



Title	Development of multi-cycle rainbow particle tracking velocimetry improved by particle defocusing technique and an example of its application on twisted Savonius turbine
Author(s)	Park, Hyun Jin; Yamagishi, Shunta; Osuka, Susumu; Tasaka, Yuji; Murai, Yuichi
Citation	Experiments in fluids, 62(4), 71 https://doi.org/10.1007/s00348-021-03179-7
Issue Date	2021-03-19
Doc URL	http://hdl.handle.net/2115/84443
Rights	This is a post-peer-review, pre-copyedit version of an article published in Experiments in fluids. The final authenticated version is available online at: http://dx.doi.org/10.1007/s00348-021-03179-7 .
Type	article (author version)
File Information	Manuscript.pdf



[Instructions for use](#)

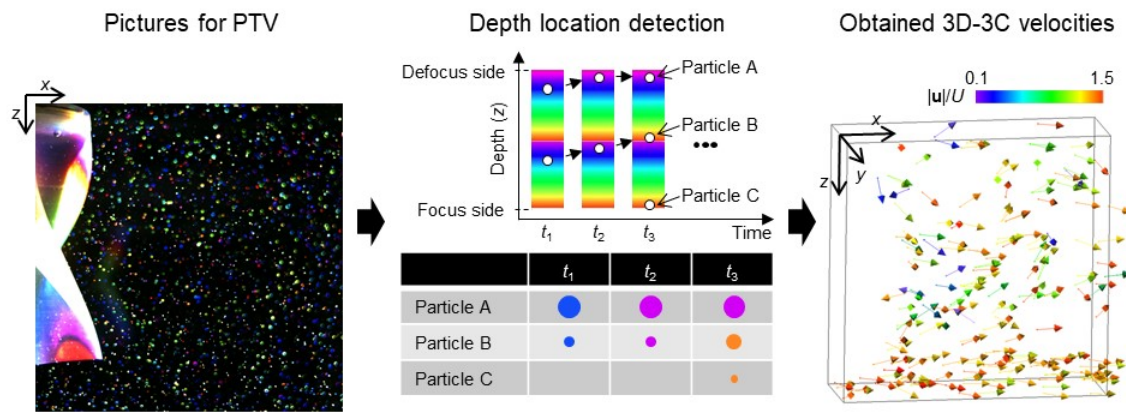
1 **Development of multi-cycle rainbow particle tracking velocimetry improved by particle**
 2 **defocusing technique and an example of its application on twisted Savonius turbine**

3 Hyun Jin Park*, Shunta Yamagishi, Susumu Osuka, Yuji Tasaka, Yuichi Murai

4 Laboratory for Flow Control, Division of Mechanical and Aerospace Engineering, Faculty of
 5 Engineering, Hokkaido University, N13 W8, Kita-ku, Sapporo 060-8628, Japan

6 * Corresponding author. Tel./fax: +81 11 706 6373. *E-mail address*: park@eng.hokudai.ac.jp.

7 **Graphical abstract**



9 **Abstract**

10 Rainbow particle tracking velocimetry (PTV) is a PTV method that enables three-dimensional (3D)
 11 three-component flow measurement using a single camera. Despite the advantage of its simple setup,
 12 the accuracy of the particle depth is restricted due to false color caused by image sensor arrays, such
 13 as Bayer arrangement. Since the false color occurs near sharp edges in the color gradient of in-focus
 14 individual particle images, we here introduced a defocusing technique to rainbow PTV to remove these
 15 false colors. Defocusing led to moon-shaped distorted particle images, which we applied an adaptive
 16 mask correlation technique to detect. Multi-cycle rainbow illumination was realized as an additional
 17 improvement on the defocusing technique. In particular, individual particle coordinates were obtained
 18 by a combination of the color and constitution of pixels. This dramatically increased the depth
 19 resolution of the 3D particle tracking. The feasibility of the proposed method was demonstrated by a
 20 flow driven by rotating impellers and a wake behind a twisted Savonius turbine. By the demonstration,
 21 it is confirmed that the twisted turbine suppresses the loss of kinetic energy by shedding streamwise
 22 vortices in the wake.

23 **Keywords**

24 Particle tracking velocimetry, color image processing, 3D flow measurement, twisted Savonius turbine

25 **1 Introduction**

26 Over the past two decades, particle image velocimetry (PIV) and particle tracking velocimetry (PTV)
27 have advanced from planar velocimetry to volumetric velocimetry that can measure three-
28 dimensional (3D) three-component (3C) velocity vector fields in fluid flows. Such a full 3D–3C flow
29 measurement has contributed to experimental fluid mechanics as well as fluid engineering applications.
30 It has also enabled direct comparison with direct numerical simulation (DNS) results. To realize
31 volumetric PIV/PTV, a number of different optical principles have been proposed to date, including
32 multi-camera 3D PTV (Walpot et al. 2006), tomographic PIV (Scarano 2013), plenoptic PIV
33 (Fahringer et al. 2015), defocusing PTV (Barnkob et al. 2015), holographic PIV (Lee et al. 2019), and
34 rainbow PTV (Xiong et al. 2017). It should be noted that there have been many other publications on
35 these individual techniques in various journals depending on the measurement target. Overall, we can
36 classify these techniques into two groups: those using multiple cameras to capture 3D particle
37 positions and those using a single camera with additional optical characteristics introduced to estimate
38 the particle depth coordinate. In the former group, tomographic PIV is regarded as the best example
39 in the present generation of tools. This method uses more than three cameras to accurately reconstruct
40 3D particle positions. One drawback to it is the difficulty in setting up the optical configuration for
41 complex measurement targets, such as those in fluid machinery. For instance, all the elementary
42 procedures of PIV need to be controlled precisely for all the cameras, considering the depth of field,
43 refraction, reflection, seeding, and illumination at different angles for each camera. Another option is
44 to use an approach from the latter group of single-camera techniques. Since these approaches deal
45 with a single image, time and cost both for the hardware and software components are significantly
46 reduced. Even though accuracy and precision are limited to a lower level compared with those obtained
47 by tomographic PIV, the development of single-camera volumetric PIV/PTV is desirable in fluid
48 engineering applications where multi-directional optical access is highly restrained.

49 In this study, we focused on two PTV techniques, color PTV and defocusing PTV, to develop a
50 single-camera volumetric PTV technique with higher accuracy and precision than the other current
51 methods. Color PTV is a method based on single-camera volumetric velocimetry. In particular, it
52 makes use of the color-coded volumetric illumination of tracer particles captured by a color camera
53 with three charged coupled devices (CCD) or a complementary metal–oxide–semiconductor (CMOS)
54 sensors. This idea has a long history of being examined (Post et al. 1994; Brucker 1996; Gogineni et
55 al. 1998). Because of simplicity in setting up, many past researchers adopted several different kinds
56 of color PIV/PTV to examine their measurement performances of 3D velocity vector fields. In the
57 present setup, we use the experimental instruments similarly to that used for conventional 2D–2C

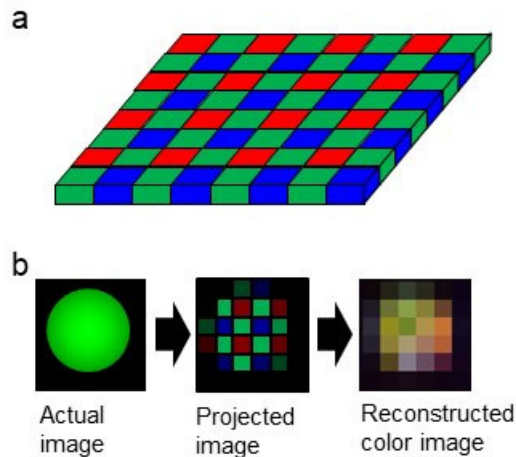
58 planer PIV/PTV systems. Difference from them is employing of a color illumination device and a
59 color camera. This setup for 3D-3C velocimetry allows a larger measurement volume compared with
60 the case of using multiple cameras. It enables to utilize a full range of depth of field of a single camera.
61 Kanda et al. (2007) tried to investigate 3D-3C velocity vector field of wind blowing on a tennis court
62 using soap bubbles and a color liquid crystal display (LCD) projector as a demonstration of color PTV
63 for a large-scale flow. However, color PIV/PTV has not yet become a widespread tool because
64 sensitivity and image size are considerably limited to resolve exact color of the particles. Brightness
65 of color particle image must normally be maintained at a darker than that of monochrome particle
66 image due to a need to avoid saturation in RGB components. This dark recording condition can
67 conserve hue information, i.e. linear sensitivity to the three primary colors is kept only in dark
68 brightness level. Monochrome PIV/PTV does not require such a condition since linearity of brightness
69 level does not matter for implementing particle tracking or image correlation analysis. In early stage
70 of color PTV trials in 1990s, the selection of methodologies for color-to-depth conversion was severely
71 restricted by videotape recording of an analog TV signal. Based on these limitations, development was
72 limited in those days and the academic spotlight moved away from color PIV/PTV until there caused
73 widespread use of digital cameras. For example, in the famous review by Adrian (2005), he did not
74 mention color PIV/PTV. However, there was still the possibility to overcome its limitations, and the
75 next year the review by Prenel and Bailly (2006) discussed the potential of color volumetric
76 velocimetry. Currently, the availability of highly sensitive high-speed color digital cameras with
77 megapixel resolutions has overcome these issues and allowed for quantitative analysis with reliable
78 reproducibility. Our group has previously reported the effective use of color-coded volumetric
79 illumination for 3D-3C PTV (Watamura et al. 2013) and the 3D location detection of microbubbles
80 (Park et al. 2019). Our understanding is that the development of color PTV is now in a revival stage,
81 as made evident by the obvious increase in publications on the topic since 2010. For example, to
82 perform color PTV, Matsushita et al. (2004) and McGregor et al. (2007) used prism-split rainbow
83 illumination, Bendickes et al. (2011) used color-painted particles, Tien et al. (2014) used color-coded
84 pinholes, Xiong et al. (2017) used rainbow color coupled with diffractive optical element (DOE)-lens
85 imaging, Wang et al. (2018) used a two-camera color-coded sequence, Menser et al. (2018) used a 3C
86 LED with time chart control, and Schultz et al (2019) proposed the generation of multi-cycle rainbow
87 illumination using a Sanderson prism. There have even been reports aimed at color PTV using a single-
88 lens-reflex (SLR) camera (Funatani et al. 2013) or a smartphone (Aguirre-Pablo et al. 2017).

89 Another technique for single-camera 3D-3C velocimetry is defocusing PTV, the first example of
90 this being reported by Willart and Gharib (1992). This method measures shape distortion and size
91 variation of defocused particle images to estimate the depth coordinate with a controlled depth of focus
92 in the measurement volume. To judge the exact particle positions with regard to depth, tracer particles
93 with uniform shape and size are required. However, particles have some distribution in their shape and

94 size, which can lead to poor accuracy and precision in the depth of defocusing PTV. Although the
95 accuracy and precision have been much improved by the help of large imaging sizes (Barnkob et al.
96 2015, Barnkob and Rossi 2020), these limitations remain in the present generation of tools.

97 In the present study, color PTV and defocusing PTV are combined to improve two aspects on
98 3D–3C vector acquisition realized by a single camera: enlargement of the measurable depth and
99 improvement of the estimation accuracy of particles' depth coordinates. First, we extend the
100 measurable depth by including the particle images that exist outside the depth of field. Such
101 defocused particles are also collected in the labeling process of PTV by considering the defocusing
102 principle of the lens optics. Next, aperture on camera lens is fully opened in the present approach to
103 intentionally defocus the particles so that color components can be stably captured with large number
104 of pixels. The judging of color is relatively easy on these particle images comparing to in-focus
105 particle images. In particular, we use the color and size information of particle images simultaneously
106 so that the uncertainty of the depth coordinate is significantly reduced. In this paper, the improvement
107 of the estimation accuracy is precisely discussed. Among various color-coding patterns proposed for
108 color PTV, we apply a rainbow-type volumetric illumination with gradually changing hue in the depth
109 coordinate. Here, hue is defined as one of color appearance parameters such as with brightness,
110 chroma, and saturation. It expresses color as a degree from 0° to 360° . For example, red, green and
111 blue are expressed as 0° ($= 360^\circ$), 120° and 240° , respectively. In principle, continuous change of
112 hue like a rainbow allows a high spatial resolution in the depth direction compared with that of
113 stepwise or split color patterns. Such a way is called rainbow PTV as a nick name of color PTV using
114 a rainbow-type illumination. This should be clearly distinguished from three-layer color PTV that
115 uses only three primary colors. Rainbow PTV deals with many intermediate colors (mixed from RGB
116 components) to determine the particles' depth coordinates. In an ideal situation, the spatial resolution
117 of rainbow PTV is excellent, as hue is given continuously in the depth coordinate. For example, when
118 three primary colors are resolved as three 8-bit signals (one for each), the hue resolution becomes
119 $360^\circ/(3 \times 2^8) \sim 0.47^\circ$, and the measurement volume is divided by 768 layers in the depth direction.
120 Unfortunately, this resolution cannot be achieved because of false colors in actual optical
121 configurations caused by the following five factors: (i) light source characteristics for rainbow
122 illumination, (ii) wavelength-dependent light scattering characteristics of tracer particles, (iii)
123 overlapping of particles in the imaging plane, (iv) color contamination in RGB sensors, and (v) digital
124 compression of the image/movie. Among these factors, color contamination has the greatest effect
125 and depends on the image sensor array adopted in the digital camera (Busin et al. 2008, Pick and
126 Lehmann 2009; Charonko et al. 2014). The concept of color contamination is briefly explained using
127 Fig. 1. The color sensor array most commonly used on cameras is the so-called Bayer sensor (Fig.
128 1(a)). Since the sensor has a one-color receptor for each pixel, the color of the pixel is interpolated
129 using information given by the receptors around the pixel to form color images. This interpolation

130 generally causes no problems for human vision but causes a problem in the case of color PTV, which
131 requires quantified colors. The interpolation leads to false color, especially in regions with high-
132 gradient RGB components, i.e., near the edge of individual particles (Fig. 1(b)). Since PTV can only
133 be used to analyze particle images composed of 5–20 pixels, most of the particles have a false color
134 that deviates significantly from the true one.



135

136 **Fig. 1.** Cause of false color on the Bayer sensor. (a) RGBG mosaic-type Bayer sensor normally used
137 in a digital camera. (b) Process of false color generation on a particle caused by the Bayer sensor. The
138 color in the reconstructed image is modified to be a different color. This effect is called color
139 contamination.

140 Watamura et al. (2013) attempted to solve this problem using a saturation-weighted average of
141 hue in individual particle images. They also introduced two kinds of rainbow illumination switching
142 alternatively in time for a commercial liquid-crystal display (LCD) projector. With this technique, a
143 depth resolution equivalent to 256 divisions of a single measurement volume was successfully
144 achieved. Aguirre-Pablo et al. (2019) reported the use of time–space structured illumination, realizing
145 single-camera 3D PTV. They applied four kinds of illumination in cyclic repetition by an LCD
146 projector. However, the switching frequency for the LCD projector was lower than 60 Hz, and
147 therefore the measurement was limited to very slow flows. This can be overcome in future with the
148 latest LCD projectors, which realize a projection frame rate higher than 1000 fps (Kagami and
149 Hashimoto 2018, Ishikawa 2019). Until further development, the brightness of projection images from
150 these high-speed projectors will be low, and it is thus difficult to actually use them for rainbow PTV.

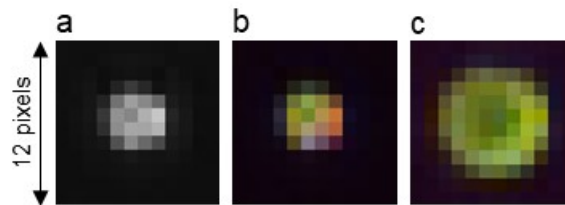
151 As a method to improve the accuracy of hue recognition by removing the false color on particle
152 images and improve the spatial precision in the depth direction by multi-cycle rainbow lighting
153 without switching, the defocusing technique is in this paper applied to rainbow PTV (called defocusing

154 rainbow PTV). To make this principle applicable, we examine how the defocused particle images are
155 generated on the imaging plane and propose a method to accurately detect various kinds of particle
156 information with high accuracy (i.e., in-plane coordinate, defocused size, and effective hue). The
157 methodology of defocusing rainbow PTV is explained in the next section, and the technique is then
158 demonstrated in Section 3.

159 **2 Color particle imaging**

160 **2.1 Defocusing to remove false colors**

161 False colors are generated at the edges the individual particle images due to the Bayer sensor
162 arrangement. Defocusing can suppress this effect so that the correct colors can be extracted. Figs. 2(a)
163 and (b) show in-focus particle images, while (c) shows a defocused particle image illuminated by
164 green-color illumination. These images were taken by a high-speed color digital video camera
165 (FASTCAM Mini AX50, Photron) having Bayer sensor with resolving each primary color as 12-bit,
166 i.e. 4096 levels. Each 12×12 pixels image is enlarged for the sake of comparison. In the in-focus image,
167 the corresponding color information of the green particle is contaminated by orange, red, magenta,
168 and cyan pixels around the edges of the particle. In the defocused condition, approximately pure green
169 pixels exist within the particle image.



170
171 **Fig. 2.** Image of a scattered particle with green illumination. (a) Focused particle image in grayscale.
172 (b) Focused particle image generating false colors. (c) Defocused particle image in which the false
173 colors are reduced.

174 The most significant information used in rainbow PTV is the hue of the particle images
175 (McGregor et al. 2007; Watamura et al. 2013; Xiong et al. 2017). To examine how much the precision
176 of color recognition is improved by the defocusing technique, the hue of the particle images
177 illuminated by volumetric color-coded light was measured, as shown in Figs. 3(a) and (b). The
178 illumination light, which changes hue from 0° to 360° over time, was generated by an LCD projector
179 (EB-W420, Epson) and refracted by a convex lens to irradiate parallel to the x axis. Particles (HP20,
180 Mitsubishi Chemical Co.) $300\text{--}700\ \mu\text{m}$ in diameter and $1020\ \text{kg/m}^3$ in density were suspended
181 neutrally in a transparent viscoelastic fluid (0.2wt% polyacrylamide aqueous solution), which enabled
182 them to maintain their initial positions.

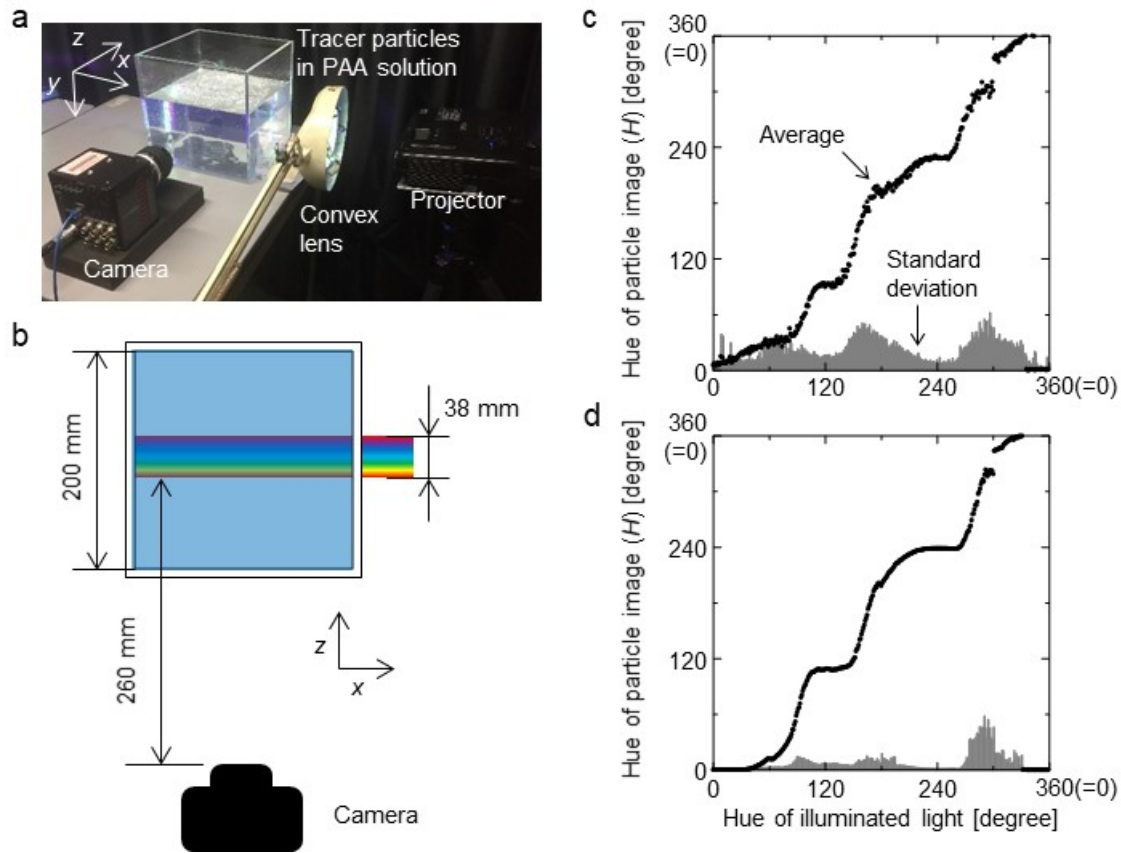
183 For estimation of the hue, we adopted a saturation-weighted averaged hue inside the particle
 184 images, defined as follows:

$$185 \quad \bar{H} = \arctan\left(\frac{\sum S \cos H}{\sum S \sin H}\right), \quad (1)$$

186 where H and S are the hue and saturation in each pixel of the image, respectively. The effectiveness of
 187 this formula for rainbow PTV has been confirmed by Watamura et al. (2013). The relationship between
 188 the illuminated and measured color in terms of hue is plotted in Fig. 3(c) for the in-focus condition
 189 and Fig. 3(d) for the defocused condition. The plots reveal a single meandering curve caused by the
 190 different sensitivity spectrums among the RGB sensors. The flat regions around 0° ($= 360^\circ$; red), 120°
 191 (green), and 240° (blue) in the illuminated hue are caused by overlapping of the spectra among the
 192 three bands. Similar results have also been reported by Park et al. (2019) for microbubbles illuminated
 193 by rainbow color. Although the curves are not approximated by a linear function, they maintain
 194 monotonic functions based on the increase of the illuminated hue. This deterministically achieves
 195 regression of the illuminated hue from the measured hue. However, its accuracy is determined by the
 196 standard deviation of the plots as applied to rainbow PTV, which requires the hues of individual
 197 particles hue but not an average. The resolvable number M of the depth coordinate by a single rainbow
 198 illumination is estimated by the following:

$$199 \quad M = \frac{360}{\tilde{\sigma}}, \quad \tilde{\sigma} = \left(\frac{1}{360} \int_0^{360} \frac{1}{\sigma(\theta)} d\theta \right)^{-1}, \quad (2)$$

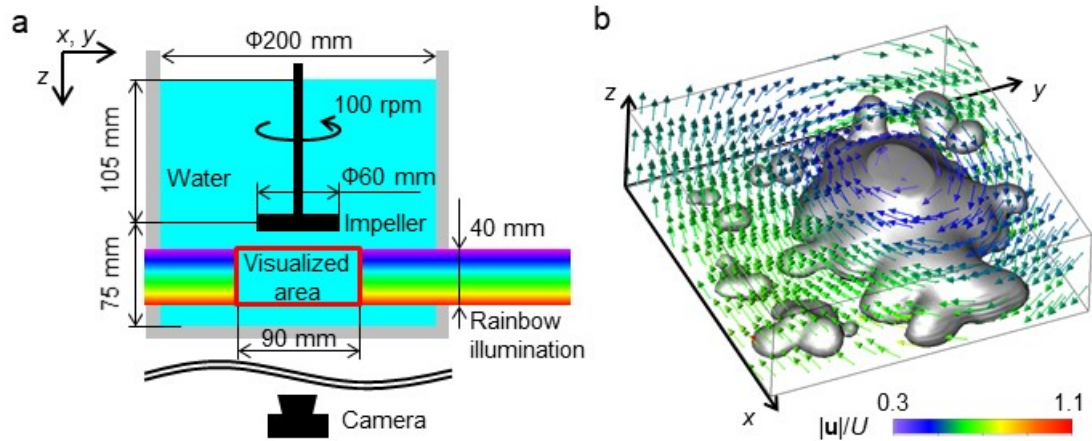
200 where $\sigma(\theta)$ is the standard deviation as a function of the illuminated hue. M becomes a function of the
 201 harmonic mean of $\sigma(\theta)$, with small deviations in $\sigma(\theta)$ dominantly contributing to the mean value. Based
 202 on the data of the standard deviations shown in the bottom profile in Figs. 3(c) and (d), the resolvable
 203 number is calculated to be $M = 15$ for the in-focus image and $M = 75$ for defocused image.
 204 Approximately five-times improved accuracy can be confirmed.



205

206 **Fig. 3.** Improved identification of particle scattering colors achieved by defocusing. (a) Picture and
 207 (b) schematic diagram of experimental setup for hue calibration. (c–d) Hue calibration curves with (c)
 208 focused images and (d) defocused images, where gray error bars indicate the standard deviation.

209 As one of the demonstrations of the rainbow PTV incorporating the defocusing technique, we
 210 measured a flow under a rotating impeller in a rectangular water container, as shown in Fig. 4(a). A
 211 volumetric light with gradually changing hue in the z direction was irradiated parallel to the horizontal
 212 x – y plane. In this setup, we produced a single-cycle rainbow color, and all the particles were equally
 213 defocused to remove false colors (note that we will introduce multi-cycle rainbows in Section 2.3).
 214 The number of instantaneous 3D velocity vectors had an average of 150 when a two-frame nearest
 215 neighbor search was applied for particle tracking. A sample of the velocity vector field is shown in
 216 Fig. 4(b), to which Laplace equation rearrangement (LER; Ido et al. 2002) was applied in spatio-
 217 temporal 4-D domain to obtain the flow on a regular grid system. Here, U stands for the tip speed of
 218 the impeller. We will not elaborate the flow structure in this paper. However, a change in the swirling
 219 flow in the z direction was reliably measured, as highlighted by the iso-surface of the vorticity at $|\text{rot}$
 220 $(\mathbf{u}/U)| = 0.1$, for example.



221

222 **Fig. 4.** Demonstration of rainbow volumetric PTV with defocusing technique. (a) Experimental setup.

223 (b) Instantaneous 3D-3C velocity vector field, where the gray surface is the iso-surface indicating $|\text{rot}$

224 $(\mathbf{u}/U)| = 0.1$

225 2.2 Detection of particle positions from distorted particle images

226 Particle images under defocusing conditions are unavoidably distorted (Barnkob et al. 2015). The
 227 distortion becomes significant in the region away from the center of the imaging plane due to lens
 228 characteristics. This worsens the accuracy of particle detection as well as the identification of particles
 229 in comparison with a focused image. To predict how significant distortion occurs, we simulated
 230 particle images using a ray analysis for a simple single-lens geometry, as illustrated in Fig. 5(a). In the
 231 ray analysis, the defocusing effect was realized by an imaging plane shifted towards the lens at a small
 232 distance, l_d . Light sources were distributed on the object surface, which radiated rays in all 3D
 233 directions. Only the rays that reached the lens contributed to the formation of images. Table 1 shows
 234 the parameters used for the ray analysis.

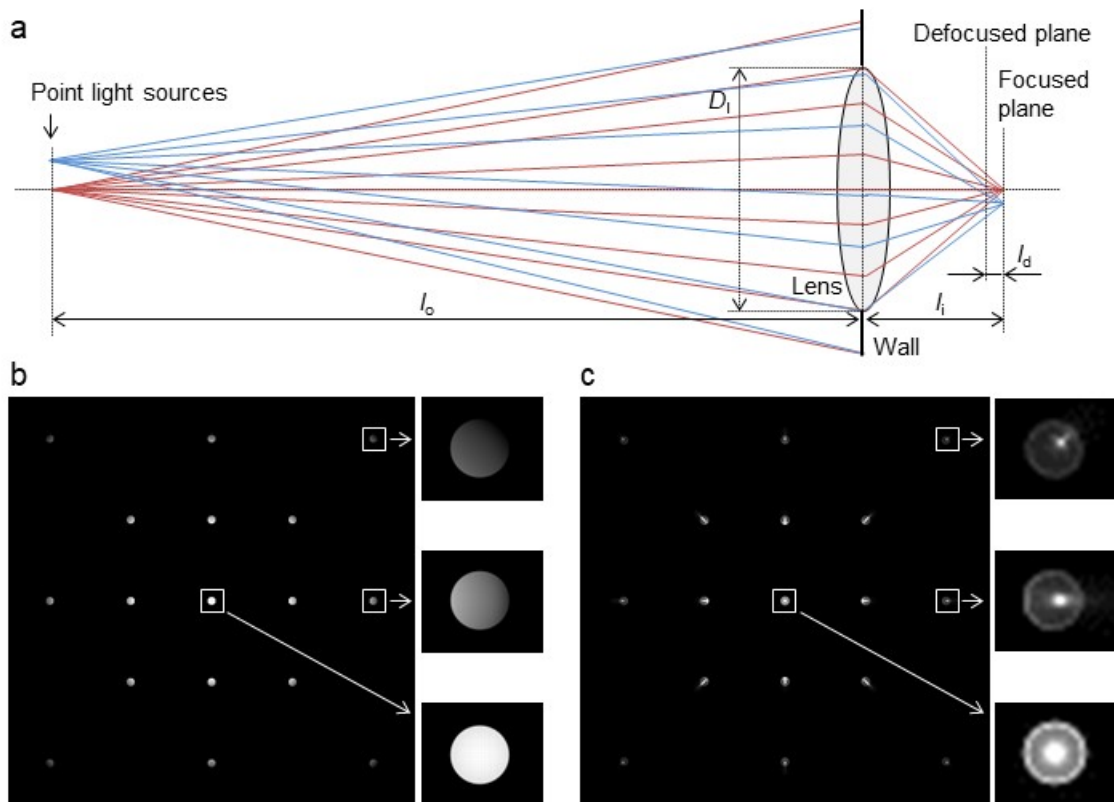
235 First, we show a simulated result without considering any optical aberration (Fig. 5(b)). In the
 236 figure, three characteristics can be identified: the finite size of the light source image, local brightness
 237 gradients in individual particle images, and a global brightness gradient in the imaging plane. Here,
 238 the former two characteristics originate from defocusing, while the latter is independent of the
 239 defocusing effect. When the light source was located far from the lens axis, the number of rays
 240 reaching the lens decreased, and the average brightness became lower outside of the imaging plane.
 241 This was caused by the use of a lens with a finite size regardless of focusing control. The other two
 242 characteristics appeared only in the defocused situation. The finite size of the light source results
 243 in rays not accumulating at a single point, as illustrated in Fig. 5(a). This causes both a local brightness
 244 gradient and a global brightness gradient. The imaging plane was on the front side of the focus in this
 245 simulation, and therefore the brightness in the image was darker toward the outside from the center of

246 the image. In the case that the imaging plane was located on the back side, a reverse brightness gradient
 247 was produced.

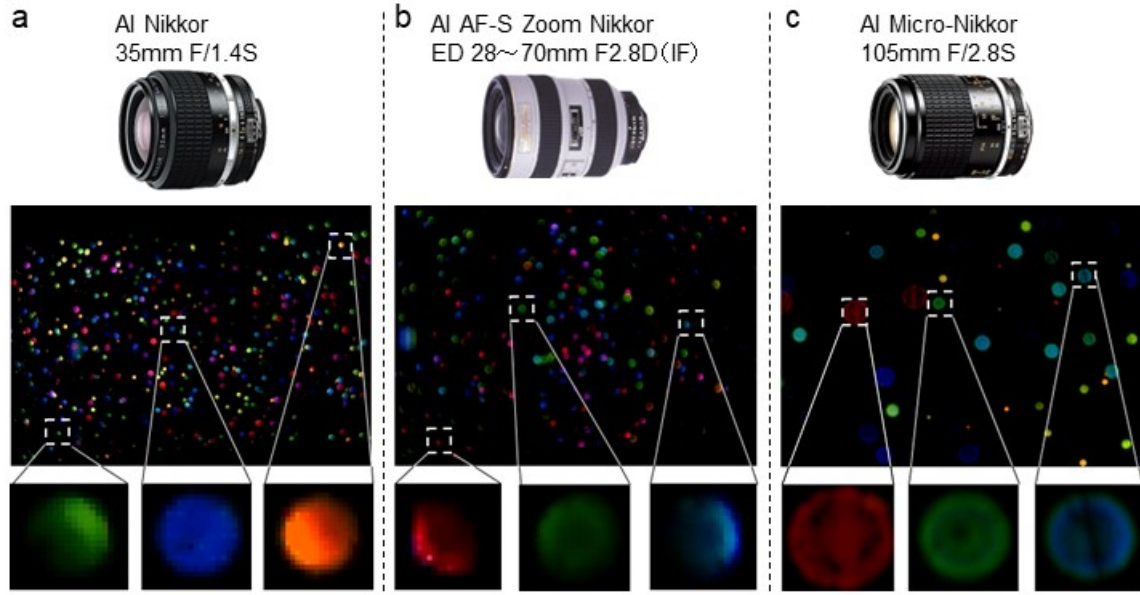
248 Next is an explanation of particle image distortion, which is mainly caused by aberrations of the
 249 lens. To simulate the effects of aberration, we added spherical aberration in the ray analysis. Because
 250 the influence of aberrations varies depending on the lens and the cindering, aberrations make it difficult
 251 to conduct ray analysis. Thus, we selected spherical aberration as the simplest case. In particular, a
 252 spherical glass lens following Snell's law was considered. That is, only the refraction of light on the
 253 lens surface was computed. A simulated result is shown in Fig. 5(c). The particle images are distorted
 254 to have asymmetric brightness patterns, including bright spots with outward tails and circular rims. If
 255 other types of aberration were added in the ray analysis, the particle shape would be changed. In real
 256 cameras composed of multiple lenses, the particle shape in the defocused condition becomes much
 257 more complex. As for this demonstration, we examined three kinds of commercially available cameras,
 258 shown in Fig. 6. Light was projected from the right side in each picture and the aperture of lens was
 259 fully opened. In these lens-mounting units, multiple lenses are combined in line. The particles were
 260 illuminated by volumetric rainbow light and recorded in the defocused condition. In these results, the
 261 particle shape and local gradient varied significantly depending on the unit. An inward gradient was
 262 found for unit (a), an outward gradient for unit (b), and split circles for unit (c). This suggests that
 263 particle images will be analytically unpredictable using simple ray analysis, and that we thus need to
 264 apply an adaptive algorithm in the detection of the particles.

265 Table 1. Characteristics of PIV techniques.

Object distance (l_o)	300 mm
Image distance (l_i)	20 mm
Moving length for the defocusing (l_d)	0.1 mm
Diameter of the lens (D_l)	200 mm
Size of imaging plane	50×50 mm ²
Vertical and horizontal distances from the lens axis	0, 150, 300 mm



266
 267 **Fig. 5.** Particle images distorted by defocusing. (a) Schematic diagram of ray tracing with a convex
 268 lens and simulated particle images at the defocused plane. (b) Images without consideration of the
 269 aberration caused by the lens. (c) Images with consideration of spherical aberration.



270

271

272

273

Fig. 6. Shape-dependence of defocused particle images on a lens with fully opened aperture. The upper, middle, and bottom of the figure show the lenses used for the visualization, pictures taken of the particles, and enlarged samples of the particle images from the pictures, respectively.

274

Tracer particles and their centers have to be accurately detected for PTV. The Gaussian mask algorithm is well-known and widely applied for this purpose (e.g., Takehara and Etoh 1999). However, in the case of defocused images the applicability of the mask algorithm is limited because of the distortion leading to large deviations from Gaussian brightness patterns. Further, the relationship between the center of the particle image and the actual center position of the particle needs to be investigated. For these reasons, we employed a pattern-adaptive mask algorithm for particle detection.

280

281

282

283

284

285

286

287

288

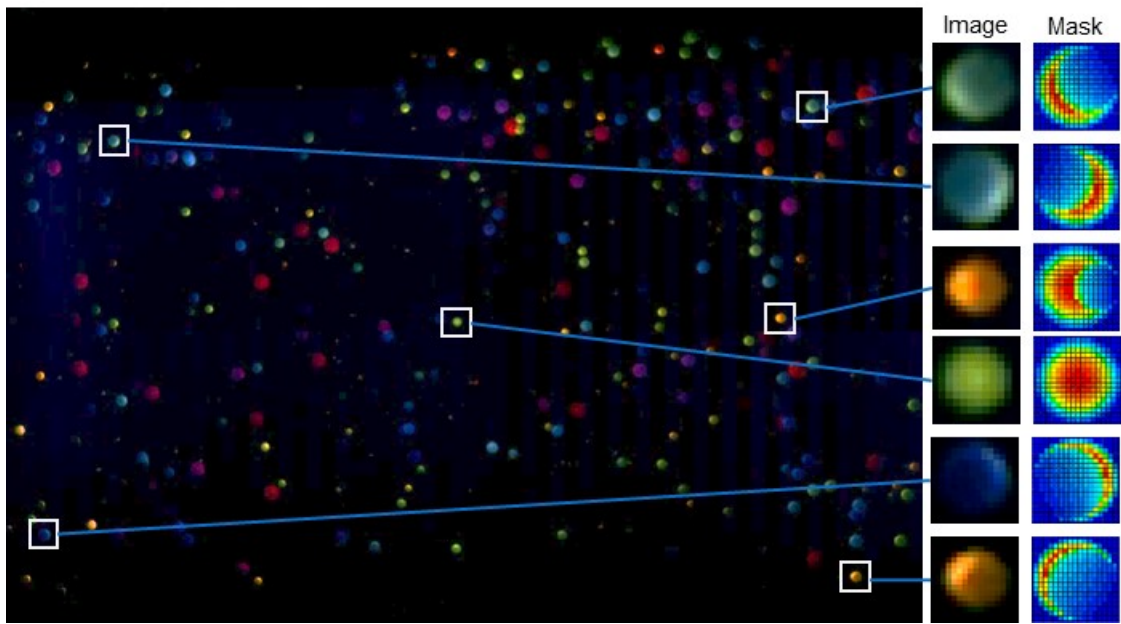
289

First, a picture of particle images illuminated by rainbow color is shown in Fig. 7. The picture was taken using the experimental setup shown in Fig. 3(a) and one of lenses (AI Nikkor 35mm F/1.4S, Nikon) introduced in Fig. 6(a). In the picture, although light was projected from the right side, the particle images in the picture have moon-phase patterns and orientation dependent on the location of the particle in the image. From Figs. 6 and 7, we found that there is little effect of the lighting direction when size of particles is sufficiently small and they are observed as spherical particle images on a focused picture. The particles located in the center of the picture are projected as a full moon (i.e., a circular shape), while the particles on the outer edges become crescent shapes with a loss of brightness on their outer sides. We modeled these shapes as masks to detect particle images. The variety of moon-shaped masks are defined by subtracting a small circular mask from a large circular mask as follows:

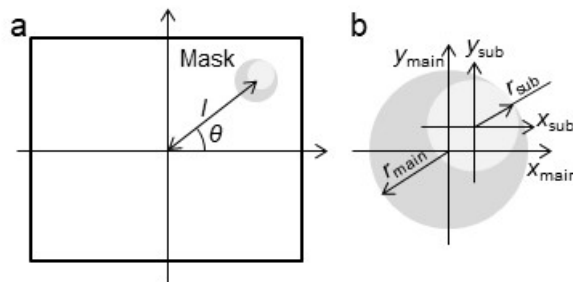
290

$$I_{\text{moon}} = aI_{\text{main}} - I_{\text{sub}}, \quad I_{\text{main or sub}} = \sqrt{r^2 - (x^2 + y^2)}, \quad a = 1.25, \quad I \geq 0. \quad (3)$$

291 Here, I and a are the intensity of the mask and a coefficient for intensity control, respectively. As
 292 shown in Fig. 8, the center locations of the masks are described as follows:
 293 $(x_{\text{main}}, y_{\text{main}}) = (l \cos \theta, l \sin \theta)$, $(x_{\text{sub}}, y_{\text{sub}}) = ((l + r_{\text{sub}}) \cos \theta, (l + r_{\text{sub}}) \sin \theta)$, (4)
 294 where l and θ are a length from the center of the picture and angle from the horizontal axis of the
 295 picture, respectively. In the case of the presented example, the radii of the masks are set as a constant
 296 $r_{\text{main}} = 9$ pixel. Here, the radius of the subtraction mask is given by $r_{\text{sub}} = r_{\text{main}} // l_{\text{max}}$. For these moon-
 297 shaped masks, distorted particle images were robustly captured by searching for the maximum cross-
 298 correlation between the target particle image and the mask properties.

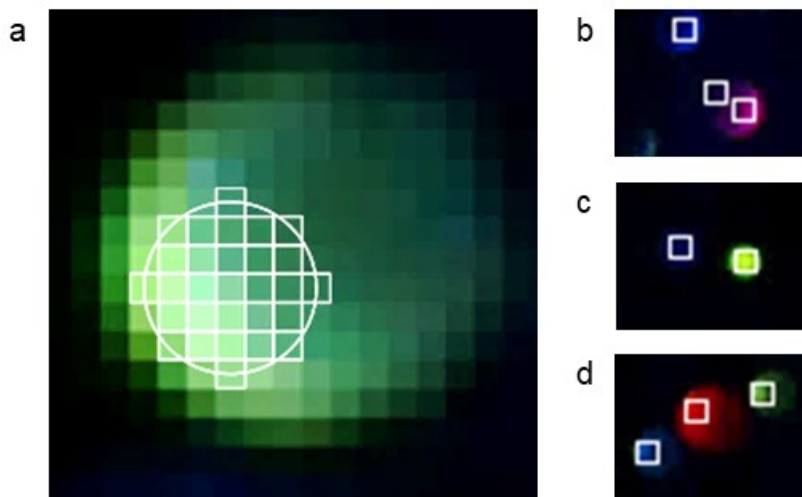


299
 300 **Fig. 7.** Distorted particle images obtained by defocusing and moon-shaped masks imitating distorted
 301 images for detecting the center coordinates of each image.

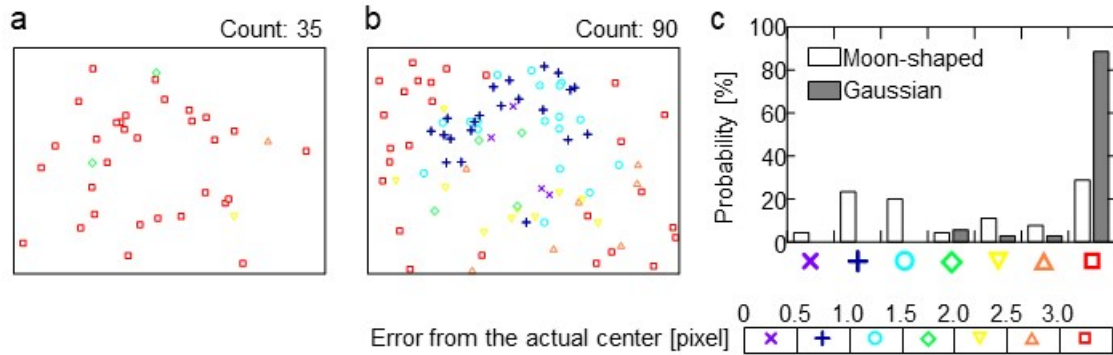


302
 303 **Fig. 8.** Parameters for generation of the moon-shaped mask. (a) The mask in a picture. (b) Coordinates
 304 of each mask forming the moon-shaped mask.

305 Figure 9 shows a defocused image of a single particle, with the white square representing the
306 actual center location of the particle. The actual center was detected from a different picture taken
307 under in-focus conditions obtained by minimizing the aperture. As seen in the figure, the brightest
308 points of individual particle images are displaced from the actual centers with a deviation that depends
309 on the position in the picture. In this experimental case, the direction is toward the center of the picture
310 but not affected by the direction of the illumination light. To realize accurate particle tracking, the
311 particle center needs to be defined within the mask region. Based on the figure, it can be confirmed
312 that the center of the outer circular rim does not represent the actual particle center. Instead, the particle
313 center is located close to the highest intensity area. Figure 10 shows to what extent the particle
314 detection ability and accuracy of center identification were improved by the moon-shaped mask,
315 whose center position was modified. Note that this figure is not taken from Fig. 7 but is taken from a
316 different picture for evaluating statistics. The Gaussian mask and moon-shaped mask were adopted
317 for particle image detection and center identification, respectively, in the sample picture. Symbols are
318 used to indicate the error in the distance between the actual center and the center identified based on
319 the Gaussian and moon-shaped masks. The number of particles detected in the case of the Gaussian
320 mask was approximately 60% lower than that detected for the moon-shaped mask because the
321 Gaussian mask does not match the shape of the particle image. By using the moon-shaped mask, the
322 accuracy of center identification was improved by 40% compared with the Gaussian mask.



323
324 **Fig. 9.** The actual center location in a defocused particle image. (a) A defocused particle image and a
325 focused image described by white cells, where this particle image is located on the right-upper corner
326 on the full picture. Other particle images in the right column are sampled on the (b) left, (c) center,
327 and (d) right of the picture, with white squares indicating the central location of each particle.



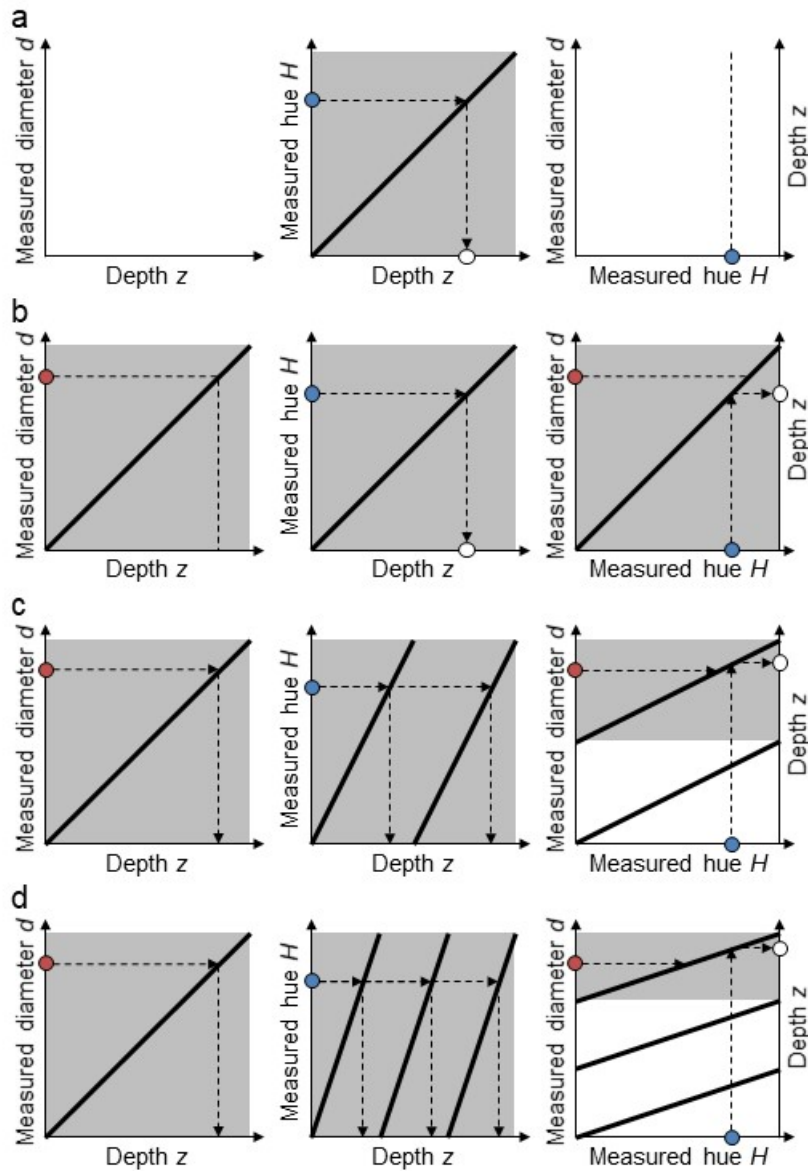
328
 329 **Fig. 10.** Comparison of the values calculated by the algorithm to detect particles and their center
 330 locations. (a–c) Particles detected using the (a) Gaussian and (b) moon-shaped masks. (c) The
 331 probabilities of error based on the actual particle centers.

332 In the present paper, we made subjective masks, i.e. the moon-shaped mask, for particle image
 333 detection and center identification as a test case. The shape of the particle image depends on the
 334 particular lens to utilize, thus predicting the shape before testing is generally difficult. The moon-
 335 shaped mask introduced in this paper does not cover wide situation of defocusing rainbow PTV that
 336 utilizes lens different from the present case. For example, the particle image in Fig. 6(c) is not moon-
 337 shaped and our mask does not properly work in this case. Toward the general use of defocusing
 338 rainbow PTV, it is expected to build up an automatic mask generation algorithm with help of methods
 339 such as machine learning of the defocused color image patterns.

340 **2.3 Multi-cycle rainbow illumination in the depth**

341 Employing the defocusing technique allows for the application of multi-cycle rainbow illumination in
 342 determination of particle depth coordinates. In particular, the 3D position is given by a combination
 343 of the size and color of individual particle images. Fig. 11 illustrates various possible combinations to
 344 explain this principle. In a case in which the defocusing technique is not used (Fig. 11(a)), the depth z
 345 of the particle is simply estimated recursively based on the hue of a single-cycle rainbow illumination.
 346 Fig. 11(b) shows a case in which defocusing is applied together with single-cycle rainbow illumination.
 347 We can determine the depth independently by either the measured particle diameter or the hue. In the
 348 present paper, depth is determined by hue because the precision of the hue is improved by defocusing.
 349 Further, it is difficult to estimate the size correctly since the shape of the particle image is distorted by
 350 the defocusing. If it is possible to estimate the size correctly, taking an average of these two depths
 351 will better estimate the true depth of the particle. A combination of defocusing imaging and two-cycle
 352 rainbow illumination is shown in Fig. 11(c). In this case, we cannot judge the depth using only the hue
 353 because it presents two distinct possibilities. However, because the size gives an approximation of the
 354 depth, it is possible to define the depth using the hue and size simultaneously. An advantage of this

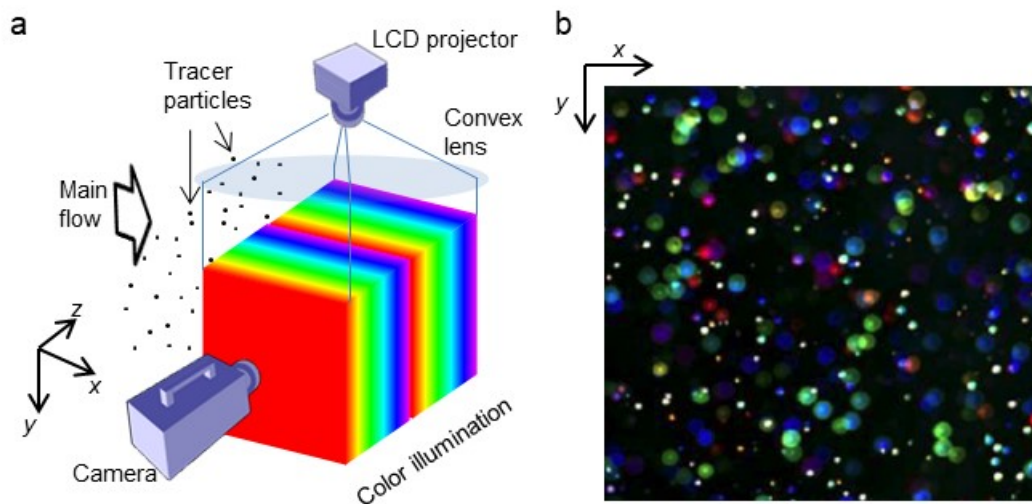
355 combination is an improvement in the accuracy of hue-to-depth conversion based on the large gradient
356 in the hue, dH/dz . This leads to errors in the hue measurement, such as random and systematic hue
357 fluctuation, being relaxed during depth estimation. Since multi-cycle rainbow illumination is easily
358 producible using a commercial LCD projector, defocusing imaging can be successfully combined with
359 it. As shown in Fig. 11(d), a case of three-cycle illumination, would further improve the spatial
360 resolution. However, its combination with defocusing technique is ineffective because defocusing has
361 a limitation to classify the size of the particle image into more than two layers. In order to increase the
362 number of cycles, it is necessary to suppress the deviation in the size distribution of tracer particles
363 and use a camera with a larger number of pixels.



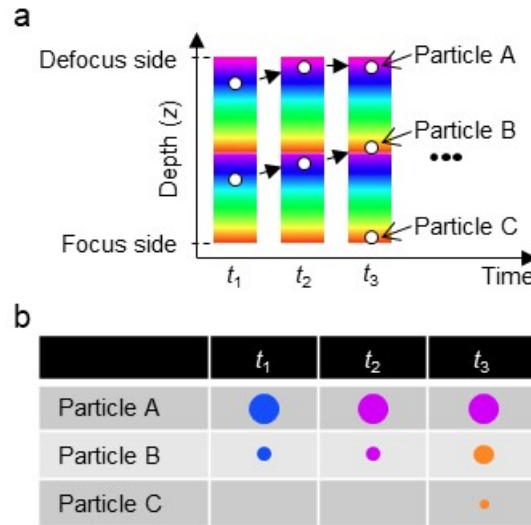
364
 365 **Fig. 11.** Possible patterns in the combination of defocusing and rainbow PTV. (a) Normal rainbow
 366 PTV. (b) Defocusing rainbow PTV with one-cycle, (c) two-cycle, and (d) three-cycle illumination.
 367 Red, blue, and white circles indicate the measured diameter, measured hue, and measured depth,
 368 respectively. Gray region indicates an effective cycle of the color, to which the particle depth belongs
 369 with information of the particle image size.

370 Two figures are presented to help in understanding this principle. First, Fig. 12(a) shows an
 371 optical setup for two-cycle rainbow illumination combined with defocusing imaging. Using this setup
 372 in a water flow seeded with particles, the color particle images shown in Fig. 12(b) were obtained.
 373 Here, particle images of the same color with different sizes can be seen; one is relatively small and the
 374 other is relatively large. Fig. 13 illustrates the algorithm used to determine the depth coordinates of

375 individual particles. For example, particles A and B make blue images at time t_1 , but their sizes have
 376 different projections. The size of particle image B is smaller than that of particle image A when the
 377 camera's focal plane is close to particle B. Small movements of these particles caused changes in color
 378 from blue to cyan at t_2 . Further motion caused emergence, disappearance, and change in size at t_3 . On
 379 the one hand, this procedure is unaffected by deviation of real particle size since the particle image
 380 size is mostly determined based on the defocusing degree. Further, the color changes sharply with the
 381 introduction of multi-cycle rainbow illumination. This combination makes the proposed technique
 382 feasible for wide flow conditions. On the other hand, the overlapping of particle images becomes
 383 frequent in defocusing imaging, restricting the upper limit of detectable particle image densities.
 384 Roughly, the upper limit is estimated to be around 200 particles/(500×500 pixels) ~ 0.001 particles per
 385 pixel (ppp). Similar issues have been reported in the defocus imaging of bubbles and droplets (Murai
 386 et al. 2001, Kawaguchi et al. 2002). Reducing the defocusing level or using an image processing which
 387 separates multiple overlapping particle images is a possible solution to raise the ppp value.



388
 389 **Fig. 12.** Two-cycle rainbow color PTV with defocusing technique. (a) Schematic diagram of facility
 390 setup, where divergence of light was eliminated by inserting convex lens. (b) Part of a picture obtained
 391 from the camera.

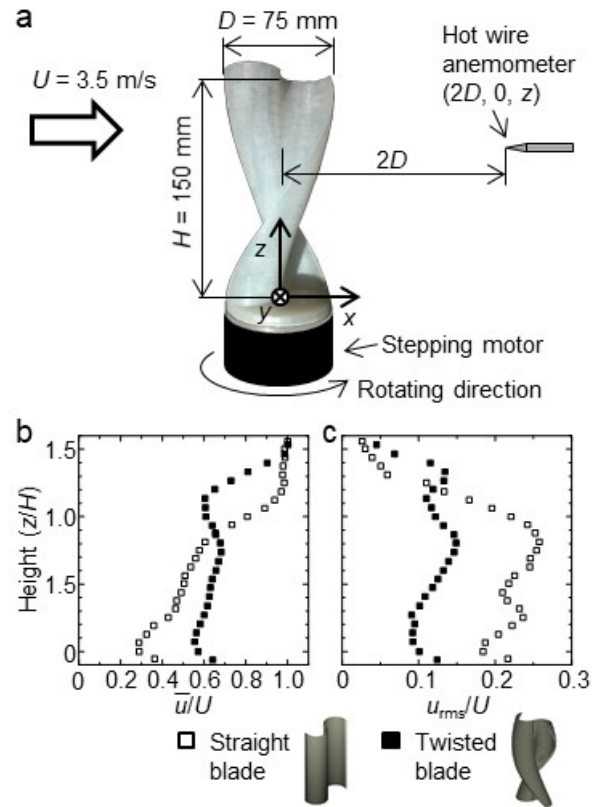


392

393 **Fig. 13.** Principle for recognizing particle location. (a) Situation in which three particles pass in the
 394 measurement area. (b) Particle images obtained at each time.

395 **3 An example application to 3D flow measurement**

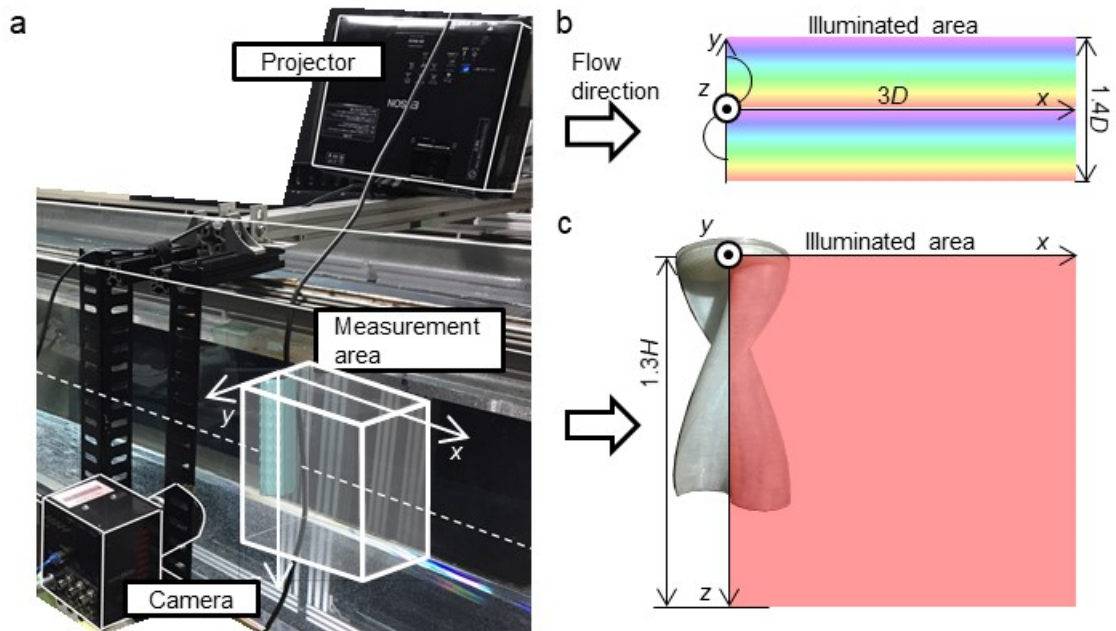
396 As an experimental demonstration, we selected the investigation of a 3D flow in the downstream
 397 region of a twisted Savonius turbine. Several researchers have reported that twisted turbines have
 398 better performance than normal straight-type Savonius turbines (Saha and Rajkumar. 2006; Damark
 399 et al. 2013). One of the reasons for this is the reduction of large periodic vortex shedding, which
 400 releases large amounts of kinetic energy downstream. Before the investigation applying the defocusing
 401 rainbow PTV, the flow was measured by a hot-wire anemometer, as shown in Fig. 14(a). A turbine 150
 402 mm in height and 75 mm in the diameter (D) with a form twisted 180° was examined. The main flow
 403 velocity in the wind tunnel was $U = 3.5$ m/s, and the tip speed ratio of the turbine was fixed at 0.4 by
 404 a stepping motor. In these experimental conditions, the Reynolds number defined by D and U was
 405 approximately 1.8×10^4 . The hot-wire anemometer was set at $2D$ in the region downstream from the
 406 turbine. Time-averaged velocity and turbulence intensity are shown in Figs. 14(b) and (c), respectively.
 407 To compare the effects of twisted blades, measurement data regarding a straight-type Savonius turbine
 408 was also plotted in the figures. The average velocity with the straight-type turbine gradually increases
 409 in the vertical z direction due to the ground effect, while that with the twisted turbine has a uniform
 410 distribution with approximately 50% of the main flow velocity in the vertical direction. We expect that
 411 this is explained by contribution to vertical flow induced by the twisted blades. The turbulence
 412 intensity of the twisted turbine was relatively low, although its average velocity was relatively high at
 413 $z/H < 0.9$. To find the answer of what was kind of 3D flow structures which modified these wake
 414 characteristics, it was sought using the present multi-cycle defocusing rainbow PTV.



415

416 **Fig. 14.** Effect of the twisted blade of a Savonius turbine on flow in the downstream region, where the
 417 tip speed ratio of the turbine is 0.4. (a) Experimental setup, where x - and z -axes are set as the
 418 streamwise direction of main flow and the rotating axis of turbine, respectively. (b) Time-averaged
 419 streamwise velocity. (c) Turbulence intensity.

420 Fig. 15 shows the experimental facilities used to measure the downstream flow structure of the
 421 twisted turbine. A towing tank containing tap water was used, in which the turbine was towed
 422 horizontally at a constant speed together with a camera and an LCD projector. The turbine was
 423 installed upside down in the towing tank, and its end plate was located at the water surface to avoid
 424 the ground effect. The towing speed was set to $U = 0.3$ m/s, and the corresponding Reynolds number
 425 was approximately $Re = UD/\nu = 1.8 \times 10^4$, where ν is the kinematic viscosity of water. The frame rate
 426 of the camera was set to 750 fps, and the spatial resolutions in the picture were 0.2 mm/pixel in the x -
 427 z plane and 0.15 mm per 1° of hue in the y direction. With a given accuracy regarding the particle
 428 center detection and a given precision regarding the hue recognition, the bias error of particle location
 429 was estimated to be within 1 mm in all directions for the 3D measurement volume.



430

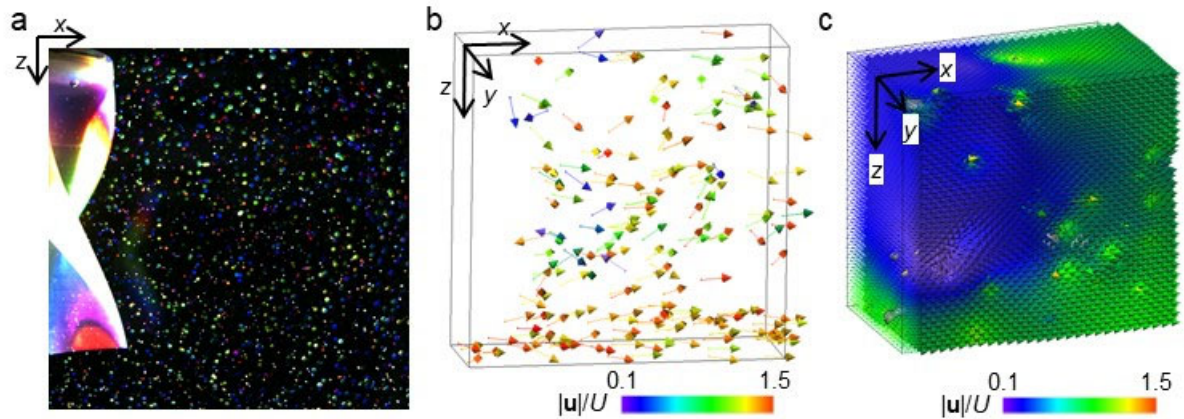
431 **Fig. 15.** Twisted Savonius turbine experiments performed in a towing tank. (a) Picture of facility setup.
 432 (b) Top and (c) side views of measurement area, where D and H are the diameter and height of the
 433 turbine, respectively.

434 Samples of the visualization results are shown in Fig. 16. In a camera picture shown in Fig. 16(a),
 435 tracer particles are projected as a variety of colors and sizes. As the first step for the PTV, particle
 436 locations in the x - z plane of the measurement volume were determined using image masking
 437 correlation based on moon-shaped masks. Then, individual particle locations in the y direction were
 438 computed using the size and hue of the color particle images. All the 3D particle coordinates were
 439 tracked in four consecutive frames to obtain an instantaneous velocity vector with three components,
 440 $\mathbf{u} = (u, v, w)$, as presented in Fig. 16(b). The number of velocity vectors captured was 120 among the
 441 ~ 500 particles identified in the original image. A reduction in the number was caused by particles'
 442 partial overlapping and unsuccessful tracking of particles due to the finite hue resolution. Considering
 443 sub-pixel processing to define particle locations, accuracy of the present velocity vectors is about
 444 $0.013U$ (Udrea et al. 1996). The instantaneous velocity vector distribution in the figure does not mean
 445 much in identifying the flow structure, however, the particle position z and the velocity component in
 446 z direction are secured. This allows the data to be interpolated to see the 3-D wake structure in more
 447 detail. For preparation of evaluating various contours inside the wake, we converted these PTV data
 448 to regular grid vector field as shown in Fig. 16(c). Here we employed Lagrangian-to-Eulerian
 449 formatting of the scattered vector field in spatio-temporal four-dimensional domain (x, y, z, t) using
 450 biquadratic ellipsoidal rearrangement (BER) algorithm proposed by Ido and Murai (2006). This
 451 interpolation allows to estimate fine individual vortices from a limited number of velocity vectors per

452 vortex. According to their paper, 12 vectors around a single vortex can reconstruct the original vortical
453 structure at 0.95 in vector cross correlation coefficient (Ido et al. 2002).

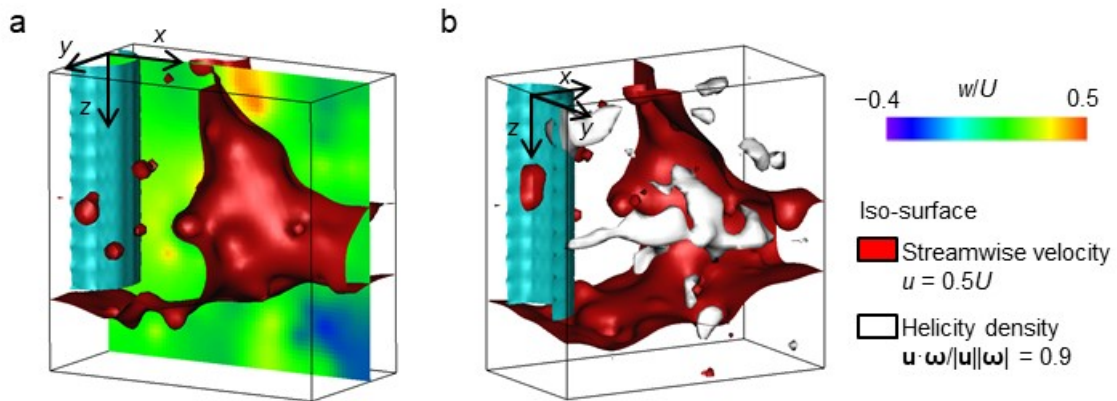
454 Figs. 17 show iso-surfaces of a scalar distribution computable from the measured velocity vector
455 distribution. Fig. 17(a) shows a vertical velocity contour at $y = 0$ and an iso-surface of $u = 0.5U$ in red
456 color. The white iso-surface in Fig. 17(b) represents helicity density at $\mathbf{u} \cdot \boldsymbol{\omega} / |\mathbf{u}| |\boldsymbol{\omega}| = 0.9$. Helicity is one
457 of the conservative quantities that can be used to visualize 3D vortical structures (Kelvin 1867; Kasagi
458 et al. 1995; Janke et al. 2017). From the results, two specific flow structures were identified to explain
459 the vertically more uniform streamwise velocity profiles recovered by the twisted turbine. One was a
460 vertical flow reaching half of the turbine's height from the upper and the bottom region, and the other
461 was a streamwise vortical structure released downstream. These do not occur in the case of a normal
462 straight turbine because the original 2D flow is maintained (Murai et al. 2007). Vertical flows supply
463 kinetic energy toward the center area, while the streamwise vortex equalizes the energy by momentum
464 transfer. As a result, velocity in the downstream region of the twisted turbine was recovered quickly
465 in this case compared with that of a normal straight Savonius turbine. This fact also tells that turbine
466 drag of the twisted turbine is smaller than the straight one while torque increases with twisting the
467 blades.

468 In more detail, unlike the case of lift-driven turbines, the twisted Savonius turbine relies on flow
469 separation behind rotating buckets in power generation. Kinetic energy loss in the wake does not
470 immediately explain the correlation to the power. To understand the reason why twisting obtains better
471 performance, 3D–3C velocity vector fields need to be investigated, from which intrinsic coherent
472 structures can be extracted as well as pressure field and torque fluctuation in the next step. Although
473 the present rainbow-defocusing PTV technique did not have significantly high accuracy and resolution
474 of velocity fields to perform such analysis, we here offered the flow structure information directly
475 obtained experimentally with the PTV technique in the demonstration. Of course, CFD simulations
476 supply 3D–3C velocity vector fields with very good quality possible to perform the analysis.
477 Simulations, however, are subject to several assumptions such as turbulent flow model and 3D
478 boundary layer resolutions along rotating bucket surfaces. Thus, it is required to confirm the validity
479 of simulations by experimental data. We expect that our findings will contribute to their validation.



480

481 **Fig. 16.** Processing to obtain 3D–3C instantaneous velocity field. (a) Snap picture of particles
 482 illuminated by two-cycle rainbow illumination in the depth direction. (b) Instantaneous velocity vector
 483 $\mathbf{u}(u, v, w)$ obtained by the PTV. (c) Interpolated velocity vector field obtained by converting PTV data
 484 to a regular grid format using the algorithm proposed by Ido and Murai (2006).



485

486 **Fig. 17.** Sample results. (a) Vertical velocity w interpolated by BER. (b) Visualized streamwise vortex,
 487 where $\boldsymbol{\omega}$ is vorticity.

488 4 Conclusion

489 In this paper, we proposed a method that combines rainbow PTV and defocusing PTV to improve the
 490 spatial resolution of 3D particle coordinates. We demonstrated that the method is able to prevent false
 491 color generation in individual particle images. This leads to a high precision in hue definition in
 492 comparison with in-focus particle imaging. Further, it allows for multi-cycle rainbow illumination, as
 493 the particle image size becomes a function of the depth coordinate. The multi-cycle technique led to a
 494 steep change in the hue of the individual particle images and improved the accuracy in the hue-to-
 495 depth recursive estimation. The combination of these two kinds of information (color and size) reduced
 496 the uncertainty of the depth coordinate so that 3D Lagrangian particle tracking could be successfully

497 realized. At the same time, distortion of the image occurred due to the defocused imaging depended
498 strongly on lens adopted on the camera. This was overcome by introducing an adaptive mask
499 correlation technique designed for the lens, with which the centers of the moon-shaped particle images
500 were reconstructed.

501 For a demonstration of the defocusing rainbow PTV, we investigated the 3D structure of a wake
502 behind a twisted Savonius turbine. 120 velocity vectors were obtained in every consecutive frame
503 using a four-frame tracking algorithm without any smoothing process applied. Helicity density and
504 other quantities revealed that the twisted turbine induced vertical flow while shedding streamwise
505 vortices in the wake, revealing the reason that the loss of kinetic energy was suppressed in comparison
506 with a straight turbine. Based on the demonstration, the feasibility of the proposed defocusing rainbow
507 PTV as a tool for experimental fluid engineering research was confirmed.

508 **Acknowledgment**

509 This work was supported by research funding from Hokkaido Gas Co., Ltd.

510 **References**

- 511 Adrian R (2005) Twenty years of particle image velocimetry. *Exp Fluids* 39: 159–169
- 512 Aguirre-Pablo AA, Alarfaj MK, Li EQ, Hernandez-Sanchez JF, Thoroddsen ST (2017)
513 Tomographic particle image velocimetry using smartphones and colored shadows. *Scientific Reports*
514 7: 3714
- 515 Aguirre-Pablo AA, Aljedaani AB, Xiong J, Idoughi R, Heidrich W, Thoroddsen ST (2019) Single-
516 camera 3D PTV using particle intensities and structured light. *Exp Fluids* 60: 25
- 517 Barnkob R, Kähler CJ, Rossi M (2015) General defocusing particle tracking. *Lab Chip* 15: 3556–
518 3560
- 519 Barnkob R, Rossi M (2020) General defocusing particle tracking: fundamentals and uncertainty
520 assessment. *Exp Fluids* 61: 110
- 521 Bendicks C, Tarlet D, Roloff Cm Bordas R, Wunderlich B, Michaelis B, Thevenin D (2011)
522 Improved 3-D particle tracking velocimetry with colored particles. *J Signal Info Processing* 2: 59–71.
- 523 Brucker C (1996) 3-D PIV via spatial correlation in a color-coded light-sheet. *Exp Fluids* 31:
524 312–314
- 525 Busin L, Vandenbroucke N, Macaire L (2008) Color spaces and image segmentation. *Adv*
526 *Imaging Electron Phys* 151: 65–168
- 527 Charonko JJ, Antoine E, Vlachos PP (2014) Multispectral processing for color particle image
528 velocimetry. *Microfluid Nanofluid* 17: 729–743
- 529 Damak A, Driss Z, Abid MS (2013) Experimental investigation of helical Savonius rotor with a
530 twist of 180, *Renew Energy* 52: 136–142

531 Fahringer TW, Lynch KP, Thurow BS (2015) Volumetric particle image velocimetry with a single
532 plenoptic camera. *Meas Sci Techol* 26: 115201

533 Funatani S, Takeda T, Toriyama K (2013) High-resolution three-color PIV technique using a
534 digital SLR camera. *J Flow Vis Image Processing* 20: 35–45

535 Gogineni S, Goss L, Pestian D, River R (1998) Two-color digital PIV employing a single CCD
536 camera. *Exp Fluids* 25: 320–328

537 Ido T, Murai Y, Yamamoto F (2002) Postprocessing algorithm for particle tracking velocimetry
538 based on ellipsoidal equations. *Exp Fluids* 32: 326–336

539 Ido T, Murai Y (2006) A recursive interpolation algorithm for particle tracking velocimetry, *Flow*
540 *Meas Instrum* 17: 267–275

541 Ishikawa M (2019) High-speed image processing devices and its applications. 2019 IEEE Int
542 Elec Dev Meet: 10.7.1–10.7.4

543 Janke T, Schwarze R, Katrin B (2017) Measuring three-dimensional flow structures in the
544 conductive airways using 3D-PTV, *Exp Fluids* 58: 133 (2017)

545 Kagami S, Hashimoto K (2018) A full-color single-chip-DLP projector with an embedded 2400-
546 fps homography warping engine. *ACM SIGGRAPH 2018 Emerging Technol*: Article No. 1

547 Kanda T, Murai Y, Tasaka Y, Takeda Y (2007) Dynamics and Optics of Bubble Tracking
548 Velocimetry for Airflow Measurement. *Proc ASME/JSME 2007 5th Joint Fluids Eng Conf, San Diego,*
549 *California, USA*: 679–686

550 Kasagi N, Sumitani Y, Suzuki Y, Iida O (1995) Kinematics of the quasi-coherent vortical structure
551 in near-wall turbulence. *Int J Heat Fluid Flow* 16: 2–10

552 Kawaguchi T, Akasaka Y, Maeda M (2002) Size measurements of droplets and bubbles by
553 advanced interferometric laser imaging technique. *Measurement Sci Tech* 13 (3) : 308–316

554 Kelvin L (1987) On vortex atoms, *Proc Royal Soc Edinburgh* 6: 94–105

555 Lee SJ, Yoon GY, Go T (2019) Deep learning-based accurate and rapid tracking of 3D positional
556 information of microparticles using digital holographic microscopy. *Exp Fluids* 60: 170

557 Matsushita H, Mochizuki T, Kaji N (2004) Calibration scheme for three-dimensional particle
558 tracking with a prismatic light. *Rev Scientific Instruments* 75: 541

559 Menser J, Schneider F, Dreier T, Kaiser SA (2018) Multi-pulse shadowgraphic RGB illumination
560 and detection for flow tracking. *Exp Fluids* 59: 90

561 McGregor TJ, Spence DJ, Coutts DW (2007) Laser-based volumetric colour-coded three-
562 dimensional particle velocimetry. *Opt Lasers Eng* 45: 882–889

563 Murai Y, Matsumoto Y, Yamamoto F (2001) Three-dimensional measurement of void fraction in
564 a bubble plume using statistic stereoscopic image processing. *Exp Fluids* 30: 11–21

565 Murai Y, Nkada T, Suzuki T, Yamamoto F (2007) Particle tracking velocimetry applied to
566 estimate the pressure field around a Savonius turbine. *Meas Sci Techol* 18: 2491–2503

567 Park HJ, Saito D, Tasaka Y, Murai Y (2019) Color-coded visualization of microbubble clouds
568 interacting with eddies in a spatially developing turbulent boundary layer. *Exp Therm Fluid Sci* 109:
569 109919

570 Pick S, Lehmann FO (2009) Stereoscopic PIV on multiple color-coded light sheets and its
571 application to axial flow in flapping robotic insect wings. *Exp Fluids* 47: 1009–1023

572 Post ME, Trump DD, Goss LP, Hancock RD (1994) Two-color particle-imaging velocimetry
573 using a single argon-ion laser. *Exp Fluids* 16: 263–272

574 Prenel JP, Bailly Y (2006) Recent evolutions of imagery in fluid mechanics: from standard
575 tomographic visualization to 3D volume velocimetry. *Optics and Lasers in Engineering* 44: 321–334

576 Saha UK, Rajkumar MJ (2006) On the performance analysis of Savonius rotor with twisted
577 blades. *Renew Energy* 31: 1776–1788

578 Scarano F (2013) Tomographic PIV: Principles and practice. *Meas Sci Technol* 24: 012001

579 Schultz J, Skews B, Filippi A (2019) Flow visualization using a Sanderson prism. *J Vis* 22: 1–13

580 Takehara K, Etoh T (1999) A study on particle identification in PTV –Particle mask correlation
581 method–, *J Vis* 1: 313–323

582 Tien WH, Dabiri D, Hove JR (2014) Color-coded three-dimensional micro particle tracking
583 velocimetry and application to micro backward-facing step flows. *Exp Fluids* 55: 1684

584 Udrea DD, Bryanston-Cross PJ, Lee WK, Funes-Gallanzi M (1996) Two sub-pixel processing
585 algorithms for high accuracy particle centre estimation in low seeding density particle image
586 velocimetry. *Opt Laser Technol* 28: 389–396

587 Walpot RJE, Rosielle PCJN, van der Geld CWM (2006) Design of a set-up for high-accuracy 3D
588 PTV measurements in turbulent pipe flow. *Meas Sci Technol* 17: 3015–3026

589 Wang H, Wang G, Li X (2018) High-performance color sequence particle streak velocimetry for
590 3D airflow measurement. *Applied Optics* 57: 1518

591 Watamura T, Tasaka Y, Murai Y (2013) LCD-projector-based 3D color PTV, *Exp. Thermal Fluid*
592 *Sci.* 47: 68–80

593 Willert CE, Gharib M (1992) Three-dimensional particle imaging with a single camera. *Exp*
594 *Fluids* 12: 353–358

595 Xiong J, Idoughi R, Aguirre-Pablo AA, Aljedaani AB, Dun X, Fu Q, Thoroddsen ST, Heidrich W
596 (2017) Rainbow particle imaging velocimetry for dense 3D fluid velocity imaging. *ACM Trans Graph*
597 (TOG) 36: 36