FINITE VOLUME METHOD ON THE UNSTRUCTURED GRID SYSTEM

Nobuyuki TANIGUCHI and Toshio KOBAYASHI

Institute of Industrial Science

University of Tokyo, Tokyo, 106 Japan

Abstract

Applying the Voronoi diagram to the cell system for the finite volume method, a new method on the unstructural grid is constructed for the incompressible steady flow simulation. In this method, the SIMPLE algorithm can be applied with little expansion. The turbulent flow around the two-dimensional vehicle model is simulated with the k-e turbulence model by this method. Comparing the calculation result with another result by the structural grid system and the experimental data, the new method is confirmed to be available for the simulation of the complex flow fields.

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1. Introduction

Flow simulation techniques on the general coordinate system, based on the finite difference method (FDM) or the finite volume method (FVM), have been expanding the applications of Computational Fluid Dynamics. The grid generation, however, becomes a serious problem in these simulations. It is often difficult to define the appropriate grid systems, not only in the realistic simulations but also in the simple and basic ones.

Otherwise, unstructural grid systems will be available for the flow simulations of complex geometries. Though the finite element method (FEM) has been applied to them [ref.1], it generally needs more computational time or memory size than the FDM or the FVM, especially in steady flow simulations. The calculation with the time-averaged turbulence model such as k-ε model or Reynolds stress model is a major problem in the recent applications of the flow simulation [ref.2]. Most of them were performed by the FDM or the FVM on the structural grids.

In this study, we propose a new conceptual method based on the FVM [ref.3], where much of the previous research using the structural grids can be directly introduced, especially in the case of the turbulent flow simulations. In this new method, a Voronoi diagram is adopted to divide the calculation domain into the control cells for the FVM, where the discretized forms are derived in the same way as the previous method on the structural grid system. Therefore, most of the concepts of algorithms or boundary conditions can be also available in the unstructural grid systems.

The above concept of the discretization has been researched in simulating heat transfer [ref.4,5] though there are few application to high Reynolds number flows.

2. Discretization on a Voronoi Diagram

A successful method based on the FVM was proposed by Patankar [ref.6] on the Cartesian coordinates and some researchers expanded it for the grid system along the general coordinates [7-10]. According to these methods, the basic equations are integrated in each control cell to derive the discretized equations from them. In the simulations of the incompressible flow, the basic equations are generally expressed as the conservation of physical values, such as mass or momenta. The integrated form for a general variable is,

$$\iiint \frac{\partial \phi}{\partial t} \, dV = \iint \mathbf{J} \cdot d\mathbf{n} + \iiint S \, dV , \qquad (1)$$

ϕ : a general variable or a component of vector,
$\mathbf{J}$ $(= \boldsymbol{v} \phi - \Gamma \nabla \phi)$: total flux of ϕ through the faces,
$\boldsymbol{v}$: velocity vector,
S : source of ϕ in the cell, Γ : diffusive coefficient,
$\iint d\mathbf{n}$: surface integration by the normal vector,
$\iiint dV$: volume integration in the cell.

It can be indicated that a new cell generation technique may produce a new discretization method because the above equation does not restrict the shapes of cells. We consider that the control cells must satisfy the following conditions in order to apply the basic concepts of the above methods.

1) The cells fill the whole of the calculation domain and are never superimposed.
2) Each cell has one calculation point.

Also, it is better to add the next conditions to keep the simplicity of the discretized form.

3) Each cell can be approximated to the convex domain.

4) Cells are automatically defined by the general algorithm.

The Voronoi diagram can be adopted on the unstructural grid systems under these conditions. The Voronoi diagram which expresses a concept in Computational Geometry defines the governing domains of the mother points distributed in the space [ref.11]. As Fig.1 shows, the face "a" is the perpendicular bisector of the line segment joining X0 to X1. In a two-dimensional case, an intersection of the surfaces of three neighboring cells is defined as the center of the circumscribed circle for their mother points.

-fig.1

Concerning the surface integrations of the Eq.(1), the discretized form in this method is derived as easily as in the method on the Cartesian coordinate system by Patankar because this cell system has the same local geometry in the respect to the neighboring mother points and their cell boundary face. For example, the total flux J through the face "a" can be estimated in the local coordinates as follows.

$$J = JD\ f(JM/JD)(\phi_1 - \phi_0) + JM\,\phi_{UP}\ , \qquad (2)$$

JM $(=u_a)$, JD $(=\Gamma/dx)$: convective and diffusive flux factors,
ϕ_1, ϕ_0 : values at X_1 and X_0, ϕ_{UP} : up-wind value,
u_a : normal velocity component on face "a",
dx : distance between X_1 and X_0,
f( ) : scheme function [ref.4].

· Central dif.	$f(P) = 1 - 0.5*\|P\|$
· Up Wind	$f(P) = 1$
· Hybrid	$f(P) = \max(0, 1-.5*\|P\|)$
· Power Law	$f(P) = \max(0, (1-.1*\|P\|)^5)$

In the same manner, the flux on each face can be estimated in its local coordinates.

The volume integrations are calculated by the values at the mother points. The weighted average techniques are adopted when estimating the gradient values. Finally, the discretized equation is written in the linearized form, that is,

$$a_0 \phi_0 = \Sigma\, a_{NB} \phi_{NB} + b \,, \tag{3}$$

a , b : coefficients and constant term,
0, NB (subscripts) : indeces of the center and neiboring points. (Suming symbol refers to NB.)

It is noted that the number of neighboring points depends on the shape of each cell, because it is defined by the Voronoi diagram instead of using any coordinate systems.

3. SIMPLE Algorithm and Calculating Procedures

The algorithm linking the continuity equation with the momentum equations has an influence on the calculation efficiency of the flow simulation method. Concerning the steady flow simulations, the SIMPLE algorithm by Patankar [ref.6] is successful in the structural grid system. The modified method for the non-staggered grid system was proposed by Rhie and Chow [ref.9]. It can be applied to the discretized equation (3) with little expansion.

The continuity equation is also discretized by the above-mentioned method on the Voronoi cell system.

$$\Sigma\, JM_{nb} = 0 \,, \tag{4}$$

JM : the same definition as in the equation (2) on the cell surfaces nb. (Suming symbol refers to nb.)

In the SIMPLE method, an equation for the corrective pressure p' is derived from Eq.(4). According to the method by Rhie and Chow, the contribution of the pressure to the mass flux JM on the face "a" is estimated directly by the values of the two points X0 and X1. The equation of p' is also written in the form (3). The iterative procedure of the SIMPLE algorithm can be applied without any corrections, and the relaxation factors are also the same values [ref.4].

The systems of equations expressed in the form (3) are able to be solved by the point-iterative procedure. The successive over relaxation (SOR) method is also effective by the optimization of the tracing order of the points for the parallel computations. In this case, the multicolors technique is adopted for ordering points [ref.12], in the concept of which the points are divided into some groups (namely colors) so that the points are not directly related each other; in other words, each point has a different color from its neighboring points in Eq.(3). As the points of each color are calculated in a step, the procedure of the SOR can be parallelized.

The whole procedure for this method is as follows.

1) Defining the calculation points
2) Creating the cell system by the Voronoi diagram
3) Calculating the geometry factors
4) Setting the list-vectors for the array processing
5) Iterative procedure of SIMPLE

The fourth process arranges the list-vectors for optimizing the last process on the multi-colors technique.

4. Applications

Based on the above considerations, we have constructed a simulation method on the unstructural grid system for two-dimensional, incompressible, and viscous flow. For the stability of calculation, the hybrid scheme [ref.6] is adopted, the form of which is identified with the second order central scheme or the first order upwind scheme according to the cell Pecret number. It can be applied to the simulations of the laminar and turbulent flow fields. In the turbulent flow cases of the present research, the standard k-ε model [ref.2] is adopted. The basic equations are shown as follows.

$$\frac{\partial}{\partial x_j}(u_j k) = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + G - \varepsilon \ ,$$

$$\frac{\partial}{\partial x_j}(u_j \varepsilon) = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) + C_1 \frac{\varepsilon}{k} G - C_2 \frac{\varepsilon^2}{k} \ ,$$

$$G = \nu_t \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \frac{\partial u_j}{\partial x_i} \ ,$$

$$\nu_t = C_\mu k^2 / \varepsilon \ , \qquad (5)$$

where the five constants are fixed as

$$C_\mu = 0.09,\ C_1 = 1.44,\ C_2 = 1.92,\ \sigma_k = 1.0,\ \sigma_\varepsilon = 1.3\ .$$

The differential equations of the turbulence energy k and the energy dissipation ε can be discretized by the method mentioned in this paper. Concerning the wall boundary condition, the wall function based on the universal velocity law can be applied in the same manner as the previous FVM on the structural grid system.

-fig.2

First in the laminar case, the flow in the square cavity with the driving wall is simulated. Two types of grids are adopted, one of which is a structural grid along the Cartesian coordinates (Fig.2a) and the other is unstructural (Fig.2b). The number of points is 1600 in both grids. In this simulation the Reynolds number is fixed as 1000 based on the width of the cavity and the wall-driving velocity.

-fig.3

-fig.4

Figures 3 and 4 are the stream lines and the velocity distributions in the center vertical section. They confirm the good agreement of the calculation results between the structured and unstructured grids. It can be supposed that the grid effects are comparably small in the present method. These results satisfactorily agree with those by the previous method based on the Cartesian grid using the same scheme and the same grid (Fig.2a) [ref.13], though the numerical diffusion may influence the calculation results in such high Reynolds number flows. We can estimate the numerical errors according to the previous method.

-fig.5

-fig.6

Next is the simulation of a more complicated flow field, which is the turbulent flow around a two-dimensional vehicle model. Composite grid techniques are more useful in such complex fields, because it is not easy to generate the appropriate grids in the whole region. They are easily applied when the unstructural grid system is permitted. In this case, the calculation grids are individually defined in the two regions, which are illustrated in Fig.5. The two grids are simply merged without the special techniques. The fields near the joint of the regions are also automatically divided into the control cells by the Voronoi diagram (Fig.6).

-fig.7

-fig.8

Figures 7 and 8 are the velocity field and the pressure contours, which indicate that the smooth solution is calculated by this method. The Reynolds number is adjusted to the experiment [ref.14], 2.1 millions based on the free stream velocity and the body length. In Fig.9, the pressure distribution predicted by the present method, is compared with that by the method on the general coordinates [ref.10], where the calculation points near the vehicle body are identified in both methods. They agree sufficiently with each other and also with the experimental data [ref.11], at least in the forward half of the body. Because the disagreement in the backward half between the calculation and the experiment is supposed to depend mainly on the turbulence model and its boundary conditions, the present method, we consider, is available for the turbulent simulation as well as the structural grid method. In addition, it should be noted that the structured grid method predicts the pseudo peek at the front corner, otherwise the present methods is free from the grid singularity problems.

-fig.9

Concerning the convergence speed, the iteration numbers in the SIMPLE algorithm and the CPU times are compared in the table 1. These calculations are performed on the high speed computer with the array processors, S820/80 produced by Hitachi, which has 2 GFLOPS maximum performance. For the method with the Voronoi diagram, the CPU time of the last process is displayed because the pre-processes (1-4) take only small computational time, less than 10 presents of the total CPU time on the scalar computation but can not be parallelized effectively. The last process was highly accelerated by the parallel computation with the multicolors technique.

-tbl.1

For the cavity flow, we refer the data of the previous method on the Cartesian grid [ref.12], which adopted the same grid as A and the SIMPLE algorithm on the staggered system. In both simulations, the criterion of convergence is when the residuals of the momentum and mass equations (Eqs.(3),(4) in the present method) becomes less than 10^{-3}. This data indicates the new method requires only the same iterations as the reference methods.

For the vehicle flow, the calculation time is compared with the data of the reference method on the general coordinate system [ref.12], which also adopted the SIMPLE. Both simulation were performed by 500 iterations of the SIMPLE algorithm, when the above criterion was almost satisfied in both cases. Considering the easiness of grid generation, the present method can be satisfactorily effective in calculational speed though it is lower than the general coordinate method. In the unstructured grid, the slowness of the data access with the list-vectors is suspected to cause the lower speed of calculation. It is noted that the present method has almost the same performance as the structured grid method in the convergence ratio to iterations.

5.Conclusion

Using the Voronoi diagram for the cell generation of the FVM, we derived the new simulation method on the unstructural grid system for incompressible steady flow. It is confirmed by the computations with the present method that it can be easily applied to the turbulent flow with complicated geometries. Its performance with regard to the convergence speed is as high as the previous methods on the structural grid. It is expected to expand the applications of the large scale and complex flow simulations by this method.

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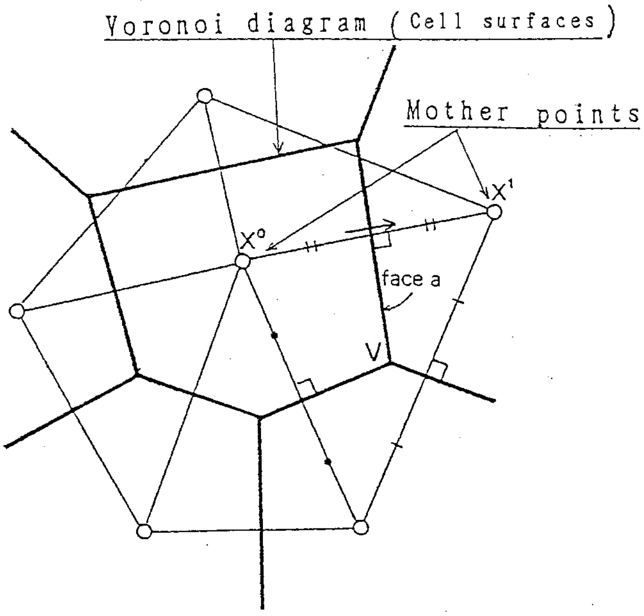
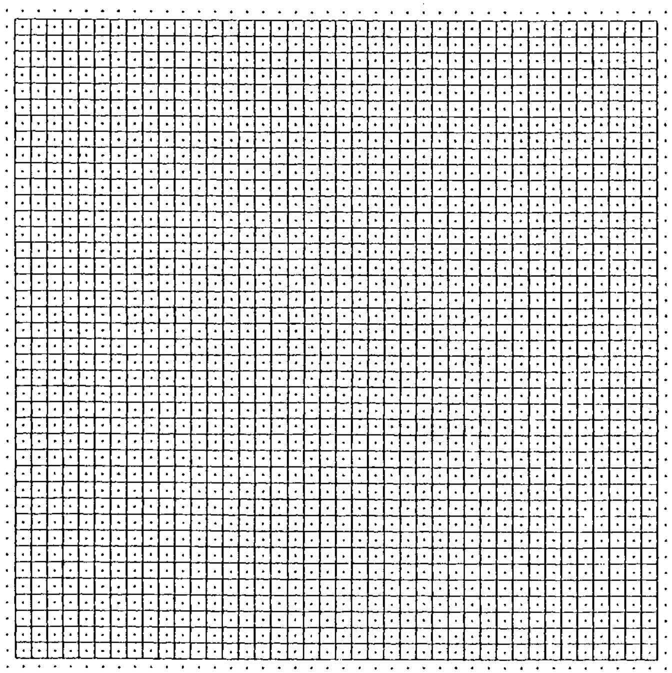
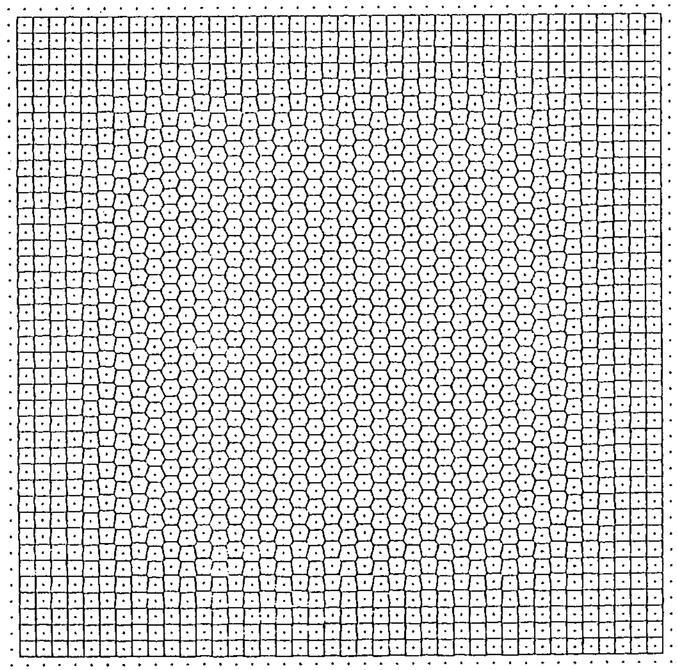


Figure 1　A concept of Voronoi diagram

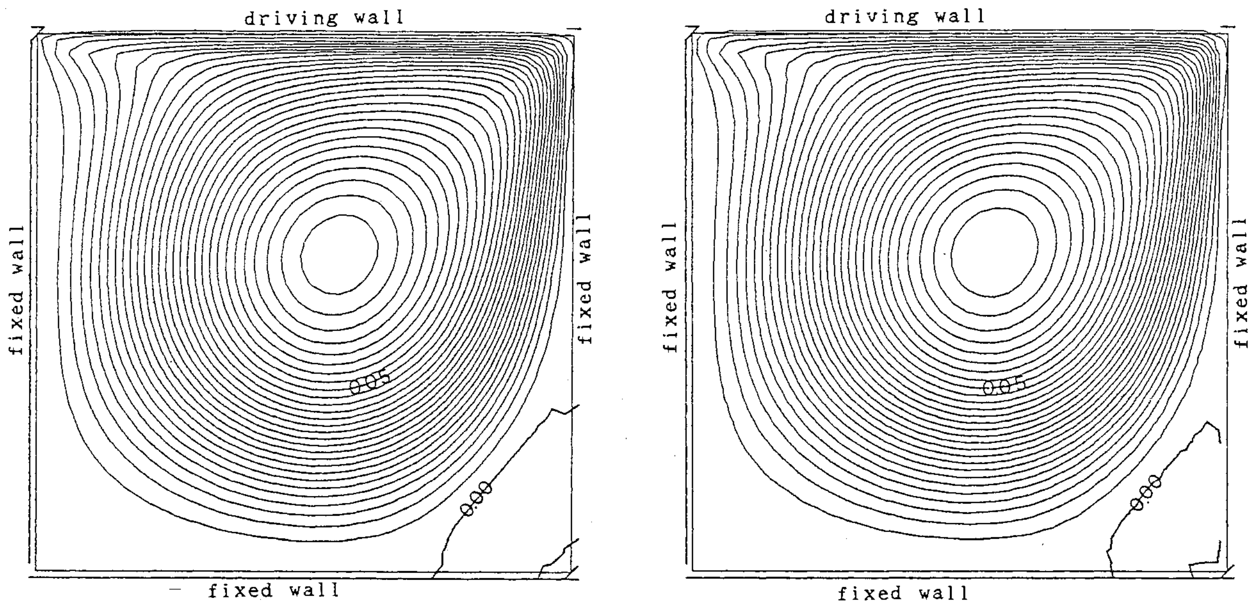


grid a



grid b

Figure 2 Calculation grids for the square cavity flow
(grid a :structural (40x40), grid b :unstructural)



grid a

grid b

Figure 3 Stream lines calculated on the structural(a) and unstructural(b) grids. (driving velocity = 1., cavity size = 1., increment per line = 0.0025)

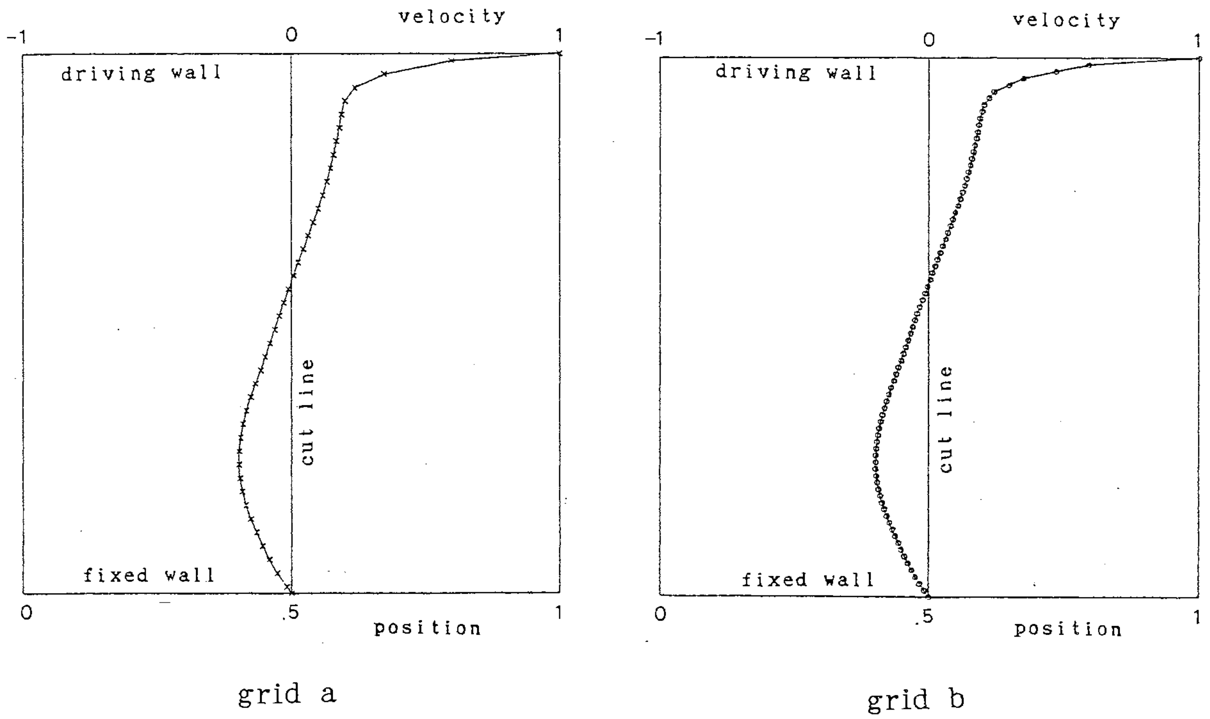


Figure 4 Velocity distribution in the center vertical section calculated on the structural(a) and unstructural(b) grids. (driving velocity =1.)

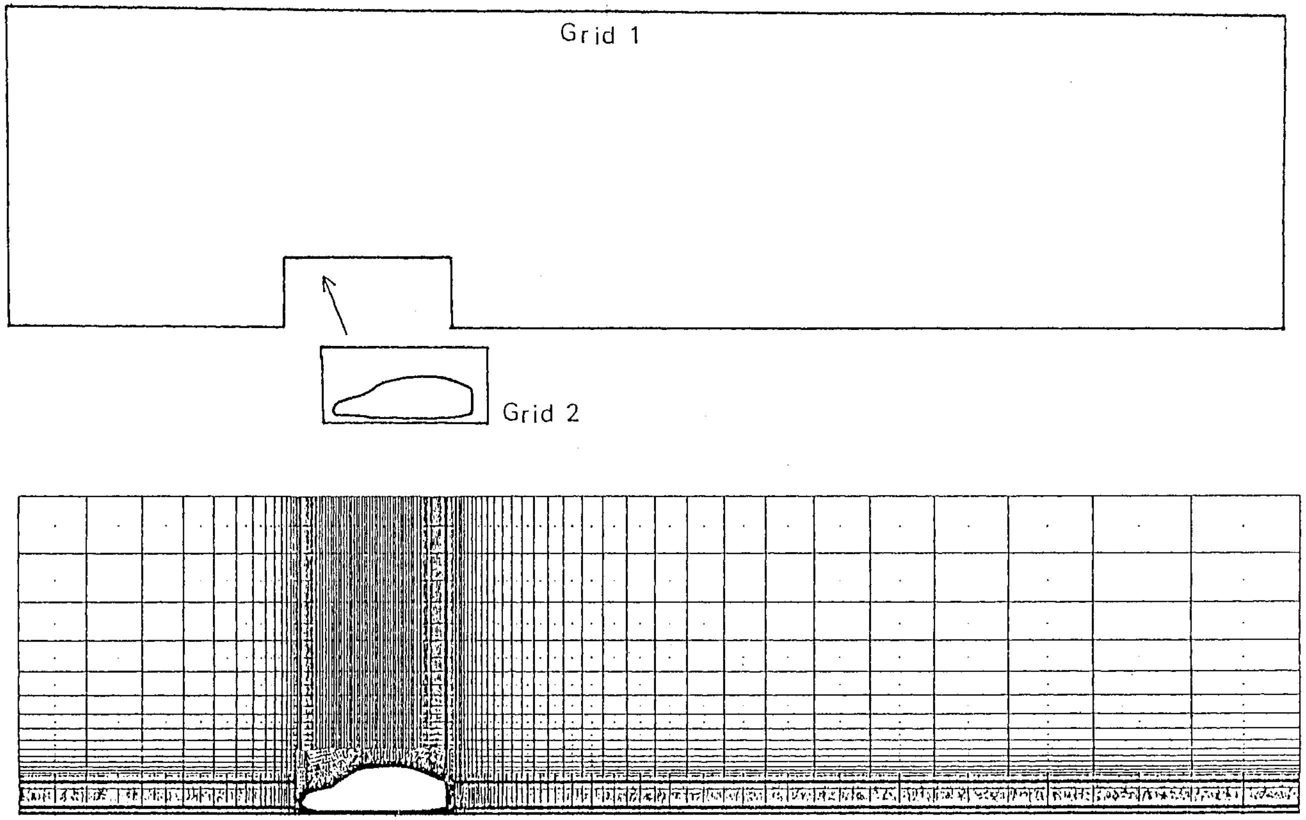


Figure 5 An ilustration of the composite grid technique for the flow field around a vehicle model.

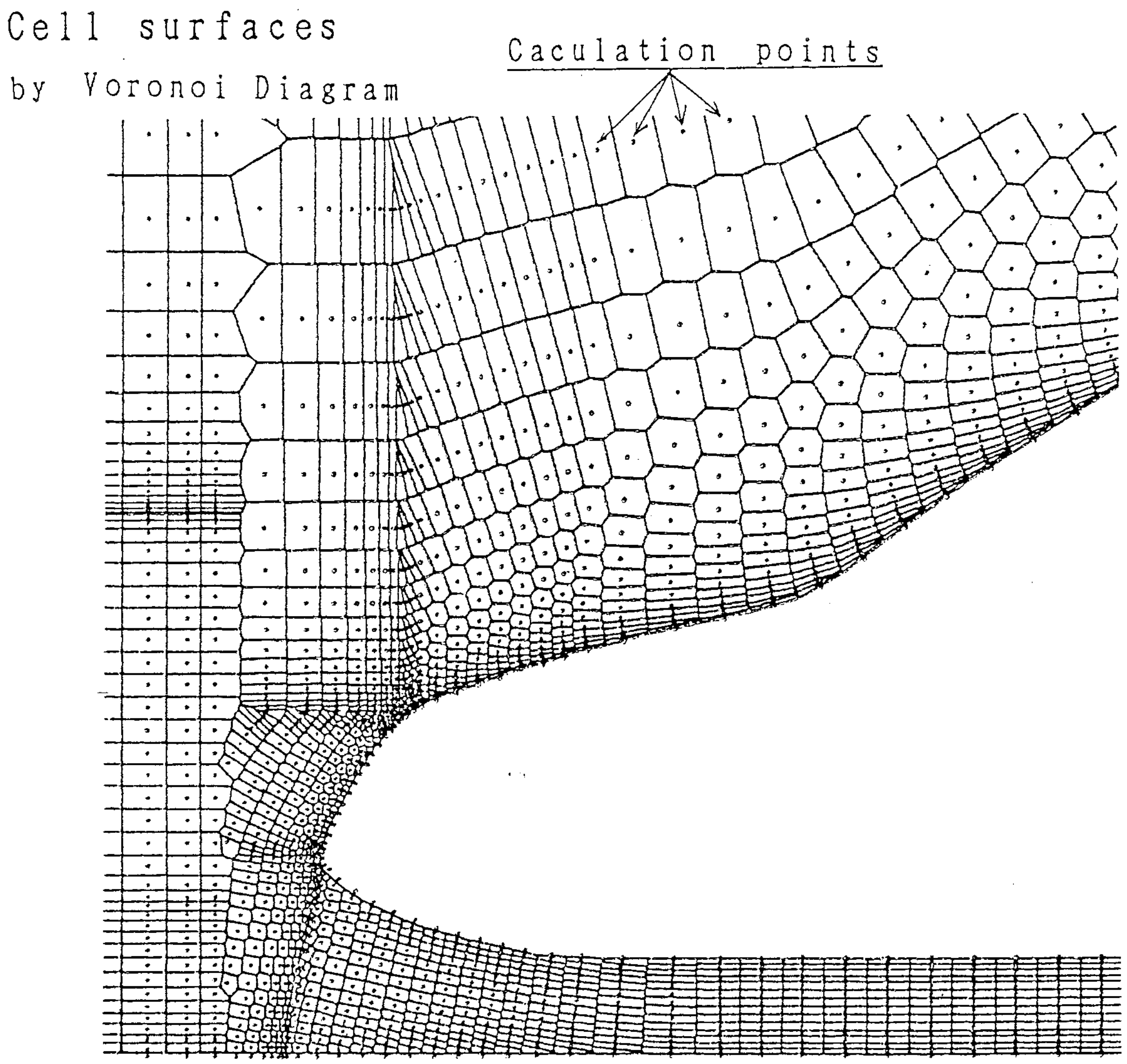


Figure 6 The cell system near the joint of grids by the Voronoi diagram.

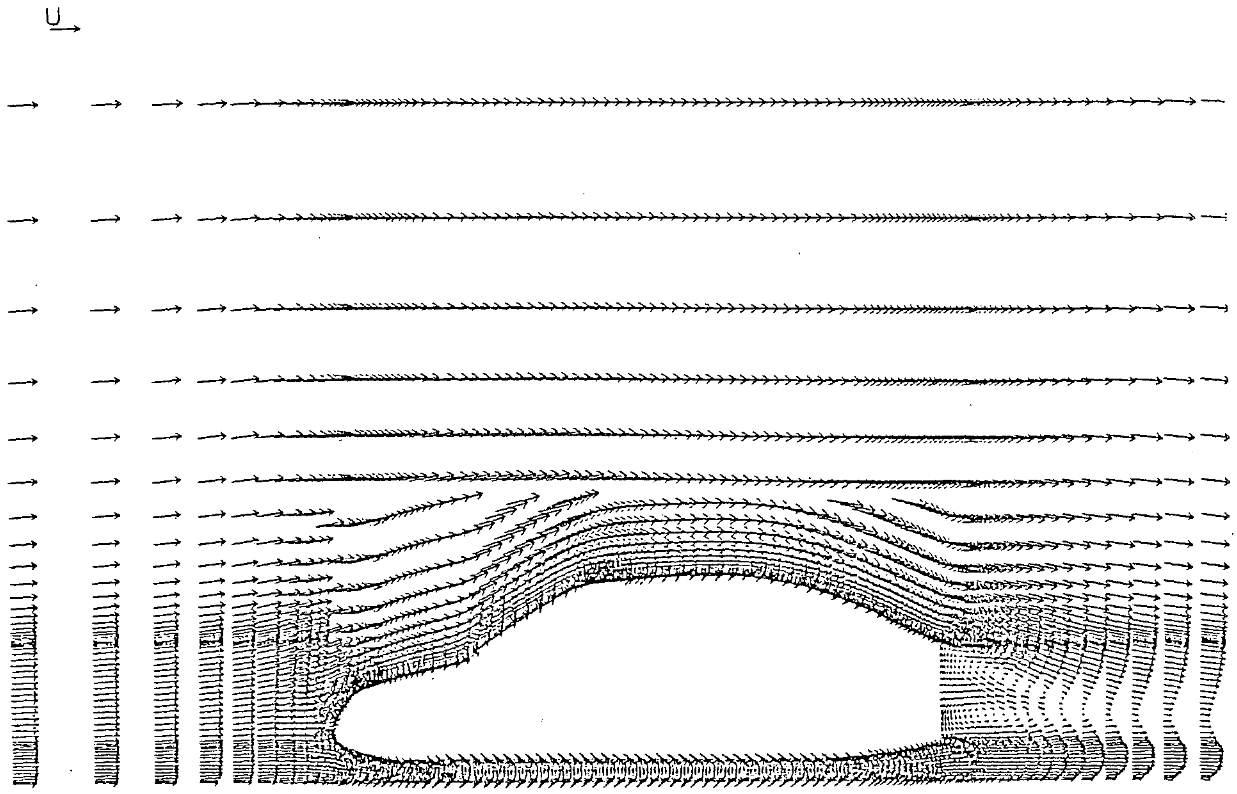


Figure 7 Velocity field around a vehicle model calculated by the present method.

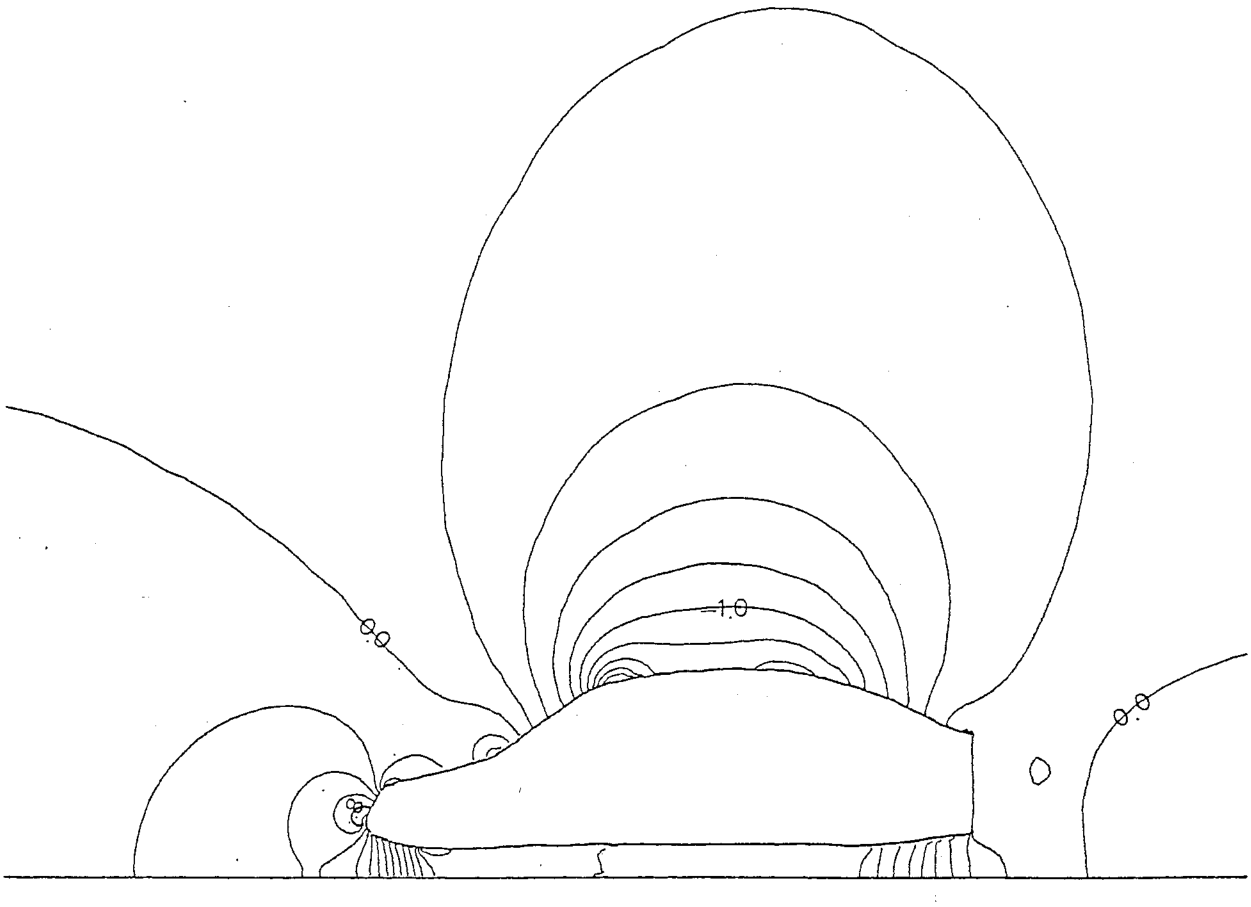


Figure 8 Contours of the pressure coefficient around a vehicle model calculated by the present method.

Table 1 The iteration numbers of SIMPLE algorithm and the CPU times in the present calculations.

	Cavity Flow			Vehicle flow	
	A	B	Ref.	Voronoi	Ref. (BFC)
Iteration No.	188	174	218	500	500
CPU time(sec)	4.2	—	—	69	12
No. of points	1600	1600	1600	6726	9375

CPU time on the HITAC-S820/80