Leaders, Followers and Equity Risk Premiums in Booms and Busts

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Abstract

We study an investment problem in which two asymmetric firms face competition and the regime characterizing the economic condition follows Markov switching. We derive the value functions and investment thresholds of a leader and follower. It is found that the option value of regime uncertainty is quite important for the investment decision of firms. We also show the relationship between the equity risk premium and the economic cycle **that is not possible in previous studies that proxy economic conditions by the level of demand or other state variables.**

Key words: Real options, Competition, Timing game, Regime uncertainty

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1. Introduction

The real options approach studies an investment problem in which the value of an investment opportunity is uncertain in the future and the cost of investment incurs some irreversibility. As Dixit and Pindyck (1994) point out, it is becoming more and more important to study the investment under competition because not only it enables us to analyze a more realistic situation but also the economy is globalizing under worldwide deregulations and competition becomes fierce as a result. In this background, many theoretical studies construct a model with multiple firms in a real options framework to study the investment problem under competition.

Among them, Grenadier (1996) is regarded as a pioneering paper. He models a real estate market with two firms using a real options framework and claims that his model explains a US construction boom in 1990s. Other important theoretical papers include Huisman and Kort (1999) and Nielsen (2002). Pawlina and Kort (2006) consider the case where two firms are asymmetric in their irreversible costs and present some theoretical results. Their model has three patterns of equilibrium: preemptive, sequential and simultaneous equilibria. Takashima et al. (2008) investigate an electricity market in which two firms are asymmetric in cost parameters and operating options. Kijima and Shibata (2005) and Bouis et al. (2009) extend such approaches to the framework of three or more symmetric firms. Nishide and Yagi (2016) introduced policy uncertainty to the preemption game. As seen above, the literature on real options in competitive environments is very extensive. See, for more detail literature review, e.g. Huisman et al. (2004), Chevalier-Roignanta et al. (2011) and Azevedo and Paxson (2014).

From another viewpoint, there are several studies that introduce regime uncertainty within a real options analysis to capture the economic cycles. As we observe in the worldwide financial crisis after the failure of Lehman Brothers in September 2008, the change of regime can have a huge impact on economic circumstances. One example is the dislocations in the foreign exchange (FX) swap market between the US dollar and three major European currencies, which is empirically reported by Baba and Packer (2009). They report that almost all the FX swap deviates from the covered interest rate parity after the Lehman failure, indicating a big effect caused by the change of economic conditions.

Theoretical papers assuming regime shifts within a real options framework include Chapter 9 of Dixit and Pindyck (1994), Hassett and Metcalf (1999), Guo et al. (2005), Pawlina and Kort (2005), and Nishide and Nomi (2009). Typically, regime uncertainty is modeled in a way that parameters describing the dynamics of the state variables follow Markov switching. Among them, Driffill et al. (2013) study the investment decision of a project with Markov-modulated geometric Brownian motions. They derive a simultaneous ordinary differential equation system that can calculate an investment threshold for each regime. Their major finding is that Markov switching risk causes a delay in the expected timing of the investment.

In this paper, we consider a situation where two asymmetric firms face an investment problem under competition and the market regime is randomly switching. More concretely, we study the problem of investment timing where the cash flow is defined by the demand shock and profit coefficient. Key assumptions are that the coefficient is affected by the investment of the other firm, and that the dynamics of demand shock are modulated by a time-homogeneous Markov chain. The asymmetry of coefficients and investment costs enables us to investigate how a firm choose its optimal timing, taking into consideration the firm's advantage or disadvantage in profits and costs. Investment timing is determined by corresponding investment threshold, and if a firm's investment threshold is lower (higher) and investment timing is earlier (later) than the other's, the firm becomes a leader (follower). To the authors' best knowledge, this paper is the first attempt to combine a competitive real options model with a Markov switching regime. Not only our model is an extension of the previous studies to a more general and realistic setup, but also it enables us to describe various patterns of the competitive investment. In other words, we construct a theoretical model that can produce a wide variety of strategies in a unified framework.

The major results of this study is as follows. Each finding or implication confirms that regime uncertainty is quite important for the investment decision of firms as well as the market equilibrium.

First, our model is flexible enough to produce a wide variety of results, such that a disadvantaged firm can be a leader even if the initial demand is low. Recall that, in previous studies, if both firms wait for the investment due to the low demand, only an advantaged firm has an incentive to invest earlier and always becomes a leader when the demand reaches a certain level. This means that existing theoretical studies cannot explain the fact that a less profitable firm sometimes enters a new and developing market before a more profitable firm, while our model can do so.

Following Pawlina and Kort (2006), we analyze the condition for which type of equilibrium to occur. The second result is the finding that a preemptive equilibrium, which represents a competitive situation among firms, is more likely to occur in a boom than in a bust, and this result is most remarkable when the intensity of regime transition takes a moderate value. Intuitively, uncertainty of the demand evolution is higher in a bust and both the leader and follower have an incentive to wait for investment, resulting in a sequential or simultaneous equilibrium. The second result says that this situation is less likely to happen when the transition probability is extremely high or low. As we discuss later, we have an implication from the result that both firms takes the option value of regime uncertainty into consideration.

Third, unlike other previous studies such as Carlson et al. (2014), the equity risk premium can be non-monotonic with respect to the level of demand between leader's and follower's investment thresholds. [1] The reason is that both firms take the possibility of a regime change into account in our model. **More specifically, potential investment caused by a sudden regime change vanishes the option value and the risk premium in a bust changes the shape drastically at that point. Therefore, the risk premium in a bust has a kink and the non-monotonicity.**

Fourth, we show that the firm's beta in a bust is higher than that in a boom. Aguerrevere (2009) finds that when the demand is low, firms in competitive industries are riskier, whereas firms in concentrated industries

[1] Lambrecht et al. (2015) show that a decrease in demand level increases a firm's stock beta due to operating leverage in downturns as in Carlson et al. (2004). However when the firm switches between different procurement options, a non-monotonic behavior in the firm's beta is shown as in this work.

are riskier when demand is high. At first glance, our study replicates the result of Aguerrevere (2009), but it is not true in that our study shows the negative relationship between the beta and the economic growth. Many empirical papers such as Chen (1991) and Hoberg and Phillips (2010) suggest that the time-varying beta is negatively associated with the economic growth rate or the market return, not the absolute level of state variables. In other words, our result with regime switching model theoretically describes the relationship in a more precise way than in Aguerrevere (2009). **Intuitively lower economic growth rate reduces the investment opportunity due to decrease in the option value. Thus assets in place amount to relatively large fraction of the firm value when the economic growth rate is low. In addition assets in place in competitive market become riskier because firms' cash flows are more sensitive to demand dynamics. This corresponds to the results of Chen (1991) and Hoberg and Phillips (2010), i.e., there exists the negative relation between beta and the rate of economic growth.**

The remaining part of the paper is organized as follows. In the next section, we concisely review the model and results of Pawlina and Kort (2006) as a benchmark case. Section 3 presents our model that introduces Markov regime switching. In Section 4, we implement a numerical analysis and show how each firm chooses its investment threshold, depending on the regime. Following the analysis in Pawlina and Kort (2006), we examine in Section 5 the conditions for which type of equilibrium to occur in each regime and show the effect of regime uncertainty on the investment decisions of both firms and the market equilibrium. We discuss in Section 6 how effectively our model explains the behavior of a firm's beta in relation to the economic cycles. Section 7 provides some concluding remarks. The appendices following Section 7 present the glossary of the notation used in the paper, and supplementary results.

2. The Model

2.1. Cash Flow and Market Settings

Consider a situation where two firms compete in a product market. The demand shock in the market is denoted by P_t. Superscript $i \in \{1, 2\}$ denotes the identity of a firm. Each firm has a single investment opportunity to increase their profits. Prior to making an investment, firm i generates the cash flow $D^i_{00} P_t$. We assume that P_t follows a stochastic differential equation as

$$\mathrm{d}P_t = \mu_{\epsilon(t)} P_t \mathrm{d}t + \sigma_{\epsilon(t)} P_t \mathrm{d}z_t,$$

with initial value $P_0 = P$. Here, the expected growth rate μ and the volatility σ are dependent on $\epsilon(t)$, the regime at time t. We assume that there are only two regimes in the economy, so that we have

$$(\mu_\epsilon, \sigma_\epsilon) = \begin{cases} (\mu_1, \sigma_1), & \text{if } \epsilon = 1, \\ (\mu_2, \sigma_2), & \text{if } \epsilon = 2. \end{cases}$$

The key assumption is that the regime $\{\epsilon(t)\}$ follows a stationary Markov chain as

$$\begin{aligned} &1 \to 2, \quad \text{with intensity } \lambda_1, \\ &2 \to 1, \quad \text{with intensity } \lambda_2. \end{aligned}$$

In later discussions, we regard regime 1 as a good state (boom) and regime 2 as a bad one (bust).

Suppose that firm i currently receives the instantaneous cash flow $D^i_{00}P$ and considers an investment in the new technology. The investment incurs an irreversible cost K^i for firm i. Let τ^i_L denote the investment timing of firm i when the firm is a leader of the investment, and τ^i_F the timing in the case of a follower. If firm i becomes a leader, the firm receives an instantaneous cash flow $D^i_{10}P_t$ until the other firm invests. After the investment by the other firm, the cash flow of firm i changes to $D^i_{11}P_t$. On the other hand, if firm i becomes a follower, the firm receives $D^i_{01}P_t$ after the other firm's investment, and then $D^i_{11}P_t$ after the firm's own investment. Here, to examine how the preemption of a leader firm affects the investment timing of both firms, we assume that the deterministic profit coefficient $D^i_{N_iN_j}$ has the relative magnitude relation

$$\begin{array}{ccc} D^i_{10} & > & D^i_{00} \\ \vee & & \vee \\ D^i_{11} & > & D^i_{01}, \end{array} \tag{1}$$

where

$$N_k = \begin{cases} 0, & \text{if firm } k \in \{i,j\} \text{ has not invested,} \\ 1, & \text{if firm } k \in \{i,j\} \text{ has invested.} \end{cases}$$

The inequalities $D^i_{10} > D^i_{00}$ and $D^i_{11} > D^i_{01}$ imply that firm's investment increases its profit regardless whether the other firm has invested or not. On the other hand, $D^i_{11} < D^i_{10}$ and $D^i_{01} < D^i_{00}$ imply that the investment of the other firm causes a decrease in the cash flow due to the product obsolescence.[2] Thus the instantaneous cash flow of firm i in the case of a leader can be expressed as

$$\mathbf{1}_{\{t<\tau_L^i\}} D_{00}^i P_t + \mathbf{1}_{\{\tau_L^i \le t < \tau_F^j\}} D_{10}^i P_t + \mathbf{1}_{\{t \ge \tau_F^j\}} D_{11}^i P_t, \tag{2}$$

where $j = 3 - i$. When firm i decides to be a follower, the firm receives the instantaneous cash flow $D_{11}^i P_t$ after the investment. The cash flow in this case is written as

$$\mathbf{1}_{\{t<\tau_L^j\}} D_{00}^i P_t + \mathbf{1}_{\{\tau_L^j \le t < \tau_F^i\}} D_{01}^i P_t + \mathbf{1}_{\{t \ge \tau_F^i\}} D_{11}^i P_t. \tag{3}$$

Finally, the discount rate r is assumed to be constant for simplicity.[3]

2.2. The Asymmetric Case without Regime Shift

In this subsection, we quickly review the investment problem of asymmetric firms without regime switching, considered by Pawlina and Kort (2006). The setup corresponds to the case $\mu \equiv \mu_1 = \mu_2$ and $\sigma \equiv \sigma_1 = \sigma_2$.[4]

Suppose first that firm i is a follower and let V_F^i and τ_F^i denote the value function and the investment timing of firm i, respectively. The optimal investment timing takes the form of a first hitting time as

$$\tau_F^i = \inf\{t \ge 0;\ P_t \ge \bar{P}_F^i\}.$$

[2]By imposing $D_{00}^i = D_{01}^i = 0$, we can consider the market entry model as in Grenadier (1996), Nielsen (2002), Takashima et al. (2008), and so on.

[3]We do not consider the case where the discount rate r is modulated by a Markov chain because it produces no qualitative difference.

[4]Pawlina and Kort (2006) consider the case where only cost parameters $\{K^i\}$ are asymmetric. The results in this subsection are essentially the same as theirs despite the difference.

Let G_L^i denote the net present value of the project for firm i as a leader for $t < \tau_F^j$.[5] If we assume the equilibrium notion of Fudenberg and Tirole (1985), firm i has an incentive to invest in the project when $G_L^i(P) - K^i \geq V_F^i(P)$. In other words, denoting by $\bar{P}_L^i$ the investment threshold of firm i as a leader, $\bar{P}_L^i$ satisfies the equation

$$G_L^i(\bar{P}_L^i) - K^i = V_F^i(\bar{P}_L^i). \tag{4}$$

Throughout the following analysis, we lose no generality in assuming that $\bar{P}_F^1 < \bar{P}_F^2$. Hereafter we say that firms 1 and 2 are advantaged and disadvantaged, respectively, if this inequality holds. In what follows we consider only the case where $\bar{P}_L^1 < \bar{P}_L^2$ in addition to $\bar{P}_F^1 < \bar{P}_F^2$.[6]

In some cases, both firms are willing to invest simultaneously, even though each firm knows that the other firm invests at the same time. Although the firms compete in the market, it results in a noncooperative outcome, which is often referred to as tacit collusion. Let V_S^i denote the value function of firm i's simultaneous investment. Simultaneous investment occurs if and only if

$$G_L^i(x) - K^i \leq V_S^i(x), \qquad \forall x. \tag{5}$$

The following proposition describes the strategies of both firms, depending on the three cases.

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[5] The closed form expressions of V_F^i, G_L^i and $\bar{P}_F^i$ are obtained by Pawlina and Kort (2006).

[6] The sufficient conditions for $\bar{P}_L^1 < \bar{P}_L^2$ and $\bar{P}_F^1 < \bar{P}_F^2$ are that

$$\frac{D_{10}^1 - D_{00}^1}{K^1} \geq \frac{D_{10}^2 - D_{00}^2}{K^2} \text{ and } \frac{D_{11}^1 - D_{01}^1}{K^1} > \frac{D_{11}^2 - D_{01}^2}{K^2},$$

which are always assumed throughout this paper.

Proposition 1 (Pawlina and Kort, 2006)**.** *In the case of asymmetric firms and no regime switch, each firm takes the following strategy, depending on parameters, especially* $\bar{P}_L^2$ *and the initial value of* P*.*

(i) Simultaneous investment: If (5) *holds, both firms invest at the same time.*

(ii) Preemptive investment: Suppose that (5) *does not hold and there exist two real numbers* $\bar{P}_L^2$ *and* $\tilde{P}_L^2$ *that satisfy* (4) *with* $\bar{P}_L^2 < \tilde{P}_L^2$*. Only for* $\bar{P}_L^2 \leq P < \tilde{P}_L^2$*, both firms have an incentive of immediate investment. Otherwise, firm 2 has no incentive to invest.*

(iii) Sequential investment: Otherwise, the strategy of each firm is described by the following:[7] *For all* P*, only firm 1 has an incentive to be a first investor.*

Remark 1. In this paper, we focus on which strategy for each firm to take and what the market is like as a consequence. Equivalently, we pay no attention to which firm actually becomes a leader. We also exclude the case of coordination failure in which both firms simultaneously invest although it is not optimal. On the timing game and the results, refer to Fudenberg and Tirole (1985) for a general concept and Huisman and Kort (1999) for related topics in a real options analysis.

Hereafter, firm 2 is said to be fully disadvantaged if there exists no real

[7] If (4) has exactly one solution for firm 2, firm 2 is at this point indifferent between being the leader and the follower and strictly prefers being the follower for the remaining values of P. Therefore, it always weakly prefers to be the follower.

number that satisfies (4) for $i = 2$. In other words, firm 2 has no incentive to become a leader if firm 2 is fully disadvantaged. Otherwise, we call firm 2 partly disadvantaged.

We observe from Proposition 1 that firm 1 is always a leader when the state variable starts at a low level. In other words, if investments in a newly developing market are considered within this setup, a firm that is profitable or has an advanced technology in costs can always invest first and increase its profit before the other. However, in actual markets, there are some cases in which a firm that seems less profitable invests before an advantaged firm. For example, in the thin-film transistor-liquid crystal display (TFT-LCD) industry, various firms including followers have invested in a boom by following an economic cycle in the industry, it is called "crystal cycle" (Mathews, 2005). As a result Korean and Taiwanese companies as Samsung and LG Display, which are previous follower companies, account for more than 80% of the TFT-LCD market. In the next section, we present a model that can explain this fact. That is, a disadvantaged firms may invest and increase its profit before an advantaged firm in our model.

3. The Asymmetric Case with Markov Regime Switching

In this section, we propose our original model that introduces a Markov switching regime into Pawlina and Kort (2006), and show how results are different from the case of no regime switch. As in the previous section, we assume that firm 1 is advantageous for all regimes.

3.1. Follower's Problem

We firstly consider the problem of the follower's investment decision. Denote by $V^i_{F\epsilon}$ the value function of firm i in regime ϵ and by $G^i_{F\epsilon}$ the net present value of an immediate investment.

Recall that many papers, e.g. Bloome (2009), report the negative relationship between uncertainty and the economic conditions. Following this empirical finding, we here assume $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$, implying that regimes 1 and 2 represent a boom and a bust, respectively. Other variables such as $D^i_{N_iN_j}$ are assumed to be independent of the regime. When the parameters μ and σ are modulated by a Markov chain with two possible states, there are two thresholds $\bar{P}^i_{F1}$ and $\bar{P}^i_{F2}$ with $\bar{P}^i_{F1} < \bar{P}^i_{F2}$.[8] Suppose that $\bar{P}^i_{F1} \leq P < \bar{P}^i_{F2}$ and the regime shifts from 2 to 1. Then the follower firm has an incentive to invest in the project all at once. Note that an investment is irreversible in the sense that the firm cannot cancel the project if the regime becomes 2 again. Figure 1 describes how the project values changes, depending on the value of P and the regime.

[Figure 1 is inserted around here.]

We need to take the possibility of a regime shift into account to derive the value function for each regime. The derivation procedure is exactly the same as Driffill et al. (2013) and thus we refer to their paper for a detailed discussion.

First suppose that $P \geq \bar{P}^i_{F2}$. Firm i immediately invest in the project

[8] From numerical implementation with a wide variety of parameter settings, $\bar{P}^i_{F1}$ is always lower than $\bar{P}^i_{F2}$ if $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$.

regardless of the realized regime. Hence the value function $V^i_{F\epsilon}$ is equal to the net present value of the project minus the cost, or $V^i_{F\epsilon} = G^i_{F\epsilon} - K^i$. It is easily confirmed from Ito's formula that $\{G^i_{F\epsilon}\}_{\epsilon=1,2}$ satisfy the following simultaneous ordinary differential equation (ODE hereafter) system:

$$\begin{cases} \dfrac{\sigma_1^2}{2}P^2\dfrac{\mathrm{d}^2G^i_{F1}}{\mathrm{d}P^2} + \mu_1 P\dfrac{\mathrm{d}G^i_{F1}}{\mathrm{d}P} - rG^i_{F1} + \lambda_1(G^i_{F2} - G^i_{F1}) + D^i_{11}P = 0, \\ \dfrac{\sigma_2^2}{2}P^2\dfrac{\mathrm{d}^2G^i_{F2}}{\mathrm{d}P^2} + \mu_2 P\dfrac{\mathrm{d}G^i_{F2}}{\mathrm{d}P} - rG^i_{F2} + \lambda_2(G^i_{F1} - G^i_{F2}) + D^i_{11}P = 0. \end{cases} \tag{6}$$

The last terms of (6) represent the received cash flow of the follower in regime ϵ because both firms have already invested, and the fourth terms represent the possibility of a regime shift from one to the other.

Since $G^i_{F\epsilon}$ evidently includes no option value, we conjecture that the function takes a linear form

$$G^i_{F\epsilon}(P) = \pi_\epsilon D^i_{11}P.$$

Substituting it into the simultaneous ODEs, we have

$$\pi_\epsilon = \frac{r + \lambda_\epsilon + \lambda_{\hat{\epsilon}} - \mu_{\hat{\epsilon}}}{(r + \lambda_\epsilon - \mu_\epsilon)(r + \lambda_{\hat{\epsilon}} - \mu_{\hat{\epsilon}}) - \lambda_\epsilon\lambda_{\hat{\epsilon}}}, \tag{7}$$

where $\hat{\epsilon} = 3 - \epsilon$.

Second, we consider the case $\bar{P}^i_{F1} \leq P < \bar{P}^i_{F2}$. When $\epsilon = 1$, the follower firm immediately invests in the project and value function is equal to $\pi_1 D^i_{11}P - K^i$ with coefficient π_1 given by (7). On the other hand, the value function in regime 2, which includes the value of a potential investment in the future, satisfies the following ODE:

$$\frac{\sigma_2^2}{2}P^2\frac{\mathrm{d}^2V^i_{F2}}{\mathrm{d}P^2} + \mu_2 P\frac{\mathrm{d}V^i_{F2}}{\mathrm{d}P} - rV^i_{F2} + \lambda_2(G^i_{F1} - K^i - V^i_{F2}) + D^i_{01}P = 0.$$

We conjecture that the candidate function takes the form

$$V_{F2}^i(P) = b_{21}^i P^{\alpha_1} + b_{22}^i P^{\alpha_2} + b_{23}^i P + b_{24}^i. \tag{8}$$

The first two terms of (8) represent the option value to wait for the investment in the project, while the last two terms are the net present value of the cash flow after investment due to a sudden regime shift. Substituting it into the ODE, we obtain

$$b_{23}^i = \frac{D_{01}^i + \lambda_2 \pi_1 D_{11}^i}{r + \lambda_2 - \mu_2}, \qquad b_{24}^i = -\frac{\lambda_2}{r + \lambda_2} K^i$$

and find that α_1 and α_2 are the roots of the quadratic equation

$$\frac{\sigma_2^2}{2}\alpha(\alpha - 1) + \mu_2 \alpha - (r + \lambda_2) = 0. \tag{9}$$

Note also that the value function in regime 2 must satisfy

$$V_{F2}^i(\bar{P}_{F2}^i) = G_{F2}^i(\bar{P}_{F2}^i) - K^i,$$

and

$$\lim_{P \uparrow \bar{P}_{F2}^i} \frac{\mathrm{d}V_{F2}^i}{\mathrm{d}P}(P) = \lim_{P \downarrow \bar{P}_{F2}^i} \frac{\mathrm{d}G_{F2}^i}{\mathrm{d}P}(P)$$

as value-matching and smooth-pasting conditions, respectively.

Third, for $P < \bar{P}_{F1}^i$, the value functions satisfy the following ODE system:

$$\begin{cases} \dfrac{\sigma_1^2}{2} P^2 \dfrac{\mathrm{d}^2 V_{F1}^i}{\mathrm{d}P^2} + \mu_1 P \dfrac{\mathrm{d}V_{F1}^i}{\mathrm{d}P} - rV_{F1}^i + \lambda_1 (V_{F2}^i - V_{F1}^i) + D_{01}^i P = 0, \\ \dfrac{\sigma_2^2}{2} P^2 \dfrac{\mathrm{d}^2 V_{F2}^i}{\mathrm{d}P^2} + \mu_2 P \dfrac{\mathrm{d}V_{F2}^i}{\mathrm{d}P} - rV_{F2}^i + \lambda_2 (V_{F1}^i - V_{F2}^i) + D_{01}^i P = 0. \end{cases} \tag{10}$$

The candidate function of $V_{F\epsilon}^i$ is conjectured to be

$$V_{F\epsilon}^i(P) = c_{\epsilon 1}^i P^{\gamma_1} + c_{\epsilon 2}^i P^{\gamma_2} + c_{\epsilon 3}^i P, \qquad \epsilon = 1, 2. \tag{11}$$

In contrast to (8), (11) does not contain a constant term associated with the cost K^i since a sudden regime shift does not induce an immediate investment. Substituting (11) into (10) leads to the particular solution

$$c^i_{\epsilon 3} = \pi_\epsilon D^i_{01},$$

and the four equations:

$$\begin{cases} \left(\frac{\sigma_1^2}{2}\gamma_1(\gamma_1 - 1) + \mu_1\gamma_1 - (r + \lambda_1)\right) c^i_{11} + \lambda_1 c^i_{21} = 0, \\ \left(\frac{\sigma_1^2}{2}\gamma_2(\gamma_2 - 1) + \mu_1\gamma_2 - (r + \lambda_1)\right) c^i_{12} + \lambda_1 c^i_{22} = 0, \\ \left(\frac{\sigma_2^2}{2}\gamma_1(\gamma_1 - 1) + \mu_2\gamma_1 - (r + \lambda_2)\right) c^i_{21} + \lambda_2 c^i_{11} = 0, \\ \left(\frac{\sigma_2^2}{2}\gamma_2(\gamma_2 - 1) + \mu_2\gamma_2 - (r + \lambda_2)\right) c^i_{22} + \lambda_2 c^i_{12} = 0. \end{cases}$$

Since $\lim_{P\downarrow 0} V^i_{F\epsilon}(P) = 0$, γ_1 and γ_2 must be the positive roots of the following quartic equation:

$$\left[\frac{\sigma_1^2}{2}\gamma(\gamma - 1) + \mu_1\gamma - (r + \lambda_1)\right]\left[\frac{\sigma_2^2}{2}\gamma(\gamma - 1) + \mu_2\gamma - (r + \lambda_2)\right] = \lambda_1\lambda_2. \tag{12}$$

The threshold in regime 1, denoted by $\bar{P}^i_{F1}$, satisfies

$$V^i_{F1}(\bar{P}^i_{F1}) = G^i_{F1}(\bar{P}^i_{F1}) - K^i$$

and

$$\lim_{P\uparrow \bar{P}^i_{F1}} \frac{\mathrm{d}V^i_{F1}}{\mathrm{d}P}(P) = \lim_{P\downarrow \bar{P}^i_{F1}} \frac{\mathrm{d}G^i_{F1}}{\mathrm{d}P}(P)$$

as value-matching and smooth-pasting conditions. Similarly, in regime 2, the continuity and high-contact conditions are given by

$$\lim_{P\uparrow \bar{P}^i_{F1}} V^i_{F2}(P) = \lim_{P\downarrow \bar{P}^i_{F1}} V^i_{F2}(P)$$

and

$$\lim_{P \uparrow \bar{P}^i_{F1}} \frac{\mathrm{d}V^i_{F2}}{\mathrm{d}P}(P) = \lim_{P \downarrow \bar{P}^i_{F1}} \frac{\mathrm{d}V^i_{F2}}{\mathrm{d}P}(P),$$

respectively.

We now summarize the result as a proposition.

Proposition 2. *The value function of firm i in the case of a follower for regime 1 is given by*

$$V^i_{F1}(P) = \begin{cases} \pi_1 D^i_{11} P - K^i, & \text{for } P \geq \bar{P}^i_{F1}, \\ c^i_{11} P^{\gamma_1} + c^i_{12} P^{\gamma_2} + \pi_1 D^i_{01} P, & \text{for } P < \bar{P}^i_{F1} \end{cases}$$

and for regime 2 by

$$V^i_{F2}(P) = \begin{cases} \pi_2 D^i_{11} P - K^i, & \text{for } P \geq \bar{P}^i_{F2}, \\ b^i_{21} P^{\alpha_1} + b^i_{22} P^{\alpha_2} & \\ \quad + \frac{D^i_{01} + \lambda_2 \pi_1 D^i_{11}}{r + \lambda_2 - \mu_2} P - \frac{\lambda_2}{r + \lambda_2} K^i, & \text{for } \bar{P}^i_{F1} \leq P < \bar{P}^i_{F2}, \\ \ell_1 c^i_{11} P^{\gamma_1} + \ell_2 c^i_{12} P^{\gamma_2} + \pi_2 D^i_{01} P, & \text{for } P < \bar{P}^i_{F1}, \end{cases}$$

where

$$\ell_k = \frac{r + \lambda_1 - \mu_1 \gamma_k - \frac{\sigma_1^2}{2} \gamma_k (\gamma_k - 1)}{\lambda_1}, \qquad k = 1, 2.$$

The coefficients and the investment thresholds are determined by the system of six simultaneous equations (22)–(27) *in Appendix B.*

The formulae of value functions are same as in Driffill et al. (2013) because a follower no longer competes the other firm. Since the system has totally six unknowns $\bar{P}^i_{F1}$, $\bar{P}^i_{F2}$, b^i_{21}, b^i_{22}, c^i_{11} and c^i_{12} and has six equations at the same time, it is theoretically solvable. However, it seems hard to obtain a closed-form solution. Therefore, we shall numerically calculate the simultaneous equations to solve and derive the investment thresholds.

3.2. Leader's Problem

In this subsection, we consider the investment decision of firm i as a leader. Let $G^i_{L\epsilon}$ denote the net present value (NPV herearter) of the project for a leader in regime ϵ after investment. Note that the function $G^i_{L\epsilon}$ is dependent on the thresholds of the follower firm $\bar{P}^j_{F\epsilon}$, since the cash flow is affected by whether the other firm invests or not. Taking this into consideration, the NPVs of an immediate investment by the leader are described as Figure 2.

[Figure 2 is inserted around here.]

We derive the functions $G^i_{L\epsilon}$ by noting these relations.

Consider first the case $P \geq P^j_{F2}$. In this situation the other firm is willing to immediately invest regardless of a regime, and we have $G^i_{L\epsilon}(P) = G^i_{F\epsilon}(P) = \pi_\epsilon D^i_{11} P$, where π_ϵ are given by (7).

For $\bar{P}^j_{F1} \leq P < \bar{P}^j_{F2}$, the other firm immediately invests and receives the cash flow in regime 1, implying that $G^i_{L1}(P) = G^i_{F1}(P) = \pi_1 D^i_{11} P$. On the other hand, G^i_{L2}, the NPV of firm i in regime 2 as a leader, satisfies the following ODE

$$\frac{\sigma_2^2}{2} P^2 \frac{\mathrm{d}^2 G^i_{L2}}{\mathrm{d}P^2} + \mu_2 P \frac{\mathrm{d}G^i_{L2}}{\mathrm{d}P} - rG^i_{L2} + \lambda_2 (G^i_{F1} - G^i_{L2}) + D^i_{10} P = 0. \tag{13}$$

Note that (13) includes G^i_{F1} and that it is already solved in the previous discussions. The last term of (13) represents the current cash flow of firm i as a leader. Let the candidate function of G^i_{L2} be conjectured as

$$G^i_{L2}(P) = e^i_{21} P^{\alpha_1} + e^i_{22} P^{\alpha_2} + e^i_{23} P. \tag{14}$$

The first two terms describe the (negative) option value which represents the future entry by the other firm, while the last term is equal to the net present value of the cash flow in the future. Substituting the particular solution $e^i_{23}P$ into the ODE yields

$$e^i_{23} = \frac{D^i_{10} + \lambda_2\pi_1 D^i_{11}}{r + \lambda_2 - \mu_2}.$$

In the case of a leader firm, only the value-matching condition at $\bar{P}^j_{F2}$ holds, i.e.,

$$G^i_{L2}(\bar{P}^j_{F2}) = G^i_{F2}(\bar{P}^j_{F2})$$

and any smooth-pasting condition is not necessary. See Driffill et al. (2013) for the discussion on this issue.

For $P < \bar{P}^j_{F1}$, the ODEs of $G^i_{L\epsilon}$ are given by

$$\begin{cases} \dfrac{\sigma_1^2}{2}P^2\dfrac{\mathrm{d}^2G^i_{L1}}{\mathrm{d}P^2} + \mu_1 P\dfrac{\mathrm{d}G^i_{L1}}{\mathrm{d}P} - rG^i_{L1} + \lambda_1(G^i_{L2} - G^i_{L1}) + D^i_{10}P = 0, \\ \dfrac{\sigma_2^2}{2}P^2\dfrac{\mathrm{d}^2G^i_{L2}}{\mathrm{d}P^2} + \mu_2 P\dfrac{\mathrm{d}G^i_{L2}}{\mathrm{d}P} - rG^i_{L2} + \lambda_2(G^i_{L1} - G^i_{L2}) + D^i_{10}P = 0. \end{cases} \tag{15}$$

The candidate function of $G^i_{L\epsilon}$ is conjectured to be

$$G^i_{L\epsilon}(P) = h^i_{\epsilon 1}P^{\gamma_1} + h^i_{\epsilon 2}P^{\gamma_2} + h^i_{\epsilon 3}P. \tag{16}$$

We can provide an interpretation for (16) in a similar way to the one for (8). Substituting the particular solution $h^i_{\epsilon 3}P$ into the ODEs, we obtain

$$h^i_{\epsilon 3} = \pi_\epsilon D^i_{10}.$$

In regime 1, the value-matching condition at $\bar{P}^j_{F1}$ is given by

$$G^i_{L1}(\bar{P}^j_{F1}) = G^i_{F1}(\bar{P}^j_{F1}).$$

In regime 2, we have continuity and high-contact conditions as

$$\lim_{P \uparrow \bar{P}^j_{F1}} G^i_{L2}(P) = \lim_{P \downarrow \bar{P}^j_{F1}} G^i_{L2}(P)$$

and

$$\lim_{P \uparrow \bar{P}^j_{F1}} \frac{\mathrm{d}G^i_{L2}(P)}{\mathrm{d}P} = \lim_{P \downarrow \bar{P}^j_{F1}} \frac{\mathrm{d}G^i_{L2}(P)}{\mathrm{d}P},$$

respectively.[9]

The following proposition summarizes the case of a leader.

Proposition 3. *The NPV of cash flow for firm i as a leader is given by*

$$G^i_{L1}(P) = \begin{cases} \pi_1 D^i_{11} P, & \text{for } P \geq \bar{P}^j_{F1}, \\ h^i_{11} P^{\gamma_1} + h^i_{12} P^{\gamma_2} + \pi_1 D^i_{10} P, & \text{for } P < \bar{P}^j_{F1} \end{cases}$$

in regime 1 and

$$G^i_{L2}(P) = \begin{cases} \pi_2 D^i_{11} P, & \text{for } P \geq \bar{P}^j_{F2}, \\ e^i_{21} P^{\alpha_1} + e^i_{22} P^{\alpha_2} + \frac{D^i_{10} + \lambda_2 \pi_1 D^i_{11}}{r + \lambda_2 - \mu_2} P, & \text{for } \bar{P}^j_{F1} \leq P < \bar{P}^j_{F2}, \\ \ell_1 h^i_{11} P^{\gamma_1} + \ell_2 h^i_{12} P^{\gamma_2} + \pi_2 D^i_{10} P, & \text{for } P < \bar{P}^j_{F1} \end{cases}$$

in regime 2. The coefficients and the investment thresholds are determined by the system of four simultaneous equations (28)–(31) *in Appendix B. The threshold of firm i as a leader in regime ϵ, which denotes $\bar{P}^i_{L\epsilon}$, can be obtained by the condition $G^i_{L\epsilon}(\bar{P}^i_{L\epsilon}) - K^i = V^i_{F\epsilon}(\bar{P}^i_{L\epsilon})$.*

The formulae of the NPV cash flow for a leader are different from Driffill et al. (2013) unlike that of value functions for a follower. We remark the difference by the decomposition of $G^i_{L\epsilon}$.

[9] The function G^i_{L2} must be of C^1 except for $P = \bar{P}^j_{F2}$.

Remark 2. As in Carlson et al. (2014), we can give what each term in the function $G^i_{L\epsilon}$ represents as follows:

$$\underbrace{\pi_\epsilon D^i_{10} P}_{\text{assets in place}} \quad + \quad \underbrace{h^i_{\epsilon 1} P^{\gamma_1} + h^i_{\epsilon 2} P^{\gamma_2}}_{\text{rival-value adjustment}}$$

for $P < \bar{P}^j_{F1}$, and

$$\underbrace{\frac{D^i_{10} + \lambda_2 \pi_1 D^i_{11}}{r + \lambda_2 - \mu_2} P}_{\text{assets in place + cash flow for regime change}} \qquad + \qquad \underbrace{e^i_{21} P^{\alpha_1} + e^i_{22} P^{\alpha_2}}_{\text{rival-value adjustment}} \tag{17}$$

for $\bar{P}^j_{F1} \geq P < \bar{P}^j_{F2}$ and $\epsilon = 2$.[10] The first term in (17) includes the NPV associated with a sudden regime changes from 2 to 1. The rival-value adjustment reflects the effect of competitor expansion and is always negative.

3.3. Simultaneous Investment

Let $\tau_{S\epsilon} = \inf\{t \geq 0;\ P_t \geq \bar{P}_{S\epsilon}\}$ denote the simultaneous investment timing of both firms in regime ϵ. Thus the instantaneous cash flow of firm i's simultaneous investment can be expressed as

$$\mathbf{1}_{\{t<\tau_{S\epsilon}\}} D^i_{00} P_t + \mathbf{1}_{\{t\geq\tau_{S\epsilon}\}} D^i_{11} P_t, \tag{18}$$

[10] More formally,

$$\pi_\epsilon D^i_{10} P = \mathbb{E}_{(\epsilon,P)}\left[\int_0^\infty \mathrm{e}^{-rt} D^i_{10} P_t \mathrm{d}t\right],$$
$$\frac{D^i_{10} + \lambda_2 \pi_1 D^i_{11}}{r + \lambda_2 - \mu_2} P = \mathbb{E}_{(\epsilon=2,P)}\left[\int_0^{T_1} \mathrm{e}^{-rt} D^i_{10} P_t \mathrm{d}t + \int_{T_1}^\infty \mathrm{e}^{-rt} D^i_{11} P_t \mathrm{d}t\right],$$

where $T_1 = \inf\{t \geq 0; \epsilon(t) = 1\}$.

which means that the value function of simultaneous investment is given by replacing D^i_{01} with D^i_{00} in the value function of the follower.

Since firm 1 has advantage in profit and cost, the optimal investment threshold of firm 1 is always lower than that of firm 2. Firm 2 reluctantly follows firm 1's timing and only firm 1 can maximize the value of simultaneous investment. Therefore, the smooth-pasting condition is satisfied for only firm 1, implying that

$$V^i_{S\epsilon}(\bar{P}_{S\epsilon}) = G^i_{F\epsilon}(\bar{P}_{S\epsilon}) - K^i,$$

for $i = 1, 2$ and

$$\lim_{P \uparrow \bar{P}_{S\epsilon}} \frac{\mathrm{d}V^1_{S\epsilon}}{\mathrm{d}P}(P) = \lim_{P \downarrow \bar{P}_{S\epsilon}} \frac{\mathrm{d}G^1_{F\epsilon}}{\mathrm{d}P}(P).$$

We now summarize the result for the simultaneous investment as a proposition.

Proposition 4. *The value function of a simultaneous investment in regime 1 is given by*

$$V^i_{S1}(P) = \begin{cases} \pi_1 D^i_{11} P - K^i, & \textit{for } P \geq \bar{P}_{S1}, \\ q^i_{11} P^{\gamma_1} + q^i_{12} P^{\gamma_2} + \pi_1 D^i_{00} P, & \textit{for } P < \bar{P}_{S1} \end{cases}$$

and in regime 2 by

$$V^i_{S2}(P) = \begin{cases} \pi_2 D^i_{11} P - K^i, & \textit{for } P \geq \bar{P}_{S2}, \\ m^i_{21} P^{\alpha_1} + m^i_{22} P^{\alpha_2} & \\ \quad + \frac{D^i_{00} + \lambda_2 \pi_1 D^i_{11}}{r + \lambda_2 - \mu_2} P - \frac{\lambda_2}{r + \lambda_2} K^i, & \textit{for } \bar{P}_{S1} \leq P < \bar{P}_{S2}, \\ \ell_1 q^i_{11} P^{\gamma_1} + \ell_2 q^i_{12} P^{\gamma_2} + \pi_2 D^i_{00} P, & \textit{for } P < \bar{P}_{S1}. \end{cases}$$

The coefficients and the investment thresholds are determined by the system of six simultaneous equations (32)–(37) *in Appendix B.*

The formulae of value functions are same as in Proposition 2 because the value function of simultaneous investment is given by replacing D^i_{01} with D^i_{00} in the value function of the follower. Note that the system for firm 2 has only four simultaneous equations (32)–(35) and unknowns m^i_{21}, m^i_{22}, q^i_{11} and q^i_{12} since firm 2 can not determine the investment thresholds.

4. Investment Strategies

In this section, we study with numerical examples how each firm chooses its investment strategy, depending on the strategy of the other firm. We present three example to show that our model is rich and flexible enough to explain many actual situations within a unified framework.

4.1. Case 1: Benchmark case

The parameter values in Table 1 are used for the numerical analysis as a benchmark case. With these parameter values, we obtain thresholds in Table 2. The numerical results actually shows that $\bar{P}^1_{F\epsilon} < \bar{P}^2_{F\epsilon}$ for $\epsilon = 1, 2$. Note that firm 2 is partly disadvantaged in regime 1 but fully disadvantaged in regime 2.

Table 3 summarizes the investment strategies that each firm chooses, depending on the range of the state variable P. Numbers in the table represent the label of the investing firm, and a blank cell indicates that both firms wait for an investment. The situation where both firms have an incentive to invest and only one of them can become a leader is represented by $\times$. For

Table 1: Parameter setting in the benchmark case.

μ_1	μ_2	σ_1	σ_2	r	λ_1	λ_2	K^1	K^2
0.05	0	0.2	0.5	0.1	0.2	0.2	10	12
D_{00}^1	D_{00}^2	D_{01}^1	D_{01}^2	D_{10}^1	D_{10}^2	D_{11}^1	D_{11}^2	
0.5	0.5	0.25	0.25	1.5	1.4	1	0.9	

Table 2: Thresholds of the firms for the benchmark case.

$\bar{P}_{L1}^1$	$\bar{P}_{L2}^1$	$\bar{P}_{F1}^1$	$\bar{P}_{F2}^1$	$\bar{P}_{S1}$	$\bar{P}_{S2}$
0.8635	1.1383	2.1706	3.4224	3.2558	5.1336
$\bar{P}_{L1}^2$	$\tilde{P}_{L1}^2$	$\bar{P}_{F1}^2$	$\bar{P}_{F2}^2$		
1.6159	1.8054	3.0054	4.7387		

Table 3: Investment strategies of each firm in case 1.

$\epsilon = 1$	1	1	$\times$	1	1	1, 2	1, 2	1, 2
$\epsilon = 2$		1	1	1	1	1	1	1, 2
	$\bar{P}_{L1}^1$	$\bar{P}_{L2}^1$	$\bar{P}_{L1}^2$	$\tilde{P}_{L1}^2$	$\bar{P}_{F1}^1$	$\bar{P}_{F1}^2$	$\bar{P}_{F2}^1$	$\bar{P}_{F2}^2$

example, for $\bar{P}_{L1}^1 \leq P < \bar{P}_{L2}^1$, firm 1 can become a leader and firm 2 can not in regime 1, while both firms wait for investing in regime 2.[11] For $P \geq \bar{P}^2_{F2}$, both firms immediately and simultaneously invest.

In this case, firm 1 always has an incentive to become a leader for $P \geq \bar{P}^1_{L2}$. However, firm 2 has the incentive only for $\bar{P}^2_{L1} \leq P < \tilde{P}^2_{L1}$ in regime 1 and can never become a leader in regime 2. We observe that in this parameter setting, only firm 1 can be a leader when the state variable starts at a lower level like previous theoretical papers.

4.2. Case 2: Unknown winner

In this case, we choose $K^2 = 11$, $D^2_{10} = 1.5$ and assume that the other parameters remain the same. The thresholds under this parameter setting are calculated as Table 4. A major difference from case 1 is that firm 2 is partly disadvantaged in both regimes 1 and 2.

Table 4: Thresholds of the firms in case 2.

$\bar{P}^1_{L1}$	$\bar{P}^1_{L2}$	$\bar{P}^1_{F1}$	$\bar{P}^1_{F2}$	$\bar{P}_{S1}$	$\bar{P}_{S2}$
0.8812	1.1772	2.1706	3.4224	3.2558	5.1336
$\bar{P}^2_{L1}$	$\tilde{P}^2_{L1}$	$\bar{P}^2_{L2}$	$\tilde{P}^2_{L2}$	$\bar{P}^2_{F1}$	$\bar{P}^2_{F2}$
1.1121	2.0805	1.6371	3.2653	2.7549	4.3438

Table 5 presents the investment strategies that each firm takes in each

[11]Note that investment timing of firm 1 as the result is determined by optimization of firm 1 as a leader. See, for detail, Appendix C. We focus on the incentive to become a leader in this section.

regime. A novel observation is as follows. Suppose that the current regime

Table 5: Investment strategies of each firm in case 2.

$\epsilon = 1$	1	$\times$	$\times$	$\times$	1	1	1, 2	1, 2	1, 2	1, 2
$\epsilon = 2$			1	$\times$	$\times$	$\times$	$\times$	1	1	1, 2
	$\bar{P}^1_{L1}$	$\bar{P}^2_{L1}$	$\bar{P}^1_{L2}$	$\bar{P}^2_{L2}$	$\tilde{P}^2_{L1}$	$\bar{P}^1_{F1}$	$\bar{P}^2_{F1}$	$\tilde{P}^2_{L2}$	$\bar{P}^1_{F2}$	$\bar{P}^2_{F2}$

is a bust ($\epsilon = 2$) and the current level of demand P_0 lies in $[\bar{P}^2_{L1}, \bar{P}^1_{L2})$. Then both firms do not invest immediately and wait until the demand becomes higher as long as the current regime continues. However, when the regime suddenly changes from 2 to 1, both firms have an incentive to invest as a leader.[12]

This result shows a stark contrast to Pawlina and Kort (2006). That is, in their model without regime switching in the economic condition, a firm that is more profitable than the other always becomes a leader and enters the market before the other when the initial value of P is low. On the contrary, our model produces a situation where a disadvantaged firm may be a leader in a newly developing market, by simply introducing a Markov chain in the exogenous parameters.

4.3. Case 3: Simultaneous investment

In this case, we choose $K^2 = 10.5$, $D^1_{10} = D^2_{10} = 1.45$, $D^2_{11} = 1$ and set the other parameters to be the same as the benchmark. With these parameter

[12]More formally, both firms takes mixed strategies and optimally choose the probability of investment.

values, we obtain the investment thresholds as in Table 6. We verify from the calculation that firm 1 prefers simultaneous investment to preempt firm 2 and being a leader in regime 2 since $V_{S2}^1 \geq G_{L2}^1 - K^1$ for all $P < \bar{P}_{S2}$, while both firms have an incentive to become a leader in regime 1. An important difference from case 2 is that all thresholds except for $\bar{P}_{S2}$ are ignored in regime 2.

Table 6: Thresholds of the firms in case 3.

$\bar{P}_{L1}^1$	$\bar{P}_{L2}^1$	$\bar{P}_{F1}^1$	$\bar{P}_{F2}^1$	$\bar{P}_{S1}$	$\bar{P}_{S2}$
0.9872	1.3858	2.1706	3.4224	3.2558	5.1336
$\bar{P}_{L1}^2$	$\tilde{P}_{L1}^2$	$\bar{P}_{L2}^2$	$\tilde{P}_{L2}^2$	$\bar{P}_{F1}^2$	$\bar{P}_{F2}^2$
1.0810	2.1659	1.5622	3.4150	2.2791	3.5935

Table 7 presents the investment strategies that each firm takes in each regime. For $\bar{P}_{L1}^2 \leq P < \tilde{P}_{L1}^2$, we obtain the same situation as in case 1.

Table 7: Investment strategies of each firm in case 3.

$\epsilon = 1$	1	$\times$	1	1	$1, 2$	$1, 2$
$\epsilon = 2$						$1, 2$
	$\bar{P}_{L1}^1$	$\bar{P}_{L1}^2$	$\tilde{P}_{L1}^2$	$\bar{P}_{F1}^1$	$\bar{P}_{F1}^2$	$\bar{P}_{S2}$

Another novel observation is the following. Suppose that the current regime is a bust ($\epsilon = 2$) and that $\bar{P}_{F1}^2 \leq P < \bar{P}_{S2}$. Then, both firms wait for simultaneous investment until the state variable becomes higher. However, when the regime changes from 2 to 1, both firms do not care about the decision of the other and simultaneously invest in the project. The result is an extreme version of case 2. Such a simultaneous investment is not tacit collusion but caused by a sudden regime shift. Namely, there are two different types of simultaneous investment, depending on the presence of a tacit collusion. Recall again that previous theoretical papers of competitive real options approach cannot give such a scenario.

In summary, we have found from the numerical examples that our model is quite rich and flexible to explain many actual situations within a unified framework.

5. Equilibrium Types

Pawlina and Kort (2006) examine the conditions for each type of equilibrium to occur, depending on the parameter setting. They call a preemptive equilibrium if one of the firms is partially disadvantaged and can have an incentive to invest as a leader, and a sequential equilibrium if one of the firms is fully disadvantaged and always become a follower. The other type of equilibrium is a simultaneous equilibrium, where both firms invest at the same point. In what follows, we follow their analysis and examine the conditions.

To compare our result to Pawlina and Kort (2006), we suppose that $D_{N_iN_j} := D^1_{N_iN_j} = D^2_{N_iN_j}$, meaning that asymmetry lies only in the investment cost.[13] We define

$$u = \frac{D_{10} - D_{00}}{D_{11} - D_{00}},$$
$$v = \frac{D_{11} - D_{01}}{D_{11} - D_{00}}$$

and

$$w = \frac{D_{10} - D_{01}}{D_{11} - D_{01}}.$$

The first-mover advantage and cost asymmetry are defined by D_{10}/D_{11} and $\kappa = K^2/K^1$, respectively. Pawlina and Kort (2006) show in their model with constant (μ, σ) that a simultaneous equilibrium happens if $\kappa < \kappa^{**}$, where

$$\kappa^{**} = \max\left\{ v \left(\frac{\theta(u-1)}{u^\theta - 1} \right)^{\frac{1}{\theta-1}}, 1 \right\},$$
$$\theta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}}.$$

A sequential equilibrium happens if $\kappa > \kappa^*$, where

$$\kappa^* = \left(\frac{w^\theta - 1}{\theta(w-1)} \right)^{\frac{1}{\theta-1}}.$$

Otherwise, a preemptive equilibrium happens and a disadvantaged firm can be a leader. While a closed-form expressions of κ^* and κ^{**} are obtained in the one-regime case, κ^*_ϵ and κ^{**}_ϵ in our model need to be found numerically.[14]

[13] In this analysis, as in Pawlina and Kort (2006), we consider an asymmetric situation in which each firm has different investment costs. For example, in the power industry there exist some cases where firms invest power generations of distinct technologies for same capacities such as peak and base or renewable and non-renewable.

[14] We numerically calculate the functions $V^i_{S\epsilon}$ and $G^i_{L\epsilon}$ to check the magnitude relationship.

We use the base case parameter set in Table 1 again, except for $D_{10}^1 = D_{10}^2$ and $D_{11}^1 = D_{11}^2 = 1$.

Figure 3 depicts the regions of equilibria as a function of the first-mover advantage D_{10}/D_{11} and the investment cost asymmetry κ in our model. To simplify the analysis, we only investigate the case $\lambda_1 = \lambda_2$.[15]

[Figure 3 is inserted around here.]

Our calculation shows that κ^*'s in regime 1 are higher than in the one regime case, while κ^*'s in regime 2 are lower than those in the one regime case. On the other hand, κ^{**}'s in regime 1 are lower than in the one regime case, while κ^{**}'s in regime 2 are higher than those in the one regime case. In other words, a preemptive equilibrium is more likely to occur in a boom than in a bust. Intuitively, the booms create large investment opportunities, which make firms' preemption strategy relatively more attractive. By contrast, the busts lead to decreases in the investment opportunities, which induces firms to prefer sequential or simultaneous investments. This corresponds to the result of Pawlina and Kort (2006), i.e., market uncertainty delays investment by making the firms switch across equilibria. In this work, however the equilibria are also dependent on the switching intensity, that is, the regime uncertainty.

Note in Figure 3 that the above result is more remarkable especially when

[15]We use

$$\mu \equiv \frac{\lambda_1\mu_1 + \lambda_2\mu_2}{\lambda_1 + \lambda_2} = \frac{\mu_1 + \mu_2}{2},$$
$$\sigma \equiv \frac{\lambda_1\sigma_1 + \lambda_2\sigma_2}{\lambda_1 + \lambda_2} = \frac{\sigma_1 + \sigma_2}{2},$$

for the expected growth rate and the volatility in the one-regime model, respectively.

λ_1 and λ_2 are higher. Intuitively, we would conjecture that the line of κ_1^* in regime 1 is located farer from the line of κ_2^* in regime 2 when λ is low, and then converges to that of κ^* in a one-regime case as λ goes to infinity, and that a similar argument can be given for κ^{**}. But the numerical result shows the conjecture is not true.

To examine the observation in more depth, we present Figure 4, plotting κ^* and κ^{**} in both regimes for different values of λ with other parameter values fixed.[16]

[Figure 4 is inserted around here.]

The above figure show that in regime 1, κ_1^* (κ_2^* in regime 2, respectively) is increasing (decreasing, respectively) for a small λ and then turn decreasing (increasing, respectively) afterwards. The opposite shapes can be found for κ^{**}'s.

Regarding the observation in Figure 4, we can give the following theoretical explanations. Suppose first that λ is small. In this situation, the probability of a regime change is negligible and both firms do not need to take a regime change into account for the investment decision. Therefore, an equilibrium type should be the same as the one-regime case. In the case where λ is moderately high, both firms actually consider the effect of regime change, and hasten to invest in a boom while hesitate to invest in a bust, leading to the situation where a preemptive equilibrium is more likely to occur in a boom and it is less likely in a bust. If λ is extremely high, then

[16]Unfortunately, numerical calculations for $\lambda > 3$ are unstable and unable to be presented.

the regime easily switches from one to another and both firms regard the economic condition as a one-regime setting with $\mu \equiv (\lambda_2\mu_1+\lambda_1\mu_2)/(\lambda_1+\lambda_2)$ and $\sigma \equiv (\lambda_2\sigma_1+\lambda_1\sigma_2)/(\lambda_1+\lambda_2)$. The above explanation effectively describes how regime uncertainty affects the investment decision of both firms.

Put it differently, both firms takes the option value to wait and see the future evolution of a regime into account, especially when the regime is bad for investment and the intensity of a sudden regime shift is moderate. In the real options literature, the option value of wait is extensively studied by many papers but is usually related to the volatility of demand. The effect of regime uncertainty is analyzed by Guo et al. (2005) and other papers but the option value of a regime change is not deeply discussed in the literature. The current study sheds new light on the investment theory by presenting the importance of regime uncertainty in a different way from other theoretical studies.

6. Equity Risk Premium

In this section, we present a numerical analysis on the equity risk premium. To this end, $G^i_{L\epsilon}$ is not appropriate and we should calculate $V^i_{L\epsilon}$, the value function of a leader firm including the option value like of a follower. We derive $V^i_{L\epsilon}$ in Appendix C.

Following Carlson et al. (2004) and Aguerrevere (2009), we define the beta of firm i's equity as a leader in regime ϵ to be

$$\beta^i_{L\epsilon}(P) = \frac{\mathbb{C}_{P,\epsilon}[(\mathrm{d}P/P),(\mathrm{d}V^i_{L\epsilon}/V^i_{L\epsilon})]}{\mathbb{V}_{P,\epsilon}[(\mathrm{d}P/P)]} = \frac{P}{G^i_{L\epsilon}(P)}{V^i_{L\epsilon}}'(P), \tag{19}$$

that as a follower to be

$$\beta^i_{F\epsilon}(P) = \frac{\mathbb{C}_{P,\epsilon}[(\mathrm{d}P/P), (\mathrm{d}V^i_{F\epsilon}/V^i_{F\epsilon})]}{\mathbb{V}_{P,\epsilon}[(\mathrm{d}P/P)]} = \frac{P}{V^i_{F\epsilon}(P)} V^{i\prime}_{F\epsilon}(P) \tag{20}$$

and that in simultaneous investment to be

$$\beta^i_{S\epsilon}(P) = \frac{\mathbb{C}_{P,\epsilon}[(\mathrm{d}P/P), (\mathrm{d}V^i_{S\epsilon}/V^i_{S\epsilon})]}{\mathbb{V}_{P,\epsilon}[(\mathrm{d}P/P)]} = \frac{P}{V^i_{S\epsilon}(P)} V^{i\prime}_{S\epsilon}(P), \tag{21}$$

where $\mathbb{V}_{P,\epsilon}$ and $\mathbb{C}_{P,\epsilon}$ are the variance and covariance operators conditional on (P, ϵ), respectively.

In this analysis, the parameter values are chosen based on Bhamra et al. (2009) except for K^i and $D_{N_iN_j}$ to match the actual economic environment. Table 8 presents the values of exogenous parameters. Note that firm 1 is advantaged in cost.

Table 8: Parameter values. We follow Bhamra et al. (2009) except for K and D.

μ_1	μ_2	σ_1	σ_2	r	λ_1	λ_2	K^1	K^2	D_{00}	D_{01}	D_{10}	D_{11}
0.0782	-0.0401	0.0834	0.1334	0.1	0.2718	0.4928	10	12	0.5	0.25	1.5	1

The thresholds of both firms as a leader and follower under this setup are given in Tables 9.[17] As conjectured, the thresholds of firm 1 in regimes 1 and 2 are lower than the counterparts of firm 2.

First we present Figures 5 and 6, depicting the relationship between β and P in a one-regime case as a benchmark.

[Figure 5 is inserted around here.]

[17] $\bar{P}^{1*}_{L\epsilon}$ is defined in Appendix C and necessary to calculate leader's value functions.

Table 9: Thresholds of the firms.

$\bar{P}^1_{L1}$	$\bar{P}^{1*}_{L1}$	$\bar{P}^1_{L2}$	$\bar{P}^{1*}_{L2}$	$\bar{P}^1_{F1}$	$\bar{P}^1_{F2}$	$\bar{P}_{S1}$	$\bar{P}_{S2}$
0.6050	1.1067	0.6964	1.1609	1.4934	1.6980	2.2401	2.5470
$\bar{P}^2_{L1}$	$\tilde{P}^2_{L1}$	$\bar{P}^2_{L2}$	$\tilde{P}^2_{L2}$	$\bar{P}^2_{F1}$	$\bar{P}^2_{F2}$		
0.8599	1.4398	0.9908	1.6522	1.7921	2.0376		

[Figure 6 is inserted around here.]

Figures 5 and 6 plot betas of leader, follower and simultaneous investment for firms 1 and 2, respectively. Both figures almost reproduce the results of Carlson et al. (2014). The beta for the leader discontinuously increases when P is equal to the investment thresholds and finally decreases for a larger value of P. On the other hand, the beta for the follower increases when P is smaller than the follower's thresholds and decreases afterwards. The beta for the simultaneous investment is similar to that for the follower except that the beta of firm 2 discontinuously increases at the investment threshold. This is because firm 1 invests simultaneously and optimally, while firm 2 reluctantly invests simultaneously at the same point.

The difference from Carlson et al. (2014) is seen at leader's investment threshold. Figures 5 and 6 show that the beta for the leader (follower) discontinuously increases (does not change) at that point, while the beta for the leader (follower) discontinuously decreases (increases) in Carlson et al. (2014). This difference is caused by the difference in the model setting, i.e., the leader's investment is not optimal due to preemption and the follower's option value is independent from the leader's investment in our setting. We also observe that the leader's beta is more volatile than the follower's. The reason is that an actual investment is irreversible and a decrease of P after investment has a big impact on the leader's value. Note finally that the betas in a bust is more volatile than in a boom, which comes from the fact that the volatility of P is higher in a bust.

Now we show the betas in our regime-switching model. Figures 7 and 8 plot the betas of the advantaged and disadvantaged firms in the two regimes under the benchmark parameters, respectively.

[Figure 7 is inserted around here.]

[Figure 8 is inserted around here.]

We observe that the difference of the beta between two regimes in Figures 7 and 8 is quite less than that in Figures 5 and 6, which means that introducing the regime switch can prevent from underestimating the beta in a boom and overestimating the beta in a bust. An important observation from Figures 7 and 8 is that the graph of beta in regime 1 is similar to Figures 5 and 6 but the graph in regime 2 is different. More concretely, the beta for the leader in regime 2 is not monotonic for a small P and has a kink at $P = \bar{P}^2_{L1}$. The reason is that the beta in regime 2 reflects the possibility of a sudden change to regime 1, which leads to an immediate investment and makes the decision irreversible. And then, the option value of the leader vanishes and the value of the leader includes only the NPV of an immediate investment. Therefore, the beta for the leader in regime 2 changes the shape drastically at $\bar{P}^2_{L1}$. Similarly, the beta for the leader i in regime 2 has an inflection point at $\bar{P}^j_{F1}$. This is because follower j will invest at $\bar{P}^j_{F1}$ when the regime changes from 2 to 1. However, the impact of the possibility of a regime change at this point is less than that at $\bar{P}^2_{L1}$ since the option value of the leader has already vanished at $\bar{P}^2_{L1}$. These theoretical findings are new in the literature and can be obtained only in our regime-switching model.

We also verify from Figures 7 and 8 that the risk premium in regime 1 is tend to be lower than the one in regime 2. Our study replicates the result of Aguerrevere (2009) that describes the business cycle by the level of the state variable. However many empirical papers such as Chen (1991) and Hoberg and Phillips (2010) report that the time-varying beta is negatively associated with the economic growth rate or the market return, not the absolute level of demand or the market size. **By means of considering changes in the expected growth rate this study provides explanations to empirical facts about the relationship between the economic cycle and risk premium that are not possible in previous studies that proxy economic conditions by the level of demand or other state variables.**

7. Conclusion

In this study, we introduce a Markov switching regime as Driffill et al. (2013) into the model of Pawlina and Kort (2006) to consider the investment problem of asymmetric firms with regime uncertainty. In the case of no regime switch, a profitable firm always becomes a leader in the investment, and a disadvantaged firm never has an incentive to become a leader in a newly developing market. However, if there is uncertainty in regime, there are some parameter settings in which both firms can be a leader even when the initial state variable is in a lower level. This finding shows a stark contrast to Pawlina and Kort (2006) in that our model can provide richer results within a unified framework.

From the numerical calculations, we conclude that regime uncertainty can have a big impact on the investment decision and the market equilibrium. When there is a regime switching structure in the economy, each firm needs to take the probability and effect of a regime change into account, which can cause a shift of the equilibrium type. In addition, the equity risk premium is tend to be higher when the expected growth rate is low. This theoretical result descries previous empirical findings in a more precise way than other extant studies.

As a future study, it seems important to consider the changes of profitability and cost invoked by the regime. It is natural that the firm's profitability and cost are better in a boom than a bust. By doing this, we will be able to explain more complicated economic behavior of the firms facing the entry race under uncertainty.

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A. Glossary

The glossary of the notation used in the paper is presented for the readers' convenience.

P_t	the level of demand at time t
μ_ϵ	expected growth rate of P in regime ϵ
σ_ϵ	volatility of P in regime ϵ
r	discount rate to calculate the net present value
λ_ϵ	transition intensity from regime ϵ to the other regime
$D^i_{N_i N_j}$	contribution parameter to the profit of firm i, where $N_k = 1$ if firm $k \in \{i, j\}$ has invested and $N_k = 0$ otherwise
K^i	firm i's investment cost
τ^i_L (τ^i_F)	firm i's investment timing if it is a leader (follower)
$\bar{P}^i_{L\epsilon}$ ($\bar{P}^i_{F\epsilon}$)	firm i's investment threshold in regime ϵ if it is a leader (follower)
$\bar{P}^{1*}_{L\epsilon}$	firm 1's optimal investment threshold in regime ϵ if it is a leader
$\tilde{P}^2_{L\epsilon}$	the value which relates the incentive to be a leader for firm 2 in regime ϵ
$\bar{P}_{S\epsilon}$	both firm's investment threshold in regime ϵ for the case of a simultaneous equilibrium
$G^i_{L\epsilon}$ ($G^i_{F\epsilon}$)	firm i's net present value for an immediate investment in regime ϵ if it is a leader (follower)
$V^i_{L\epsilon}$ ($V^i_{F\epsilon}$)	firm i's value including the option value in regime ϵ if it is a leader (follower)
$V^i_{S\epsilon}$	firm i's value function including the option value of the future investment in regime ϵ for the case of a simultaneous equilibrium
κ^*_ϵ	parameter determining if a sequential equilibrium occurs in regime ϵ
κ^{**}_ϵ	parameter determining if a simultaneous equilibrium occurs in regime ϵ

B. Boundary Conditions in Propositions

In this appendix, we provide boundary conditions in Propositions 2–4. First, boundary conditions for a follower in Proposition 2 are as follows:

$$\pi_2 D^i_{11} \bar{P}^i_{F2} - K^i$$

$$= b_{21}^i(\bar{P}_{F2}^i)^{\alpha_1} + b_{22}^i(\bar{P}_{F2}^i)^{\alpha_2} + \frac{D_{01}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}\bar{P}_{F2}^i - \frac{\lambda_2}{r+\lambda_2}K^i, \quad (22)$$

$$\pi_2 D_{11}^i = \alpha_1 b_{21}^i(\bar{P}_{F2}^i)^{\alpha_1-1} + \alpha_2 b_{22}^i(\bar{P}_{F2}^i)^{\alpha_2-1} + \frac{D_{01}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}, \quad (23)$$

$$c_{11}^i(\bar{P}_{F1}^i)^{\gamma_1} + c_{12}^i(\bar{P}_{F1}^i)^{\gamma_2} + \pi_1 D_{01}^i P = \pi_1 D_{11}^i \bar{P}_{F1}^i - K^i, \quad (24)$$

$$\gamma_1 c_{11}^i(\bar{P}_{F1}^i)^{\gamma_1-1} + \gamma_2 c_{12}^i(\bar{P}_{F1}^i)^{\gamma_2-1} + \pi_1 D_{01}^i = \pi_1 D_{11}^i, \quad (25)$$

$$\ell_1 c_{11}^i(\bar{P}_{F1}^i)^{\gamma_1} + \ell_2 c_{12}^i(\bar{P}_{F1}^i)^{\gamma_2} + \pi_2 D_{01}^i P$$
$$= b_{21}^i(\bar{P}_{F1}^i)^{\alpha_1} + b_{22}^i(\bar{P}_{F1}^i)^{\alpha_2} + \frac{D_{01}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}\bar{P}_{F1}^i - \frac{\lambda_2}{r+\lambda_2}K^i, \quad (26)$$

$$\gamma_1\ell_1 c_{11}^i(\bar{P}_{F1}^i)^{\gamma_1-1} + \gamma_2\ell_2 c_{12}^i(\bar{P}_{F1}^i)^{\gamma_2-1} + \pi_2 D_{01}^i$$
$$= \alpha_1 b_{21}^i(\bar{P}_{F1}^i)^{\alpha_1-1} + \alpha_2 b_{22}^i(\bar{P}_{F1}^i)^{\alpha_2-1} + \frac{D_{01}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}. \quad (27)$$

(22) and (23) ((24) and (25)) are the value-matching and the smooth-pasting conditions at $\bar{P}_{F2}^i$ ($\bar{P}_{F1}^i$), respectively. (26) and (27) are the continuity and high-contact conditions, respectively.

Second, we provide boundary conditions for a leader in Proposition 3:

$$e_{21}^i(\bar{P}_{F2}^j)^{\alpha_1} + e_{22}^i(\bar{P}_{F2}^j)^{\alpha_2} + \frac{D_{10}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}\bar{P}_{F2}^j = \pi_2 D_{11}^i \bar{P}_{F2}^j, \quad (28)$$

$$h_{11}^i(\bar{P}_{F1}^j)^{\gamma_1} + h_{12}^i(\bar{P}_{F1}^j)^{\gamma_2} + \pi_1 D_{10}^i \bar{P}_{F1}^j = \pi_1 D_{11}^i \bar{P}_{F1}^j, \quad (29)$$

$$\ell_1 h_{11}^i(\bar{P}_{F1}^j)^{\gamma_1} + \ell_2 h_{12}^i(\bar{P}_{F1}^j)^{\gamma_2} + \pi_2 D_{10}^i \bar{P}_{F1}^j$$
$$= e_{21}^i(\bar{P}_{F1}^j)^{\alpha_1} + e_{22}^i(\bar{P}_{F1}^j)^{\alpha_2} + \frac{D_{10}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}P_{F1}^j, \quad (30)$$

$$\gamma_1\ell_1 h_{11}^i(\bar{P}_{F1}^j)^{\gamma_1-1} + \gamma_2\ell_2 h_{12}^i(\bar{P}_{F1}^j)^{\gamma_2-1} + \pi_2 D_{10}^i$$
$$= \alpha_1 e_{21}^i(\bar{P}_{F1}^j)^{\alpha_1-1} + \alpha_2 e_{22}^i(\bar{P}_{F1}^j)^{\alpha_2-1} + \frac{D_{10}^i + \lambda_2\pi_1 D_{11}^i}{r+\lambda_2-\mu_2}. \quad (31)$$

(28) and (29) are the value-matching conditions at $\bar{P}_{F2}^i$ and $\bar{P}_{F1}^i$, respectively. (30) and (31) are the continuity and high-contact conditions, respectively. Note that smooth-pasting conditions do not exist for leader's problem.

Finally, boundary conditions for simultaneous investment in Proposition 4 are given by

$$\pi_2 D_{11}^i \bar{P}_{S2} - K^i$$
$$= m_{21}^i (\bar{P}_{S2})^{\alpha_1} + m_{22}^i (\bar{P}_{S2})^{\alpha_2} + \frac{D_{00}^i + \lambda_2 \pi_1 D_{11}^i}{r + \lambda_2 - \mu_2} \bar{P}_{S2} - \frac{\lambda_2}{r + \lambda_2} K^i, \quad (32)$$

$$q_{11}^i (\bar{P}_{S1})^{\gamma_1} + q_{12}^i (\bar{P}_{S1})^{\gamma_2} + \pi_1 D_{00}^i P = \pi_1 D_{11}^i \bar{P}_{S1} - K^i, \quad (33)$$

$$\ell_1 q_{11}^i (\bar{P}_{S1})^{\gamma_1} + \ell_2 q_{12}^i (\bar{P}_{S1})^{\gamma_2} + \pi_2 D_{00}^i P$$
$$= m_{21}^i (\bar{P}_{S1})^{\alpha_1} + m_{22}^i (\bar{P}_{S1})^{\alpha_2} + \frac{D_{00}^i + \lambda_2 \pi_1 D_{11}^i}{r + \lambda_2 - \mu_2} \bar{P}_{S1} - \frac{\lambda_2}{r + \lambda_2} K^i, \quad (34)$$

$$\gamma_1 \ell_1 q_{11}^i (\bar{P}_{S1})^{\gamma_1 - 1} + \gamma_2 \ell_2 q_{12}^i (\bar{P}_{S1})^{\gamma_2 - 1} + \pi_2 D_{00}^i$$
$$= \alpha_1 m_{21}^i (\bar{P}_{S1})^{\alpha_1 - 1} + \alpha_2 m_{22}^i (\bar{P}_{S1})^{\alpha_2 - 1} + \frac{D_{00}^i + \lambda_2 \pi_1 D_{11}^i}{r + \lambda_2 - \mu_2}, \quad (35)$$

$$\pi_2 D_{11}^1 = \alpha_1 m_{21}^1 (\bar{P}_{S2})^{\alpha_1 - 1} + \alpha_2 m_{22}^1 (\bar{P}_{S2})^{\alpha_2 - 1} + \frac{D_{00}^1 + \lambda_2 \pi_1 D_{11}^1}{r + \lambda_2 - \mu_2}, \quad (36)$$

$$\gamma_1 q_{11}^1 (\bar{P}_{S1})^{\gamma_1 - 1} + \gamma_2 q_{12}^1 (\bar{P}_{S1})^{\gamma_2 - 1} + \pi_1 D_{00}^1 = \pi_1 D_{11}^1. \quad (37)$$

(32) and (33) ((36) and (37)) are the value-matching and the smooth-pasting conditions at $\bar{P}_{S2}$ ($\bar{P}_{S1}$), respectively. (34) and (35) are the continuity and high-contact conditions, respectively. Note that the smooth-pasting conditions hold for only firm 1 because of its advantage.

C. Derivation of Leader's Value Function

In this appendix, we drive the value function of both firms as a leader for investment, to calculate their β's. To this end, we need to consider the magnitude relationship between leader's optimal investment threshold of firm 1 and leader's investment threshold of firm 2.

C.1. The case of firm 1's optimization

First, we consider the case where firm 1 can surely become a leader, and let $\bar{P}^{1*}_{L\epsilon}$ denote leader's optimal investment threshold of firm 1 in regime ϵ. If $\epsilon(t) = \epsilon$ and $P \geq \bar{P}^{1*}_{L\epsilon}$, the optimal decision of firm 1 is to invest immediately and $V^1_{L\epsilon} = G^1_{L\epsilon} - K^1$.

Suppose that $\bar{P}^{1*}_{L1} \leq P < \bar{P}^{1*}_{L2}$ and $\epsilon(t) = 2$. In this situation, firm 1 invests in the new project immediately after the regime changes from 2 to 1. Therefore, the value function V^1_{L2} satisfies the ODE given by

$$\frac{\sigma_2^2}{2}P^2\frac{\mathrm{d}^2V^1_{L2}}{\mathrm{d}P^2} + \mu_2 P\frac{\mathrm{d}V^1_{L2}}{\mathrm{d}P} - rV^1_{L2} + \lambda_2(G^1_{L1} - K^1 - V^1_{L2}) + D^1_{00}P = 0 \tag{38}$$

where G^1_{L1} appears in Proposition 3, and the boundary condition is given by

$$\lim_{P\uparrow\bar{P}^{1*}_{L2}} V^1_{L2}(P) = \lim_{P\downarrow\bar{P}^{1*}_{L2}} G^1_{L2}(P) - K^1.$$

We conjecture that the functional form of (38) is

$$V^1_{L2}(P) = e^{L1}_{21}P^{\alpha_1} + e^{L1}_{22}P^{\alpha_2} + \hat{e}^{L1}_{23}P^{\gamma_1} + \hat{e}^{L1}_{24}P^{\gamma_2} + \hat{e}^{L1}_{25}P + \hat{e}^{L1}_{26}, \tag{39}$$

where γ_1 and γ_2 are the positive roots of (12) and α_1 and α_2 are the roots of the quadratic equation (9). Plugging (39) into (38), we obtain

$$\hat{e}^{L1}_{23} = \frac{\lambda_2 h^1_{11}}{r + \lambda_2 - \mu_2\gamma_1 - \frac{\sigma_2^2}{2}\gamma_1(\gamma_1 - 1)}, \tag{40}$$

$$\hat{e}^{L1}_{24} = \frac{\lambda_2 h^1_{12}}{r + \lambda_2 - \mu_2\gamma_2 - \frac{\sigma_2^2}{2}\gamma_2(\gamma_2 - 1)}, \tag{41}$$

$$\hat{e}^{L1}_{25} = \frac{D^1_{00} + \lambda_2\pi_1 D^1_{10}}{r + \lambda_2 - \mu_2} \tag{42}$$

and

$$\hat{e}^{L1}_{26} = -\frac{\lambda_2}{r + \lambda_2}K^1, \tag{43}$$

where h^1_{11} and h^1_{12} are given in Proposition 3. The coefficients e^{L1}_{21} and e^{L1}_{22} are derived later.

Next we suppose that $P < \bar{P}^{1*}_{L1}$. In this situation, firm 1 does not invest at the time of a regime change. Therefore $\{V^1_{L\epsilon}\}_{\epsilon=1,2}$ must satisfy the simultaneous ODEs

$$\frac{\sigma_\epsilon^2}{2}P^2\frac{\mathrm{d}^2V^1_{L\epsilon}}{\mathrm{d}P^2} + \mu_\epsilon P\frac{\mathrm{d}V^1_{L\epsilon}}{\mathrm{d}P} - rV^1_{L\epsilon} + \lambda_\epsilon(V^1_{L\hat{\epsilon}} - V^1_{L\epsilon}) + D^1_{00}P \tag{44}$$

for $\epsilon = 1, 2$. The boundary conditions are

$$\lim_{P\uparrow\bar{P}^{1*}_{L1}} V^1_{L1}(P) = \lim_{P\downarrow\bar{P}^{1*}_{L1}} G^1_{L1}(P) - K^1$$

and

$$\begin{aligned} \lim_{P\uparrow\bar{P}^{1*}_{L1}} V^1_{L2}(P) &= \lim_{P\downarrow\bar{P}^{1*}_{L1}} V^1_{L2}(P), \\ \lim_{P\uparrow\bar{P}^{1*}_{L1}} V^{1\prime}_{L2}(P) &= \lim_{P\downarrow\bar{P}^{1*}_{L1}} V^{1\prime}_{L2}(P). \end{aligned} \tag{45}$$

The function V^1_{L2} is of C^1 except for $P = \bar{P}^{1*}_{L2}$, implying that the high contact condition (45) holds. The conjectured functions of (44) are

$$V^1_{L\epsilon}(P) = h^{L1}_{\epsilon 1}P^{\gamma_1} + h^{L1}_{\epsilon 2}P^{\gamma_2} + \pi_\epsilon D^1_{00}P. \tag{46}$$

The unknown parameters are given in the following proposition.

Proposition C.1. *Suppose that $\bar{P}^{1*}_{L\epsilon} < \bar{P}^2_{L\epsilon}$. Then firm 1 can surely become a leader and the value function of firm 1 for regime 1 is given by*

$$V^1_{L1}(P) = \begin{cases} G^1_{F1}(P) - K^1, & \text{for } P \geq \bar{P}^2_{F1}, \\ G^1_{L1}(P) - K^1, & \text{for } \bar{P}^{1*}_{L1} \leq P < \bar{P}^2_{F1}, \\ h^{L1}_{11}P^{\gamma_1} + h^{L1}_{12}P^{\gamma_2} + \pi_1 D^1_{00}P, & \text{for } P < \bar{P}^{1*}_{L1} \end{cases} \tag{47}$$

and the function for regime 2 is

$$V_{L2}^1(P) = \begin{cases} G_{F2}^1(P) - K^1, & \textit{for } P \geq \bar{P}_{F2}^2, \\ G_{L2}^1(P) - K^1, & \textit{for } \bar{P}_{L2}^{1*} \leq P < \bar{P}_{F2}^2, \\ e_{21}^{L1} P^{\alpha_1} + e_{22}^{L1} P^{\alpha_2} + \hat{e}_{23}^{L1} P^{\gamma_1} + \hat{e}_{24}^{L1} P^{\gamma_2} & \\ \quad + \hat{e}_{25}^{L1} P + \hat{e}_{26}^{L1}, & \textit{for } \bar{P}_{L1}^{1*} \leq P < \bar{P}_{L2}^{1*}, \\ \ell_1 h_{11}^{L1} P^{\gamma_1} + \ell_2 h_{12}^{L1} P^{\gamma_2} + \pi_2 D_{00}^1 P, & \textit{for } P < \bar{P}_{L1}^{1*}, \end{cases} \tag{48}$$

where $\hat{e}_{23}^{L1}$, $\hat{e}_{24}^{L1}$, $\hat{e}_{25}^{L1}$ *and* $\hat{e}_{26}^{L1}$ *are given in* (40)–(43). *The unknown parameters* $(\bar{P}_{L1}^{1*}, \bar{P}_{L2}^{1*}, e_{21}^{L1}, e_{22}^{L1}, h_{11}^{L1}, c_{12}^{L1})$ *are the solution of the simultaneous equation*

$$\begin{aligned} & h_{11}^1 (\bar{P}_{L1}^{1*})^{\gamma_1} + c_{12}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_2} + \hat{c}_{13}^{L1} \bar{P}_{L1}^{1*} - K^1 = h_{11}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_1} + h_{12}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_2} + \hat{h}_{13}^{L1} \bar{P}_{L1}^{1*}, \\ & \gamma_1 h_{11}^1 (\bar{P}_{L1}^{1*})^{\gamma_1 - 1} + \gamma_2 c_{12}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_2 - 1} + \hat{c}_{13}^{L1} \\ & \quad = \gamma_1 h_{11}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_1 - 1} + \gamma_2 h_{12}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_2 - 1} + \hat{h}_{13}^{L1}, \end{aligned} \tag{49}$$

$$\begin{aligned} & c_{21}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_1} + c_{22}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_2} + \hat{c}_{23}^{L1} \bar{P}_{L2}^{1*} - K^1 \\ & \quad = e_{21}^{L1} (\bar{P}_{L2}^{1*})^{\alpha_1} + e_{22}^{L1} (\bar{P}_{L2}^{1*})^{\alpha_2} + \hat{e}_{23}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_1} + \hat{e}_{24}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_2} + \hat{e}_{25}^{L1} \bar{P}_{L2}^{1*} + \hat{e}_{26}^{L1}, \\ & \gamma_1 c_{21}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_1 - 1} + \gamma_2 c_{22}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_2 - 1} + \hat{c}_{23}^{L1} \\ & \quad = \alpha_1 e_{21}^{L1} (\bar{P}_{L2}^{1*})^{\alpha_1 - 1} + \alpha_2 e_{22}^{L1} (\bar{P}_{L2}^{1*})^{\alpha_2 - 1} \\ & \qquad \qquad + \gamma_1 \hat{e}_{23}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_1 - 1} + \gamma_2 \hat{e}_{24}^{L1} (\bar{P}_{L2}^{1*})^{\gamma_2 - 1} + \hat{e}_{25}^{L1}, \\ & e_{21}^{L1} (\bar{P}_{L1}^{1*})^{\alpha_1} + e_{22}^{L1} (\bar{P}_{L1}^{1*})^{\alpha_2} + \hat{e}_{23}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_1} + \hat{e}_{24}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_2} + \hat{e}_{25}^{L1} \bar{P}_{L1}^{1*} + \hat{e}_{26}^{L1} \\ & \quad = \ell_1 h_{11}^{L1} (\bar{P}_{L1}^{1*})^{\gamma} + \ell_2 h_{12}^{L1} (\bar{P}_{L1}^{1*})^{\gamma_2} + \pi_2 D_{00}^1 \bar{P}_{L1}^{1*}, \\ & e_{21}^{L1} \alpha_1 (\bar{P}_{L1}^{1*})^{\alpha_1 - 1} + e_{22}^{L1} \alpha_2 (\bar{P}_{L1}^{1*})^{\alpha_2 - 1} + \hat{e}_{23}^{L1} \gamma_1 (\bar{P}_{L1}^{1*})^{\gamma_1 - 1} + \hat{e}_{24}^{L1} \gamma_2 (\bar{P}_{L1}^{1*})^{\gamma_2 - 1} + \hat{e}_{25}^{L1} \\ & \quad = \ell_1 h_{11}^{L1} \gamma_1 (\bar{P}_{L1}^{1*})^{\gamma_1 - 1} + \ell_2 h_{12}^{L1} \gamma_2 (\bar{P}_{L1}^{1*})^{\gamma_2 - 1} + \pi_2 D_{00}^1. \end{aligned} \tag{50}$$

If $P < \bar{P}^2_{L\epsilon}$, firm 2 does not have incentive to become a leader. Therefore, firm 1 can surely become a leader optimally at $\bar{P}^{1*}_{L\epsilon}$ in this case. Smooth-pasting conditions (49) and (50) reflect firm 1's optimization.

C.2. The case of firm 1's preemption

Second, we consider the case of $\bar{P}^2_{L\epsilon} \leq \bar{P}^{1*}_{L\epsilon}$. In this case, firm 2 has incentive to become a leader before firm 1's optimal investment threshold, so firm 1 reluctantly invests at $\bar{P}^2_{L\epsilon}$ in order to preempt firm 2. We can summarize the result in case of $\bar{P}^2_{L\epsilon} \leq \bar{P}^{1*}_{L\epsilon}$ by replacing $\bar{P}^{1*}_{L\epsilon}$ with $\bar{P}^2_{L\epsilon}$ and omitting smooth-pasting conditions in Proposition C.1.

Proposition C.2. *Suppose that $\bar{P}^2_{L\epsilon} \leq \bar{P}^{1*}_{L\epsilon}$. Then firm 1 preempts firm 2 and becomes a leader at $\bar{P}^2_{L\epsilon}$. The value function of firm 1 for regime 1 is given by*

$$V^1_{L1}(P) = \begin{cases} G^1_{F1}(P) - K^1, & \text{for } P \geq \bar{P}^2_{F1}, \\ G^1_{L1}(P) - K^1, & \text{for } P^2_{L1} \leq P < \bar{P}^2_{F1}, \\ h^{L1}_{11} P^{\gamma_1} + h^{L1}_{12} P^{\gamma_2} + \pi_1 D^1_{00} P, & \text{for } P < \bar{P}^2_{L1} \end{cases} \tag{51}$$

and the function for regime 2 is

$$V^1_{L2}(P) = \begin{cases} G^1_{F2}(P) - K^1, & \text{for } P \geq \bar{P}^2_{F2}, \\ G^1_{L2}(P) - K^1, & \text{for } \bar{P}^2_{L2} \leq P < \bar{P}^2_{F2}, \\ e^{L1}_{21} P^{\alpha_1} + e^{L1}_{22} P^{\alpha_2} + \hat{e}^{L1}_{23} P^{\gamma_1} + \hat{e}^{L1}_{24} P^{\gamma_2} & \\ \quad + \hat{e}^{L1}_{25} P + \hat{e}^{L1}_{26}, & \text{for } \bar{P}^2_{L1} \leq P < \bar{P}^2_{L2}, \\ \ell_1 h^{L1}_{11} P^{\gamma_1} + \ell_2 h^{L1}_{12} P^{\gamma_2} + \pi_2 D^1_{00} P, & \text{for } P < \bar{P}^2_{L1}, \end{cases} \tag{52}$$

The unknown parameters $(e_{21}^{L1}, e_{22}^{L1}, h_{11}^{L1}, c_{12}^{L1})$ *are the solution of the simultaneous equation*

$$h_{11}^{1}(\bar{P}_{L1}^{2})^{\gamma_1} + c_{12}^{L1}(\bar{P}_{L1}^{2})^{\gamma_2} + \hat{c}_{13}^{L1}\bar{P}_{L1}^{2} - K^1 = h_{11}^{L1}(\bar{P}_{L1}^{2})^{\gamma_1} + h_{12}^{L1}(\bar{P}_{L1}^{2})^{\gamma_2} + \hat{h}_{13}^{L1}\bar{P}_{L1}^{2},$$

$$\begin{aligned}
&c_{21}^{L1}(\bar{P}_{L2}^{2})^{\gamma_1} + c_{22}^{L1}(\bar{P}_{L2}^{2})^{\gamma_2} + \hat{c}_{23}^{L1}\bar{P}_{L2}^{2} - K^1 \\
&\quad = e_{21}^{L1}(\bar{P}_{L2}^{2})^{\alpha_1} + e_{22}^{L1}(\bar{P}_{L2}^{2})^{\alpha_2} + \hat{e}_{23}^{L1}(\bar{P}_{L2}^{2})^{\gamma_1} + \hat{e}_{24}^{L1}(\bar{P}_{L2}^{2})^{\gamma_2} + \hat{e}_{25}^{L1}\bar{P}_{L2}^{2} + \hat{e}_{26}^{L1}, \\
&e_{21}^{L1}(\bar{P}_{L1}^{2})^{\alpha_1} + e_{22}^{L1}(\bar{P}_{L1}^{2})^{\alpha_2} + \hat{e}_{23}^{L1}(\bar{P}_{L1}^{2})^{\gamma_1} + \hat{e}_{24}^{L1}(\bar{P}_{L1}^{2})^{\gamma_2} + \hat{e}_{25}^{L1}\bar{P}_{L1}^{2} + \hat{e}_{26}^{L1} \\
&\quad = \ell_1 h_{11}^{L1}(\bar{P}_{L1}^{2})^{\gamma} + \ell_2 h_{12}^{L1}(\bar{P}_{L1}^{2})^{\gamma_2} + \pi_2 D_{00}^{1}\bar{P}_{L1}^{2}, \\
&e_{21}^{L1}\alpha_1(\bar{P}_{L1}^{2})^{\alpha_1-1} + e_{22}^{L1}\alpha_2(\bar{P}_{L1}^{2})^{\alpha_2-1} + \hat{e}_{23}^{L1}\gamma_1(\bar{P}_{L1}^{2})^{\gamma_1-1} + \hat{e}_{24}^{L1}\gamma_2(\bar{P}_{L1}^{2})^{\gamma_2-1} + \hat{e}_{25}^{L1} \\
&\quad = \ell_1 h_{11}^{L1}\gamma_1(\bar{P}_{L1}^{2})^{\gamma_1-1} + \ell_2 h_{12}^{L1}\gamma_2(\bar{P}_{L1}^{2})^{\gamma_2-1} + \pi_2 D_{00}^{1}.
\end{aligned}$$

The value function of firm 2 as a leader for investment is given as in Proposition C.2 regardless of the magnitude relationship between $\bar{P}_{L\epsilon}^{1*}$ and $\bar{P}_{L\epsilon}^{2}$ as long as $\bar{P}_{L\epsilon}^{2}$ exists. In case of $\bar{P}_{L\epsilon}^{1*} < \bar{P}_{L\epsilon}^{2}$ and $\bar{P}_{L\hat{\epsilon}}^{2} \leq \bar{P}_{L\hat{\epsilon}}^{1*}$, the result can be given as the mix of Propositions C.1 and C.2.

References

Aguerrevere, F. L., 2009. Real options, product market competition, and asset returns. Journal of Finance 64 (2), 957–983.

Azevedo, A., Paxson, D., 2014. Developing real option game models. European Journal of Operational Research 237 (3), 909–920.

Baba, N., Packer, F., 2009. From turmoil to crisis: Dislocations in the FX swap market before and after the failure of Lehman Brothers. Journal of International Money and Finance 28 (8), 1350–1374.

Bhamra, H. S., Kuehn, L.-A., Strebulaev, I. A., 2009. The levered equity risk premium and credit spreads: A unified framework. Reveiw of Financial Studies 23 (2), 645–703.

Bloome, N., 2009. The impact of uncertainty shocks. Econometrica 77 (3), 623–685.

Bouis, R., Huisman, K. J. M., Kort, P. M., 2009. Investment in oligopoly under uncertainty: The accordion effect. International Journal of Industrial Organization 27 (2), 320–331.

Carlson, M., Dockner, E. J., Fisher, A., Giammarino, R., 2014. Leaders, followers, and risk dynamics in industry eequilibrium. Journal of Financial and Quantitative Analysis 49 (2), 321–349.

Carlson, M., Fisher, A., Giammarino, R., 2004. Corporate investment and asset price dynamics: Implications for the cross-section of returns. Journal of Finance 59 (6), 2577–2603.

Chen, N.-F., 1991. Financial investment opportunities and the macroeconomy. Journal of Finance 46 (2), 529–554.

Chevalier-Roignanta, B., Flath, C. M., Huchzermeier, A., Trigergis, L., 2011. Strategic investment under uncertainty: A synthesis. European Journal of Operational Research 215 (3), 639–650.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainty. Princeton University Press, Princeton, NJ.

Driffill, J., Kenc, T., Sola, M., 2013. Real options with priced regime-switching risk. International Journal of Theoretical and Applied Finance 16 (5), 1350028.

Fudenberg, D., Tirole, J., 1985. Preemption and rent equalization in the adoption of new technology. Review of Economic Studies 52 (3), 383–401.

Golder, P. N., Tellis, G. J., 1993. Pioneer advantage: Marketing logic or marketing legend? Journal of Marketing Research 30 (2), 158–170.

Grenadier, S. R., 1996. The strategic exercise of options: Development cascades and overbuilding in real estate markets. Journal of Finance 51 (5), 1653–1679.

Guo, X., Miao, J., Morellec, E., 2005. Irreversible investment with regime shifts. Journal of Economic Theory 122 (1), 37–59.

Hassett, K. A., Metcalf, G. E., 1999. Investment with uncertain tax policy: Does random tax policy discourage investment? Economic Journal 109 (457), 372–393.

Hoberg, G., Phillips, G., 2010. Real and financial industry booms and busts. Journal of Finance 65 (1), 45–86.

Huisman, K. J. M., Kort, P. M., 1999. Effect of strategic interactions on the option value of waiting, centER Discussion Paper 9992, Tilburg University.

Huisman, K. J. M., Kort, P. M., Pawlina, G., Thijssen, J. J. J., 2004. Strategic investment under uncertainty: A survey of game theoretic real options models. Journal of Financial Transformation 13, 111–118.

Kijima, M., Shibata, T., 2005. Real options in an oligopoly market. Kyoto Economic Review 74 (1), 47–64.

Lambrecht, B. M., Pawlina, G., Teixeira, J. C. A., 2015. Making, buying, and concurrent sourcing: Implications for operating leverage and stock beta. Review of Finance 20 (3), 1013–1043.

Mathews, J. A., 2005. Strategy and the crystal cycle. California Management Review 47 (2), 6–32.

Nielsen, M. J., 2002. Competition and irreversible investments. International Journal of Industrial Organization 20 (5), 731–743.

Nishide, K., Nomi, E. K., 2009. Regime uncertainty and optimal investment timing. Journal of Economic Dynamics and Control 33 (10), 1796–1807.

Nishide, K., Yagi, K., 2016. Investment under regime uncertainty: Impact of competition and preemption. International Journal of Industrial Organization 45, 47–58.

Pawlina, G., Kort, P. M., 2005. Investment under uncertainty and policy change. Journal of Economic Dynamics and Control 29 (7), 1193–1209.

Pawlina, G., Kort, P. M., 2006. Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage? Journal of Economics and Management Strategy 15 (1), 1–35.

Takashima, R., Goto, M., Kimura, H., Madarame, H., 2008. Entry into the electricity market: Uncertainty, competition, and mothballing options. Energy Economics 30 (4), 1809–1830.

Figures

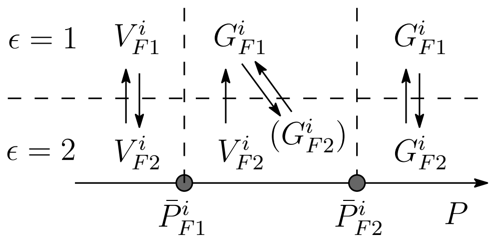


Figure 1: Regime shifts and the value functions for the follower firm.

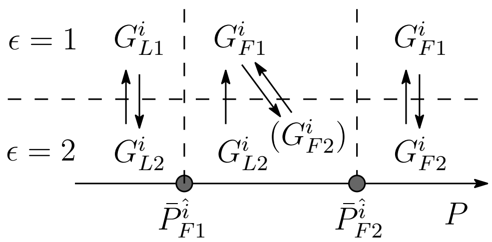


Figure 2: Regime shifts and the NPV of the leader firm.

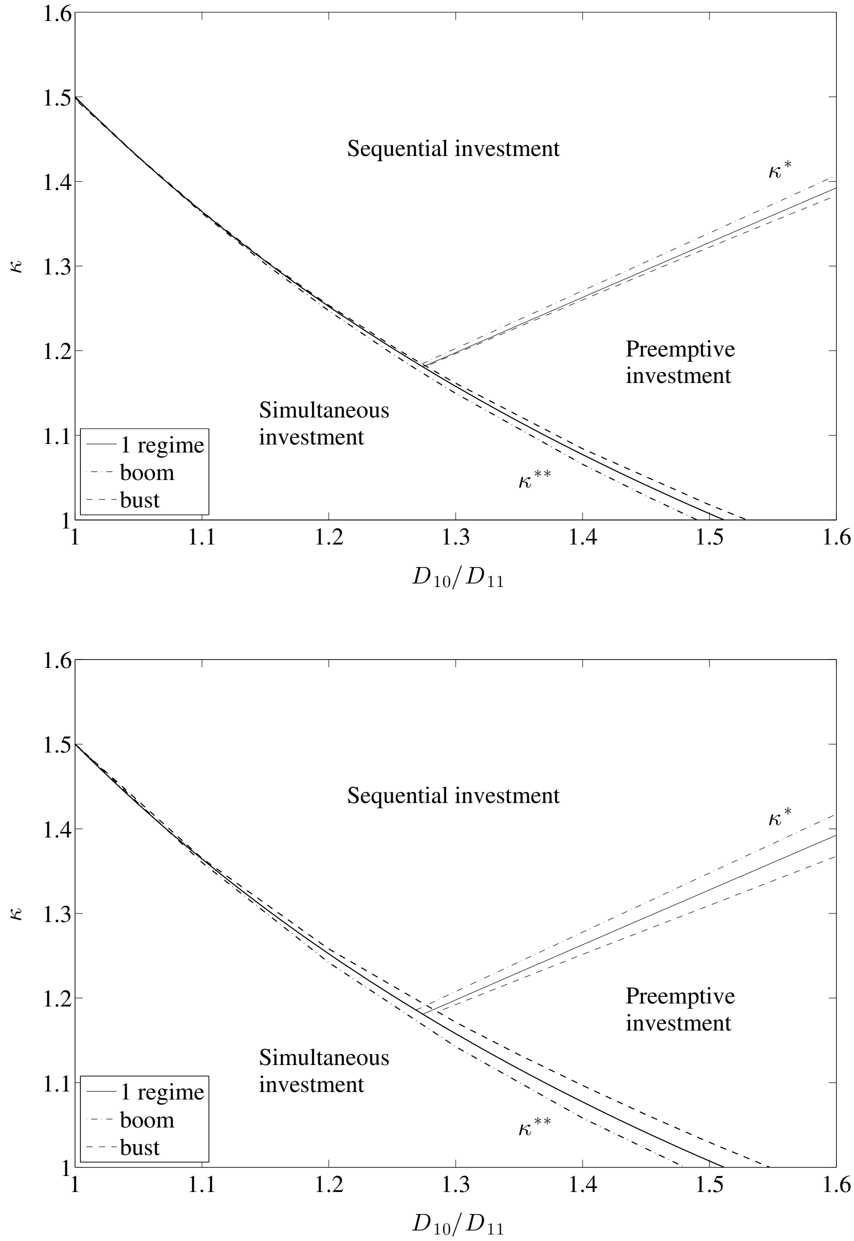


Figure 3: Regions of sequential, preemptive and simultaneous investment for the benchmark case except for $D^1_{10} = D^2_{10}$ and $D^1_{11} = D^2_{11} = 1$. The intensities are $\lambda_1 = \lambda_2 = 0.2$ in the upper and $\lambda_1 = \lambda_2 = 0.8$ in the lower.

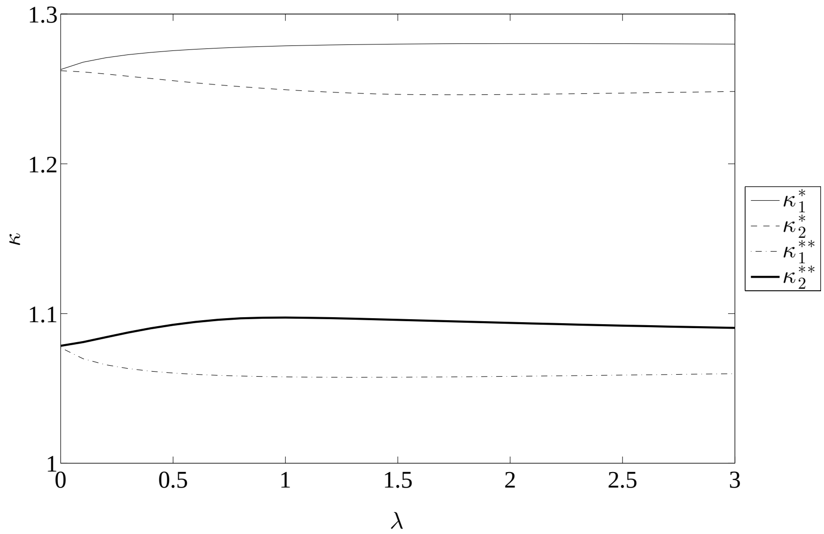


Figure 4: Comparative statics of κ's with respect to λ for the benchmark case except for $D_{10}^1 = D_{10}^2 = 1.4$ and $D_{11}^1 = D_{11}^2 = 1$.

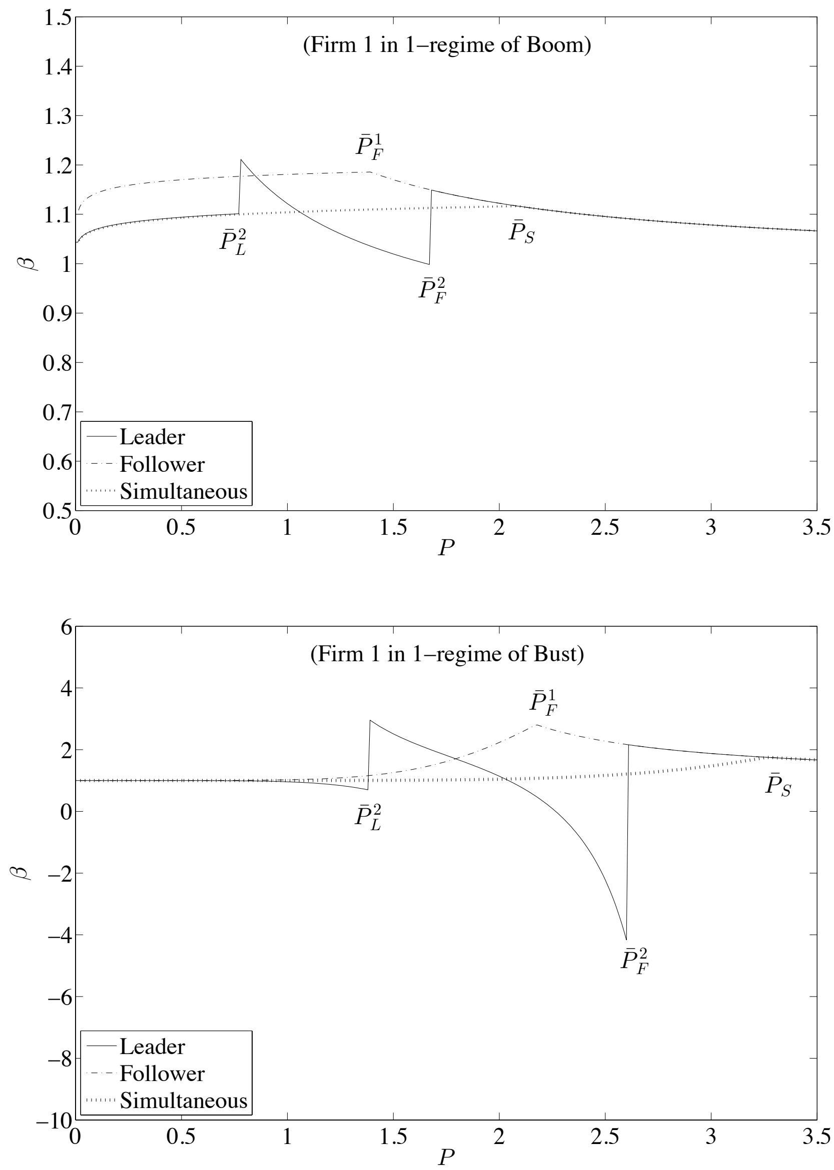


Figure 5: Betas of leader, follower and simultaneous investment for firm 1 in boom (upper) and bust (lower) without a regime shift.

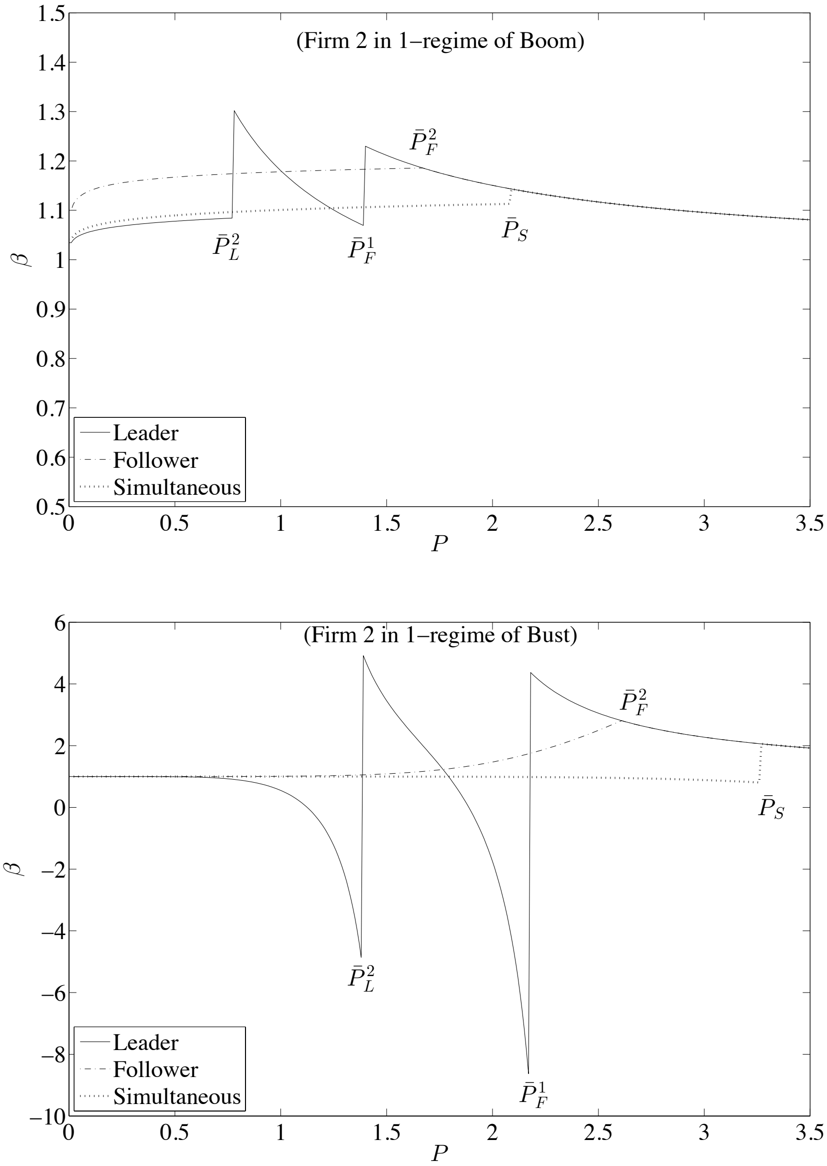
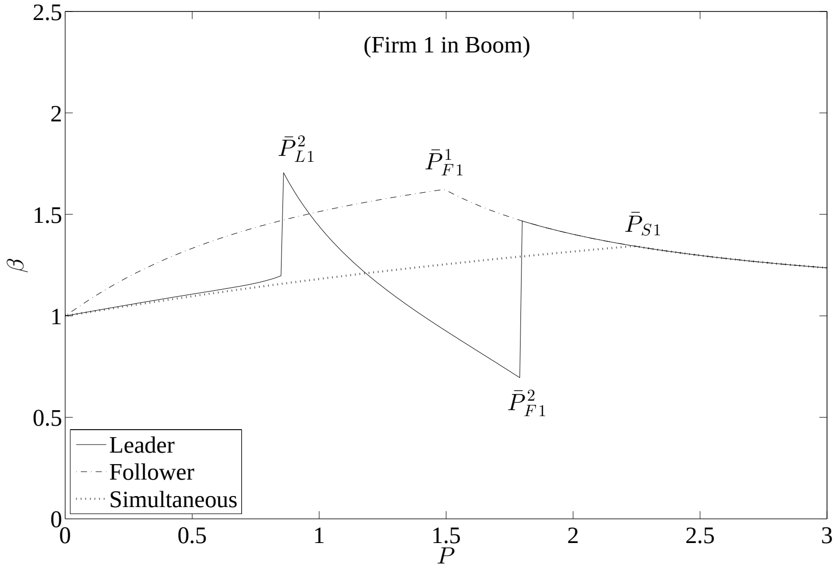


Figure 6: Betas of leader, follower and simultaneous investment for firm 2 in boom (upper) and bust (lower) without a regime shift.



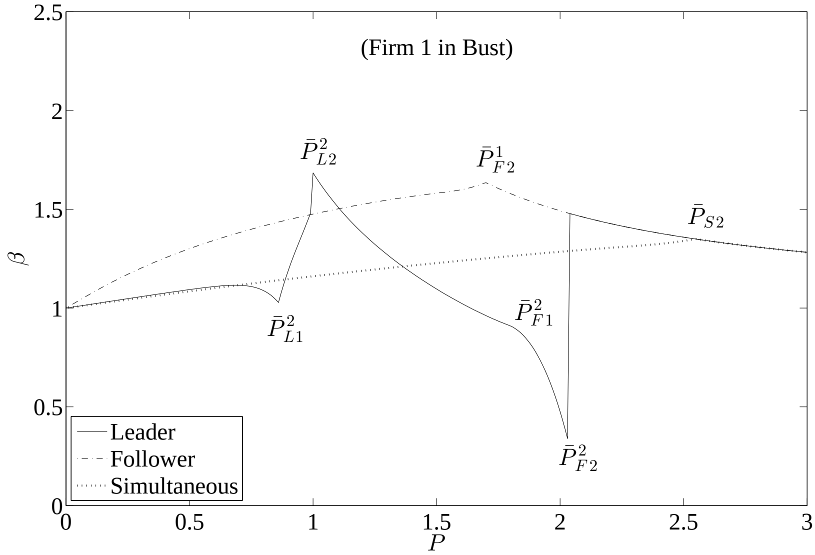


Figure 7: Betas of leader, follower and simultaneous investment for firm 1 in boom (upper) and bust (lower) with a regime shift.

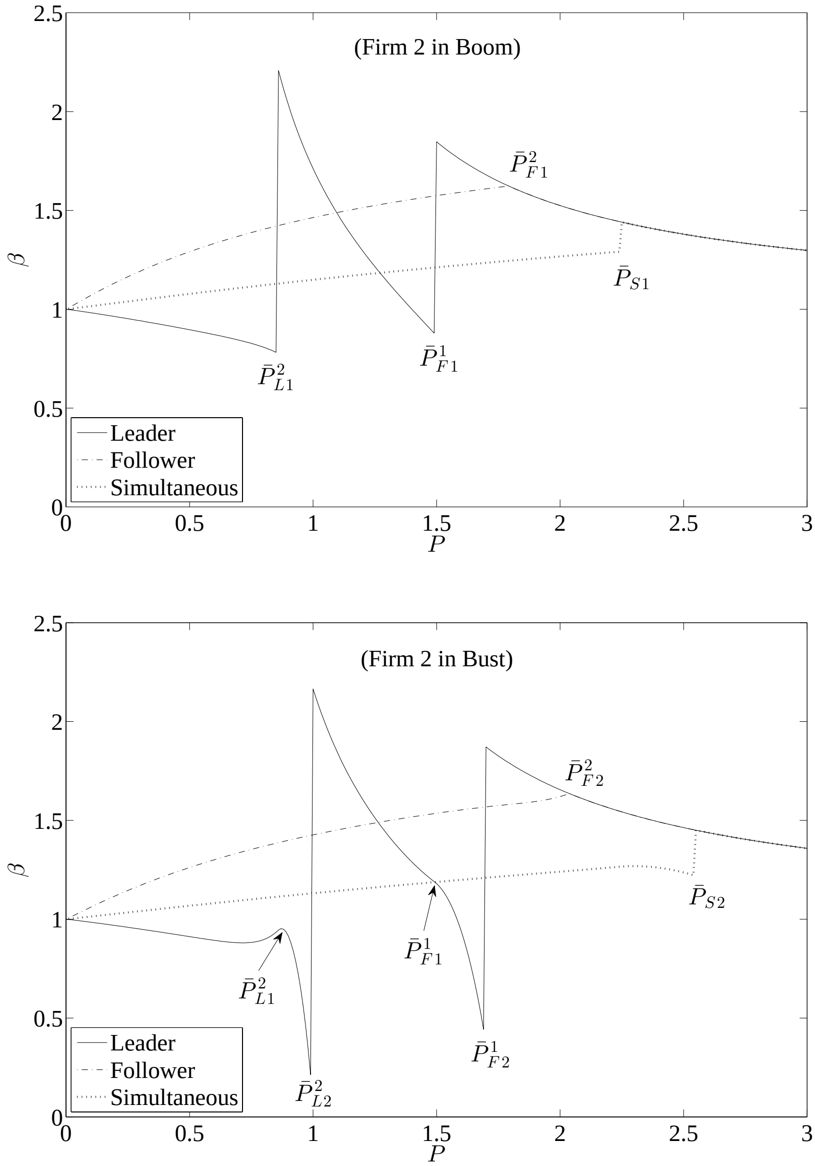


Figure 8: Betas of leader, follower and simultaneous investment for firm 2 in boom (upper) and bust (lower) with a regime shift.