Graph Games and Logic Design

Johan van Benthem

Amsterdam, Stanford and Tsinghua

Graph games model interesting social scenarios when normal behavior gets disrupted, or (perhaps beneficially) nudged away from its ordinary course. At the same time, these games offer interesting interfaces with old and new logics. In this survey talk, I present some classical results on the sabotage game and its modal logic, then move to a range of new results obtained recently by students, and I end with a general discussion of the logic design/game design interface, including the pressing challenge of bringing in more informational/epistemic aspects.

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Johan van Benthem & Fenrong Liu, 2019, Graph Games and Logic Design, Journal of Tsinghua University (Philosophy and Social Sciences), 34:2, 131139.

GRAPH GAMES AND LOGIC DESIGN

Johan van Benthem

28 February 2022

6th International SOCREAL Workshop on

Philosophy and Logic of Social Reality,

University of Hokkaido, Sapporo

Logic and Games

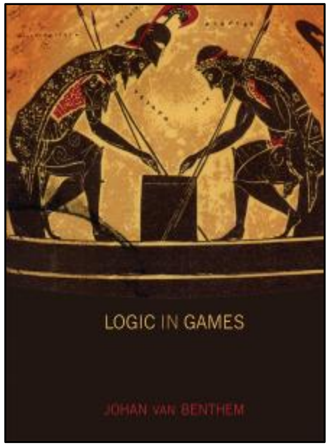
Logic as Games

evaluation of formulas in models

comparing models for similarity/expressive power

constructing models, SAT

proof search, argumentation and dialogue



Part IV

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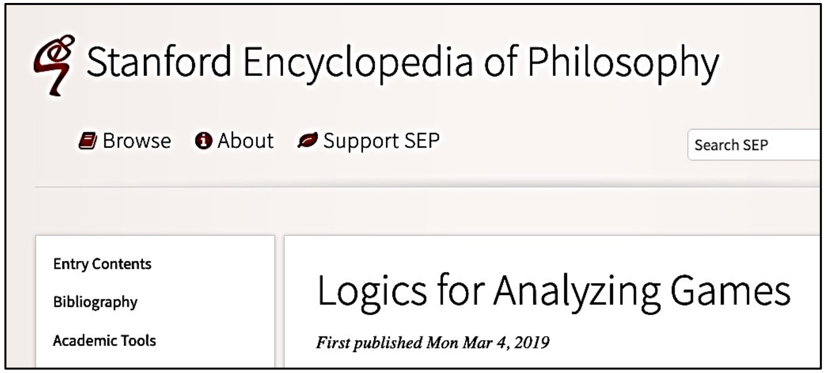
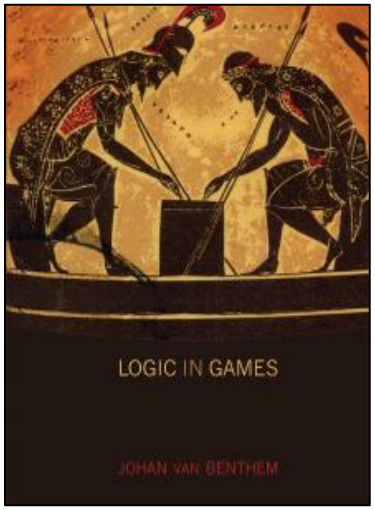
Logic of Games

logical analysis of game structure

links with game theory, computer science,

cognitive and social sciences





I, II, III

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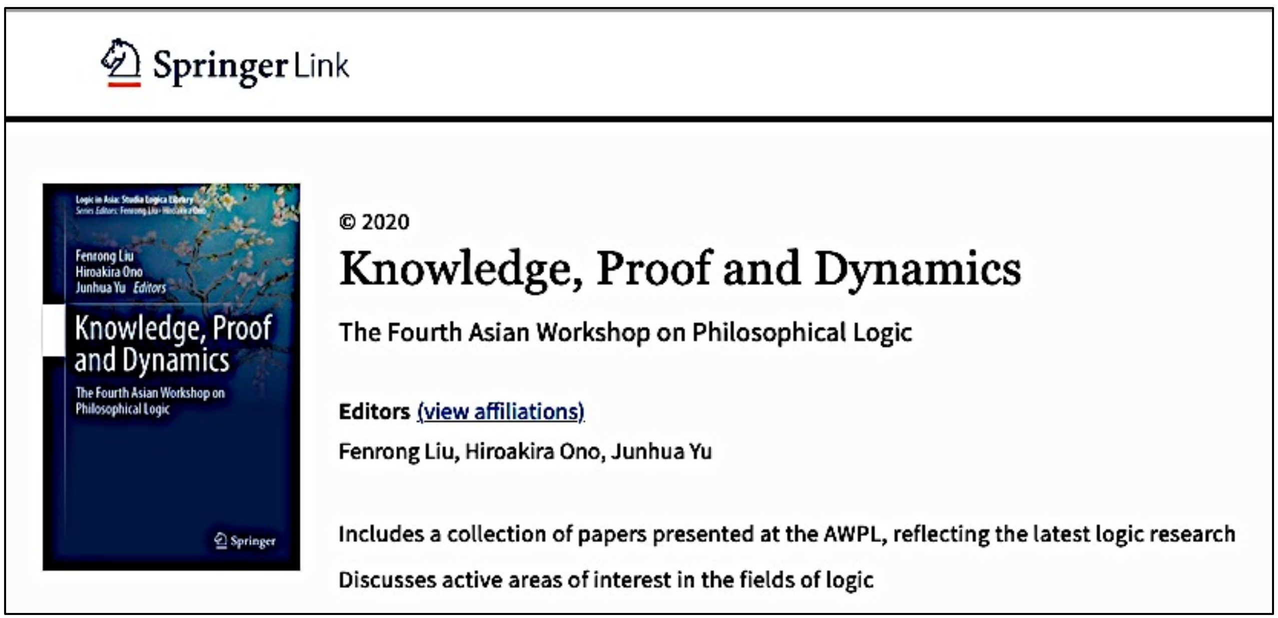
Today: In the Middle

games designed in tandem with logical considerations

broader program in:

J. van Benthem & F. Liu, **Graph Games and Logic Design**



Recent Dissertation (2021)

社会结构中的形式脉络: 多主体交互的逻辑动态研究

申请清华大学-阿姆斯特丹大学联合授予
博士学位论文

李大柱

二〇二一年七月

Formal Threads in the Social Fabric: Studies in the Logical Dynamics of Multi-Agent Interaction

Dissertation Submitted to
Tsinghua University and University of Amsterdam
in partial fulfillment of the requirement
for a joint doctorate degree

by

Dazhu Li

July, 2021

further student contributions:

later in this talk and in talk Fenrong Liu

Graph Games

Graph Games

Graph games (many versions)

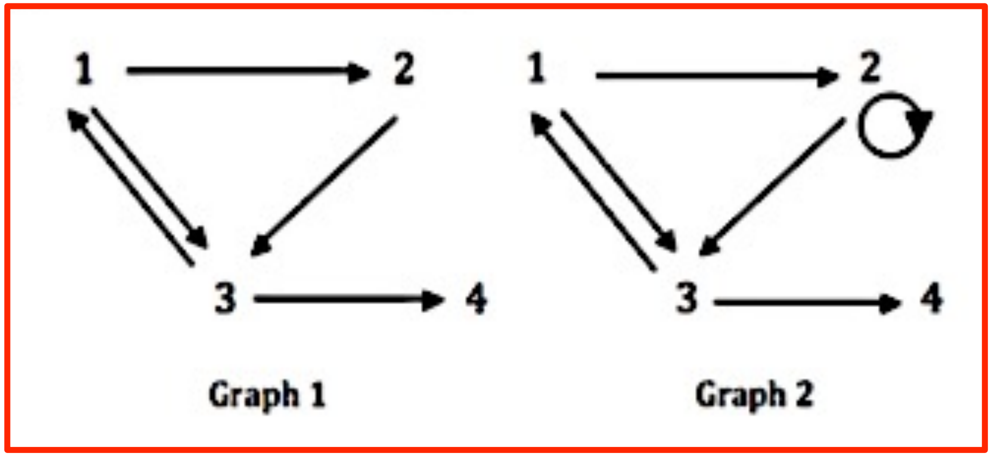
analyze complexity of computation, logic, argumentation

Intermediate between logic games and game logic

A first simple example

Travel game Players alternate in following links in a given graph, Traveler wins if she can always make a step, whatever Demon chooses. Zermelo: one of the two players has a winning strategy.

Travel Game and Modal Logic



Graph 1 Traveler has winning strategy starting in 1

Demon has w.s. when Traveler starts in 2

Graph 2 Traveler has w.s. starting in 2

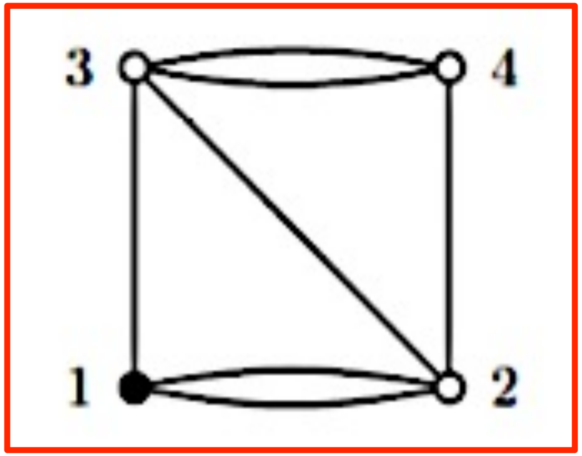
Game design and logic design Travel game described by standard modal logic, response pattern □◊ for Traveler, generic definition across models in μ-calculus: $\nu p \bullet \Box\Diamond p$.

The Sabotage Game

original motivation Dutch railways 2004

Sabotage game

In each round, Demon cuts a link, then Traveler makes a step.
Can Traveler reach her goal region? The game is determined.
For instance, Traveler starts in point *1*, goal region is *{4}*.



The Sabotage Game

algorithmic tasks under disturbance

complexity jumps for perturbed Graph Search problem

in general **larger goal regions**

Zermelo: game is **determined**

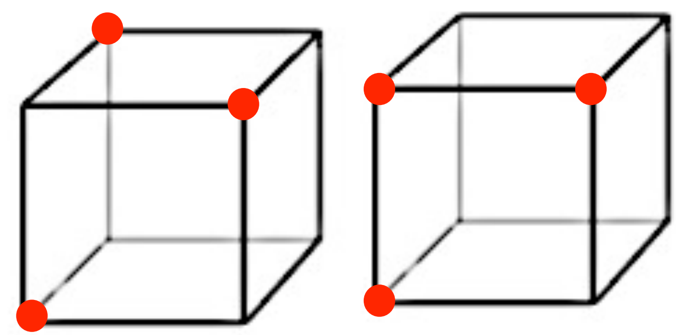
solution complexity **Pspace-complete**

interesting special cases

trees [best cut links **locally**], grids

geometrical solids: winning positio

with goal region sizes on **Cube**



Modal Logics for Graph Games

Sabotage Modal Logic

Modal language for studying the game via its graph invariants

Logical operator $\Diamond\varphi$ for travel, new operator $\blacksquare\varphi$: 'after each link deletion'.

Traveler has a winning strategy: $(\blacksquare\Diamond)^k\gamma$, for k large enough in finite model.

Sabotage μ-calculus $\nu p\bullet\ (\gamma \vee \Box\bot \vee \blacksquare\Diamond p)$. Tricky system, unexplored.

Validities Not closed under substitution: 'ripple effects', e.g.,

$\Box p \rightarrow \blacksquare(\Diamond T \rightarrow \Diamond p)$ is valid, but not in general $\Box\varphi \rightarrow \blacksquare(\Diamond T \rightarrow \Diamond\varphi)$

truth values of complex formulas can change after deletion

dynamic logic of graph change

Counter-Example

Validities Not closed under substitution: 'ripple effects', e.g.,

$\Box p \to \blacksquare(\Diamond T \to \Diamond p)$ is valid, but not in general $\Box \varphi \to \blacksquare(\Diamond T \to \Diamond \varphi)$

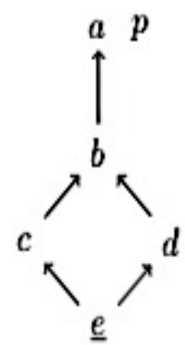


FIGURE 1. A model showing that the general schema $\Box\varphi \to \blacksquare(\Diamond \mathrm{T} \to \Diamond\varphi)$ fails in SML. Let $\varphi \triangleq \Diamond\Diamond p$, let $V(p) = \{a\}$, and let the evaluation point be e.

Open problem Axiomatize the schematic validities of SML

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Some Theory

computational complexity

model checking Pspace-complete (for modal logic: Ptime)

SAT undecidable (for modal logic: Pspace-complete)

first-order translation exists, but needs **memory**

validities axiomatizable in principle, no PAL\DEL-style reduction

sabotage bisimulation and invariance

Student contributions: Aachen, Stanford, Beijing (Li Dazhu, Zhang Tianwei)

axiomatizing validities JvB, Shi Chenwei, Lilei, Yin Haoxuan

to appear in Logic and Computation

Local Sabotage, Graded Modal Logic

Recall our sabotage game Assume for concreteness that in each round, Demon first cuts and then makes a step along some edge. For an illustration, if Demon stands at the top left, he can still win as follows. First cut one link to the right, then go to the exit point E. Now, whatever Traveler does, Demon can cut a connection at E preventing Traveler from reaching the exit point from where he stands, and he has time to cut the third link if Traveler tries somewhere else. However, if Demon is localized at the bottom right when the game starts, it is Traveler who has the winning strategy.

Clearly, this new game restricts the powers of Demon in the original game. Locality changes things considerably. [42] gives a polynomial time solution algorithm for local sabotage games, in contrast with the Pspace solution complexity for the global sabotage game. Interestingly, on the logical side, locality induces a significant change in the earlier modal fixed-point definition

$$\nu p \cdot (\gamma \vee \Box\bot \vee \blacksquare \Diamond p)$$

for Traveler's winning positions in the sabotage game. Instead of using the dynamic link deletion modality, we can make do with the following formula:

$$\nu p \cdot (\gamma \vee \Box\bot \vee \langle \geqslant 2 \rangle\, p)$$

Here $\langle \geqslant 2 \rangle \varphi$ is a static graded modality stating the existence of at least two successors of the current point satisfying φ. Proving the equivalence between these two modal fixed-point formulas requires care with comparing winning strategies for Traveler at the same point in different graphs, and the argument typically seems to fail for the global sabotage game, since the solution complexity for the latter is Pspace, [30].

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Zero-One Law and Random Graph

Zero-One Law Given any first-order sentence φ, as finite domain size n goes to infinity, the percentage of models with *n* objects that satisfy φ goes to either 1 or 0. Moreover, it is decidable which case will occur. New significant statistical property of logical system. Discovered independently in US and (some years earlier) in the USSR.

Proof: analyze truth on the **countable random graph**

Also holds for modal logic, by its translation into FOL.

Even for first-order fixed-point logic **LFP(FO)** (Week 6)

that defines players' winning powers in sabotage game

Fact (Chris Mierzewski) **Probability 1 for** $\mathbf{WIN_T}$

The sabotage game is massively in favor of Traveler!

Intermezzo: Some General Issues

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Graph Logics and Game Logics

graph logics describe the graph and game invariants

that depend only on graph positions (e.g., winning positions)

game logics describe game structure: extensive form (procedure, temporal unfolding), strategic form/powers (control over outcomes)

when is a transition needed?

(preferences, imperfect information, agents)

In between game logics and logic games? Adequacy theorems for evaluation games describe board invariants. *Fact* Travel game = evaluation game for its solution formula $\nu p \bullet \Box\Diamond p$. Changing winning conditions changes logics (cf. parity games for μ-calculus).

Game Logics

Stanford Encyclopedia of Philosophy

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Entry Contents

Bibliography

Academic Tools

Logics for Analyzing Games

First published Mon Mar 4, 2019

Journal of Philosophical Logic
https://doi.org/10.1007/s10992-018-9489-7

CrossMark

A New Game Equivalence, its Logic and Algebra

Johan van Benthem[1,2,3] · Nick Bezhanishvili[1] · Sebastian Enqvist[2,4]

Issue: Mismatch?

graph logic overgenerates

very few formulas correspond with natural game assertions (?)

worse here than with logical systems in general?

paradoxette: **complexity mismatch**

the logic of the graphs can be complex

while the game itself is simple to play

explanation: foundations of mathematics logical theories

of regular simple structures can be complex, and nice versa

Issues: Architecture

When are two graph games the same?

'supergame' instead of graph game?

reductions between games/between logics

Comparisons With

Other Dynamic Logics

Contrast with DEL-style Update Logic

Analogies with Week 1 topic, but also contrasts:

Dynamic-epistemic logics of graph change Graph changes in DEL definable, e.g., cutting all links between φ- and $\neg\varphi$-points $(|\varphi)$. This operation validates recursion axioms for $[|\varphi]\Diamond\psi$ that reduce out dynamic modalities to base logic: decidability. Key difference with SML: stepwise cutting.

Review of Symbolic Logic 2020, JvB, KM & F Z-B

Turn PAL into logic of **stepwise object removal**

Logic of stepwise removal New modality $\langle -\varphi \rangle\psi$: existential quantification without replacement. *MLSR* is undecidable. With universal modality, define grid: $U(\langle E \rangle T \wedge \langle -T \rangle [E]\bot)$, same for converse of *E*, *N* and its converse, and $U\langle -T \rangle(\langle E \rangle [N]\bot \,\&\, \langle N \rangle [E]\bot)$. Axiomatizable using mix of laws for $\langle !\varphi \rangle$, $\langle -\varphi \rangle$: e.g., $(E(n \wedge \varphi) \wedge \langle !\neg n \rangle \psi) \rightarrow \langle -\varphi \rangle \psi$. Fragments: MLSR(MFOL) = MFOL-with-identity.

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From PAL to MLSR

The Review of Symbolic Logic

ASL

Article   Metrics

Accepted manuscript July 2020 , pp. 1-28

The Modal Logic of Stepwise Removal

Johan van Benthem, Krzysztof Mierzewski and Francesca Zaffora Blando

DOI: https://doi.org/10.1017/S1755020320000258   Published online by Cambridge University Press: 21 July 2020

Extensions, Variations

Other Graph Games

Game Design and Taxonomy

Variations Many natural changes possible in winning conditions in Travel Game.

Meet/avoid game original sabotage paper How to avoid unpleasant people at receptions.

Current project Fenrong Liu, Sujata Ghosh

Likewise in sabotage games. We already saw the local variant, also definable variants, etc.

Poison games: localize Demon, make nodes he visits losing for Traveler. Perfect kernels. Various logics, some undecidable, Areces, Mierzewski & Zaffora 2018, Grossi & Rey 2018.

Occupation games, various winning conditions (barriers, jumping in Chinese Checkers).

Berge, Chen: From graph games to *hypergraph games*, and modal neighborhood logic.

Taxonomy Natural parameters variation: moves, goals – perfect/imperfect information.

Issues: General results about change of game-theoretic properties, equivalence of games.

Meet Avoid Game & Logical Surprises

Variations Many natural changes possible in winning conditions in Travel Game.

Meet/avoid game original sabotage paper How to avoid unpleasant people at receptions.

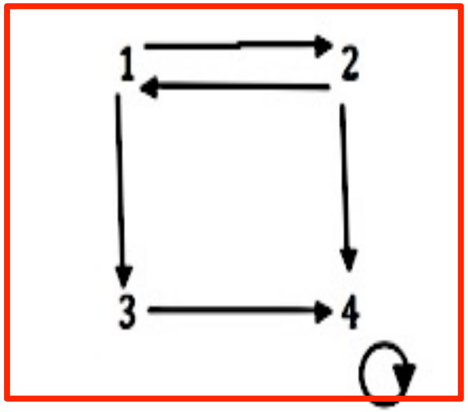
Current project Fenrong Liu, Sujata Ghosh, students

Undecidable logic, Talk Fenrong Liu

Imperfect Information

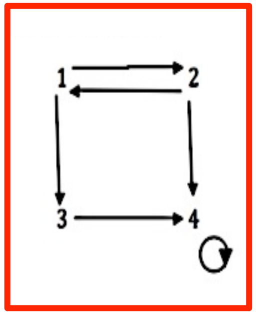
Major challenge: imperfect information Limited observation (war games). Short-sight versions of all graph games (Grossi & Turrini; Liu, Su et al.). Simple example:

Traveler invisible, moves one step at a time, has to answer Yes/No to conjectures by Demon. What is the optimal number of questions for catching Traveler?



In general, games like this will need **probabilistic equilibria**

Information and Questions



Here Observer has a winning strategy. How many questions does he need? Say, he asks in the following order: 1, 4, 3. The answer to the first question might be negative, so Traveler is in {2, 3, 4} and the next hidden move then takes Traveler into {1, 4}. Suppose the answer to the second question is negative again: then Traveler is at 1, and his next move takes him into {2, 3}. If the answer to the third question is negative once more, Traveler is at 2, so his next move takes him into {1, 4}. Observer still does not know where Traveler is.

However, there is a much better strategy for Observer: just ask *twice* whether Traveler is at 1. As is easy to see, this will pinpoint Traveler's location.

general information-theoretic analysis

number of questions needed in dynamic search?

Extended Epistemic-Temporal Graph Logics?

This question scenario looks like a standard information-theoretic one, but the dynamics of moving in graph games add an unexpected flavor. This can be described in a logical framework combining a modal logic for travel steps with an *epistemic logic* for what Observer knows. Worlds in the models for this epistemic logic consist of the above graph with various locations for Traveler. Information flow can then be modeled by dynamic-epistemic techniques, [9].

Another example: logics with **short sight in the graph**

tree of graph positions, action links for possible moves,

epistemic links for what cannot be distinguished

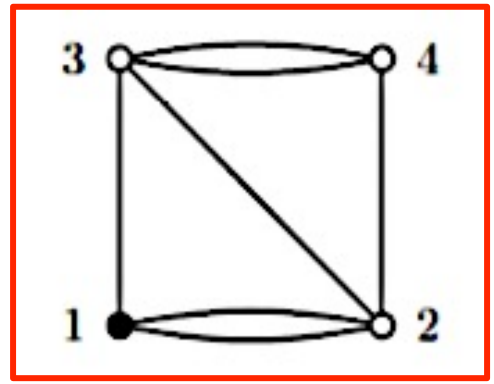
Traveler and Demon may have different views

Sabotage Game With Short Sight in Graph

Traveler only sees links

at the current point

Demon sees everything



Game tree **nodes**: graphs with Traveler position

moves: link cut, travel step. Epistemic links for Traveler

Travel steps can reveal which links were cut

Epistemic modal logic, forcing modalities

Real Games

Game design and real games Links with real parlor games: Halma, Scotland Yard, Settlers of Catan. Issues: fixed graph size, probabilistic moves, bounded agency. The sabotage game as a board game for children: game design in cognitive experiments.

ongoing research at the ILLC