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Electric Quadrupolar Contributions in the Magnetic Phases of UNi₄B

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We present acoustic signatures of the electric quadrupolar degrees of freedom in the honeycomb-layer compound UNi₄B. The transverse ultrasonic mode C_{66} shows softening below 30 K both in the paramagnetic phase and antiferromagnetic phases down to ~ 0.33 K. Furthermore, we traced magnetic field-temperature phase diagrams up to 30 T and observed a highly anisotropic elastic response within the honeycomb layer. These observations strongly suggest that $\Gamma_6(E_{2g})$ electric quadrupolar degrees of freedom in localized $5f^2$ ($J = 4$) states are playing an important role in the magnetic toroidal dipole order and magnetic-field-induced phases of UNi₄B, and evidence some of the U ions remain in the paramagnetic state even if the system undergoes magnetic toroidal ordering.

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The multipole formulation and its foundational concept in solid-state physics have been developed by intensive research on f -electron systems [1,2]. Recently, new theories based on the common language of “multipoles” [3,4] and “augmented multipoles” [5–9], which are spatially extended multipoles, have been evoked to construct a new framework for understanding various physical phenomena that are related to spin-orbit interactions beyond the differences in electron orbitals. In particular, the odd-parity augmented multipoles, including magnetic-electric and toroidal ones [10–12], have recently been extensively studied. Recent academic advances in understanding augmented multipoles have been preceded by theory rather than experiment. Therefore, it is necessary to demonstrate whether the new framework allows for a unified understanding of spontaneous spatial inversion symmetry breaking in metallic and insulating compounds. A major experimental challenge is to demonstrate odd-parity multipole ordering by observing cross-correlation phenomena and spontaneous spatial inversion-symmetry breaking in a suitable compound [5,8]. Among them we focus on the U-based honeycomb-layer compound UNi₄B [13–16], which is considered to be a good candidate for studying

augmented odd-parity multipoles, magnetic toroidal dipoles (MTDs), and the interplay with magnetoelectric phenomena [12,17].

UNi₄B crystallizes in an orthorhombic structure (Space group; $Cmcm$, D_{2h}^{17} , No. 63) as shown in Fig. 1(a) [18]. Below $T_N = 20.4$ K, this compound orders antiferromagnetically (AFM) in a magnetic structure where the magnetic moments are carried by two-thirds of the U ions [U_{AFM} sites in Fig. 1(b) form vortices in each pseudo-honeycomb plane], and one-third of the U ions [U_{PM1} or U_{PM2} in Fig. 1(b)]

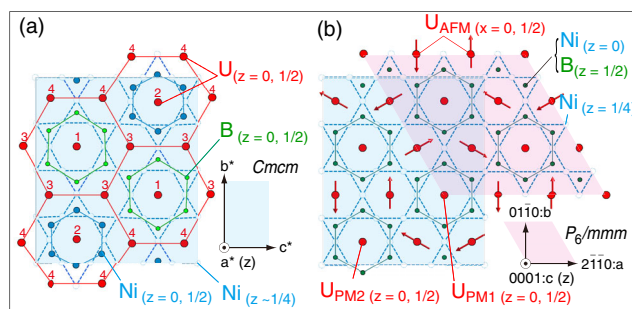


FIG. 1. Crystal and magnetic structure of UNi₄B, reported for $B = 0$ [16,18]. (a) The pseudo-honeycomb network consists of two-thirds of U ions with $Cmcm$ lattice. Red, blue, and green circles indicate U and Ni or B on the layer at $z = 0$, $1/2$, respectively. Dashed lines with open circles show the (pseudo) Kagome layer of Ni atoms at $z \sim 1/4$. (b) Colored backgrounds with pink-rhomboidal and blue-rectangular shapes denote antiferromagnetic unit cells in hexagonal (P_6/mmm) and orthorhombic ($Cmcm$) symmetry, respectively.

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remain in a paramagnetic (PM) state. [13,15,16] Assuming a hexagonal crystal structure ($P6/mmm$, D_{6h}^1 , No. 191), an exotic magnetic structure was proposed from neutron scattering experiments in earlier studies [16]. Since a slight deformation of the crystal structure from hexagonal to orthorhombic symmetry and different site occupations of Ni and B atoms have recently been reconfirmed by neutron and resonant x-ray scattering studies as well as by ^{11}B -NMR measurements [19,20], the previously proposed magnetic structure should be reconsidered based on the orthorhombic space group.

On the other hand, Hayami *et al.* has pointed out that such vortex-type magnetic structure in the (pseudo) honeycomb arrangement in UNi_4B can be understood in the framework of MTD order (see Fig. S1 in the Supplemental Material) [12,21]. Their theory has also predicted a new magnetoelectric effect: current-induced magnetization, which can occur in ferro-toroidal ordered metallic compounds, which has been experimentally confirmed in UNi_4B [17]. Recently, Yatsushiro and Hayami have reported on a theoretical investigation of an atomic scale MTD by taking into account orbital degrees of freedom with different parity [11]. This theory predicts that the orbital degrees of freedom in an interorbital space play an important role in stabilizing MTD order by odd-parity hybridization. However, the theory deals with the tetragonal point group C_{4v} , and the contribution of the orbital degrees of freedom such as even-parity electric multipolar moments to the noncollinear magnetic order in UNi_4B has not been investigated.

Another fascinating point and also an open question for this compound is a specific-heat anomaly at $T^* \sim 0.33$ K of unknown origin [22]. Previous studies have explained that the narrowness of the 0.33 K anomaly in UNi_4B may be an indication of glassy behavior caused by the geometrical frustration of the paramagnetic U spins and their Kondo screening by the conduction electrons, since ac magnetic susceptibility and μSR measurements have shown no changes in the magnetic structure below T^* [22]. Other possibilities, which have not been verified yet, are a non-magnetic multipolar order of the PM-1/3 U ions and/or Schottky peak due to level splitting of the degenerate CEF ground state with low orthorhombicity. In order to study the electrical multipole contribution, it is useful to measure the elastic constants by ultrasound [23]. The elastic constants reflect the coupling of the strain field caused by the ultrasonic wave to the electric multipolar moments, which are described by orbital degrees of freedom of the CEF state. In this study, we show evidence of an electric multipolar ground state for UNi_4B based on ultrasound results. We further analyzed the possible contributions of electric quadrupoles to the noncollinear magnetic order and to the low-temperature specific-heat anomaly (See Secs. (c)–(g) in the Supplemental Material [21] for experimental and analysis details).

Figure 2 shows the measured elastic constants of UNi_4B as a function of temperature. Here, the four ultrasonic

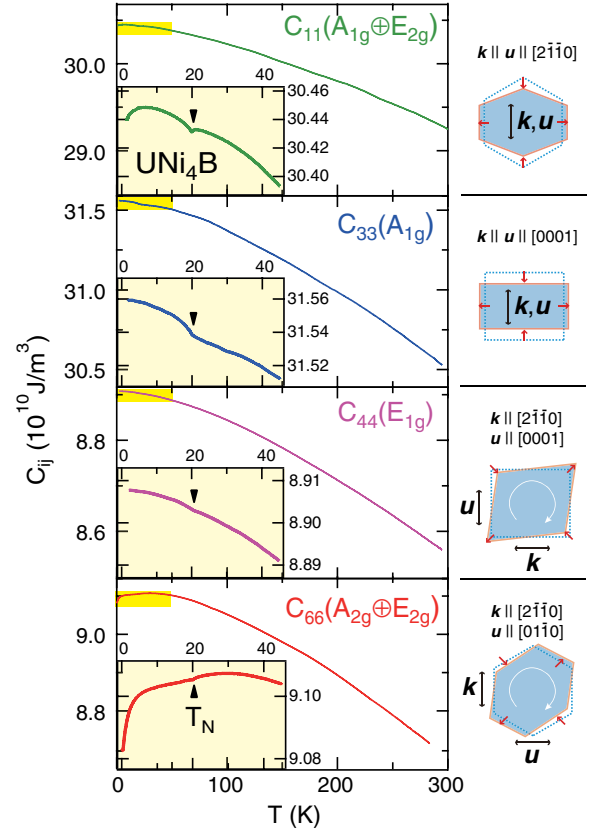


FIG. 2. Elastic constants of UNi_4B as a function of temperature. The insets in each panel show enlarged views of the data below 50 K. Illustrations of the distorted hexagon and/or rectangle indicate the lattice strain, which is induced by the respective ultrasonic mode (See Table SI in the Supplemental Material) [21,25,26].

modes are symmetrized using the hexagonal point group D_{6h} . The ultrasound induces local strain and rotation fields [21] in the solid sample as shown by the schematic illustrations in each panel of Fig. 2. The local strain and rotation field behave as conjugate fields for electric quadrupole or electric hexadecapole moments [24]. Responses of the multipoles can be observed as sound-velocity change and ultrasonic attenuation via electron-phonon interaction. By comparing the temperature dependence of the four ultrasonic modes, it becomes obvious that only the transverse ultrasonic mode C_{66} exhibits a softening below 30 K in the PM phase with a kink at T_N and keeps decreasing in the antiferromagnetic (AFM) phase down to ~ 0.33 K [Fig. 3(b)].

The other ultrasonic modes, the longitudinal C_{11} , C_{33} , and transverse C_{44} modes, do not show such softening particularly below T_N . From the selection rules within the category of even-parity multipoles, the results indicate that an electric quadrupole with $\Gamma_6(E_{2g})$ symmetry (in the hexagonal point group) is active in UNi_4B . In the first stage of the analysis, we consider the conventional localized $5f$ -electronic states with even-parity CEF levels and

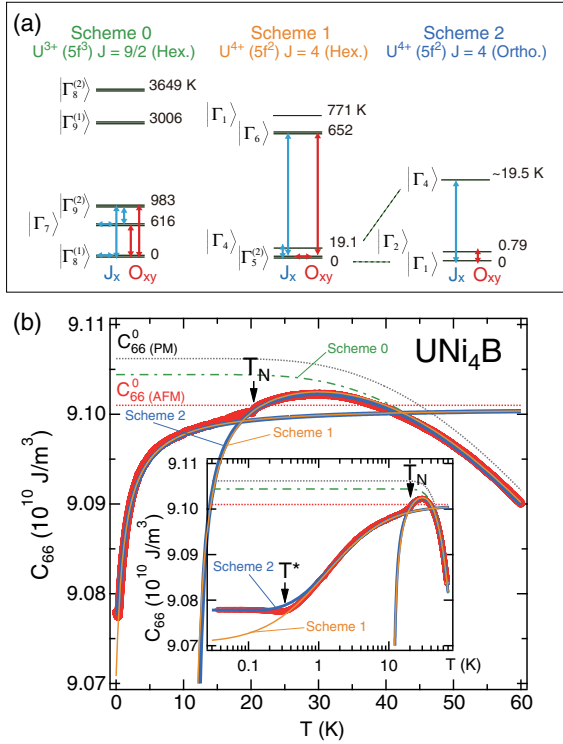


FIG. 3. (a) CEF level schemes used for the present analysis (see Table SIII, SIV, and SV in the Supplemental Material [21]). Blue and red arrows indicate selection rules where finite matrix elements exist for magnetic dipole J_x and electric quadrupole O_{xy} , respectively. (b) Temperature dependence of the elastic constant C_{66} , compared with calculations based on the quadrupolar susceptibility using the CEF level schemes shown in panel (a): Scheme 0 (green), scheme 1 (orange), and scheme 2 (blue) as shown by dashed and solid curves. The dotted black and red curves show the background of the elastic constant C_{66}^0 in the PM and AFM phases, respectively. The inset shows an enlarged view of the low-temperature region on a log- T scale.

multipoles for the analysis of the elastic responses. Here, we do not take into account the contribution from odd-parity multipoles, because such effect only couples to the elastic strain through cross-correlation with the application of appropriate external fields with odd-parity space-inversion symmetry [6,8]. As shown below, the current analysis mimics the experimental results well that the odd-parity mixing inherent in the CEF is negligible. The softening in C_{66} in the paramagnetic phase could, however, not be reproduced by the previously proposed CEF level scheme [scheme 0 in Fig. 3(a)] [27] with the localized $5f$ -electronic state of U^{3+} having a hexagonal symmetry because the $5f^3$ ($J = 9/2$) state only shows Γ_6 -quadrupolar excitation [$O_{xy} = \sqrt{3}(J_x J_y + J_y J_x)/2$] in the off-diagonal elements between the ground-state Kramers doublet and the excited levels that are separated by an energy gap of over 600 K, as shown in Fig. 3(a) [for matrix elements, see Sec. (f) in the Supplemental Material [21]].

In order to reproduce the softening in the C_{66} mode at higher temperature than T_N , we propose a different CEF model [scheme 1 in Fig. 3(a)] with a localized $5f^2$ ($J = 4$) state of U^{4+} , which has a pseudo-triplet ground state. The CEF parameters of CEF scheme 1 (Table SIV in the Supplemental Material [21]) are set to reproduce simultaneously the elastic softening in C_{66} , no elastic softening in C_{44} , and also the magnetic susceptibility below 50 K at the same time (for details see Fig. S3 in the Supplemental Material [21]). The coupling constant g'_Γ of quadrupolar inter-site interactions is described by the Hamiltonian $\mathcal{H}_{MM} = -\sum_\alpha g'_{\Gamma_6} \langle O_{xy} \rangle O_{xy}^\alpha$ for sublattices α and a Γ_6 symmetry quadrupolar moment O_{xy} [see Eqs. (17)–(21) in the Supplemental Material] [21,23,28]. Our analysis reveals a positive value $g'_{\Gamma_6(\text{PM})} = +0.42$ K, which strongly suggests the presence of a weak but finite ferro-type quadrupolar interaction in the PM phase of UNi_4B . A possible relationship between the ferro-type MTD order of this system and the ferro-quadrupolar interaction is non-trivial and remains an open question. The softening of C_{66} in the AFM phase can also be analyzed using the same CEF parameters, since the PM-1/3 U ions (on the U_{PM1} or U_{PM2} sites) located in the center of the pseudo-honeycomb plane are not affected by the on-site electric or magnetic fields formed by the MTD moment. Although the global inversion symmetry on the U_{PM1} and U_{PM2} sites is broken due to the MTD order, the odd-parity CEF states and multipole will, however, not be active on the PM-1/3 U sites when only considering the $J = 4$ Hilbert space and assuming weak orbital coupling between the U and Ni or B ions.

The inset in Fig. 3(b) shows the temperature dependence of C_{66} on a log- T scale. Calculations of the quadrupolar susceptibility of the PM-1/3 U ions using the CEF scheme 1 (orange curve) [21] with hexagonal symmetry well reproduces the softening in the AFM phase down to ~ 0.33 K. Here, the contribution from the AFM ordered-2/3 U ions is assumed as constant background and the CEF level scheme of the PM-1/3 U ions is not changed from the PM phase. Note that the intersite quadrupolar interaction for the AFM phase obtained by our analysis is negative with $g'_{\Gamma_6(\text{AFM})} = -0.045$ K, which means the presence of antiferro-quadrupolar (AFQ) interaction in the AFM phase. In order to reproduce the temperature dependence down to T^* in the present analysis, it is unlikely that $g'_{\Gamma_6(\text{AFM})}$ takes a positive value. Therefore, we can conclude that in the ordered phase there is an antiferro-quadrupolar interaction between the U_{PM} 's [in Fig. 1(b)], which is fundamentally the opposite to that for U_{AFM} in the PM phase.

The calculated result using scheme 1 deviates from the experimental data below $T^* \sim 0.33$ K. This deviation indicates a small CEF splitting of the ground-state doublet. Such splitting might occur for two reasons: (i) symmetry lowering due to ordering of the remaining PM-1/3 U multipolar moments or (ii) the crystal structure essentially

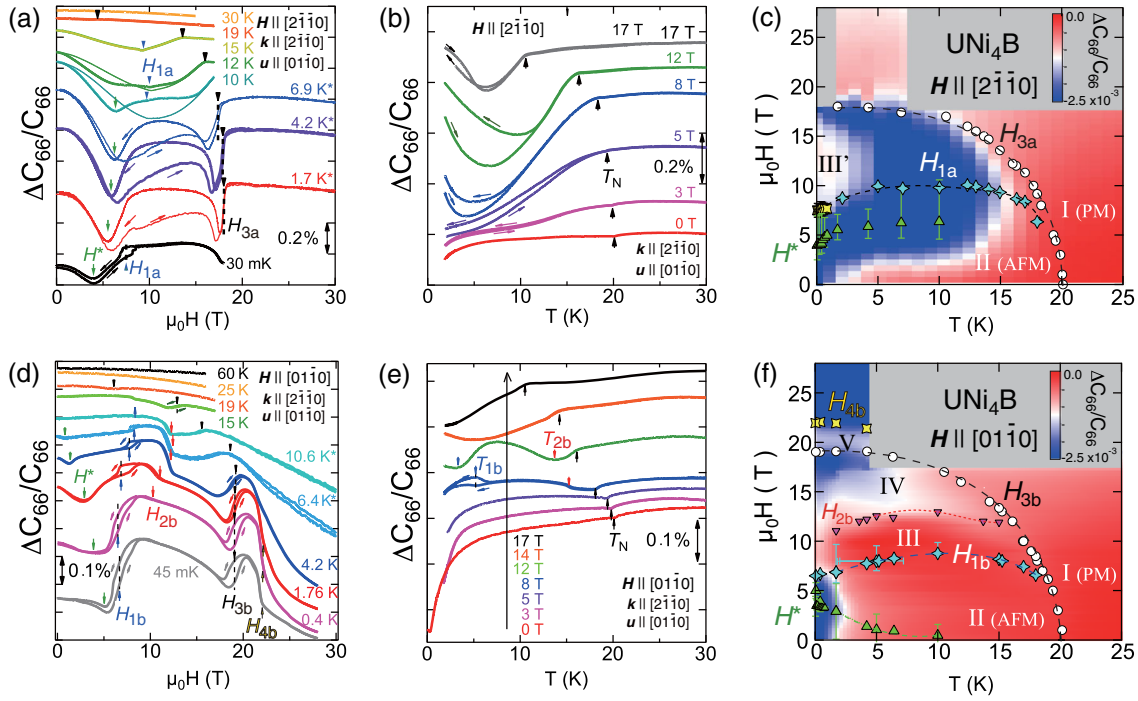


FIG. 4. Magnetic-field dependence of C_{66} for (a) $H \parallel [2\bar{1}\bar{1}0]$ and (d) $H \parallel [01\bar{1}0]$ of UNi_4B at selected temperatures. C_{66} vs temperature under various magnetic fields for (b) $H \parallel [2\bar{1}\bar{1}0]$ and (e) $H \parallel [01\bar{1}0]$. In each panel, vertical arrows indicate elastic anomalies, which correspond to phase boundaries in the magnetic field-temperature phase diagram of UNi_4B for (c) $H \parallel [2\bar{1}\bar{1}0]$ and (f) $H \parallel [01\bar{1}0]$. Dotted and dashed lines are guides for the eyes. The relative change of the elastic constant C_{66} is highlighted by the color map from zero (red) to large negative (blue) values.

having lower symmetry. Since it has been confirmed that the point group of the U ions in the present system has orthorhombic symmetry, we have modified the CEF scheme 1 by adding low orthorhombicity [as CEF parameter B_2^2 and B_4^2 for Steven's operator O_2^2 and O_4^2 [see Eqs. (3)–(11) in the Supplemental Material [21]] [3,21,29,30], respectively, have finite values] to split the ground-state non-Kramers doublet with a gap of $\Delta \sim 0.79$ K. This CEF scheme 2 [Fig. 3(a), Table SV in the Supplemental Material [21]] reproduces the leveling off of C_{66} (blue curve) as well as a Schottky-type specific-heat peak at $\Delta/2.398 \sim 0.33$ K [22]. We, therefore, conclude that the mentioned T^* transition, found in specific heat, is a Schottky anomaly due to the orthorhombicity of the crystal. It should be noted that we determined the magnetic-field dependence of T^* and its anisotropy for $H \parallel [2\bar{1}\bar{1}0]$ and $[01\bar{1}0]$ in the elastic constant C_{66} . Our results are roughly consistent with those of earlier studies [16] (see Fig. S8 in the Supplemental Material [21]).

The temperature and magnetic-field dependence of the elastic constant C_{66} are shown in Figs. 4(a) and 4(b) for $H \parallel [2\bar{1}\bar{1}0]$ and in Figs. 4(d) and 4(e) for $H \parallel [01\bar{1}0]$. In Fig. 4(a), the data with an asterisk are obtained in pulsed-magnetic-field measurements up to 60 T (see Fig. S9 in the Supplemental Material [21]). We obtained C_{66} vs $H \parallel [01\bar{1}0]$ up to 28 T, shown in Fig. 4(d), in static magnetic fields using the cryogen-free hybrid magnet system equipped with a dilution refrigerator [31]. Several elastic anomalies are

observed, which are indicated by arrows; H^* , H_{1b} , and H_{3b} for $H \parallel [2\bar{1}\bar{1}0]$, H^* , H_{1a} , H_{2a} , H_{3a} , and H_{4a} for $H \parallel [01\bar{1}0]$. The data display both up and down sweeps of the magnetic field. We observe hysteretic regions below H_{3a} . The elastic responses in C_{66} show a large in-plane (0001) anisotropy for $H \parallel [2\bar{1}\bar{1}0]$ and $H \parallel [01\bar{1}0]$, while the magnetization does not show such strong anisotropies [16]. The positions of the elastic anomalies are indicated as well in the magnetic field-temperature (H - T) phase diagrams as shown in Figs. 4(c) and 4(f). Here, a number of phases are distinguishable for $H \parallel [01\bar{1}0]$; PM phase I, AFM phase II, the spin-reoriented AFM phase III, the spin-flop phase IV with hysteresis, which was previously evidenced by magnetization data [16], and a newly found unknown phase V. The obtained phase boundaries are consistent with the previously reported phase diagram [16] except for the high-magnetic-field region. On the other hand, the H - T phase diagram for $H \parallel [2\bar{1}\bar{1}0]$ is quite different with less phases and completely different elastic responses. The background red-white-blue color code in Figs. 4(c) and 4(f) represents the relative changes in C_{66} from larger to smaller stiffness. Remarkably, the contour plot shows a significant difference in stiffness for $H \parallel [01\bar{1}0]$ and $[2\bar{1}\bar{1}0]$, though no difference was detected in magnetization. This new observation clearly indicates a possible contribution of electric quadrupoles of the PM-1/3 U ions, which modifies the spin-reorientation process as well.

In the low-magnetic field and low-temperature regions for both $H\parallel[01\bar{1}0]$ and $H\parallel[2\bar{1}\bar{1}0]$, the blue color indicates enhanced contributions from the $\Gamma_6(E_{2g})$ -electric quadrupoles, i.e., this system has incoherent fluctuations of the electric quadrupoles due to the CEF pseudo-doublet ground state. In general, the ground-state doublet (including non-Kramers doublet) splits due to the Zeeman effect (with mixing of excited-level wave functions) in an external magnetic field or in the internal fields produced by magnetic order, and the quadrupole degrees of freedom become inactive, resulting in a hardening of C_{66} . Remarkably, C_{66} experiences a softening in phase IV for $H\parallel[01\bar{1}0]$ and in the intermediate temperature range of phase III' for $H\parallel[2\bar{1}\bar{1}0]$ compared with the changes in the other phases. This fact suggests a reactivation of the quadrupole degrees of freedom with $\Gamma_6(E_{2g})$ symmetry above ~ 12 T in the low-temperature region for $H\parallel[01\bar{1}0]$.

In summary, we conclude that the electric-quadrupole degrees of freedom play a crucial role in the low-temperature properties of UNi_4B , leading to anisotropic H - T phase diagrams and a newly revealed field-induced phase V. The observed softening of the C_{66} elastic constant can be well explained by quadrupolar-strain interactions. The corresponding CEF analysis results in a new level scheme (scheme 2) taking into account the established orthorhombic symmetry and the $5f^2$ ($J = 4$) state of uranium ions. Furthermore, in this level scheme the puzzling specific-heat anomaly at ~ 0.33 K can be understood as a Schottky anomaly due to a small level splitting of the non-Kramers ground-state doublet. Moreover, our results confirm that some of the U ions stay disordered in the MTD ordered phase. Further theoretical considerations would be of interest to clarify the quadrupolar contributions in the toroidal order and anisotropic elastic response in the newly established magnetic-field-temperature phases.

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- [21] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.126.157201> for (a) schematic illustration of the magnetic toroidal order, (b) indexing a hexagonal system with four axes, (c) experimental details, (d) formulation of multipolar susceptibility, (e) definition of multipolar moments and equivalent operator expressions, (f) matrix elements of magnetic dipole and electric quadrupoles, (g) consistency test for CEF calculations of the magnetic susceptibility, (h) elastic constant C_{66} and calculated quadrupolar susceptibility at low temperatures (< 1 K), (i) anisotropy of the H - T phase diagram and the 330 mK anomaly, and (j) elastic response of the C_{66} mode under pulsed magnetic fields above 30 T.
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