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Amplification Factor of the Electromagnetic Recorder having no Electric Amplifier

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A new expression of the amplification factor is derived. Amplitude characteristics of the recorder are soon anticipated with the calculated diagram. This paper is a synthesis of considerations of those problems which have been published in Japanese by the present author.

1. At the time of the GEOPHYSICAL YEAR many seismic observatories are being equipped with the electromagnetic seismometer. But the amplification factor hitherto derived is not beneficial for understanding total actions of the recorder. This implies that the design and the use of the seismometer are often unsuitable[1] for one's purpose. In spite of COULOMB's recent publication[2], therefore, it is proposed to derive another new expression of the amplification which will be, the author believes, the final one for the electromagnetic recorder having no electric amplifier.

2. At first let there be assumed, for simplicity, second order linear differential equations for the respective motions of the pendulums of the seismometer as well as of the galvanometer, when the electrical circuit is open between them. In this case one has the next two sets of simultaneous equations[3].

$$\left.\begin{aligned} F_1 &= -A_1 I_1 + z_1 V_1\,, &(1)\\ E_1 &= \quad Z_1 I_1 + A_1 V_1\,. &(2)\end{aligned}\right\} \qquad \left.\begin{aligned} F_2 &= -A_2 I_2 + z_2 V_2\,, &(3)\\ E_2 &= \quad Z_2 I_2 + A_2 V_2\,. &(4)\end{aligned}\right\}$$

In the above equations notations are employed as follows:

- F; external force,
- E; external electro-motive force,
- A; force factor, or voltage sensitivity,
- I; electric current,
- V; velocity of the pendulum,
- Z; electrical impedance,
- z; mechanical impedance of the pendulum.

And z is concretely expressed as

$$z = r + j(m\omega - s/\omega)\ , \qquad (5)$$

in which

ω; circular frequency of an external force,
m; mass of the pendulum,
r; coefficient of the liquid resistance,
s; stiffness,
j; $\sqrt{1}$.

Moreover effective complex values are used. The suffix 1 means the quantities belonging to the seismometer and the suffix 2 means the quantities belonging to the galvanometer.

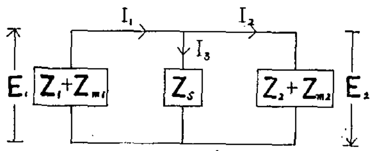


Fig. 1. The electrical circuit of the present problem.

When the seismometer is connected to the galvanometer through an electric shunt Z_s as shown in Fig. 1, several relations hold among the above quantities.

$$F_2 = 0\ , \quad E_1 = -Z_s I_s = -E_2\ , \quad I_1 = I_s + I_2\ . \qquad (6)$$

Eliminating V_1 and V_2 from eqs. (1)～(4), it follows that

$$E_1 = I_1\,(Z_1 + A_1^2/z_1) + A_1 F_1/z_1\ , \qquad (7)$$

$$E_2 = I_2\,(Z_2 + A_2^2/z_2)\ . \qquad (8)$$

Again let E_1, E_2 and I_s be eliminated from (6)～(8), and one obtains

$$I_1\,(Z_1 + A_1^2/z_1) + A_1 F_1/z_1 = -I_2(Z_2 + A_2^2/z_2)\ , \qquad (9)$$

$$I_1 = I_2\left\{(1/Z_s)\,(Z_2 + A_2^2/z_2) + 1\right\}\ . \qquad (10)$$

Therefore

$$I_2\left[(Z_1 + A_1^2/z_1)\left\{1 + (1/Z_s)(Z_2 + A_2^2/z_2)\right\} + (Z_2 + A_2^2/z_2)\right] = -A_1 F_1/z_1\ , \qquad (11)$$

or from (3), (6) and (11)

$$V_2(z_1 + z_{m1})(z_2 + z_{m2})\left[1 - \sigma(z_{m1}z_{m2})\Big/\left\{(z_1 + z_{m1})(z_2 + z_{m2})\right\}\right] = -F_1\sigma^{\frac{1}{2}}(z_{m1}z_{m2})^{\frac{1}{2}} \qquad (12)$$

where

$$z_m = A^2/\bar{Z}, \quad \bar{Z}_1 = Z_1 + Z_2/(Z_2/Z_s+1), \quad \bar{Z}_2 = Z_2 + Z_1/(Z_1/Z_s+1)$$

and $\quad \sigma = 1\Big/\Big\{(Z_1/Z_s+1)(Z_2/Z_s+1)\Big\}\ .$ (13)

If the seismometer is connected to the galvanometer through not a mere shunt but through any attenuater, for example Π-type attenuater as shown in Fig. 2, the above expression of σ must be replaced by

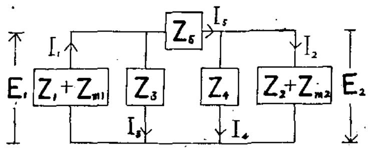


Fig. 2. An electric shunt is replaced by a Π-type attenuater.

$$\sigma = \frac{(Z_3 Z_4)^2}{\{Z_1(Z_3+Z_4+Z_5)+Z_3(Z_4+Z_5)\}\ \{Z_2(Z_3+Z_4+Z_5)+Z_4(Z_3+Z_5)\}}\ ,$$

(14)[4]

the final expression (12) being kept the same as before.

3. The form of eq. (12) can be converted to one familiar to seismologists. For this purpose the following notations will be used:

- n; natural circular frequency of the pendulum,
- u; n/ω ,
- h; damping factor which includes in general two factors, h_0 and h_e, due respectively to the liquid resistance and to the electromagnetic reaction.

Thus $\quad h = h_0 + h_e\ , \quad h_0 = r/2mn\ , \quad h_e = z_m/2mn\ .$ (15)

Employing these notations, one can rewrite the mechanical impedances as follows;

$$(z_{m1} z_{m2})\Big/\Big\{(z_1+z_{m1})(z_2+z_{m2})\Big\}$$
$$= \Big\{(h_{e1}/h_1)(h_{e2}/h_2)\Big\}\Big[1\Big/\Big\{1+j(1-u_1^2)/(2h_1u_1)\Big\}\Big]\cdot\Big[1\Big/\Big\{1+j(1-u_2^2)/(2h_2u_2)\Big\}\Big]\ ,$$

$$(z_{m1} z_{m2})^{\frac{1}{2}}\Big/\Big\{(z_1+z_{m1})(z_2+z_{m2})\Big\}$$
$$= \Big\{(h_{e1}/h_1)(h_{e2}/h_2)\Big\}^{\frac{1}{2}}\Big[1\Big/\Big\{1+j(1-u_1^2)/(2h_1u_1)\Big\}\Big]^{\frac{1}{2}}\cdot\Big[1\Big/\Big\{1+j(1-u_2^2)/(2h_2u_2)\Big\}\Big]^{\frac{1}{2}}$$
$$\times\ \omega^{-1}(m_1m_2)^{-\frac{1}{2}}\Big[1\Big/\Big\{2h_1u_1+j(1-u_1^2)\Big\}\Big]^{\frac{1}{2}}\Big[1\Big/\Big\{2h_2u_2+j(1-u_2^2)\Big\}\Big]^{\frac{1}{2}}\ .$$

Thus eq. (12) becomes

$$\frac{X_2}{X_0}=(1-S)^{-1}\left(\frac{m_1}{m_2}\right)^{\frac{1}{2}}\sigma^{\frac{1}{2}}\left(\frac{h_{e1}}{h_1}\cdot\frac{h_{e2}}{h_2}\right)^{\frac{1}{2}}\cdot$$
$$\times\left[\prod_{i=1,2}\left\{\frac{1}{\sqrt{(u_i^2-1)^2+4h_i^2u_i^2}}\cdot\frac{1}{\sqrt{\left(\frac{u_i^2-1}{2h_iu_i}\right)^2+1}}\cdot e^{j(\pi-2\delta_i)}\right\}\right]^{\frac{1}{2}} \tag{16}$$

where $\tan\delta_i=(2h_iu_i)/(u_i^2-1)$, $i=1$ and 2 ,

and

$$S=-\sigma\left(\frac{h_{e1}}{h_1}\cdot\frac{h_{e2}}{h_2}\right)\left[\prod_{i=1,2}\left\{\frac{1}{\sqrt{\left(\frac{u_i^2-1}{2h_iu_i}\right)^2+1}}\cdot e^{-j\delta_i}\right\}\right] \tag{17}$$

means the proportion of coupling of the two pendulums, of the seismometer and the galvanometer. Moreover in (16) the following notations and a relation are used:

X_2 ; displacement of the pendulum of the galvanometer,
X_0 ; displacement of the earth,
V_2/F_1 ; $(j\omega X_2)/(-\omega^2X_0m_1)$.

In the above expression (16), the right hand sides members will be taken separately as follows:

$(1-S)^{-1}$; coefficient of correction owing to coupling effects,
$Q(h_{e1}h_{e2}/h_1h_2)=\sigma^{\frac{1}{2}}(m_1/m_2)^{\frac{1}{2}}(h_{e1}h_{e2}/h_1h_1)$; statical amplification,
$\bar{P}=(\prod_{i=1,2}P_i)^{\frac{1}{2}}$; dynamical characteristics,

where

$$P_i=\frac{1}{\sqrt{(u_i^2-1)^2+4h_i^2u_i^2}}\cdot\frac{1}{\sqrt{\left(\frac{u_i^2-1}{2h_iu_i}\right)^2+1}}\cdot e^{j(\pi-2\delta_i)} .$$

Until now the pendulum having no rotation has been treated. If the pendulum of the galvanometer rotates on its axis, the expression of Q must be changed to

$$Q=L_2\sigma^{\frac{1}{2}}(m_1/k_2)^{\frac{1}{2}} ,\quad z_{m2}=2h_{e2}k_2n_2=(A_2a_2)^2/\bar{Z}_2 . \tag{18}$$

If the two pendulums rotate on their respective axis,

$$Q=l_1^{-1}L_2\sigma^{\frac{1}{2}}(k_1/k_2)^{\frac{1}{2}},\quad z_{m1}=2h_{e1}k_1n_1=(A_1a_1)^2/\bar{Z}_1 . \tag{19}$$

In these expressions several new notations are entered as follows:

k; moment of inertia for the axis of rotation,
l; equivalent length of the pendulum,
L; length of the recording lever which is twice as great as the distance from the mirror to the photographic paper in an optical galvanometer,
a; distance between the axis of rotation and the coil.

In practice the expressions (18) and (19) will be the cases, though eq. (16) will be sufficient from a theoretical point of view.

The electromagnetic recorder in general has no liquid damper, and h_0 is so smaller than h_e. In such a case

$$h_1 \approx h_{e1} \text{ and } h_2 \approx h_{e2} .$$

Therefore eq. (16) becomes

$$X_2/X_0 = (1-S)^{-1}Q\bar{P} \tag{20}$$

where

$$S = -\sigma\left[\prod_{i=1,2}\left\{\frac{1}{\sqrt{\left(\frac{u_i^2-1}{2h_iu_i}\right)^2+1}}\cdot e^{-j\delta_i}\right\}\right];$$

It may be somewhat strange that the factors A_1 and A_2 do not enter explicitly in expression (16) and also a_1 does not in (19). But all of these factors have influence upon amplification through damping factors h_{e1} and h_{e2}. Another noticeable result in (20) is that the proportion of the coupling effects does not depend explicitly on m_1/m_2 but merely on σ, u_i and h_i.

4. It will be desired that the recorder may be used eliminating the coupling effects. In order to do so, S in (17) or (20) must be so smaller than unity. In this case eq. (20) becomes simply

$$X_2/X_0 = Q\bar{P} . \tag{21}$$

where

$$|\bar{P}| = (\prod_{i=1,2}|P_i|)^{\frac{1}{2}} ; \quad |P_i| = \frac{1}{\sqrt{(u_i^2-1)^2+1}}\cdot\frac{1}{\sqrt{\left(\frac{u_i^2-1}{2h_iu_i}\right)^2+1}}$$

and
$$\arg\bar{P} = \pi-(\delta_1+\delta_2) .$$

Now $\arg P_i$, that is δ_i, has already been calculated by many authorities, for example by HAGIWARA[5]. Thus $|\bar{P}|$ alone will be discussed here. From the relation in (21), one has

$$\log|\bar{P}| = \frac{1}{2}(\log|P_1| + \log|P_2|) . \tag{22}$$

It is seen in (21) that the relation of $|P_i|$ versus u_i alone will be enough for the calculation of (22). This relation is tabulated in the Table and also is shown in Fig. 3, being the parameter h. At the discussion of $|P_i|$ the suffix i will be omitted.

The present discussions have now been finished, but the usage of Fig. 3 will be explained. If the ratio of the natural frequencies of the two pendulums is α, $n_2 = \alpha n_1$ and $u_2 = \alpha u_1$. At first $|P_1|$ curve can be chosen from Fig. 3 for the given h_1, and the curve traced on a tracing paper. As u_1 may be considered equal to u, the abscissa $u_1=1$ is correspondent to the abscissa $u=1$. Next let $|P_2|$ curve be chosen from Fig. 3 for the given h_2, and the curve be traced on the above tracing paper. But in the later case the abscissa $u_2=1$ must correspond to the abscissa $u_1=1/\alpha$. In these treatments every ordinate must be kept the same. Then the amplitude characteristics will be obtained as the mean values of the ordinates of the two curves, because Fig. 3 is drawn on log-log scales.

In Fig. 3 $|P|$ gets maximum value

$$\left\{\frac{1}{\sqrt{(u^2-1)^2+4h^2u^2}} \cdot \frac{1}{\sqrt{\left(\frac{u^2-1}{2hu}\right)^2+1}}\right\}_{\text{max.}} = \frac{1}{2} \cdot \frac{1}{u^2+1} \cdot \left(\frac{3u^2+1}{|u^2-1|}\right)^{\frac{1}{2}}$$

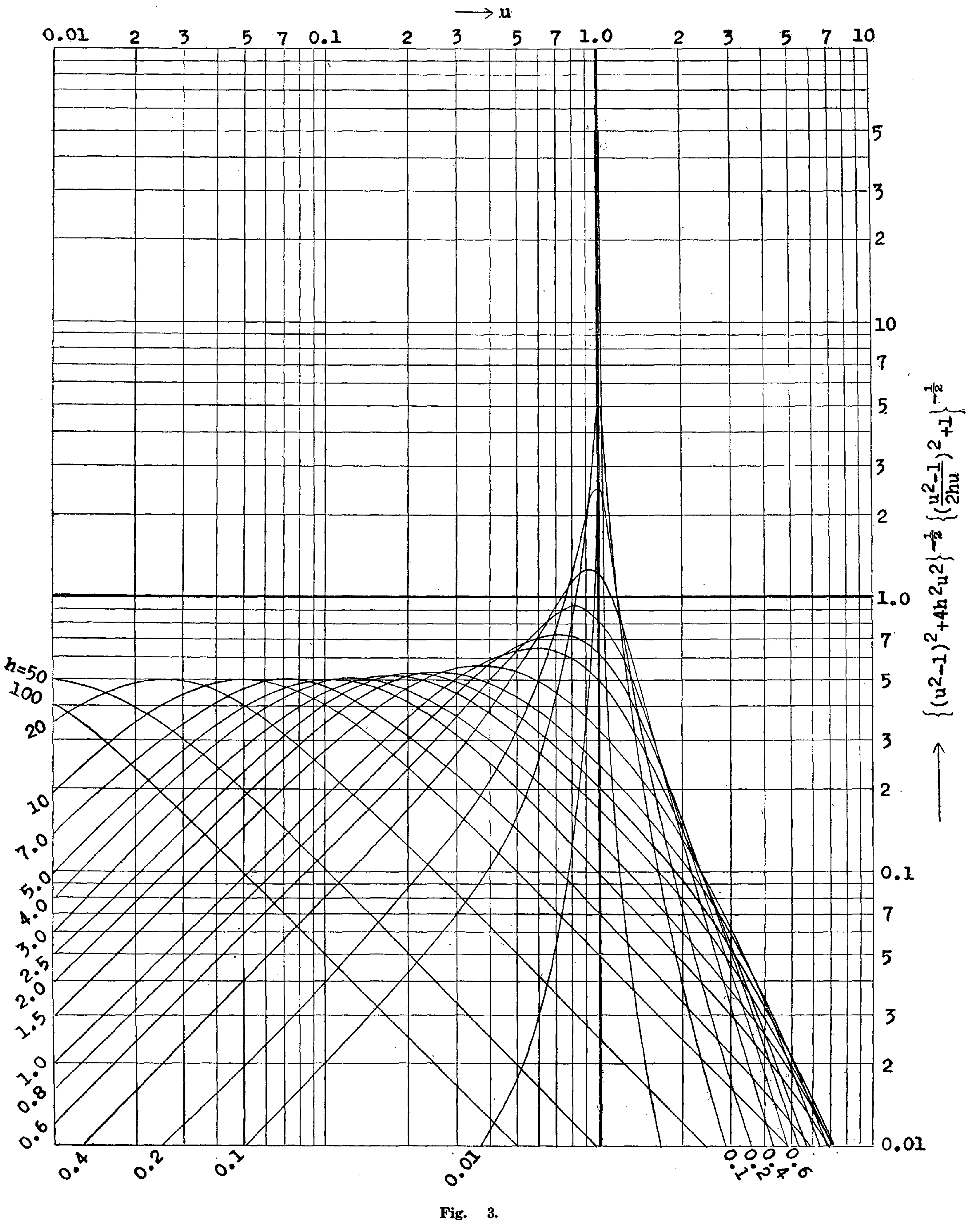
when $$u^2 = (1/3)\left\{-(2h^2-1) \pm \sqrt{(2h^2-1)^2+3}\right\} .$$

If h is smaller than unity in this case, u is necessarily samller than unity and $|P|_{\text{max.}}$ will approach 1/2.

5. Every curve in Fig. 3 will be taken as a straight line, departing from each maximum. In these parts one has

$$\log|P| = a\log u + \log b , \qquad \therefore \quad |P| = bu^a .$$

If $a=1$, then, $|P|$ is proportional to u. In this case the straight line in Fig. 3 crosses the abscissa at an angle of 45°. When the cross angle is $-45°$, the amplification is proportional to the velocity of the earth. The former condition ($a=1$) may be got when $h \approx 1$ and u is rather small; on the contrary the latter condition ($a=-1$)



→ u
0.01 2 3 5 7 0.1 2 3 5 7 1.0 2 3 5 7 10
5 3 2 10 7 5 3 2 1.0 7 5 3 2 0.1 7 5 3 2 0.01
$\left\{(u^2-1)^2+4h^2u^2\right\}^{-\frac{1}{2}}\left\{\left(\frac{u^2-1}{2hu}\right)^2+1\right\}^{-\frac{1}{2}}$ →
h=50
100
20
10
7.0
5.0
4.0
3.0
2.5
2.0
1.5
1.0
0.8
0.6
0.4
0.2
0.1
0.01
0.001
0.002
0.004
0.006

Fig. 3.

TABLE $\dfrac{1}{\sqrt{(u^2-1)^2+4h^2u^2}} \cdot \dfrac{1}{\sqrt{\left(\dfrac{u^2-1}{2hu}\right)^2+1}}$

h \ u	0.01	0.02	0.03	0.05	0.07	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0	30.0
0.01	0.0002	0.0004	0.0006	0.0010	0.0014	0.0020	0.0043	0.0072	0.0178	0.0583	50.0000	0.0044	0.0010	0.0002	0.0000	—	—	—
0.1	0.0020	0.0040	0.0060	0.0100	0.0141	0.0204	0.0433	0.0721	0.1509	0.5005	5.0000	0.0437	0.0093	0.0017	0.0006	0.0000	—	—
0.2	0.0040	0.0080	0.0120	0.0201	0.0283	0.0407	0.0862	0.1424	0.3319	0.8277	2.5000	0.0832	0.0183	0.0034	0.0011	0.0004	0.0000	—
0.3	0.0060	0.0120	0.0180	0.0301	0.0423	0.0610	0.1282	0.2091	0.4598	0.9622	1.6667	0.1148	0.0266	0.0051	0.0018	0.0006	0.0001	0.0000
0.4	0.0080	0.0160	0.0240	0.0401	0.0564	0.0811	0.1689	0.2710	0.5536	0.9760	1.2500	0.1384	0.0343	0.0068	0.0022	0.0008	0.0001	0.0000
0.5	0.0100	0.0200	0.0300	0.0501	0.0703	0.1010	0.2080	0.3268	0.6154	0.9332	1.0000	0.1538	0.0411	0.0080	0.0028	0.0010	0.0001	0.0000
0.6	0.0120	0.0240	0.0360	0.0600	0.0842	0.1222	0.2450	0.3759	0.6504	0.8698	0.8332	0.1627	0.0468	0.0096	0.0033	0.0011	0.0002	0.0000
0.7	0.0140	0.0280	0.0420	0.0700	0.0980	0.1401	0.2801	0.4183	0.6654	0.8033	0.7142	0.1663	0.0512	0.0112	0.0039	0.0014	0.0002	0.0001
0.8	0.0160	0.0319	0.0480	0.0799	0.1117	0.1591	0.3124	0.4537	0.6655	0.7397	0.6249	0.1660	0.0552	0.0128	0.0046	0.0016	0.0002	0.0001
0.9	0.0180	0.0360	0.0539	0.0897	0.1252	0.1756	0.3425	0.4826	0.6561	0.6822	0.5555	0.1638	0.0577	0.0135	0.0051	0.0018	0.0002	0.0002
1.0	0.0200	0.0400	0.0599	0.0995	0.1387	0.1961	0.3700	0.5048	0.6402	0.6307	0.5000	0.1600	0.0750	0.0148	0.0056	0.0014	0.0002	0.0002
1.5	0.0299	0.0598	0.0894	0.1475	0.2031	0.2803	0.4684	0.5495	0.5328	0.4496	0.3333	0.1332	0.0603	0.0187	0.0076	0.0028	0.0004	0.0002
2.0	0.0399	0.0796	0.1183	0.1932	0.2622	0.3509	0.5125	0.4862	0.4384	0.3449	0.2500	0.1168	0.0576	0.0204	0.0091	0.0035	0.0005	0.0002
2·5	0.0499	0.0991	0.1471	0.2365	0.3134	0.4065	0.5207	0.4874	0.3670	0.2797	0.2000	0.0920	0.0519	0.0208	0.0099	0 0041	0.0006	0.0002
3.0	0.0598	0.1184	0.1747	0.2765	0.3602	0.4477	0.5082	0.4424	0.3138	0.2344	0.1668	0.0784	0.0464	0.0203	0.0103	0.0045	0.0007	0.0002
4.0	0.0795	0.1561	0.2277	0.3463	0.4297	0.4938	0.4595	0.3643	0.2415	0.1771	0.1250	0.0603	0.0374	0.0184	0.0103	0.0050	0.0009	0.0003
5.0	0.0990	0.1924	0.2757	0.4016	0.4729	0.5050	0.4064	0.3051	0.1956	0.1421	0.1000	0.0488	0.0311	0.0163	0.0097	0.0051	0.0010	0.0004
7.0	0.1373	0.2599	0.3577	0.4713	0.5025	0.4761	0.3194	0.2275	0.1412	0.1017	0.0714	0.0353	0.0230	0.0128	0.0082	0.0048	0.0012	0.0004
10	0.1923	0.3451	0.4418	0.5012	0.4744	0.4016	0.2364	0.1629	0.0995	0.0713	0.0500	0.0249	0.0164	0.0095	0.0064	0.0040	0.0013	0.0005
20	0.3451	0.4880	0.4921	0.4004	0.3170	0,2355	0.1232	0.0829	0.0500	0.0357	0.0250	0.0125	0.0083	0.0049	0.0034	0.0024	0.0010	0.0005
50	0.5000	0.4006	0.3001	0.1924	0.1400	0.0991	0.0499	0.0333	0.0200	0.0143	0.0100	0.0050	0.0033	0.0020	0.0014	0.0000	—	—
100	0.4001	0.2354	0.1621	0.0991	0.0709	0.0498	0.0250	0.0166	0.0100	0.0071	0.0050	0.0025	0.0017	0.0010	0.0007	0.0000	—	—

will be attained when $h \gg 1$ and u is rather large. Thus the dynamical amplification proportional to the displacement of the earth can be constructed, equipping two pendulums having respectively the above different characters.

One may feel difficulty in constructing a vertical pendulum having a long period, e. g., longer than 1 second. It will be easy, on the other hand, to construct a horizontal pendulum having a longer period. Therefore the vertical displacement of the earth may be well recorded with the combination (n_1, h_1=large and n_2, h_2 =small), since the usual galvanometer has a horizontal pendulum and its natural circular frequency n_2 can be very small.

Appendix 1

Estimation of the Coupling Coefficient

When $(h_{e1}/h_1)(h_{e2}/h_2)$ and σ are both smaller than unity, S in (17) may be so smaller than unity, because the absolute value in { } is always smaller than unity. Then the coupling effects will be out of question in practice. In this section, therefore, it will be assumed that $\sigma(h_{e1}/h_1)(h_{e1}/h_2)$ is equal to unity.

As one has from (17)

$$\sin\delta_i = 1\Big/\sqrt{\left(\frac{u_i^2-1}{2h_i u_i}\right)^2+1}\,,$$

he has

$$1-S = 1+\sin\delta_1\sin\delta_2\cos(\delta_1+\delta_2) - j\sin\delta_1\sin\delta_2\sin(\delta_1+\delta_2)\,.$$

$$\therefore\quad |1-S| = \frac{\sin\delta_1\sin\delta_2\sin(\delta_1+\delta_2)}{\sin(\delta_1+\delta_2-\delta_{12})} = \frac{1+\sin\delta_1\sin\delta_2\cos(\delta_1+\delta_2)}{\cos(\delta_1+\delta_2-\delta_{12})}$$

and

$$\arg(1-S) = -(\delta_1+\delta_2-\delta_{12})$$

where

$$\tan\delta_{12} = \tan\delta_1+\tan\delta_2\,.$$

Assuming, moreover, $h=h_1=h_2$, the absolute values of $(1-S)$ for several values of α can easily be estimated as shown in Fig. 4. Argument of $(1-S)$ is also shown in Fig. 5.

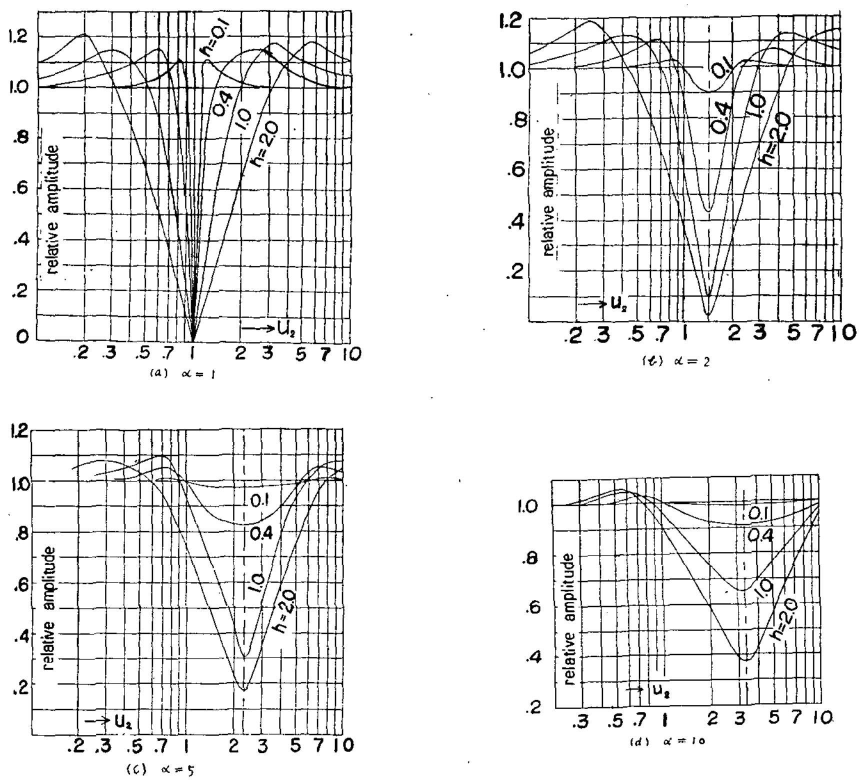


Fig. 4. Coefficients of coupling effect with respect to amplitude.

It must be remarked in Fig. 4 that the coupling effects may reach considerable values for somewhat large values of h, as in the usual case of critical damping. If it is desired to neglect coupling effects for every external frequency, $\sigma(h_{e1}/h_1)(h_{e2}/h_2)$ must be designed to be so smaller than unity. Further as to α, the more different it is from unity the better.

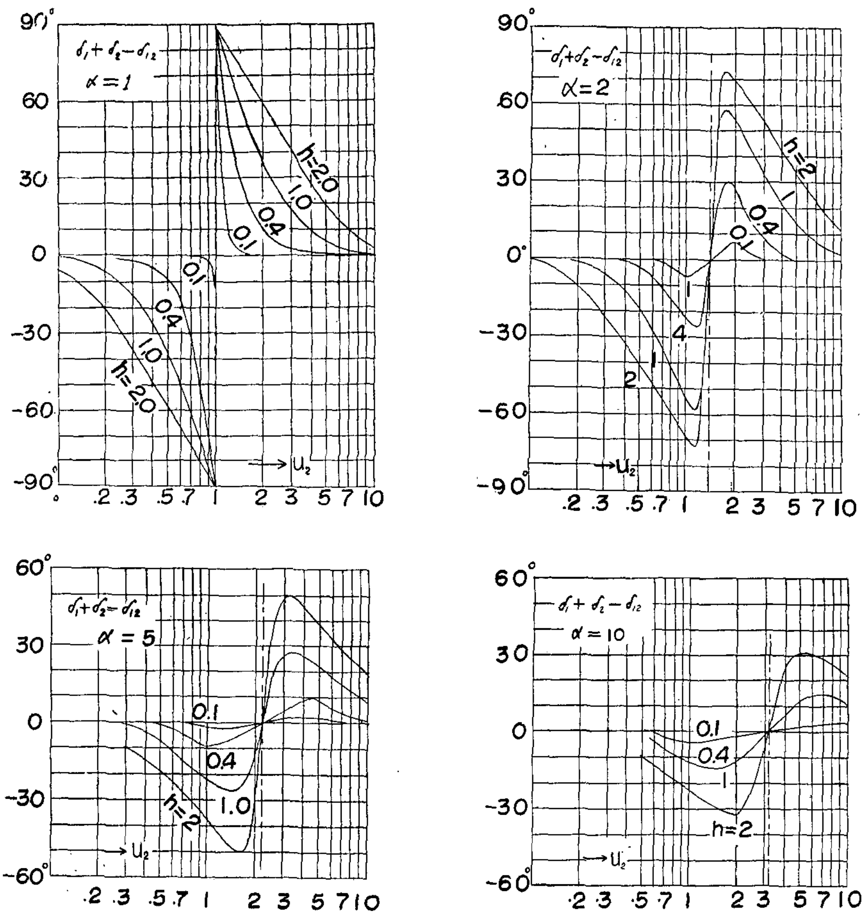


Fig. 5. Phase lag by coupling effect.

Appendix 2

Test of Dynamical Stability of the Pendulum

The second order linear differential equation[6] was assumed for the vibration of the pendulum at the beginning of this paper. If the vibration of the pendulum becomes unstable, the present simple discussion will be of little use. Theoretical investigations of unstability of the pendulum have been described indeed by many authors[7], but those are too complicated to be applied to a practical seismometer. Then it will be a matter of concern whether the pendulum at hand may be stable within certain frequency range or not. Shaking tables may be used for this purpose, but they have several disadvantages. As the seismometer including its base is set on them, the shaking plate must be solid enough and enormous shaking powers must be prepared. These circumstances will be more severe, the higher the external frequency.

Fortunately, however, the motional impedance method can be applied to the present test, overcoming these disadvantages.

In eq. (7), $Z_m = A^2/z$ is called motional impedance[8]; it can be expressed with the above notations as follows:

$$Z_m = \frac{A^2}{r + j(m\omega - s/\omega)} = \frac{A^2}{mn} \cdot \frac{1}{2h_0} \cdot \frac{1}{\sqrt{\left(\frac{u^2-1}{2h_0 u}\right)^2 + 1}} \cdot e^{j\left(\frac{\pi}{2} - \delta_0\right)},$$

$$\tan \delta_0 = 2h_0 u/(u^2 - 1) . \qquad (23)$$

Then construct an electrical circuit as in Fig. 6 corresponding to the circuit shown in Fig. 1, and it is seen, in (13), Z_{*1} is the equivalent electrical impedance from the terminals of the seismometer.

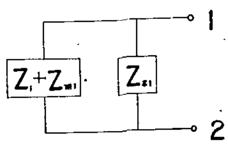


Fig. 6.

An electrical circuit applied to the motional impedance method.

$$\bar{Z}_1 = Z_1 + Z_{*1} \quad \text{and} \quad 1/Z_{*1} = 1/Z_2 + 1/Z_s . \qquad (24)$$

If the following impedance is considered

$$Z_x = \frac{A^2}{mn} \cdot \frac{1}{2h} \cdot \frac{1}{\sqrt{\left(\frac{u^2-1}{2hu}\right)^2 + 1}} \cdot e^{j\left(\frac{\pi}{2} - \delta\right)}, \quad \tan\delta = 2hu/(u^2-1), \qquad (25)$$

this electrical impedance may have formally the next relation

$$1/Z_{xi} = 1/Z_{mi} + 1/(Z_i + Z_{*i}) , \qquad (i = 1, 2), \qquad (26)$$

though physical meaning of the above electrical circuit (26) will be obscure, because Z_{mi} must be observed in seres with Z_i.

But the free impedance Z_{bi} can be observed from the terminals (1 and 2) in Fig. 6 as follows

$$1/Z_{bi} = 1/(Z_i + Z_{mi}) + 1/Z_{*i} \qquad (27)$$

and the clamped impedance Z_{ai} can be observed, the pendulum being clamped, as

$$1/Z_{ai} = 1/Z_i + 1/Z_{*i} \,. \tag{28}$$

Thus from (26), (27) and (28) it follows

$$Z_{xi} = (1 + Z_i/Z_{*i})^2 (Z_{bi} - Z_{ai}) = \left\{ Z_{*i}/(Z_{*i} - Z_{ai}) \right\}^2 (Z_{bi} - Z_{ai}) \,. \tag{29}$$

In spite of the curious relation (26), it may now be seen that Z_x can be observed in practice.

If h_0 is so smaller than h_e as before, from (25) and (29) one has at last

$$\frac{1}{\sqrt{\left(\dfrac{u^2-1}{2hu}\right)^2+1}} \cdot e^{j\left(\frac{\pi}{2}-\delta\right)} = \frac{Z_b - Z_a}{Z_* - Z_a} \,. \tag{30}$$

When a more practical expression is required, eq. (30) can be separated into the following two parts,

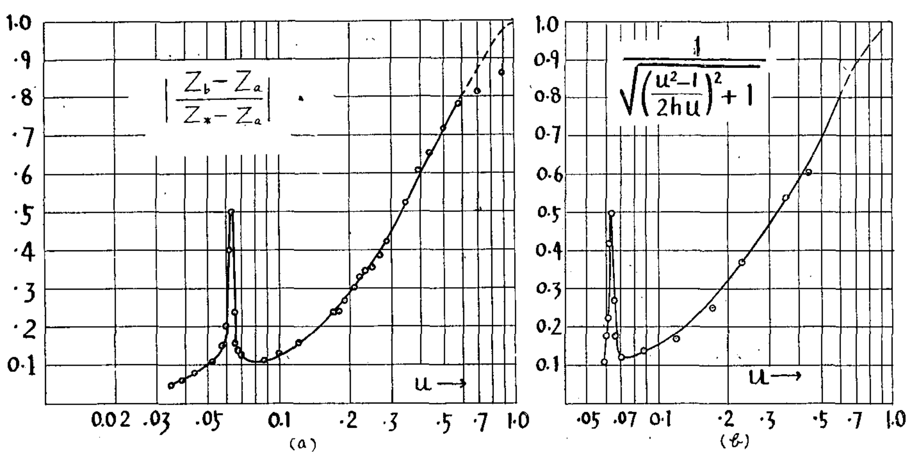


Fig. 7. Frequency characteristics by the motional impedance method (a) and that with a shaking table (b).

$$\frac{1}{\sqrt{\left(\dfrac{u^2-1}{2hu}\right)^2+1}} = \frac{\{(R_b - R_a)^2 + (X_b - X_a)^2\}^{\frac{1}{2}}}{\{(R_* - R_a)^2 + X_a^2\}^{\frac{1}{2}}} \quad \text{and} \quad \delta = \pi/2 - (\theta + \gamma) \tag{31}$$

where

$$\tan\theta=(X_b-X_a)/(R_b-R_a),\quad \tan\gamma=X_a/(R_*-R_a)\quad\text{and}\quad Z=R+jX.$$

Real and imaginary parts of the electrical impedances can be observed with a usual A. C. bridge.

A frequency characteristic of a moving-coil type seismometer observed by this motional impedance method is illustrated in Fig. 7 in comparison with that observed with a shaking table. In Fig. 7 it may be seen that the results obtained by the different methods are well in coincidence with each other, and this seismometer has been found to get unstable near 60 c. p. s., its natural frequencies being about 3 c. p. s.

The present author expresses his thanks to Miss Miyako MUROTA for her calculations in the Table and her drawings in Fig. 3.

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