimental data obtained from ex vivo beef phantoms. By the results of inversion using simulated data and experimental data, we verified that our proposed strategy is fast and robust against the choice of initial guesses.

Chapter 8. Conclusion and Remark. We summarized the thesis and gave several remarks.