An Optimal Configuration Method of Superconducting Magnet with Iron Shield Using Model Order Reduction

So Noguchi, Hiroshi Ueda, Tomonori Watanabe, Shigeo Nagaya, Atsushi Ishiyama, and Mitsuhiro Fukuda

Abstract—**We have been developing a cyclotron accelerator for radioisotope production of medical use. The features of developed cyclotron system are compactness, light weight, and multi energy outputs. To achieve such features, coils are wound with rare-earth barium copper oxide (REBCO) tapes and the magnet has no iron core to generate an azimuthal varying field (AVF).**

To install the developed cyclotron accelerator into hospitals, an iron shield is needed for protection of leakage radiation and magnetic field. In this paper, to optimally design main coils with iron shield, a fast field computation method called "Model Order Reduction (MOR)" is adopted. Using MOR technique, the field computation is accelerated ~50 times. The error of MOR field computation is sufficiently small. Hence, the MOR is effective in the optimal configuration design of coils with iron shield. The optimized configuration of a miniaturized cyclotron accelerator magnet is also shown.

Index Terms—**Cyclotron accelerator, iron shield, model order reduction, optimal design, REBCO magnet.**

I. Introduction

Radioisotope (RI) nuclear medicine is one of future promising cancer treatments. In particular, it is expected that targeted α-particle therapy will put into practical use [1], [2]. The targeted α-particle therapy is less burdensome to cancer patients. Some clinical therapy trials using ^{211}At, ^{223}Ra, or ^{225}Ac are underway around the world. In near future, we expect that it will be necessary to mass-produce α-ray radionuclide of ^{211}At, because only ^{211}At is easily employed according to the Japanese law.

We have been developing a compact iron-coreless high-temperature superconducting (HTS) cyclotron accelerator, named "HTS skeleton cyclotron," for RI mass-production [3], [4]. To aim a developed cyclotron accelerator to be installed into a hospital, it is desired that it has the following features: 1) compactness, 2) light weight, and 3) energy variation. On hospitals, multi purposes are desired for one RI accelerator: 1) ^{211}At production for targeted α-particle therapy, 2) various-energy-RI production for positron emission tomography (PET), 3) boron neutron capture therapy (BNCT), and 4) ^{225}Ac production. To achieve such multi purposes, an iron core to create an azimuthally varying field (AVF) is inconvenient to change a field intensity with a short time because of the hysteresis magnetic property of iron. Previously, we have optimally designed the configuration of a rare-earth barium copper oxide (REBCO) magnet for HTS skeleton cyclotron [3]. We have investigated the effect of screening current upon the magnetic field on particle acceleration plane [4]. Recently, we have also investigated the mechanical strength of YOROI-structured non-circular REBCO pancake coils, which will be used as sector coils to create the AVF, in a high magnetic field [5].

Meanwhile, although the HTS skeleton cyclotron has no iron core to generate an AVF, an iron shield of radiation is needed surrounding an iron-coreless REBCO magnet. The iron shield can also be used for protection of a leakage magnetic field. Hence, an optimal design method of REBCO coil configuration with iron shield needs to be developed, because a conventional field computation takes a long time due to solving a great number of simultaneous equations and the nonlinear magnetic characteristics of iron. Moreover, in an optimization procedure, a field computation must be repeated tens of million times in general [6]. In recent years, a fast computation method, called "Model Order Reduction (MOR) technique," have been proposed and developed [7]–[9]. To accelerate a field computation, we employed the method of snapshots, one of MOR techniques [7]. In this paper, the method of snapshots is presented, and an optimal configuration of miniaturized HTS skeleton cyclotron with iron shield, which has been manufactured in order to demonstrate the feasibility of HTS skeleton cyclotron, is also shown. The method of snapshots shortened the computation time by 98% by reducing the model order (degrees of freedom; DOFs) from 52080 to 4 in this paper.

Manuscript receipt and acceptance dates will be inserted here. This work was in part supported by JSPS Grant-in-Aid for Scientific Research(S) from the Ministry of Education, Science, Sports, and Culture (No. 18H05244). *(Corresponding author: So Noguchi.)*

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II. Optimization with Model Order Reduction

A. Magnetic Field Computation

Although the finite element method (FEM) is the most popular among field computation methods, we employed the magnetic moment method [10], [11] because it is necessary to mesh only iron shields. The governing equation is given as

$$\frac{1}{\chi}\boldsymbol{M}(\boldsymbol{r}_{\mathrm{f}})+\frac{1}{4\pi}\int \nabla_{\mathrm{o}}\cdot\boldsymbol{M}(\boldsymbol{r}_{\mathrm{o}})\frac{\boldsymbol{r}_{\mathrm{f}}-\boldsymbol{r}_{\mathrm{o}}}{|\boldsymbol{r}_{\mathrm{f}}-\boldsymbol{r}_{\mathrm{o}}|^{3}}\mathrm{d}V -\frac{1}{4\pi}\int \boldsymbol{M}(\boldsymbol{r}_{\mathrm{o}})\cdot\boldsymbol{n}\frac{\boldsymbol{r}_{\mathrm{f}}-\boldsymbol{r}_{\mathrm{o}}}{|\boldsymbol{r}_{\mathrm{f}}-\boldsymbol{r}_{\mathrm{o}}|^{3}}\mathrm{d}S=\boldsymbol{H}(\boldsymbol{r}_{\mathrm{f}}) \tag{1}$$

where χ, $\boldsymbol{M}$, $\boldsymbol{r}_{\mathrm{f}}$, $\boldsymbol{r}_{\mathrm{o}}$, $\boldsymbol{n}$, and $\boldsymbol{H}$ are the susceptibility, the magnetic moment, the position vectors of the field and the source points, the unit vector perpendicular to the integral surface S, and the magnetic strength generated by REBCO coils, respectively. The subscripts 'f' and 'o' mean the field and the source points. Equation (1) can be simply expressed by

$$A\boldsymbol{M}=\boldsymbol{H} \tag{2}$$

where A is the system matrix with the size of $n \times n$. The magnetic moment $\boldsymbol{M}$ in the cylindrical coordination is variable, and the number of elements and variables in the case of this paper is 17360 and 52080, respectively, as shown in Fig. 1.

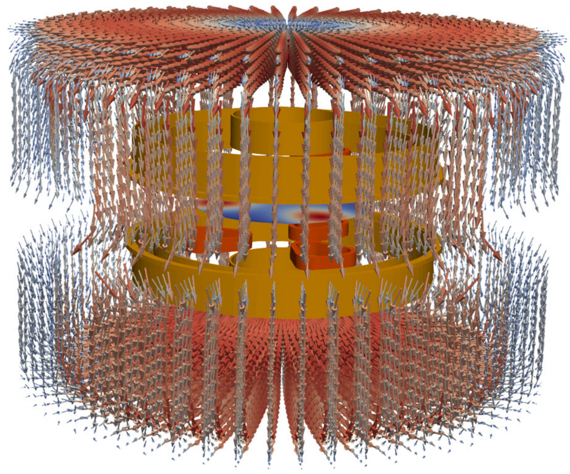

Fig. 1. Example of magnetic moment map with REBCO coils. The number of magnetic moment elements is 17360.

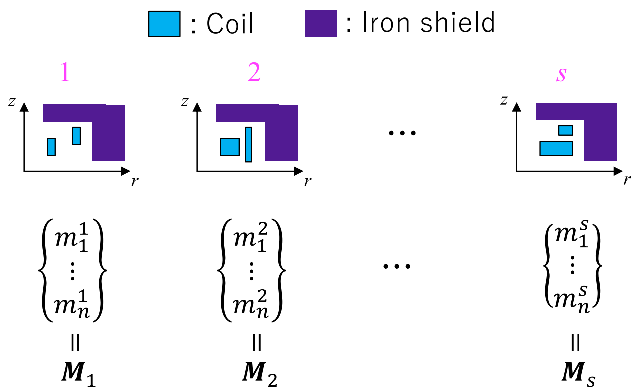

Fig. 2. As a pre-computation, the magnetic moments of s different coil shapes are computed. Here, m, $\boldsymbol{M}$, n, and s are the magnetic moment on element, a set of magnetic moments, the number of variables (DOFs), and the number of snapshots, respectively.

B. Method of Snapshots

In the optimization process, tens of millions of iterative computations of (1) are required, and it is impractical to optimize the coil configurations by solving large-scale simultaneous equations. Therefore, we employed the method of snapshots to shorten the computation time of (1). In the method of snapshots, the principal component analysis to multiple similar solutions is conducted, and then the dimension of simultaneous equations is significantly reduced to consider only the principal components [7]. The procedure of the method of snapshots is presented below.

To obtain the snapshots, (1) is solved to obtain the magnetic moment $\boldsymbol{M}_i$, where $0 \le i \le s$, for s different coil configurations, as shown in Fig. 2. Here, s is the number of snapshots ($n >> s$, where n is the number of variables, so called the degrees of freedom). The matrix X is computed from

$$\mathrm{X}=[\boldsymbol{M}_1-\boldsymbol{\mu} \quad \boldsymbol{M}_2-\boldsymbol{\mu} \quad \cdots \quad \boldsymbol{M}_s-\boldsymbol{\mu}] \tag{3}$$

where $\boldsymbol{\mu}$ is the mean vector of $\boldsymbol{M}$. The singular value decomposition of X is

$$\mathrm{X}=\sigma_1\boldsymbol{u}_1\boldsymbol{v}_1^{\mathrm{t}}+\sigma_2\boldsymbol{u}_2\boldsymbol{v}_2^{\mathrm{t}}+\cdots+\sigma_s\boldsymbol{u}_s\boldsymbol{v}_s^{\mathrm{t}} \tag{4}$$

where σ is the singular value which corresponds to the square root of eigenvalue of XX^{t}. $\boldsymbol{u}$ and $\boldsymbol{v}$ are the eigenvectors of the variance-covariance matrix XX^{t} and $\mathrm{X}^{\mathrm{t}}\mathrm{X}$, respectively. Here, t means the transpose. Using the dominant eigenvectors $\boldsymbol{u}$ as the basis vectors for the reduced system, the transfer matrix W is derived as follows:

$$\mathrm{W}=[\boldsymbol{u}_1 \quad \boldsymbol{u}_2 \quad \cdots \quad \boldsymbol{u}_k] \tag{5}$$

where k eigenvectors from the one with the largest eigenvalues are used. An arbitrary magnetic moment vector $\boldsymbol{M}_p$ can be expressed with W:

$$\boldsymbol{M}_p=\mathrm{W}\boldsymbol{y}_p \tag{6}$$

where $\boldsymbol{y}$ is the reduced unknown vector with size of k. Using (6), the system of equations (2) to be solved can change to the following form:

$$\mathrm{W}^{\mathrm{t}}\mathrm{AW}\boldsymbol{y}=\mathrm{W}^{\mathrm{t}}\boldsymbol{H} \tag{7}$$

where the size of reduced coefficient matrix $\mathrm{W}^{\mathrm{t}}\mathrm{AW}$ is $k \times k$.

In this paper, the numbers of variables (DOFs), snapshots, and reduced size are $n = 52080$, $s = 50$, and $k = 4$, respectively. Eventually, in the optimal design process, the system of 4 equations needs to be solved instead of 52080. Fast computation can be achieved using the reduced system.

C. Solving Non-linear Problem

Since the iron shield has a non-linear magnetic property, (7) must be applied to a non-linear solver, the Newton-Raphson method.

By applying the Newton-Raphson method to (8) derived from (7), we can obtain the linearized equation (9):

$$\boldsymbol{f}' = \mathrm{W}^{\mathrm{t}}\boldsymbol{H} - \mathrm{W}^{\mathrm{t}}\mathrm{A}\mathrm{W}\boldsymbol{y} = \boldsymbol{0} \tag{8}$$

$$\frac{\partial \boldsymbol{f}'}{\partial \boldsymbol{y}}\delta\boldsymbol{y} = -\boldsymbol{f}' \tag{9}$$

The coefficient matrix of (9) can be derived from

$$\frac{\partial \boldsymbol{f}'}{\partial \boldsymbol{y}} = \mathrm{W}^{\mathrm{t}}\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{M}}\mathrm{W} \tag{10}$$

where $\boldsymbol{f} = \boldsymbol{H} - \mathrm{A}\boldsymbol{M}$. Finally, the equation to be solved for the Newton-Raphson method is

$$\mathrm{W}^{\mathrm{t}}\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{M}}\mathrm{W}\delta\boldsymbol{y} = -\boldsymbol{f}' \tag{11}$$

where the size of coefficient matrix is also $k \times k$ (= 4×4 in this paper).

III. Optimization of Magnet Configuration

A. Design specifications and conditions

As mentioned above, we have now been developing a miniaturized HTS skeleton cyclotron magnet with the aim of completing it in FY 2022. Here, we have optimally designed the configuration of REBCO main coils so as to minimize the volume of REBCO main coils. The configurations of sector coils and iron shield are fixed, as shown in Fig. 3. The design specifications of miniaturized HTS skeleton cyclotron are as follows:

- Particle extraction radius is 200 mm.
- Output energy is 5 MeV for proton.
- Operating temperature of REBCO coils is 30 K cooled by conduction cooling.
- Number of main coils is 4.
- Operating current of sector and main coils are 540 A.
- Number of sector coils is 3 with flutter $F^2 = 0.01$.
- Span angle of sector coils is 45 deg.
- Objective of optimization is to minimize the volume of main REBCO coils. The REBCO tape of 4-mm width is supposed.
- Design variables are the inner and outer radii, the length, and the z-position of main coils.
- Permissive error of magnetic field is within ±5%. The error will be compensated by trim copper coils.

Here, the flutter F^2 is calculated from

$$F^2 = \frac{\langle B_z^2\rangle - \langle B_z\rangle^2}{\langle B_z\rangle^2} \tag{12}$$

$$\langle B_z\rangle = \frac{1}{2}\oint_0^{2\pi} B_z(r,\theta)\mathrm{d}\theta \tag{13}$$

As an optimization algorithm, we adopted the Simulate Annealing method [12]. The target magnetic field to achieve the output energy of 5 MeV is shown in Fig. 4.

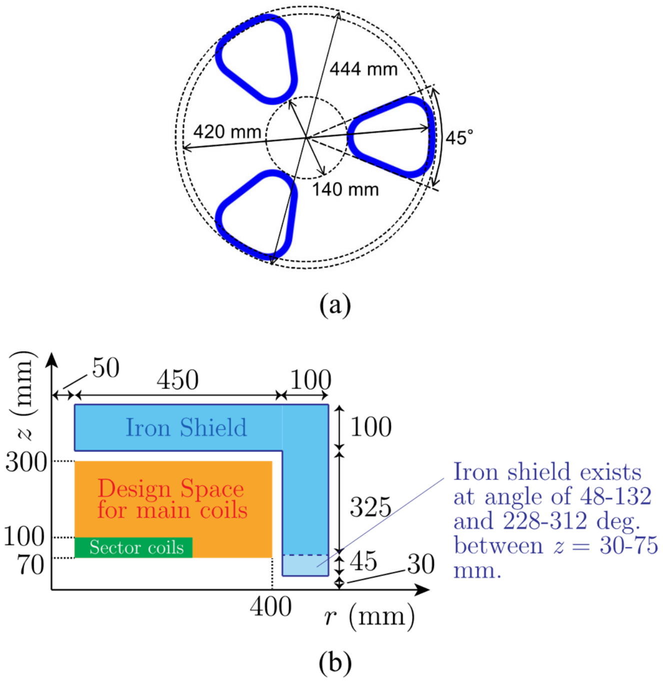

Fig. 3. (a) Top view of sector REBCO coils. (b) cross-sectional view of iron shield, sector coils, and design space for main coils. In the optimal design process, the position of the main coils to be designed is in the region of design space for main coils.

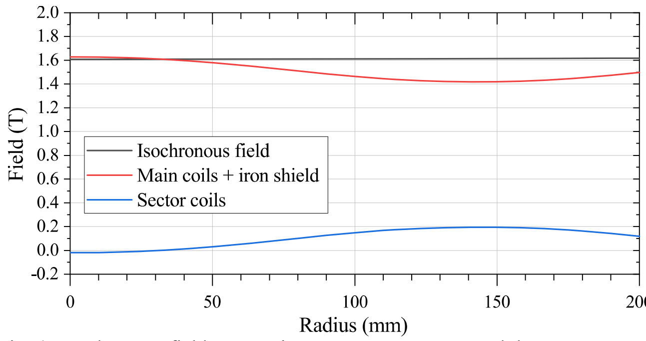

Fig. 4. Isochronous field to energize a proton to 5 MeV, and the target magnetic field to be generated by main coils and iron shield. The target magnetic field is the difference between the isochronous field and the average field generated by the sector coils.

B. Preparation of the method of snapshots

To obtain the transfer matrix W, the magnetic moments are precomputed for 50 (= s) different configurations of main coils, which are randomly produced. Fig. 5 shows the eigenvalues of the matrix X in (3) in descending order. To decide the value of k in (5), the cumulative contribution e of the vector $\boldsymbol{u}$ to the matrix X is calculated as follows:

$$e_k = \frac{\sum_{i=1}^{k}\sigma_i}{\sum_{i=1}^{50}\sigma_i} > \gamma \tag{14}$$

where γ is the threshold, and $\gamma = 0.999$ in this paper. As shown in Fig. 5, when $k = 4$, the cumulative contribution exceeds the

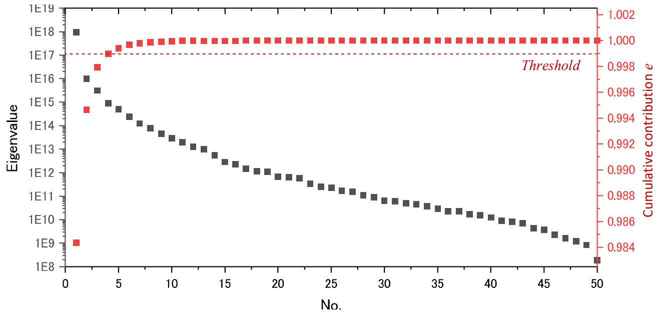

Fig. 5. Eigenvalues and cumulative contribution of eigenvalues to matrix X.

threshold ($e_k > 0.999$).

C. Optimal Design Results

Table I lists the dimension of the optimized main coils, and Fig. 6 shows the cross-sectional view of the optimized main coils with the sector coils and the iron shields. The magnetic moments inside the iron shield are shown in Fig. 1.

Fig. 7 shows the targeted isochronous field and the magnetic field generated by the optimized magnet system, and its error. The error is within ±5%, and it meets the design requirements.

Fig. 8 shows the magnetic fields computed from the reduced and full matrices together with the absolute and relative errors. Although the reduced system of 4 × 4 is used for the field computation in the optimization process, the difference from the full system of 52080 × 52080 is sufficiently small. The computation time for one configuration of main coils is ~50 s in MOR system, meanwhile ~2580 s in full-scale system. The MOR system is ~50 times faster. Although it took 1.5 days for preparation of the method of snapshot, *i.e.*, for magnetic moment computation of 50 different configurations, the optimization process took approximately 1 month using MOR technique. An optimal design with full-scale computation would be impractical.

Since the MOR computation is fast and the error is small, an optimal configuration design of main coils using the MOR method was effective.

IV. Conclusion

To develop a miniaturized HTS skeleton cyclotron for feasibility study, we have developed an optimization method for REBCO magnet with iron shield using Model Order Reduction (MOR) technique. Using the method of snapshots, one of MOR techniques, a field computation is accelerated ~50 times with a small error. The optimized configuration of main REBCO coils is also shown.

We aim to complete the miniaturized HTS skeleton cyclotron magnet in FY 2022. After the feasibility study by the miniaturized system, we will start developing the actual-size magnet system to mass-produce ^{211}At.

TABLE I
CONFIGURATION OF OPTIMIZED MAIN COILS

Main coil	#1	#2	#3	#4
Inner radius (mm)	116.8	287.4	334.7	340.6
Outer radius (mm)	122.1	291.4	338.7	357.2
Bottom z-position (mm)	129.6	78.1	75.9	73.9
Top z-position (mm)	199.6	148.1	165.9	83.9
Number of total turns	98	182	378	62
Number of double panake coils	7	7	9	1

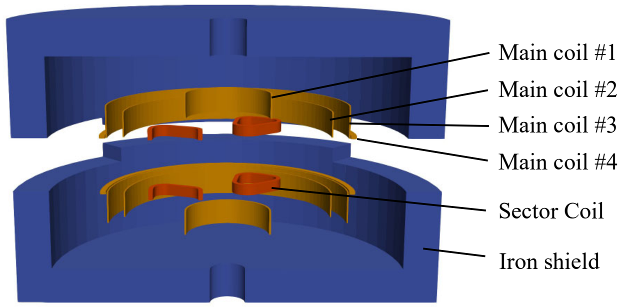

Fig. 6. Cross-sectional view of HTS skeleton cyclotron magnet system with optimized main coils.

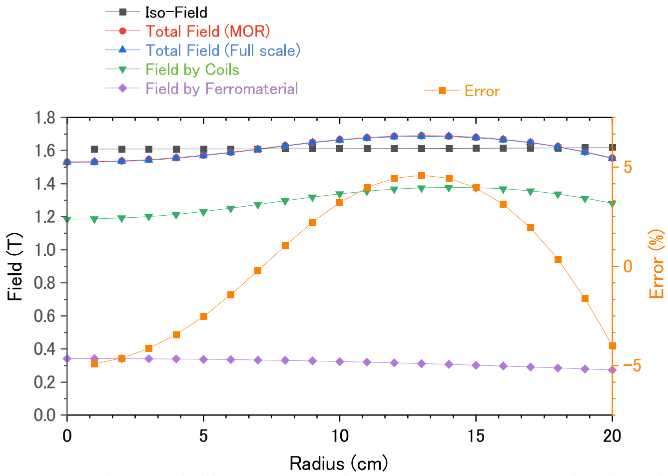

Fig. 7. Isochronous field and magnetic field generated by optimized coils, and error. "MOR" and "Full scale" means the fields computed from the reduced (k = *4*) and full (*n* = 52080) matrices. The error shows the difference between the isochronous field and the magnetic field of optimized coils.

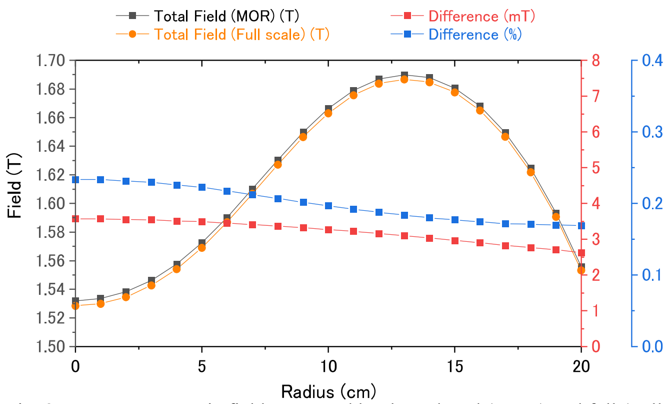

Fig. 8. Average magnetic field computed by the reduced (MOR) and full (Full scale) matrices, and the absolute and relative errors, as function of radius.

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